Structural Engineering and Mechanics, *Vol. 58, No. 4 (2016) 677-687*
DOI: http://dx.doi.org/10.12989/sem.2016.58.4.677

Yield function of the orthotropic material considering the crystallographic texture

Yaroslav A. Erisov*, Fedor V. Grechnikov[a] and Sergei V. Surudin[b]

Metal Forming Department, Samara State Aerospace University, 34 Moskovskoye shosse, Samara, 443086, Russia

(Received October 15, 2015, Revised December 15, 2015, Accepted December 17, 2015)

Abstract. On the basis of the energy approach it is reported a development of the yield function and the constitutive equations for the orthotropic material with consideration of the crystal lattice constants and parameters of the crystallographic texture for the general stress state. For practical use in sheet metal forming analysis it is considered different loading scenarios: plane stress and plane strain states. Using the proposed yield function, the influence of single ideal components on the shape of yield surface was analyzed. The six texture components investigated here were cube, Goss, copper, brass, S and rotated cube, as these components are typically observed in rolled sheets from FCC alloys.

Keywords: anisotropy; plasticity; yield function; texture; crystallographic orientation; plane stress; plane strain; yield surface; rolled sheet

1. Introduction

The current development stage of the technologies for aircraft production, automobile construction, shipbuilding and other industries is characterized by the continuous improvement of the existing constructional materials and technological processes of their forming, as well as by the research and development of new ones. The development of a rational, science-based technology in the metal forming processes is primarily concerned with the need of a detailed study of the structure (texture) formation, the material properties and the most extensive application of these properties in engineering analysis.

One of the specific characteristics inherent in the majority of materials is the anisotropy of their properties, which is caused by the crystallographic structure and texture formation under high plastic strains (Truszkowski 2001). However, the assumption of the material isotropy is still being used as one of the main hypotheses in the analysis and calculations of metal forming processes. Although this hypothesis facilitates the solution of numerous metal forming problems, it does not actually meet the real deformation conditions.

The abandonment of the assumption of the medium isotropy allows to generalize the metal

*Corresponding author, Ph.D. Candidate, E-mail: yaroslav.erisov@gmail.com
[a]Professor, E-mail: fvgr48@mail.ru
[b]Postgraduate Student, E-mail: innosam63@gmail.com

http://www.techno-press.org/?journal=sem&subpage=8 ISSN: 1225-4568 (Print), 1598-6217 (Online)

forming theory and to use the anisotropy effectively in technological processes (Hutchinson *et al.* 1989, Banabic *et al.* 2000, Engler and Hirsch 2002, Hirsch and Al-Samman 2013). Despite the fact that over the recent years greater attention is paid to the theoretical and experimental research in the field of anisotropic plastic deformations (Banabic *et al.* 2010), there is still a number of unsolved problems. These issues are related primarily to the further development of the anisotropic plasticity theory in a form suitable for the engineering and technological analysis.

In the plasticity theory for isotropic material the transition from the elastic condition to the plastic one is usually determined on the basis of the maximum shear stress criterion proposed by Tresca (1864) and developed by Saint-Venant (1870) or from the maximum distortion strain energy criterion obtained independently by Mises (1913), Huber (1904). Hershey (1954), Hosford (1972) proposed a generalized record of the criterion mentioned above, taking into account the type of crystal lattice. However, a major drawback of these criterions is that they do not consider the crystallographic texture of materials and, consequently, the anisotropy of their physical, mechanical and plastic properties.

Mises (1928) was the first to propose the anisotropic yield function, which is a quadratic stress function independent of the hydrostatic pressure. The physical interpretation of the coefficients appearing in this function was partially revealed later by Hill (1948), which proposed their determination for the orthotropic material through the yield strength under tension along the principal axes of anisotropy.

Verification of these yield function proved its applicability for steels (Woodthorpe and Pearce 1970), so for other materials, Hill (1979) proposed a non-quadratic yield criterion. However, it does not take into account the shear stresses and, therefore, it is only applicable in the case when the direction of the principal stresses coincides with the anisotropy axes. This disadvantage was eliminated by Barlat and Lian (1989), who proposed the generalized yield function that takes into account the shear stresses in the case of plane stress state.

Hereafter, Mises anisotropic yield criterion (Mises 1928) was developed by Aryshenskii *et al.* (1969, 1990), who for the case of the general stress state, expressed its coefficients through the transverse-axial strain ratios (analogue of the Poisson's ratio but in plastic field), using the equality of the invariants of the material tensor of the isotropic and anisotropic material. Then, the parameters of crystallographic texture (Aryshenskii *et al.* 1990) were embedded into the proposed yield function, using the hypothesis of proportionality of the elastic and plastic material deviators.

For the general stress state, Barlat *et al.* (1991) on the basis of the Hershey-Hosford's isotropic yield criterion developed the yield function, in which the stress tensor is expressed in terms of the coordinates parallel to the anisotropy axes. A generalization of this criterion was obtained by the stress tensor transformation using weighting factors (Karafillis and Boyce 1993).

This approach is similar to the linear transformation of stress tensor (or deviator), which allows on the basis of any isotropic yield function to obtain an anisotropic one. In this case, the anisotropy of an orthotropic material is described by the components of the fourth order material tensor. Using two linear transformations Barlat *et al.* (2003) obtained the anisotropic criterion for plane stress state for the materials with the equal yield strengths of tension and compression. Banabic *et al.* (2005) independently developed the same yield function, but written in a different form (Barlat *et al.* 2007). Also using the linear transformation Barlat *et al.* (2005), Braun and Besson (2003) proposed anisotropic yield criterions but for general stress state.

Another approach is proposed by Cazacu and Barlat (2001, 2003), who transformed Drucker's isotropic yield criterion (Drucker 1949) to orthotropic form. In this approach, initial isotropic invariants of the stress deviator were replaced by the similar generalized anisotropic invariants, derived on the basis of the theory of representations of tensor functions. The above mentioned yield criterions describe the fully anisotropic plastic strain rates and yield behavior from a single function, as required by the associated flow rule (AFR). As a result, they aren't applicable for materials with different tensile and compressive yield stresses. Contrary to this approach, Moayyedian and Kadkhodayan (2016) developed an advanced criterion based on non-AFR, which considers the strength differential effect and is dependent on structure of an anisotropic material (BCC, FCC and HCP).

Thus, the modern yield functions in contrast to the earlier ones, which were derived to describe the behavior of any metal (Mises 1928, Hill 1948, 1979, Aryshensky *et al.* 1969, 1990), allow taking into account the peculiarities of the specific materials (Barlat and Lian 1989, Barlat *et al.* 1991, 2003, 2005, 2007, Karafilis and Boyce 1993, Banabic *et al.* 2005, Braun and Besson 2003, Cazacu and Barlat 2001, 2003, Moayyedian and Kadkhodayan 2016). It should be noted that the high accuracy of the recently proposed criterions is achieved by a large amount of the anisotropy coefficients (up to 18), determination of which involves numerous mechanical tests at different stress states (Soare and Banabic 2008).

Though the applied anisotropy coefficients characterize the anisotropy of plastic deformations, they do not take into account the reason of anisotropy, i.e., the crystallographic texture. Thus, the mentioned yield functions, on the one hand, allow describing the plastic flow of anisotropic materials. On the other hand, they do not allow carrying out technological analysis of metal forming processes considering the crystallographic texture. As a result, it is impossible to determine the composition of crystallographic texture in terms of the requirements of certain metal forming operations.

In this paper, on the basis of the energy approach it is reported a development of the yield function and the constitutive equations for the orthotropic material with consideration of the crystal lattice constants and parameters of the crystallographic texture for the general stress state.

2. General anisotropic yield function

2.1 Distortion strain energy and equivalent stress

Mises yield criterion or the maximum distortion strain energy criterion is commonly used in the analysis of metal forming processes. According to it, the material starts yielding when the elastic energy of distortion U_d reaches a critical value $U_d^{\lim}$ (Hill 1950), which is determined from tension or compression tests at the yield point. In this case, the yield criterion can be written as follows

$$F = U_d - U_d^{\lim} = 0\,. \tag{1}$$

The strain energy U_0 can be divided into two parts: dilatation strain energy, U_h, that is due to change in volume, and distortion strain energy, U_d, that is responsible for change in shape. Then, the latter is defined as

$$U_d = U_0 - U_h\,. \tag{2}$$

Taking into consideration the fact that the strain energy is equal to the half of the scalar product of the stress tensor, σ_{ij}, by the strain tensor, ε_{ij}, (in the case of the dilatation energy-volumetric tensors), it is possible to transform Eq. (2) to

$$U_d = \frac{1}{2}\sigma_{ij}\varepsilon_{ij} - \frac{1}{6}\sigma_{ii}\varepsilon_{jj}. \tag{3}$$

According to generalized Hooke's law (Hosford 2005) the strain tensor is

$$\varepsilon_{ij} = S_{ijkl}\sigma_{ij}, \tag{4}$$

where S_{ijkl} is the compliance tensor with respect to the principal axes of anisotropy.

Assume that S'_{pqrs} is the compliance tensor in the case when the principal axes of anisotropy coincide with the crystallographic axes [001], [010] and [100] of the crystal lattice. For the orthotropic material with the cubic crystal lattice such tensor contains only three independent compliance constants S'_{1111}, S'_{1122} and S'_{2323}. Then, according to Adamesku (1985) the components of the tensor S_{ijkl} is expressed in terms of the components of S'_{pqrs} as

$$\begin{aligned} S_{iiii} &= S'_{1111} - 4S'_{2323}\left(A'-1\right)\Delta_i, \\ S_{iijj} &= S'_{1122} - 2S'_{2323}\left(A'-1\right)\left(\Delta_k - \Delta_i - \Delta_j\right), \\ S_{ijij} &= S'_{2323} - 2S'_{2323}\left(A'-1\right)\left(\Delta_k - \Delta_i - \Delta_j\right), \end{aligned} \tag{5}$$

where A' is the anisotropy factor of a crystal lattice, Δ_i are the parameters of the crystallographic texture. The anisotropy factor is defined as

$$A' = \frac{S'_{1111} - S'_{1122}}{2S'_{2323}}. \tag{6}$$

For a certain crystallographic orientation {*hkl*}<*uvw*> the parameters of the crystallographic texture are defined as

$$\Delta_i = \frac{h_i^2 k_i^2 + k_i^2 l_i^2 + l_i^2 h_i^2}{\left(h_i^2 + k_i^2 + l_i^2\right)^2}, \tag{7}$$

where h_i, k_i, l_i are Miller's indices, which determine the i-th direction in the crystal with respect to the principal axes of anisotropy.

Assuming that Hooke's law is valid until yielding and substituting Eq. (4)-(5) in Eq. (3) lead to

$$U_d = \frac{S'_{2323}}{15}\left(3 + 2A'\right)K_{ijkl}\sigma_{ij}\sigma_{kl}. \tag{8}$$

Here the fourth order material tensor K_{ijkl} is expressed as

$$K_{ijkl} = \begin{bmatrix} \eta_{12}+\eta_{31} & -\eta_{12} & -\eta_{31} & 0 & 0 & 0 \\ -\eta_{12} & \eta_{12}+\eta_{23} & -\eta_{23} & 0 & 0 & 0 \\ -\eta_{31} & -\eta_{23} & \eta_{23}+\eta_{31} & 0 & 0 & 0 \\ 0 & 0 & 0 & 4\left(5/2-\eta_{23}\right) & 0 & 0 \\ 0 & 0 & 0 & 0 & 4\left(5/2-\eta_{31}\right) & 0 \\ 0 & 0 & 0 & 0 & 0 & 4\left(5/2-\eta_{12}\right) \end{bmatrix}, \tag{9}$$

where the generalized anisotropy factors η_{ij} are defined as

$$\eta_{ij} = 1 - \frac{15(A'-1)}{3+2A'}\left(\Delta_i + \Delta_j - \Delta_k - \frac{1}{5}\right). \tag{10}$$

It follows from Eq. (10) that anisotropy is determined by anisotropy of the crystal lattice (A'), i.e., chemical composition of alloy, and by crystallographic texture (Δ_i), i.e., its thermo-mechanical treatment. Note that the anisotropy of materials with different factors A' differs even for materials with the same crystallographic texture. The elastic isotropic material (A'=1) will be plastic isotropic as well.

In the case under consideration, the distortion strain energy of isotropic material (Hill 1950) can be defined as

$$U_d^{iso} = \frac{2}{15} S'_{2323}\left(3+2A'\right)\sigma_{eq}^2, \tag{11}$$

where σ_{eq} is equivalent stress. Assuming isotropy as special case of anisotropy, it is feasible to equate the right hand sides of Eq. (8)-(11) and determine the equivalent stress of the orthotropic material as

$$\sigma_{eq} = \frac{1}{\sqrt{2}}\sqrt{K_{ijkl}\sigma_{ij}\sigma_{kl}} \tag{12}$$

or in the expanded form

$$\begin{aligned}\sigma_{eq} = \frac{1}{\sqrt{2}}\Bigg\{&\eta_{12}\left(\sigma_{11}-\sigma_{22}\right)^2 + \eta_{23}\left(\sigma_{22}-\sigma_{33}\right)^2 + \eta_{31}\left(\sigma_{33}-\sigma_{11}\right)^2 + \\ &+4\left[\left(\frac{5}{2}-\eta_{12}\right)\sigma_{12}^2 + \left(\frac{5}{2}-\eta_{23}\right)\sigma_{23}^2 + \left(\frac{5}{2}-\eta_{31}\right)\sigma_{31}^2\right]\Bigg\}^{1/2}.\end{aligned} \tag{13}$$

Thus, Eq. (12) contains the constants of the crystal lattice and the parameters of crystallographic texture. Note that in the case of isotropic material, when A'=1 (for example, tungsten crystal (Landolt-Bornstein 1966)) and/or Δ_i=1/5 (Adamesku 1985), i.e., η_{ij}=1, Eq. (13) simplifies to Mises yield function.

2.2 Constitutive equations and equivalent strain rate

The components of strain tensor ε_{ij} are defined by the following equation

$$\frac{d\varepsilon_{ij}}{dt} = \xi_{ij}, \tag{14}$$

where t is the time, d/dt denotes the convected derivative and ξ_{ij} is the strain rate tensor. According to associated flow rule (Hill 1950) the latter is expressed as

$$\xi_{ij} = \lambda\frac{\partial\sigma_{eq}}{\partial\sigma_{ij}}, \tag{15}$$

where λ is non-negative multiplier.

Under small plastic strains, it is feasible to assume that anisotropy does not change during deformation, i.e., $\eta_{ij}=const$. Then, differentiating Eq. (12) according to Eq. (15) yield

$$\begin{gathered}
\xi_{11}=\frac{1}{2}\frac{\lambda}{\sigma_{eq}}\left[\eta_{12}\left(\sigma_{11}-\sigma_{22}\right)-\eta_{31}\left(\sigma_{33}-\sigma_{11}\right)\right],\\
\xi_{22}=\frac{1}{2}\frac{\lambda}{\sigma_{eq}}\left[\eta_{23}\left(\sigma_{22}-\sigma_{33}\right)-\eta_{12}\left(\sigma_{11}-\sigma_{22}\right)\right],\\
\xi_{33}=\frac{1}{2}\frac{\lambda}{\sigma_{eq}}\left[\eta_{31}\left(\sigma_{33}-\sigma_{11}\right)-\eta_{23}\left(\sigma_{22}-\sigma_{33}\right)\right],\\
\xi_{12}=2\frac{\lambda}{\sigma_{eq}}\left(\frac{5}{2}-\eta_{12}\right)\sigma_{12},\quad \xi_{23}=2\frac{\lambda}{\sigma_{eq}}\left(\frac{5}{2}-\eta_{23}\right)\sigma_{23},\quad \xi_{31}=2\frac{\lambda}{\sigma_{eq}}\left(\frac{5}{2}-\eta_{31}\right)\sigma_{31}.
\end{gathered}\tag{16}$$

Using the additional condition $(\sigma_{11}-\sigma_{22})+(\sigma_{22}-\sigma_{33})+(\sigma_{33}-\sigma_{11})=0$, it is possible to transform Eq. (16) to

$$\begin{gathered}
\sigma_{11}-\sigma_{22}=2\frac{\sigma_{eq}}{\lambda}\frac{1}{\mathrm{N}\eta_{12}}\left(\frac{\xi_{11}}{\eta_{31}}-\frac{\xi_{22}}{\eta_{23}}\right),\\
\sigma_{22}-\sigma_{33}=2\frac{\sigma_{eq}}{\lambda}\frac{1}{\mathrm{N}\eta_{23}}\left(\frac{\xi_{22}}{\eta_{12}}-\frac{\xi_{33}}{\eta_{31}}\right),\\
\sigma_{33}-\sigma_{11}=2\frac{\sigma_{eq}}{\lambda}\frac{1}{\mathrm{N}\eta_{31}}\left(\frac{\xi_{33}}{\eta_{23}}-\frac{\xi_{11}}{\eta_{12}}\right),\\
\sigma_{12}=\frac{1}{2}\frac{\sigma_{eq}}{\lambda}\frac{\xi_{12}}{\frac{5}{2}-\eta_{12}},\quad \sigma_{23}=\frac{1}{2}\frac{\sigma_{eq}}{\lambda}\frac{\xi_{23}}{\frac{5}{2}-\eta_{23}},\quad \sigma_{31}=\frac{1}{2}\frac{\sigma_{eq}}{\lambda}\frac{\xi_{31}}{\frac{5}{2}-\eta_{31}},
\end{gathered}\tag{17}$$

where $N=1/\eta_{12}+1/\eta_{23}+1/\eta_{31}$.

Substituting Eq. (17) in Eq. (13), the equivalent strain rate can be expressed as

$$\begin{aligned}
\xi_{eq}=\lambda=\sqrt{2}\Bigg\{&\frac{1}{\mathrm{N}^2}\left[\frac{1}{\eta_{12}}\left(\frac{\xi_{11}}{\eta_{31}}-\frac{\xi_{22}}{\eta_{23}}\right)^2+\frac{1}{\eta_{23}}\left(\frac{\xi_{22}}{\eta_{12}}-\frac{\xi_{33}}{\eta_{31}}\right)^2+\right.\\
&\left.+\frac{1}{\eta_{31}}\left(\frac{\xi_{33}}{\eta_{23}}-\frac{\xi_{11}}{\eta_{12}}\right)^2\right]+\frac{1}{4}\left[\frac{\xi_{12}^2}{\frac{5}{2}-\eta_{12}}+\frac{\xi_{23}^2}{\frac{5}{2}-\eta_{23}}+\frac{\xi_{31}^2}{\frac{5}{2}-\eta_{31}}\right]\Bigg\}^{1/2}.
\end{aligned}\tag{18}$$

Again, in the case of isotropy Eq. (18) transforms to the classic formula for the equivalent strain rate.

3. Application to rolled sheet metals

Due to crystallographic structure and the characteristics of the rolling process, sheet metals generally exhibit a significant anisotropy of mechanical properties (Banabic 2010). In fact, the rolling process induces a particular anisotropy characterized by the symmetry of the mechanical properties with respect to three orthogonal planes, i.e., orthotropy. In the case of the rolled sheet metals, the orientation of principal anisotropy axes is as follows: rolling direction, transverse direction and normal direction. To characterize the proposed yield function as applied to rolled sheet metals, suppose the directions of normal stresses σ_{11} and σ_{22} are in the sheet plane and the direction of normal stress σ_{33} is through the thickness. Moreover, the directions of stresses σ_{11} and σ_{22} coincide with the rolling and transverse directions. From the point of sheet metal forming two cases are of interest: plane stress and plane strain states.

3.1 Plane stress state

Consider thin sheet that is acted upon only by load forces that are parallel to it. In this case, stress perpendicular to the sheet is negligible compared to those parallel to it, i.e., σ_{33}=σ_{23}=σ_{31}=0. Thus, simplifying Eq. (13), the equivalent stress is expressed as

$$\sigma_{eq}^{\sigma_{33}=0} = \frac{1}{\sqrt{2}}\sqrt{(\eta_{12}+\eta_{31})\sigma_{11}^2+(\eta_{12}+\eta_{23})\sigma_{22}^2-2\eta_{12}\sigma_{11}\sigma_{22}+4\left(\frac{5}{2}-\eta_{12}\right)\sigma_{12}^2}\,. \tag{19}$$

Using the plane stress condition and the equation of incompressibility

$$\xi_{11}+\xi_{22}+\xi_{33}=0\,, \tag{20}$$

Eq. (18) reduces and the equivalent strain rate is defined by

$$\xi_{eq}^{\sigma_{33}=0} = \sqrt{2}\sqrt{\frac{(\eta_{12}+\eta_{23})\xi_{11}^2+(\eta_{12}+\eta_{31})\xi_{22}^2+2\eta_{12}\xi_{11}\xi_{22}}{\mathrm{N}\,\eta_{12}\eta_{23}\eta_{31}}+\frac{1}{4}\frac{\xi_{12}^2}{\frac{5}{2}-\eta_{12}}}\,. \tag{21}$$

3.2 Plane strain state

In the case when the length of the sheet or plate is much greater than the other two dimensions, the strains associated with length are constrained by nearby material and are small compared to the cross-sectional strains. The plane strain assumption is used to analyze such metal forming processes as bending of wide sheets, stretch-forming, rolling, etc.

In the case under consideration, using the condition σ_{11}>σ_{22}>σ_{33}, the plane strain state is defined by ξ_{22}=ξ_{12}=ξ_{23}=0. Then, it follows from Eq. (16) that

$$\sigma_{22} = \frac{\eta_{12}\sigma_{11}+\eta_{23}\sigma_{33}}{\eta_{12}+\eta_{23}}\,. \tag{22}$$

Using the plane strain condition and Eq. (20) and (22), it is possible to transform in Eq. (13) to

$$\sigma_{eq}^{\xi_{22}=0} = \frac{1}{\sqrt{2}}\sqrt{\frac{\eta_{31}^2}{\eta_{31}-\frac{1}{\mathrm{N}}}(\sigma_{11}-\sigma_{33})^2+4\left(\frac{5}{2}-\eta_{31}\right)\sigma_{31}^2} \tag{23}$$

and Eq. (18) to

$$\xi_{eq}^{\xi_{22}=0} = \sqrt{2}\sqrt{\frac{(\eta_{12}+\eta_{23})}{\mathrm{N}\,\eta_{12}\eta_{23}\eta_{31}}\xi_{33}^{2} + \frac{1}{4}\frac{\xi_{31}^{2}}{\frac{5}{2}-\eta_{31}}}\,. \tag{24}$$

4. Illustrative example

In sheet metals, crystallographic orientations are not random but show preferred orientations around one or several components (Choi et al. 1999). After rolling, for FCC alloy sheets, these components are mainly {112}<111> (copper), {110}<112> (brass), {123}<634> (S) and {100}<011> (rotated cube) orientations, with the relative amounts dependent on rolling conditions. After annealing, depending on the process conditions, the resulting texture mainly composed of the {100}<001> (cube) and {110}<001> (Goss) recrystallization components. Using the proposed yield function, it is possible to analyze the influence of these single ideal components on the shape of yield surface.

To predict the yield surfaces of single ideal components, consider the plane stress state ($\sigma_{33}=\sigma_{23}=\sigma_{31}=0$) and assume that the principal directions of the stress tensor are coincident with the anisotropic axes, i.e., $\sigma_{11}=\sigma_1$, $\sigma_{22}=\sigma_2$ and $\sigma_{12}=0$. Then, Eq. (19) can be written in the form

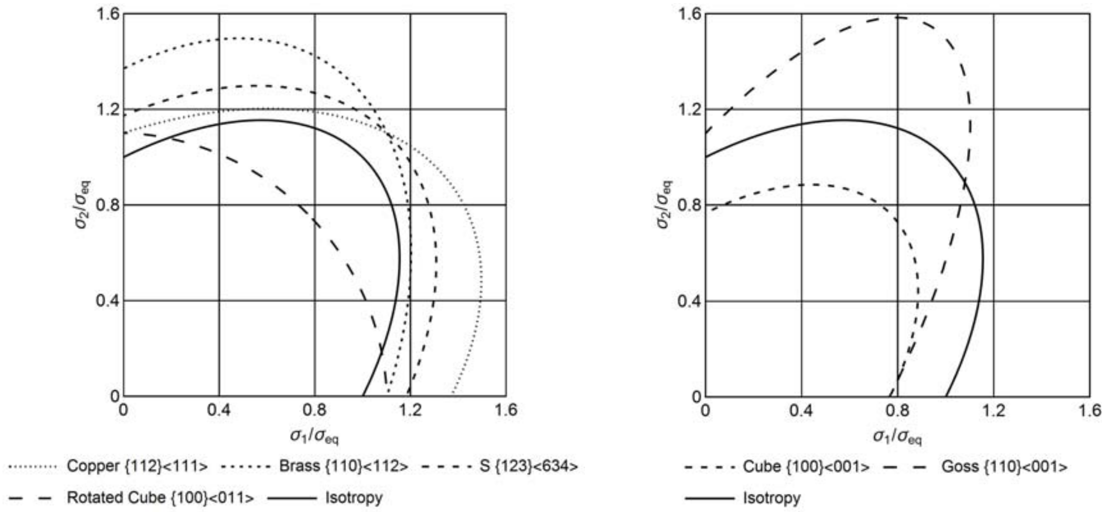
$$(\eta_{12}+\eta_{31})\left(\frac{\sigma_1}{\sigma_{eq}}\right)^2 - 2\eta_{12}\frac{\sigma_1}{\sigma_{eq}}\frac{\sigma_2}{\sigma_{eq}} + (\eta_{12}+\eta_{23})\left(\frac{\sigma_2}{\sigma_{eq}}\right)^2 - 2 = 0\,. \tag{25}$$

This equation geometrically represents the intersection of the yield surface with the $\sigma_1-\sigma_2$ plane, where $\sigma_3=0$.

Consider the sheet from copper for which components of compliance tensor S'_{pqrs} are $S'_{1111}=15.0$ TPa^{-1}; $S'_{1122}=-6.30$ TPa^{-1} and $S'_{2323}=3.33$ TPa^{-1} (Landolt-Bornstein 1966), i.e., $A'=3.203$ (Eq. (6)). The parameters of crystallographic texture and the generalized anisotropy factors calculated using Eq. (7) and (10) for stated components are listed in Table 1. The influence of

Table 1 The parameters of crystallographic texture and the generalized anisotropy factors of single ideal components

Component		The parameters of crystallographic texture			The generalized anisotropy factors		
Name	Orientation	Δ_1	Δ_2	Δ_3	η_{12}	η_{23}	η_{31}
Copper	{112}<111>	0.333	0.250	0.250	0.533	1.116	0.533
Brass	{110}<112>	0.250	0.333	0.250	0.533	0.533	1.116
S	{123}<634>	0.281	0.278	0.250	0.617	0.835	0.814
Rotated cube	{100}<011>	0.250	0.250	0.0	-0.054	1.703	1.703
Cube	{100}<001>	0.0	0.0	0.0	1.703	1.703	1.703
Goss	{110}<001>	0.0	0.250	0.250	1.703	-0.054	1.703
Isotropy	-	0.20	0.20	0.20	1.0	1.0	1.0



(a) Deformation components (b) Recrystallization components

Fig. 1 Yield curves for different texture components

these parameters upon the yield loci is demonstrated in Fig. 1.

It is seen that depending on the crystallographic texture the yield surface changes grossly. Note that the yielding of textured sheet metal can start earlier ({112}<111>, {110}<112>, {123}<634>) or later ({100}<011>, {100}<001>) in comparison with isotropic material.

The calculated curves for ideal texture components agree well with the yield surfaces received by Backofen (1972), Choi *et al.* (1999), Piehler (2009), who used the crystal plasticity theory, in particular, Taylor and Taylor-Bishop-Hill models.

5. Conclusions

The phenomenological yield criterions applied in technological analysis of the metal forming processes do not take into account the crystallographic texture of materials. The proposed yield function considers the crystal lattice constants and the parameters of crystallographic orientation of material. This function is applicable to orthotropic materials with cubic crystal lattice in case when texture does not change, for example, under small elastic-plastic strains. The main practical significance is possibility to predict the effect of crystallographic texture of rolled sheets on yield stresses and R-values. In addition, the proposed yield criterion allows determining the characteristics of the metal forming processes considering the texture of sheet metal. Consequently, it allows designing the composition of crystallographic texture depending on the requirements of the metal forming processes or products performance. Notice that an ideal texture, e.g., for improved formability of a part or minimum earing (Nakamachi and Xie 2001, Zhao *et al.* 2004), can be developed using optimization techniques on the basis of the proposed yield function (Kusiak *et al.* 2012).

References

Adamesku, P.A., Geld, R.A. and Mityshov, E.A. (1985), *Anisotropy of Physical Properties of Metals*, Mashinostroenie, Moscow, Russia. (in Russian)

Arysenskii, Yu.M., Grechnikov, F.V. and Aryshenskii, V.Yu. (1990), "The requirements determination to the sheet anisotropy depending on further forming", *Kuznechno-Shtampovochnoe Proizvodstvo*, **3**, 16-19. (in Russian)

Aryshenskii, V.Yu., Grechnikov, F.V. and Zaitsev, V.M. (1990), "The determination of the material plastic deviator of the anisotropic medium by its texture parameters", *Izvestia Akademii nauk SSSR. Metally*, **4**, 158-162. (in Russian)

Aryshenskii, Yu.M., Kaluzhskii, I.I. and Uvarov, V.V. (1969), "Some issues of the plasticity theory of orthotropic medium", *Izvestiya VUZ, Aviatsionnaya Tekhnika*, **2**, 15-18. (in Russian)

Backofen, W. (1972), *Deformation Processing*, Addison-Wesley Longman.

Banabic, D, Bunge, H.J., Pohlandt, K. and Tekkaya, A.E. (2000), *Formability Of Metallic Materials: Plastic Anisotropy, Formability Testing, Forming Limits*, Springer, Berlin, Germany.

Banabic, D. (2010), *Sheet Metal Forming Processes. Constitutive Modelling and Numerical Simulation*, Springer, Berlin, Germany.

Banabic, D., Aretz, H., Comsa, D.S. and Paraianu, L. (2005), "An improved analytical description of orthotropy in metallic sheets", *Int. J. Plasticity*, **21**, 493-512.

Banabic, D., Barlat, F., Cazacu, O. and Kuwabara, T. (2010), "Advances in anisotropy and formability", *Int. J. Mater. Form.*, **3**, 165-189.

Barlat, F. and Lian, J.A. (1989), "Plastic behavior and stretchability of sheet metals. Part 1: yield function for orthotropic sheets under plane stress conditions", *Int. J. Plasticity*, **5**, 51-66.

Barlat, F., Aretz, H., Yoon, J.W., Karabrin, M.E., Brem, J.C. and Dick, R.E. (2005), "Linear transformation-based anisotropic yield functions", *Int. J. Plasticity*, **21**, 1009-1039.

Barlat, F., Brem, J.C., Yoon, J.W., Chung, K., Dick, R.E., Lege, D.J., Pourboghrat, F., Choi, S.H. and Chu, E. (2003), "Plane stress yield function for aluminum alloy sheet. Part 1: Theory", *Int. J. Plasticity*, **19**, 1297-1319.

Barlat, F., Lege, D.J. and Brem, J.C. (1991), "A six-component yield function for anisotropic materials", *Int. J. Plasticity*, **7**, 693-712.

Barlat, F., Yoon, J.W. and Cazacu, O. (2007), "On linear transformation of stress tensors for the description of plastic anisotropy", *Int. J. Plasticity*, **23**, 876-896.

Bron, F. and Besson, J. (2003), "A yield function for anisotropic materials. Application to aluminum alloys", *Int. J. Plasticity*, **20**, 937-963.

Cazacu, O. and Barlat, F. (2001), "Generalization of Drucker's yield criterion to orthotropy", *Mathematics and Mechanics of Solids*, **6**, 613-630.

Cazacu, O. and Barlat, F. (2003), "Application of representation theory to describe yielding of anisotropic aluminum alloys", *Int. J. of Engng. Sci.*, **41**, 1367-1385.

Choi, S.H., Cho, J.H., Barlat, F., Chung, K., Kwon, J.W. and Oh, K.H. (1999), "Prediction of yield surfaces of textured sheet metals", *Metallurgical and materials transactions*, **30**(A), 377-386.

Drucker, D.C. (1949), "Relation of experiments to mathematical theories of plasticity", *J. Appl. Mech.*, **16**, 349-357.

Engler, O. and Hirsch, J., (2002) "Texture control by thermomechanical processing of AA6xxx Al-Mg-Si sheet alloys for automotive applications - a review", *Materials Science and Engineering A*, **336**, 249-262.

Hershey, A.V. (1954), "The plasticity of an isotropic aggregate of anisotropic face centered cubic crystals", *J. Appl. Mech.*, **21**, 241-249.

Hill, R. (1948), "A theory of the yield and plastic flow of anisotropic metals", *Proc. Roy. Soc. London. Ser A.*, **193**, 281-297.

Hill, R. (1950), *The Mathematical Theory of Plasticity*, Oxford University Press, New-York, USA.

Hill, R. (1979), "Theoretical plasticity of textured aggregates", *Math. Proc. Cambridge Philos. Soc.*, **85**,

179-191.
Hirsch, J. and Al-Samman, T. (2013) “Superior light metals by texture engineering: Optimized aluminum and magnesium alloys for automotive applications”, *Acta Materialia*, **61**, 818–843.
Hosford, W.F. (1972), “A generalized isotropic yield criterion”, *J. Appl. Mech. Trans.*, ASME, **39**, 607-609.
Hosford, W.F. (2005), *Mechanical Behavior of Materials*, Cambridge University Press, New-York, USA.
Huber, M.T. (1904), “Die spezifische formänderungsarbeit als Maß der anstrengung eines materials”, *Czasopismo Techniczne*, **22**, 181.
Hutchinson, W.B., Oscarsson, A. and Karlsson, A. (1989), “Control of microstructure and earing behaviour in aluminium alloy AA 3004 hot bands”, *Mater. Sci. Tech.*, **5**, 1118-1127.
Karafillis, A.P. and Boyce, M.C. (1993), “A general anisotropic yield criterion using bounds and a transformation weighting tensor”, *J. Mech. Phys. Solid.*, **41**, 1859-1886.
Kusiak, J., Szeliga, D. and Sztangret, L. (2012), “Modelling techniques for optimizing metal forming processes”, Eds. Lin, D. Balint and M. Pietrzyk, *Microstructure Evolution In Metal Forming Processes*, Woodhead Publishing, Cambridge, UK.
Landolt-Bornstein (1966), *Numerical data and functional relationships in science and technology. New Series. Group III: Crystal and solid state physics. Volume 1: Elastic, piezoelectric, piezooptic and electrooptic constants of crystals*, Springer, Berlin, Germany.
Mises, R. (1913), “Mechanik der festen Körper im plastisch deformablen Zustand”, *Göttinger Nachrichten, Mathematisch-Physikalische*, **l**(4), 582-592.
Mises, R. (1928), “Mechanik der plastischen Formanderung von Kristallen”, *ZAM*, **8**, 161-185.
Moayyedian, F. and Kadkhodayan, M. (2016), “An advanced criterion based on non-AFR for anisotropic sheet metals”, *Struct. Eng. Mech.*, **57**(6), 1015-1038.
Nakamachi, E. and Xie, C.L. (2001) “Texture design of high-strength and high-formability steel sheet by using finite element and optimization method”, Ed. K.I. Mori, *Simulation of Materials Processing: Theory, Methods and Applications*, Swets & Zeitlinger, Lisse, Netherlands.
Piehler, H.R. (2009), “Crystal-plasticity fundamentals”, *ASM Handbook*, **22A**(#05215G), 232-238.
Saint-Venant, B. (1870), “Memoire sur l’establissement des equations differentielles des mouvements interieurs operes dans les corps solides ductiles au dela des limites ou I’ elasticite pourrait les ramener a leur premier etat”, *Comptes Rendus hebdomadaire s des Seances de l’A cademie des Sciences*, **70**, 473-480.
Soare, S. and Banabic, D. (2008), “About mechanical data required to describe the anisotropy of th.in sheets to correctly predict the earing of deep-drawn cups”, *Int. J. Plasticity*, **4**, 34-37.
Tresca, H. (1864), “Sur l’Ecoulement des Corps Solides Soumis a de Fortes Pressions”, *Comptes rendus de l’Academie des Sciences*, **59**, 754.
Truszkowski, W. (2001), *The Plastic Anisotropy in Single Crystals and Polycrystalline Metals*, Springer, Netherlands.
Woodthorpe, J. and Pearce, R. (1970), “The anomalous behavior of aluminum sheet under balance biaxial tension”, *Int. J. Mech. Sci*., **12**, 341-347.
Zhao, Z., Mao, W., Roters, F. and Raabe, D. (2004), “A texture optimization study for minimum earing in aluminium by use of a texture component crystal plasticity finite element method”, *Acta Materialia*, **52**, 1003-1012.