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Dynamic behavior of the one-stage gear system with uncertainties

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Abstract. In this paper, we propose a method for taking into account uncertainties based on the projection on polynomial chaos. Due to the manufacturing and assembly errors, uncertainties in material and geometric properties, the system parameters including assembly defect, damping coefficients, bending stiffness and traction-compression stiffness are uncertain. The proposed method is used to determine the dynamic response of a one-stage spur gear system with uncertainty associated to gear system parameters. An analysis of the effect of these parameters on the one stage gear system dynamic behavior is then treated. The simulation results are obtained by the polynomial chaos method for dynamic analysis under uncertainty. The proposed method is an efficient probabilistic tool for uncertainty propagation. The polynomial chaos results are compared with Monte Carlo simulations.

Keywords: uncertainty; one-stage gear system; polynomial chaos; assembly defect, random variable; Monte Carlo simulation

1. Introduction

The gearing is the best solution to transmit rotational motions and couple which has been offered numerous advantages (Dalpiaz *et al*. 1996): it ensures a mechanical reliability. Furthermore, its mechanical efficiency is of the order of 0.96 to 0.99. But today, several applications inquire for the gearing transmissions to be more and more reliable, light and having long useful life that requires the control of the acoustic broadcast and the vibratory behavior of these gearings (Begg *et al*. 2000).

Several parametric studies have shown the great sensitivity of the dynamic behavior of gear systems. However, these parameters admit strong dispersions. Therefore, it becomes necessary to take into account these uncertainties to ensure the robustness of the analysis (Nechak *et al*. 2011, Lee *et al*. 2012). Guerine study the dynamic response of a gear system with uncertainty associated to gear parameters (Guerine *et al.* 2015a, Guerine *et al.* 2015b, Guerine *et al.* 2016a, Guerine *et al.*

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2016b). Also there are several studies in reliability for vibration structures taking into account the uncertainties (Mohsine and El Hami 2010, El Hami *et al*. 2009, Radi and El Hami 2007, El Hami and Radi 1996, El Hami *et al*. 1993).

The gear transmissions are widely used in industry. They are of a great utility when it comes to of transmitting high torques, producing of high speed of rotation, making a change in direction of rotational movement. During the operations of assemblies, the relative positioning of teeth within a real transmission is dependent on the quality of realization of all the components of the transmission. In particular, this positioning will be affected by the defect of distance between axis of the wheels. To have a smooth running of the transmission of the gearings, it is necessary that the distance between wheels axis of functioning has to be equal to the normal distance between axis of the wheels. But in reality the gap between wheels axis of functioning will be different from the normal gap between axis of the wheels.

Mitchell (Mitchell 1971) and Pearce (Pearce *et al.* 1986) consider that the distance between wheels axis is an important parameter but its influence on the transmission error has not been addressed.

Remond observed a decrease in noise when we decrease slightly the distance between wheels axis of functioning as compared to the normal distance between axis of the wheels (Remond 1991).

The distance between wheels axis is an important parameter. It acts directly on the backlash of functioning and modifies the geometry of the contact. Indeed, we consider that the distance between axis of the wheels take an uncertainty value because in reality the distance between wheels axis of functioning will be different from the normal distance between wheels axis.

Several methods are proposed in the literature. Monte Carlo (MC) simulation is a well-known technique in this field (Fishman 1996). It can give the entire probability density function of any system variable, but it is often too costly since a great number of samples are required for reasonable accuracy. Parallel simulation (Papadrakakis and Papadopoulos 1999) and proper orthogonal decomposition (Lindsley and Beran 2005) are some solutions proposed to circumvent the computational difficulties of the MC method.

Polynomial chaos (PC) gives a mathematical framework to separate the stochastic components of a system response from the deterministic ones. The stochastic Galerkin method (Babuska *et al*. 2004, Le Maître *et al*. 2001), collocation and regression methods (Babuska et al. 2007, Crestaux *et al*. 2009) are used to compute the deterministic components called stochastic modes in an intrusive and non-intrusive manner while random components are concentrated in the polynomial basis used. Non-intrusive procedures prove to be more advantageous for stochastic dynamic systems since they need no modifications of the system model, contrary to the intrusive method. In the latter, Galerkin techniques are used to generate a set of deterministic coupled equations from the stochastic system model, and then a suitable algorithm is adapted to obtain stochastic modes.

The capabilities of polynomial chaos have been tested in numerous applications, such as treating uncertainties in environmental and biological problems (Isukapalli *et al*. 1998a, Isukapalli *et al*. 1998b), nonlinear random vibration (Li and Ghanem 1998), in multibody dynamic systems Sandu *et al*. 2006a, Sandu *et al*. 2006b), solving ordinary and partial differential equations (Williams 2006, Xiu and Karniadakis 2002a, Xiu and Karniadakis 2002b), in component mode synthesis techniques (El Hami and Radi 1996, Sarsri *et al*. 2011) and parameter estimation (Saad *et al*. 2007, Blanchard *et al*. 2009, Blanchard *et al*. 2010, Smith *et al*. 2007).

The main originality of the present paper is that the uncertainty of the assembly defect in the dynamic behavior study of the one stage gear system is taken into account. The main objective is to investigate of the capabilities of the proposed approach to determine the dynamic response of a spur gear system subject to uncertain assembly defect. Other contribution is to analysis the effect of an assembly defect on the gear system dynamic behavior. So, an eight degree of freedom system modelling the dynamic behavior of a spur gear system is considered. The modelling of a one stage spur gear system is presented in Section 2. In the next section, the theoretical basis of the polynomial chaos is presented. In Section 4, the equations of motion for the eight degrees of freedom are presented. In Section 5, the modeling of an assembly defect is presented. Numerical results are presented in Section 6. Finally in Section 7, to conclude, some comments are made based on the study carried out in this paper.

2. Modelling of a one stage gear system

The global dynamic model of the one stage gear system is shown in Fig. 1. This model is composed of two blocks (j=1 to 2). Every block (j) is supported by flexible bearing which the bending stiffness is k_j^x and the traction-compression stiffness is k_j^y.

The wheels (11) and (22) characterize respectively the motor side and the receiving side. The shafts (j) admit some torsional stiffness k_j^θ.

Angular displacements of every wheel are noticed by $\theta_{(i,j)}$ with the indices j=1 to 2 designates the number of the block, and i=1 to 2 designate the two wheels of each block. Moreover, the linear displacements of the bearing noted by x_j and y_j are measured in the plan which is orthogonal to the wheels axis of rotation.

In this study, we modelled the gear mesh stiffness variation $k(t)$ by a square wave form (Fig. 2). The gear mesh stiffness variation can be decomposed in two components: an average component noted by kc, and a time variant one noted by $kv(t)$ (Walha *et al*. 2009).

The extreme values of the mesh stiffness variation are defined by

$$k_{min} = \frac{kc}{2\varepsilon^\alpha} \text{ and } k_{max} = k_{min}\frac{2-\varepsilon^\alpha}{\varepsilon^\alpha - 1} \tag{1}$$

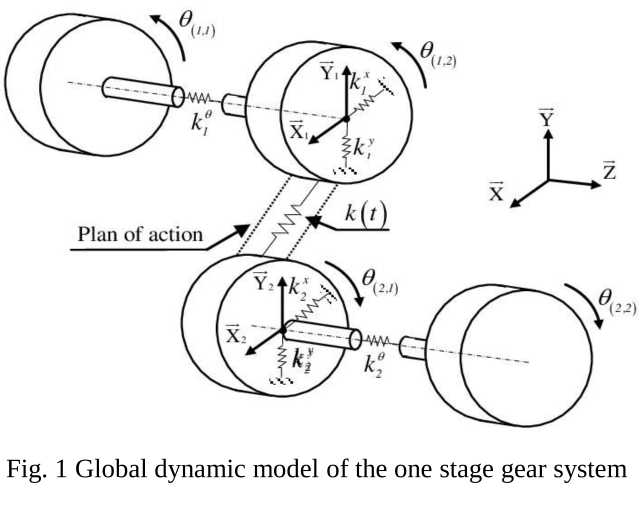


Fig. 1 Global dynamic model of the one stage gear system

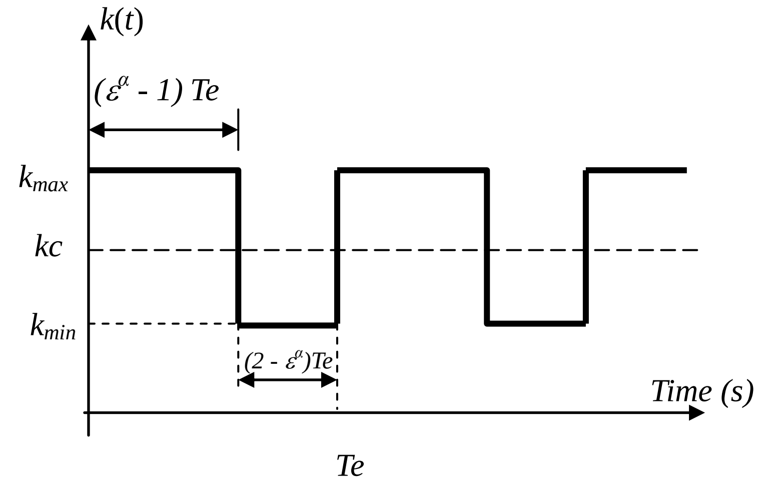


Fig. 2 Modelling of the mesh stiffness variation

ε^{α} and *Te* represent respectively the contact ratio and mesh period corresponding to the two gear meshes contacts.

3. Polynomial chaos method

In this section, we propose a new methodological method based on the projection on polynomial chaos. This method consists in projecting the stochastic desired solutions on a basis of orthogonal polynomials in which the variables are Gaussian orthonormal (Dessombz 2000). The properties of the base polynomial are used to generate a linear system of equations by means of projection. The resolution of this system led to an expansion of the solution on the polynomial basis, which can be used to calculate the moments of the random solution. With this method, we can easily calculate the dynamic response of a mechanical system.

Let us consider a multi-degrees of freedom linear system with mass and stiffness matrices $[M_T]$ and $[K_T]$ respectively. The equations of motion describing the forced vibration of a linear system are

$$\left[M_T\right]\left\{\ddot{u}_T\right\}+\left[K_T\right]\left\{u_T\right\}=\left\{f_T\right\} \tag{2}$$

Where $\{u_T\}$ is the nodal displacement vector and $\{f_T\}$ is the external excitation.

The chaotic polynomials ψ_m corresponding to the multidimensional Hermite polynomials obtained by the Eq. (3)

$$\psi_m\left(\alpha_1,\ldots,\alpha_P\right)=\left(-1\right)^P e^{\left(\frac{1}{2}{}^T\{\alpha\}\{\alpha\}\right)}\frac{\partial^P e^{\left(-\frac{1}{2}{}^T\{\alpha\}\{\alpha\}\right)}}{\partial\alpha_1\ldots\partial\alpha_P} \tag{3}$$

Where $\{\alpha\}$ is the vector grouping the random variables

$${}^T\left\{\alpha\right\}=\left\langle\alpha_1\ldots\alpha_P\right\rangle \tag{4}$$

Where P is the number of random variables.

The random matrices mass and stiffness $[M_T]$ and $[K_T]$ of the mechanical system can be written as

$$\left[M_T\right] = \left[M_T\right]_0 + \left[\tilde{M}_T\right] \tag{5}$$

$$\left[K_T\right] = \left[K_T\right]_0 + \left[\tilde{K}_T\right] \tag{6}$$

The matrices $[M_T]_0$ and $[K_T]_0$ are deterministic matrices, the matrices $[\tilde{M}_T]$ and $[\tilde{K}_T]$ correspond to the random part of the mass and stiffness matrices.

$[\tilde{M}_T]$ and $[\tilde{K}_T]$ are rewritten from an expression of type Karhunen-Loeve (Ghanem and Spanos 1991) in the following form

$$\left[\tilde{M}_T\right] = \sum_{p=1}^{P} \left[M_T\right]_p \alpha_p \tag{7}$$

$$\left[\tilde{K}_T\right] = \sum_{p=1}^{P} \left[K_T\right]_p \alpha_p \tag{8}$$

Where α_p are independent Gaussian centered reduced which may correspond to the first polynomial ψ_p, while the matrices $[M_T]_p$ and $[K_T]_p$ are deterministic.
We pose α_0=1, we can write then

$$\left[M_T\right] = \sum_{p=0}^{P} \left[M_T\right]_p \alpha_p \tag{9}$$

$$\left[K_T\right] = \sum_{p=0}^{P} \left[K_T\right]_p \alpha_p \tag{10}$$

In the same way, we can write for $\{f_T\}$

$$\left\{f_T\right\} = \sum_{p=0}^{P} \left\{f_T\right\}_p \alpha_p \tag{11}$$

The dynamic response is obtained by solving the following equation knowing that the initial conditions are predefined

$$\left[K_{eq}\right]\left\{u_T\right\}\left(t+\Delta t\right) = \left\{F_{eq}\right\} \tag{12}$$

Where

$$\left[K_{eq}\right] = \left[K_T\right] + a_0\left[M_T\right] \tag{13}$$

$$\left\{F_{eq}\right\} = \left\{f_T\right\}\left(t+\Delta t\right) + \left[M_T\right]\left(a_0\left\{u_T\right\}\left(t\right) + a_1\left\{\dot{u}_T\right\}\left(t\right) + a_2\left\{\ddot{u}_T\right\}\left(t\right)\right) \tag{14}$$

Where

$$a_0 = \frac{1}{A\Delta t^2}, \quad a_1 = \frac{B}{A\Delta t} \quad \text{and} \quad a_2 = \frac{1}{A\Delta t} \tag{15}$$

A and B are the parameters of Newmark.
$\{u_T\}(t+\Delta t)$ is decomposed on polynomials to P Gaussian random variables orthnormales

$$\left\{u_T\right\}\left(t+\Delta t\right)=\sum_{n=0}^{N}\left(\left\{u_T\right\}\left(t+\Delta t\right)\right)_n \psi_n\left(\left\{\alpha_i\right\}_{i=1}^{P}\right) \tag{16}$$

Where N is the polynomial chaos order.
$[K_{eq}]$ and $\{F_{eq}\}$ are written in the following form

$$\left[K_{eq}\right]=\sum_{p=0}^{P}\left[K_T\right]_p \alpha_p + a_0\sum_{p=0}^{P}\left[M_T\right]_p \alpha_p = \sum_{p=0}^{P}\left[K_{eq2}\right]_p \alpha_p \tag{17}$$

$$\begin{aligned}\left\{F_{eq}\right\}&=\sum_{p=0}^{P}\left(\left\{f_T\right\}\left(t+\Delta t\right)\right)_p \alpha_p+\sum_{p=0}^{P}\left[M_T\right]_p \alpha_p\left(a_0\left(\left\{u_T\right\}(t)\right)_0+a_1\left(\left\{\dot{u}_T\right\}(t)\right)_0+a_2\left(\left\{\ddot{u}_T\right\}(t)\right)_0\right)\\&=\sum_{p=0}^{P}\left\{F_{eq2}\right\}_p \alpha_p\end{aligned} \tag{18}$$

Substituting Eqs. (16), (17) and (18) into Eq. (12) and forcing the residual to be orthogonal to the space spanned by the polynomial chaos ψ_m yield the following system of linear equation

$$\sum_{p=0}^{P}\sum_{n=0}^{N}\left[K_{eq2}\right]_p\left\{u_T\right\}_n\left\langle\alpha_p\ \ \psi_n\ \ \psi_m\right\rangle=\sum_{p=0}^{P}\left\{F_{eq2}\right\}_p\left\langle\alpha_p\ \ \psi_m\right\rangle \quad m=0,1,\ldots,N \tag{19}$$

Where N is the order of Polynomial Chaos.
Where $\langle . . \rangle$ denotes the inner product defined by the mathematical expectation operator.
This algebraic equation can be rewritten in a more compact matrix form as

$$\begin{bmatrix}[D]^{(00)} & & \cdots & & [D]^{(0N)}\\ & \ddots & & & \\ \vdots & & [D]^{(ij)} & & \vdots\\ & & & \ddots & \\ [D]^{(N0)} & & \cdots & & [D]^{(NN)}\end{bmatrix}\begin{Bmatrix}\left(\left\{u_T\right\}\left(t+\Delta t\right)\right)_0\\ \vdots\\ \left(\left\{u_T\right\}\left(t+\Delta t\right)\right)_j\\ \vdots\\ \left(\left\{u_T\right\}\left(t+\Delta t\right)\right)_N\end{Bmatrix}=\begin{Bmatrix}\{f\}^{(0)}\\ \vdots\\ \{f\}^{(j)}\\ \vdots\\ \{f\}^{(N)}\end{Bmatrix}, \tag{20}$$

Where

$$[D]^{(ij)}=\sum_{p=0}^{P}\left[K_{eq2}\right]_p\left\langle\alpha_p\ \ \psi_i\ \ \psi_j\right\rangle \tag{21}$$

$$\{f\}^{(j)}=\sum_{p=0}^{P}\left\{F_{eq2}\right\}_p\left\langle\alpha_p\ \ \psi_j\right\rangle \tag{22}$$

After resolution of the algebraic system (20), the mean values and the variances of the dynamic response are given by the following relationships

$$E\left[\left\{u_T\right\}\right]=\left(\left\{u_T\right\}\left(t+\Delta t\right)\right)_0 \tag{23}$$

$$Var\left[\left\{u_T\right\}\right]=\sum_{n=1}^{N}\left(\left(\left\{u_T\right\}\left(t+\Delta t\right)\right)_n\right)^2\left(\psi_j\right)^2 \tag{24}$$

4. Equations of motion

The equation of motion describing the dynamic behavior of our system (Fig. 1) is obtained by applying Lagrange formulation and is given by

$$m\ddot{x}_1 - c^x \dot{x}_1 + sin(\alpha)c(t)\langle L^\delta \rangle \{\dot{Q}\} + k^x x_1 + sin(\alpha)\, k(t)\, \langle L^\delta \rangle \{Q\} = 0 \tag{25}$$

$$m\ddot{y}_1 - c^y \dot{y}_1 + cos(\alpha)c(t)\langle L^\delta \rangle \{\dot{Q}\} + k^y y_1 + cos(\alpha)\, k(t)\, \langle L^\delta \rangle \{Q\} = 0 \tag{26}$$

$$m\ddot{x}_2 + c^x \dot{x}_2 - sin(\alpha)c(t)\langle L^\delta \rangle \{\dot{Q}\} + k^x x_2 - sin(\alpha)\, k(t)\, \langle L^\delta \rangle \{Q\} = 0 \tag{27}$$

$$m\ddot{y}_2 + c^y \dot{y}_2 - cos(\alpha)c(t)\langle L^\delta \rangle \{\dot{Q}\} + k^y y_2 - cos(\alpha)\, k(t)\, \langle L^\delta \rangle \{Q\} = 0 \tag{28}$$

$$I\ \ddot{\theta}_{(1,1)} + k^\theta \left(\theta_{(1,1)} - \theta_{(1,2)}\right) = Cm \tag{29}$$

$$I\ \ddot{\theta}_{(2,1)} + r^b_{(1,2)}\, c(t)\langle L^\delta \rangle \{\dot{Q}\} - k^\theta \left(\theta_{(2,1)} - \theta_{(2,2)}\right) - r^b_{(2,1)}\, k(t)\, \langle L^\delta \rangle\, \{Q\} = 0 \tag{30}$$

$$I\ \ddot{\theta}_{(2,1)} - r^b_{(2,1)}\, c(t)\langle L^\delta \rangle \{\dot{Q}\} - k^\theta \left(\theta_{(2,1)} - \theta_{(2,2)}\right) - r^b_{(2,1)}\, k(t)\, \langle L^\delta \rangle\, \{Q\} = 0 \tag{31}$$

$$I\ \ddot{\theta}_{(2,2)} + k^\theta \left(\theta_{(2,1)} - \theta_{(2,2)}\right) = 0 \tag{32}$$

Where I is the moment of inertia of the wheels.
Where $\langle L^\delta \rangle$ is defined by

$$\langle L^\delta \rangle = \left[sin(\alpha) \quad -sin(\alpha) \quad cos(\alpha) \quad -cos(\alpha) \quad 0 \quad r^b_{(1,2)} \quad -r^b_{(2,1)} \quad 0 \right] \tag{33}$$

$r^b_{(1,2)}, r^b_{(2,1)}$ represent the base gears radius. α is the pressure angle.
$\{Q(t)\}$ is the vector of the model generalized coordinates, it is in the form

$$\{Q(t)\} = \left[x_1 \quad y_1 \quad x_2 \quad y_2 \quad \theta_{(1,1)} \quad \theta_{(1,2)} \quad \theta_{(2,1)} \quad \theta_{(2,2)} \right]^T \tag{34}$$

5. Modelling of an assembly defect

The distance between wheels axis represents the distance between the centers of the pinion and the wheel. In normal functioning, its value is equal to the half sum of the primal rays of the pinion and the wheel.

In an assembly operation, when we assembly the wheels that constitute the gear system we must verify the theoretical gap between wheels axis. For the first train, this theoretical gap shape e_1 is defined by

$$e_1 = r_{(1,2)} + r_{(2,1)} \tag{35}$$

$r_{(j,i)}$ represent the pitch radius of the wheels (12) and (21).

Practically, it is impossible to verify this assembly condition and we will have a defect of distance between axis (noted by the algebraic value a) of the wheels. We suppose next that the

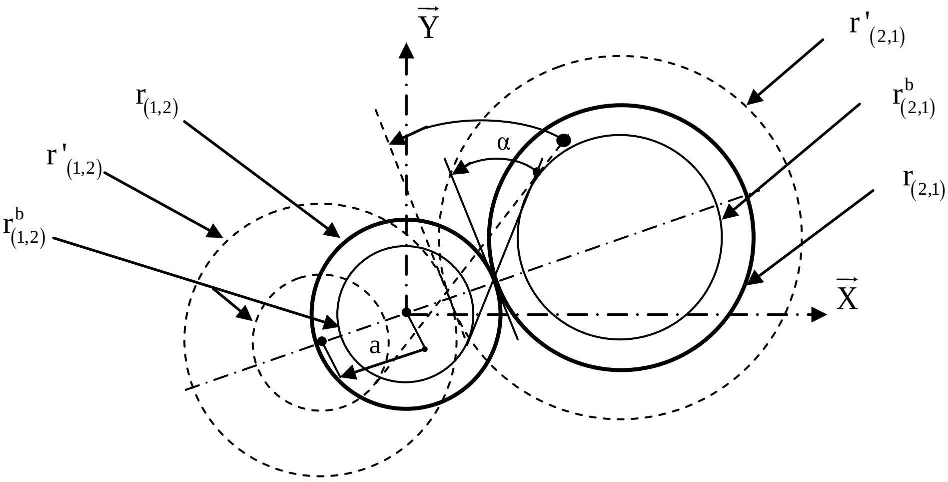


Fig. 3 Assembly defect

block 1 is shifted from its theoretical position inducing an assembly defect. As it is represented in the Fig. 3, there are numerous consequences of this defect.

Teeth gap provokes the average stiffness k_{av} reduction since the average teeth thickness decreases. We approximate this average thickness variation by a decreasing linear function. Furthermore, and according to the definitions given by the norms of the AGMA, a defect assembly in gear design is accompanied by the change of the pressure angle and the primitive radius.

The new angle of pressure can be written as

$$\alpha' = cos^{-1}\left(\frac{r^b_{(1,2)} + r^b_{(2,1)}}{e_1 + a}\right) \tag{36}$$

The new primitive radiuses are defined by

$$r'_{(1,2)} = \frac{r^b_{(1,2)}}{cos(\alpha')}, \quad r'_{(2,1)} = a + e_1 - r'_{(1,2)} \tag{37}$$

6. Numerical simulation

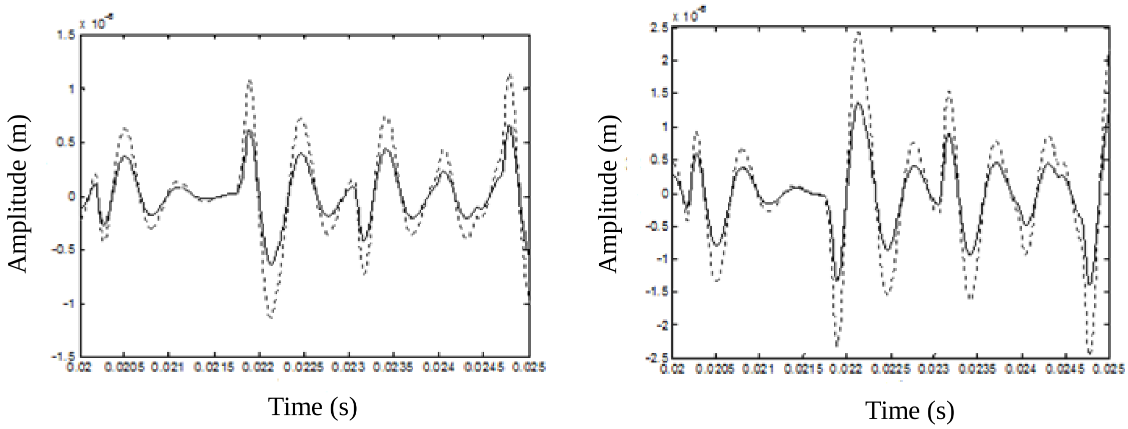
The technological and dimensional features of the one-stage gear system are summarized in the Table 1.

6.1 Dynamic response for an assembly defect

When we assembly the wheels that constitute the gear system, we must verify the theoretical gap between wheels axis. However, practically it is impossible to verify this assembly condition and we will have a defect of distance between axis of the wheels. We suppose next that the block 1

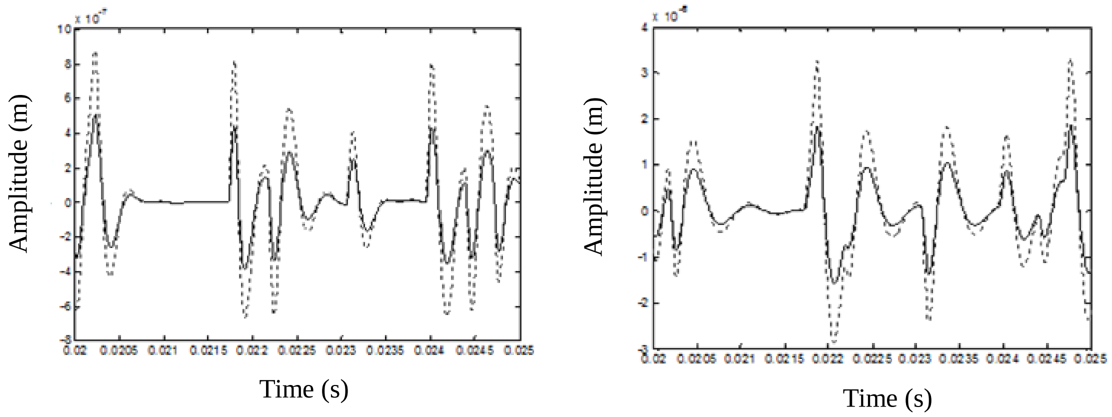
Table 1 System parameters

Material : 42CrMo4	ρ=7860 Kg/m^3
Motor torque	Cm=200 N.m
Number of teeth	Z(12)=40 ; Z(21)=50
Module of teeth	module=4.10^{-3} m
Contact ratio	ε^{α} = 1.7341
The pressure angle	α=20°



A: x_1 B: y_1

Fig. 4 Time dynamic response of the first bearing — without assembly defect; ····· with assembly defect (a=1 mm)



A: x_2 B: y_2

Fig. 5 Time dynamic response of the second bearing — without assembly defect; ····· with assembly defect (a=1 mm)

is shifted from its theoretical position inducing an assembly defect.

Fig. 4 represents the temporal features of the dynamic components of the linear displacements of the first bearing in two directions x and y. The assembly defect with amplitude a=1 mm will amplify the amplitude of the linear displacements on the first bearing. This result is the same on the second bearing. We represent on Fig. 5 the temporal features of the dynamic components of the linear displacements of the second bearing.

Fig. 6 represents the frequency responses of the linear displacement of the first bearing. We clearly see the presence of several peaks in every signal. These peaks correspond to the mesh frequency *fe*=1300 Hz with its first harmonics. We notice that an assembly defect provokes the vibratory level amplification on the spectrums associated to the temporal signatures on the eight degrees of freedom of the model.

6.2 Study with polynomial chaos

In this section numerical results are presented for the proposed approach formulations derived in the Section 4. The polynomial chaos results are compared with Monte Carlo simulations with 100000 simulations.

The assembly defect a, the coefficients of damping c^x and c^y, the bending stiffness k^x and the traction-compression stiffness k^y are supposed independent random variables and defined as follow

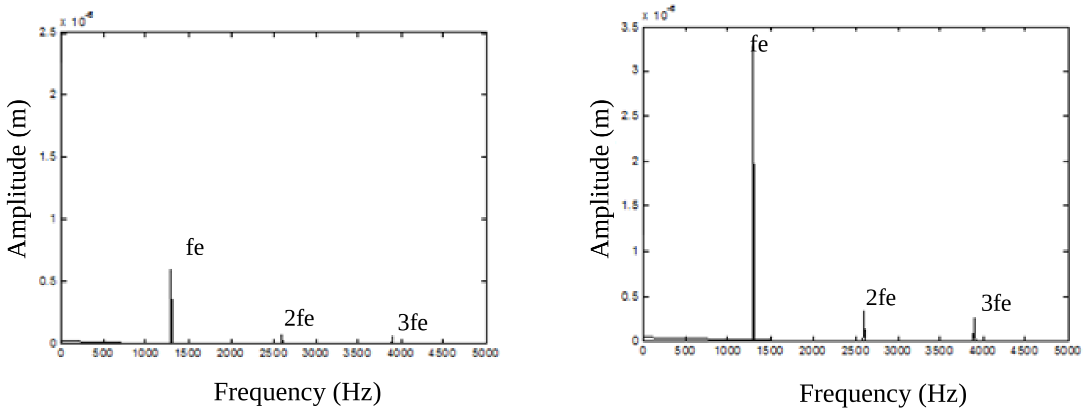
$$a = a_0 + \sigma_a \xi \quad c^x = c^x{}_0 + \sigma_{c^x} \xi \quad c^y = c^y{}_0 + \sigma_{c^y} \xi \quad k^x = k^x{}_0 + \sigma_{k^x} \xi \quad k^y = k^y{}_0 + \sigma_{k^y} \xi \tag{38}$$

Where ξ is a zero mean value Gaussian random variable, a_0, $c^x{}_0$, $c^y{}_0$, $k^x{}_0$ and $k^y{}_0$ are the mean values and σ_a, σ_{c^x}, σ_{c^y}, σ_{k^x} and σ_{k^y} are the associated standard deviations.

6.2.1 Effect analysis of the uncertain parameters

The mean value and the standard deviation of the dynamic component of the linear displacement of the first bearing in two directions x and y and the second bearing in direction x have been calculated by the polynomial chaos approach using the same order (3rd order). The obtained results are compared with those given by the Monte Carlo simulations with 100000 simulations.

Firstly, the effect of uncertain assembly defect is considered. The simulation of the mean value and the standard deviation of the dynamic component on the linear displacement of the first



a= 0.5 mm a=1 mm

Fig. 6 Frequency responses of the first bearing with assembly defect

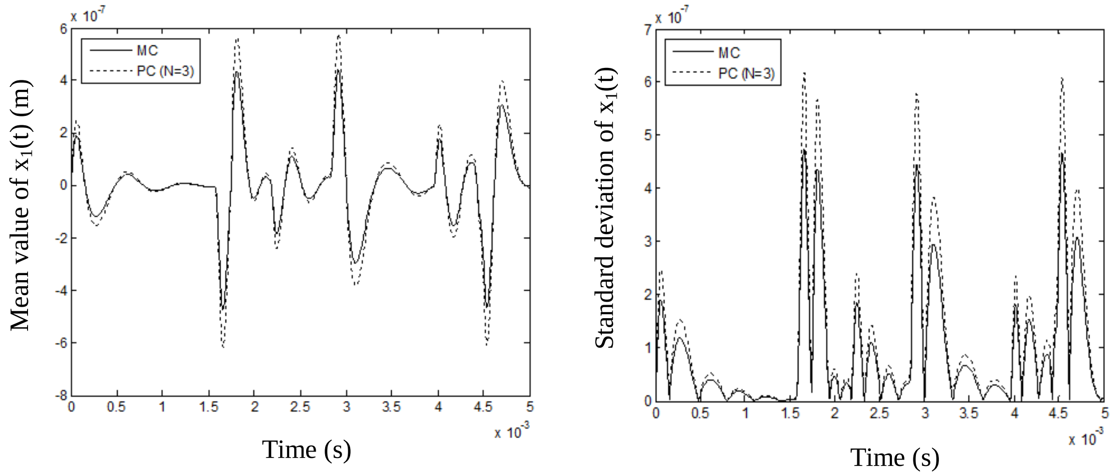


Fig. 7 Mean value and standard deviation of $x_1(t)$ considering uncertain assembly defect σ_a=3%

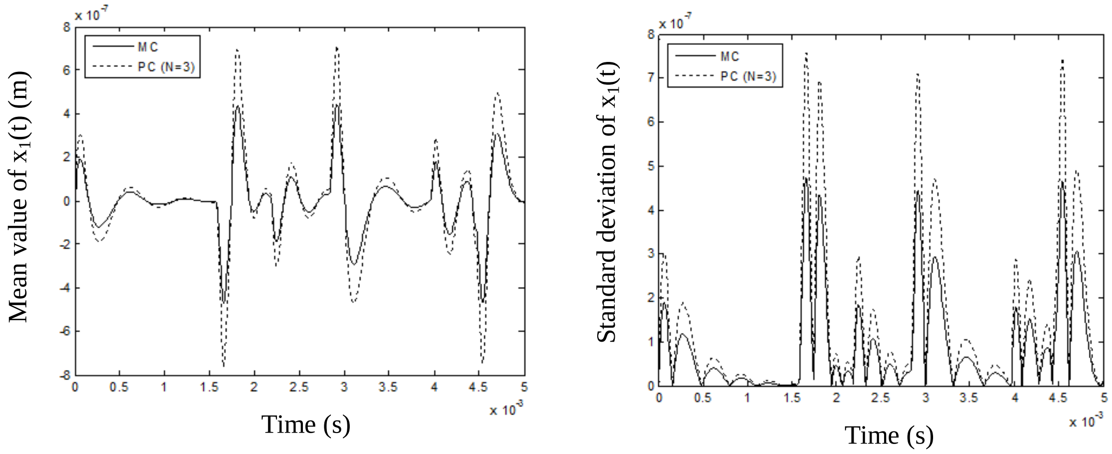


Fig. 8 Mean value and standard deviation of $x_1(t)$ considering uncertain assembly defect σ_a=7%

bearing following direction x is plotted in Figs. 7 and 8. The result shows that the uncertain assembly defect has a more significant influence on the dynamic response of the system.

Then, the effect of uncertain damping coefficients is examined. Figs. 9 and 10 present the mean value and the standard deviation of y_1. Compared with uncertain assembly defect, the effect of uncertain damping coefficients is less significant. The result shows that the polynomial chaos method can obtain high accuracy with uncertain damping coefficients.

Figs. 11 and 12 show the mean value and the standard deviation of the dynamic component of the linear displacement of the second bearing following direction x considering the uncertain bending stiffness k^x and the traction-compression stiffness k^y. The results show that as the level of uncertainty increases from $\sigma_{k^x} = \sigma_{k^y}$=3% to $\sigma_{k^x} = \sigma_{k^y}$=7%, the error using the same order (3rd order) polynomial chaos also increases.

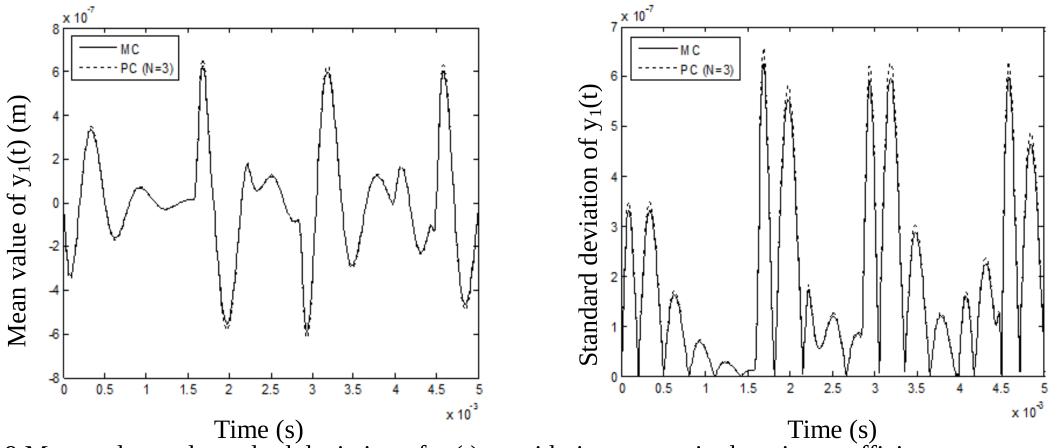


Fig. 9 Mean value and standard deviation of $y_1(t)$ considering uncertain damping coefficients $\sigma_{c^x} = \sigma_{c^y} = 3\%$

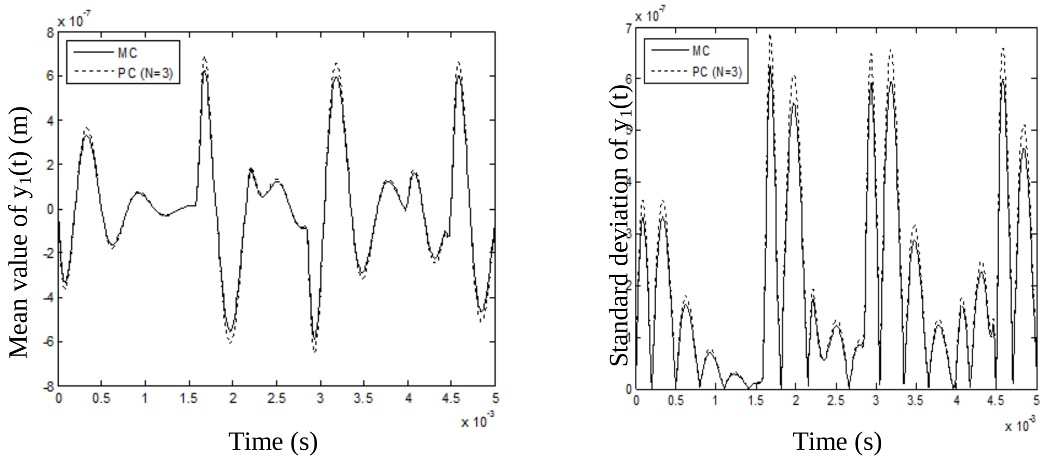


Fig. 10 Mean value and standard deviation of $y_1(t)$ considering uncertain damping coefficients $\sigma_{c^x} = \sigma_{c^y} = 7\%$

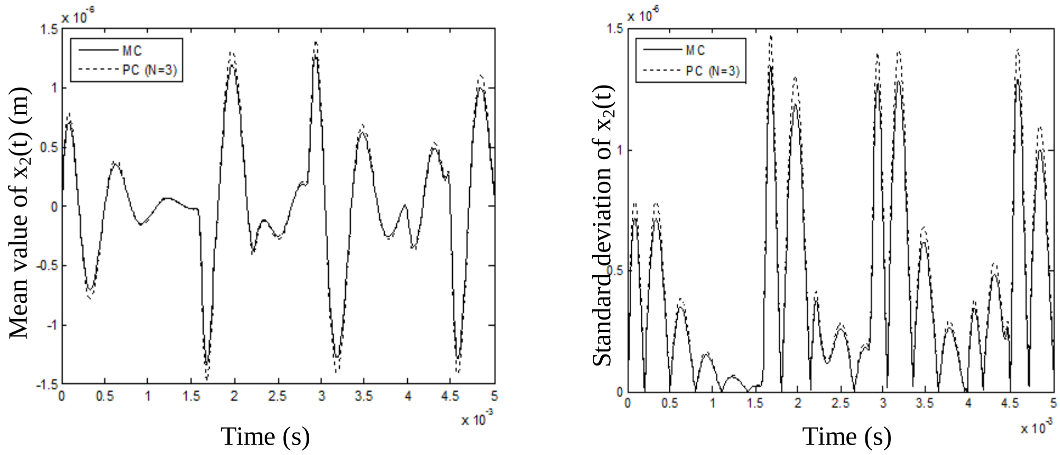


Fig. 11 Mean value and standard deviation of $x_2(t)$ considering uncertain bearings stiffness $\sigma_{k^x} = \sigma_{k^y} = 3\%$

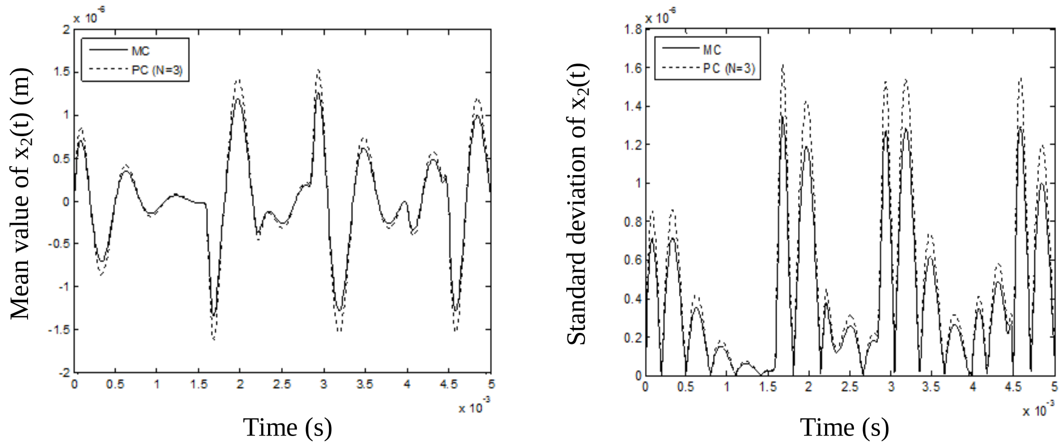


Fig. 12 Mean value and standard deviation of $x_2(t)$ considering uncertain bearings stiffness $\sigma_{k^x} = \sigma_{k^y} = 7\%$

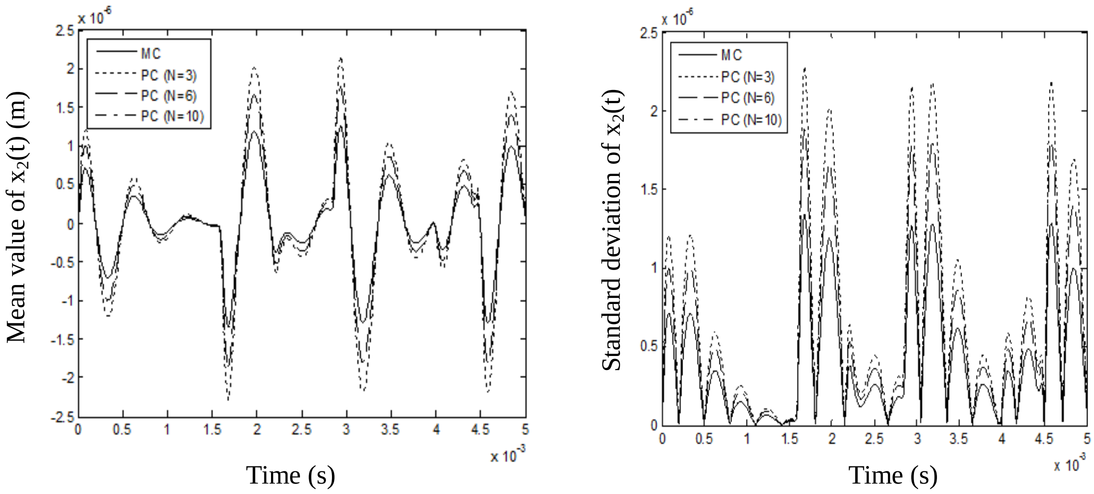


Fig. 13 Mean value and standard deviation of $x_2(t)$ considering uncertain assembly defect σ_a=15%

6.2.2 Effect analysis of the polynomial chaos order

The mean value and the standard deviation of the dynamic component of the linear displacement of the second bearing in two directions x and y are presented in Figs. 13 and 14 for σ_a=15%.

The polynomial chaos results are compared with Monte Carlo simulation with 100000 simulations. It is evident from these figures that N=3 case clearly does not have enough chaos terms to represent the output. As N increases, the results seem to become better and the error decreases with the increase of the polynomial chaos order N. With N=10, the dynamic response of the linear displacement of the second bearing with polynomial chaos values almost exactly match with the Monte Carlo simulation results and the error is fairly minimal indicating the results are very close to those of Monte Carlo. An N=10 has been used for the PC model and is seen to be enough to capture the dynamic response of the linear displacement of the system.

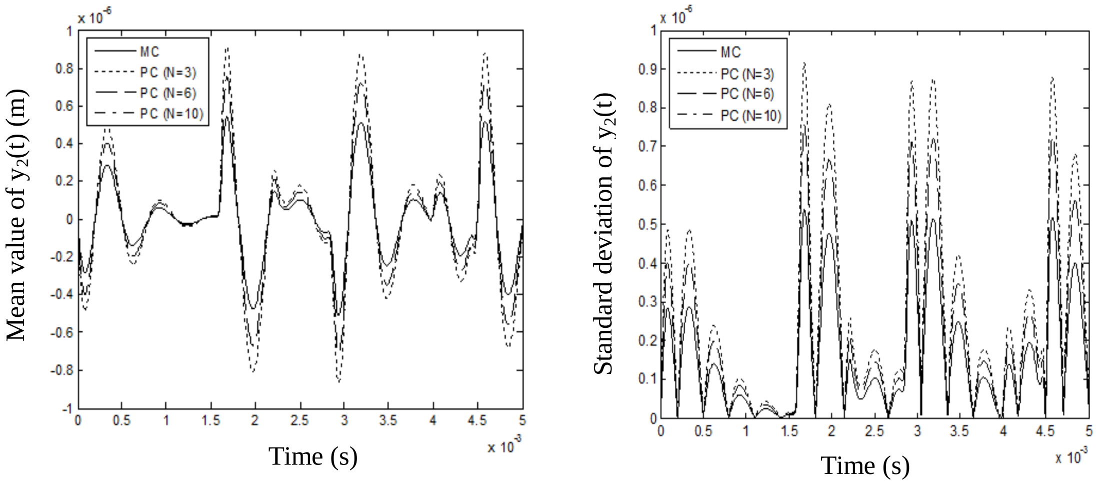


Fig. 14 Mean value and standard deviation of $y_2(t)$ considering uncertain assembly defect σ_a=15%

7. Conclusions

An approach based on the polynomial chaos method has been proposed to study the dynamic behavior of one stage gear system was modeled by eight degrees of freedom in the presence of assembly defect that admits some dispersion. The one stage gear system behavior is affected by assembly defect. This defect increases the vibratory level. The polynomial chaos method has been used to determine the dynamic response of this system. The efficiency of the proposed method compared with the Monte Carlo simulation. The main results of the present study show that the polynomial chaos may be an efficient tool to take into account the dispersions of the assembly defect in the dynamic behavior study of a spur gear system. An interesting perspective is to apply this method to a system with higher degree of freedom like epicyclic gear system. Further work in this context is in progress.

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