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Thermal stability of functionally graded sandwich plates using a simple shear deformation theory

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Abstract. In the present work, a simple first-order shear deformation theory is developed and validated for a variety of numerical examples of the thermal buckling response of functionally graded sandwich plates with various boundary conditions. Contrary to the conventional first-order shear deformation theory, the present first-order shear deformation theory involves only four unknowns and has strong similarities with the classical plate theory in many aspects such as governing equations of motion, and stress resultant expressions. Material properties and thermal expansion coefficient of the sandwich plate faces are assumed to be graded in the thickness direction according to a simple power-law distribution in terms of the volume fractions of the constituents. The core layer is still homogeneous and made of an isotropic material. The thermal loads are considered as uniform, linear and non-linear temperature rises within the thickness direction. The results reveal that the volume fraction index, loading type and functionally graded layers thickness have significant influence on the thermal buckling of functionally graded sandwich plates. Moreover, numerical results prove that the present simple first-order shear deformation theory can achieve the same accuracy of the existing conventional first-order shear deformation theory which has more number of unknowns.

Keywords: plate theory; thermal buckling; functionally graded plate; sandwich plate; volume fraction index

1. Introduction

Sandwich structures made of a core bonded to two face sheets are widely employed in the aerospace industry because of their important bending rigidity, low specific weight, excellent vibration properties and good fatigue characteristics. However, the sudden variation in the material properties from one layer to another can lead in stress concentrations which produce generally an

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interface debonding. To overcome this type of damage, the solution of functionally graded (FG) sandwich structures is developed. In such materials, two face sheets are made from isotropic FGMs while the core is made from an isotropic homogeneous material. Thanks to the smooth and continuous change in the characteristics of FGMs, the stress concentration which can be occurred in laminated sandwich structures is avoided in FG sandwich structures. FGMs are widely employed in many engineering applications such as spacecraft industry, mechanics, civil engineering, aerospace, nuclear, automotive and so on (Miamoto *et al*. 1999, Lu *et al*. 2009, Ould Larbi *et al*. 2013, Hadji *et al*. 2014, Yaghoobi *et al*. 2014, Liang *et al*. 2014, 2015, Bouguenina *et al*. 2015, Hebali *et al*. 2015, Ait Atmane *et al*. 2015, Pradhan and Chakraverty 2015, Sallai *et al*. 2015, Sofiyev and Kuruoglu 2015, Kar and Panda 2015, Bourada *et al*. 2015, Arefi 2015, Akbaş 2015, Al-Basyouni *et al*. 2015, Kirkland and Uy 2015, Ebrahimi and Dashti 2015, Hadji and Adda Bedia 2015a,b, Cunedioglu 2015, Meksi *et al*. 2015, Meradjah *et al*. 2015, Darılmaz 2015, Ait Atmane *et al*. 2016, Bellifa *et al*. 2016).

With the increased use of FG sandwich structures in the design of engineering structures, understanding their mechanical behaviors becomes an essential task (Lu *et al*. 2009, Talha and Singh 2010, Shahrjerdi *et al*. 2011, Wen *et al*. 2011, El Meiche *et al*. 2011, Jha *et al*. 2013, Chakraverty and Pradhan 2014, Mantari and Granados 2015). Indeed, in scientific literature, several researches have been reported on the bending, dynamic, and buckling analyses of sandwich plates with FG face sheets. Using an accurate higher-order shear deformation theory (HSDT), Natarajan and Manickam (2012) investigated the static and free vibration response of two types of FG sandwich plates. Bourada *et al*. (2012) proposed a new four-variable refined plate theory for thermal buckling analysis of FG sandwich plates. Based on the first-order shear deformation plate theory (FSDT), Yaghoobi and Yaghoobi (2013) studied the buckling response of sandwich plates with FG face sheets resting on elastic foundation. Kettaf *et al*. (2013) developed a new hyperbolic shear displacement model for thermal buckling response of FG sandwich plates. Tounsi *et al*. (2013) analytically studied the thermoelastic bending problem of FG sandwich plates based on the refined trigonometric shear deformation theory. Sobhy (2013) investigated the free vibration and the buckling responses of exponentially graded sandwich plates resting on Pasternak elastic foundation. Bessaim *et al*. (2013) employed a new higher-order shear and normal deformation theory for the static and free vibration behavior of sandwich plates with functionally graded isotropic face sheets. Houari *et al*. (2013) investigated the thermoelastic bending response of FG sandwich plates using a new higher order shear and normal deformation theory. Xiang *et al*. (2013) studied the free vibration behavior of FG sandwich plates by employing an *nth*-order shear deformation theory and a meshless method, while Ait Amar Meziane *et al*. (2014) examined the buckling and free vibration of FG sandwich plates using an efficient and simple refined shear deformation theory. Three-dimensional finite element simulations for investigating low velocity impact behavior of sandwich panels with a FG core were presented by Etemadi *et al*. (2009). Swaminathan and Naveenkumar (2014) proposed a higher order refined computational models for the stability analysis of FG sandwich plates. Khalfi *et al*. (2014) proposed a refined and simple shear deformation theory for thermal buckling of solar FG plates on elastic foundation. Ahmed (2014) examined the post-buckling of FG sandwich beams by employing a consistent higher order theory. Bennai *et al.* (2015) presented a new higher-order shear and normal deformation theory for FG sandwich beams. Bakora and Tounsi (2015) analyzed the thermo-mechanical post-buckling response of thick FG plates resting on elastic foundations. Bouchafa *et al*. (2015) discussed the thermal stresses and deflections of FG sandwich plates using a new refined hyperbolic shear deformation theory. Hamidi *et al*. (2015) presented a sinusoidal plate theory with 5-unknowns and stretching effect for thermo-mechanical bending behaviour of FG sandwich plates. Mahi *et al*. (2015) proposed a new hyperbolic shear deformation theory for bending and free vibration analysis of isotropic, functionally graded, sandwich and laminated composite plates. Tebboune *et al*. (2015) studied the thermal buckling response of FG plates resting on elastic foundation based on an efficient and simple trigonometric shear deformation theory. Zidi *et al*. (2014) presented the bending analysis of FGM plates under hygro-thermo-mechanical loading using a four variable refined plate theory. Ebrahimi and Habibi (2016) examined the bending and vibration behaviour of higher-order shear deformable compositionally graded porous plate. Hadji *et al*. (2016) analyzed the mechanical response of FG beam using a new first-order shear deformation theory. Bennoun *et al.* (2016) developed a novel five variable refined plate theory for vibration analysis of FG sandwich plates.

It should be signaled that HSDTs are highly computational cost because of their use of many unknowns (e.g., theories (Talha and Singh 2010) with eleven unknowns and (Natarajan and Manickam 2012) with thirteen unknowns). To reduce computational cost, HSDTs with four and five unknowns were recently proposed for FG plates (see references (Benachour *et al*. 2011, Tounsi *et al*. 2013, Bouderba *et al*. 2013, Thai and Kim 2013, Thai and Choi 2011, 2013, Yaghoobi and Fereidoon 2014, Thai *et al*. 2014, Belabed *et al*. 2014, Fekrar *et al*. 2014, Draiche *et al*. 2014, Bousahla *et al*. 2014, Mantari and Guedes Soares 2014, Nedri *et al*. 2014, Attia *et al*. 2015, Jiang *et al*. 2015, Chattibi *et al*. 2015, Ait Yahia *et al*. 2015, Sobhy 2015, Nguyen *et al*. 2015).

This work presents the thermal buckling response of FG sandwich plates composed of FG face sheets and an isotropic homogeneous core using a simple first-order shear deformation theory (FSDT). By considering a further assumption, the number of variables and governing stability equations of the present FSDT is diminished, thus makes it simple to use. Indeed, the number of unknown variables involved in the present model is only four, as opposed to five in the case of the conventional FSDT. Various boundary conditions are considered in this work. Governing equations are obtained from the principle of minimum total potential energy. Analytical solutions for thermal buckling analysis of FG sandwich plates are determined. Numerical results are presented to prove the accuracy of the present formulation.

2. Theoretical formulation

In this work, a sandwich plate composed of three layers is considered as presented in Fig. 1. Two FG face sheets are made from a mixture of a metal and a ceramic, while a core is composed of an isotropic homogeneous material. The vertical positions of the bottom surface, the two interfaces between the core and faces layers, and the top surface are denoted, respectively, by $h_0=-h/2$, h_1, h_2 and $h_3=h/2$. The total thickness of the FG plate is h, where $h=t_C=t_{FGM}$ and $t_C=t_2-h_1$. t_C and t_{FGM} are the layer thickness of the core and all-FGM layers, respectively. The material properties of FG face sheets are supposed to change continuously within the plate thickness according to a power law distribution as

$$P(z)=P_m+\left(P_c-P_m\right)V \tag{1}$$

where P denotes the effective material property such as Young's modulus P, Poisson's ratio ν, thermal expansion coefficient α; subscripts c and m represent the ceramic and metal phases, respectively; and V is the volume fraction of the ceramic phase expressed by

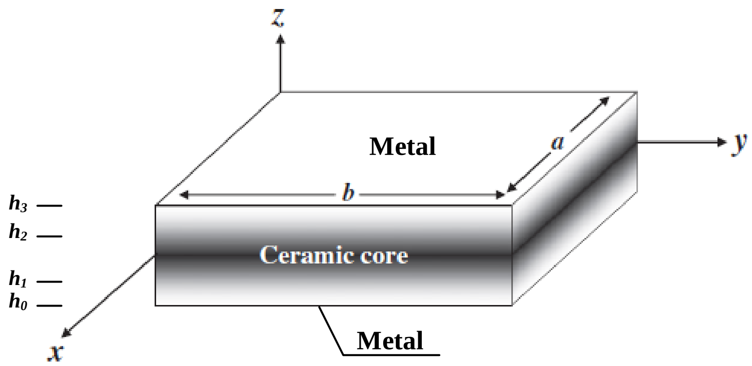

Fig. 1 Geometry of the FGM sandwich plate

$$\begin{cases} V^{(1)}(z) = \left(\dfrac{z-h_0}{h_1-h_0}\right)^p & \text{for } z \in [h_0, h_1] \\ V^{(2)}(z) = 1 & \text{for } z \in [h_1, h_2] \\ V^{(3)}(z) = \left(\dfrac{z-h_3}{h_2-h_3}\right)^p & \text{for } z \in [h_2, h_3] \end{cases} \tag{2}$$

where p is the power law index that governs the volume fraction gradation.

2.1 Kinematics and constitutive equations

The displacement field of the conventional FSDT is given by

$$\begin{aligned} u(x,y,z) &= u_0(x,y) + z\varphi_x(x,y) \\ v(x,y,z) &= v_0(x,y) + z\varphi_y(x,y) \\ w(x,y,z) &= w_0(x,y) \end{aligned} \tag{3}$$

where u_0, v_0, w_0, φ_x and φ_y are five unknown displacement functions of the midplane of the plate. Using the same methodology presented by Bouremana *et al*. (2013) in the case of beam, the displacement field of the new FSDT can be expressed in a simpler form as

$$\begin{aligned} u(x,y,z) &= u_0(x,y) - z\frac{\partial w_b}{\partial x} \\ v(x,y,z) &= v_0(x,y) - z\frac{\partial w_b}{\partial y} \\ w(x,y,z) &= w_b(x,y) + w_s(x,y) \end{aligned} \tag{4}$$

Clearly, the displacement field in Eq. (4) has only four unknowns (u_0, v_0, w_b, w_s). In fact, this reduction of the unknown variables is due to dividing the vertical displacement w into bending and shear parts (i.e., $w=w_b+w_s$) and the further assumptions given by $\varphi_x=-\partial w_b/\partial x$ and $\varphi_y=-\partial w_b/\partial y$.

The non-linear von Karman strain-displacement equations are as follows

$$\begin{aligned}
\varepsilon_x &= \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w_b}{\partial x} + \frac{\partial w_s}{\partial x}\right)^2, \\
\varepsilon_y &= \frac{\partial v}{\partial y} + \frac{1}{2}\left(\frac{\partial w_b}{\partial y} + \frac{\partial w_s}{\partial y}\right)^2, \\
\gamma_{xy} &= \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} + \left(\frac{\partial w_b}{\partial x} + \frac{\partial w_s}{\partial x}\right)\left(\frac{\partial w_b}{\partial y} + \frac{\partial w_s}{\partial y}\right), \\
\gamma_{yz} &= \frac{\partial v}{\partial z} + \left(\frac{\partial w_b}{\partial y} + \frac{\partial w_s}{\partial y}\right), \\
\gamma_{xz} &= \frac{\partial u}{\partial z} + \left(\frac{\partial w_b}{\partial x} + \frac{\partial w_s}{\partial x}\right),
\end{aligned} \tag{5}$$

On the basis of the displacement field presented in Eq. (4), Eq. (5) becomes

$$\begin{aligned}
\varepsilon_x &= \varepsilon_x^0 + z\, k_x^b \\
\varepsilon_y &= \varepsilon_y^0 + z\, k_y^b \\
\gamma_{xy} &= \gamma_{xy}^0 + z\, k_{xy}^b \\
\gamma_{yz} &= \gamma_{yz}^s \\
\gamma_{xz} &= \gamma_{xz}^s
\end{aligned} \tag{6}$$

where

$$\begin{aligned}
\varepsilon_x^0 &= \frac{\partial u_0}{\partial x} + \frac{1}{2}\left(\frac{\partial w_b}{\partial x} + \frac{\partial w_s}{\partial x}\right)^2 \\
\varepsilon_y^0 &= \frac{\partial v_0}{\partial y} + \frac{1}{2}\left(\frac{\partial w_b}{\partial y} + \frac{\partial w_s}{\partial y}\right)^2 \\
\gamma_{xy}^0 &= \frac{\partial v_0}{\partial x} + \frac{\partial u_0}{\partial y} + \left(\frac{\partial w_b}{\partial x} + \frac{\partial w_s}{\partial x}\right)\left(\frac{\partial w_b}{\partial y} + \frac{\partial w_s}{\partial y}\right) \\
k_x^b &= -\frac{\partial^2 w_b}{\partial x^2} \\
k_y^b &= -\frac{\partial^2 w_b}{\partial y^2} \\
k_{xy}^b &= -2\frac{\partial^2 w_b}{\partial x \partial y}
\end{aligned}$$

$$\gamma_{yz}^{s}=\frac{\partial w_s}{\partial y}$$
$$\gamma_{xz}^{s}=\frac{\partial w_s}{\partial x} \tag{7}$$

The linear constitutive relations of a FG sandwich plate can be written as

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{yz}\\ \tau_{xz}\\ \tau_{xy}\end{Bmatrix}^{(n)}=\begin{bmatrix}Q_{11} & Q_{12} & 0 & 0 & 0\\ Q_{12} & Q_{22} & 0 & 0 & 0\\ 0 & 0 & Q_{44} & 0 & 0\\ 0 & 0 & 0 & Q_{55} & 0\\ 0 & 0 & 0 & 0 & Q_{66}\end{bmatrix}^{(n)}\left(\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{yz}\\ \gamma_{xz}\\ \gamma_{xy}\end{Bmatrix}-\begin{Bmatrix}\alpha(z)\\ \alpha(z)\\ 0\\ 0\\ 0\end{Bmatrix}^{(n)}T(z)\right) \tag{8}$$

where (σ_x, σ_y, τ_{yz}, τ_{xz}, τ_{xy}) and (ε_x, ε_y, γ_{yz}, γ_{xz}, γ_{xy}) are the stress and strain components, respectively. $T(z)$ is the temperature difference with respect to the reference and the stiffness coefficients, Q_{ij}, can be expressed as

$$Q_{11}^{(n)}=Q_{22}^{(n)}=\frac{E^{(n)}(z)}{1-\nu^2},\ \ Q_{12}^{(n)}=\nu\, Q_{11}^{(n)} \tag{9a}$$

$$Q_{44}^{(n)}=Q_{55}^{(n)}=Q_{66}^{(n)}=G(z)^{(n)}=\frac{E^{(n)}(z)}{2(1+\nu)}, \tag{9b}$$

2.2 Stability equations

The equilibrium equations of FG sandwich plates under thermal loadings may be obtained on the basis of the stationary potential energy (Reddy, 1984). The equilibrium equations are obtained as

$$\begin{aligned}
&\delta u_0:\ \frac{\partial N_x}{\partial x}+\frac{\partial N_{xy}}{\partial y}=0\\
&\delta v_0:\ \frac{\partial N_{xy}}{\partial x}+\frac{\partial N_y}{\partial y}=0\\
&\delta w_b:\ \frac{\partial^2 M_x}{\partial x^2}+2\frac{\partial^2 M_{xy}}{\partial x\partial y}+\frac{\partial^2 M_y}{\partial y^2}+\overline{N}=0\\
&\delta w_s:\ \frac{\partial Q_x}{\partial x}+\frac{\partial Q_y}{\partial y}+\overline{N}=0
\end{aligned} \tag{10}$$

with

$$\overline{N}=\left[N_x\frac{\partial^2(w_b+w_s)}{\partial x^2}+N_y\frac{\partial^2(w_b+w_s)}{\partial y^2}+2N_{xy}\frac{\partial^2(w_b+w_s)}{\partial x\partial y}\right] \tag{11}$$

Using constitutive relations, the stress and moment resultants are defined as

$$\left(N_x, N_y, N_{xy}\right) = \int_{-h/2}^{h/2} \left(\sigma_x, \sigma_y, \tau_{xy}\right) dz \tag{12a}$$

$$\left(M_x, M_y, M_{xy}\right) = \int_{-h/2}^{h/2} \left(\sigma_x, \sigma_y, \tau_{xy}\right) z dz \tag{12b}$$

$$\left(Q_x, Q_y\right) = \int_{-h/2}^{h/2} \left(\tau_{xz}, \tau_{yz}\right) dz \tag{12c}$$

Upon substitution of Eq. (6) into Eq. (8) and the subsequent results into Eq. (12) the stress resultants are obtained in the matrix form as

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 & B_{11} & B_{12} & 0 \\ A_{12} & A_{22} & 0 & B_{12} & B_{22} & 0 \\ 0 & 0 & A_{66} & 0 & 0 & B_{66} \\ B_{11} & B_{12} & 0 & D_{11} & D_{12} & 0 \\ B_{12} & B_{2} & 0 & D_{12} & D_{22} & 0 \\ 0 & 0 & B_{66} & 0 & 0 & D_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ k_x \\ k_y \\ k_{xy} \end{Bmatrix} - \begin{Bmatrix} N_x^T \\ N_y^T \\ 0 \\ M_x^T \\ M_y^T \\ 0 \end{Bmatrix} \tag{13a}$$

$$\begin{Bmatrix} Q_x \\ Q_y \end{Bmatrix} = \begin{bmatrix} H_{55} & 0 \\ 0 & H_{44} \end{bmatrix} \begin{Bmatrix} \gamma_{xz}^s \\ \gamma_{yz}^s \end{Bmatrix} \tag{13b}$$

where (A_{ij}, B_{ij}, D_{ij}) and (H_{44}, H_{55}) are the stiffness coefficients defined by

$$\left(A_{ij}, B_{ij}, D_{ij}\right) = \int_{-h/2}^{h/2} \left(1, z, z^2\right) Q_{ij}(z) dz = \sum_{n=1}^{3} \int_{h_{n-1}}^{h_n} \left(1, z, z^2\right) Q_{ij}^{(n)}(z) dz \tag{14a}$$

$$H_{44} = H_{55} = k \int_{-h/2}^{h/2} Q_{44}(z) dz = k \sum_{n=1}^{3} \int_{h_{n-1}}^{h_n} Q_{44}^{(n)}(z) dz \tag{14b}$$

with k being the shear correction factor.

The stress and moment resultants, $N_x^T = N_y^T$ and $M_x^T = M_y^T$ to thermal loading are defined by

$$\begin{Bmatrix} N_x^T \\ M_x^T \end{Bmatrix} = \int_{-h/2}^{h/2} \frac{E(z)}{1-\nu} \alpha(z) T(z) \begin{Bmatrix} 1 \\ z \end{Bmatrix} dz = \sum_{n=1}^{3} \int_{h_{n-1}}^{h_n} \frac{E^{(n)}(z)}{1-\nu} \alpha(z)^{(n)} T(z) \begin{Bmatrix} 1 \\ z \end{Bmatrix} dz \tag{15}$$

In order to obtain the stability equations and study the thermal buckling behavior of the FG sandwich plate, the adjacent equilibrium criterion is employed (Brush and Almroth 1975). By employing this approach, the governing stability equations are determined as

$$
\begin{aligned}
&\frac{\partial N_x^1}{\partial x}+\frac{\partial N_{xy}^1}{\partial y}=0\\
&\frac{\partial N_{xy}^1}{\partial x}+\frac{\partial N_y^1}{\partial y}=0\\
&\frac{\partial^2 M_x^1}{\partial x^2}+2\frac{\partial^2 M_{xy}^1}{\partial x\partial y}+\frac{\partial^2 M_y^1}{\partial y^2}+N_x^0\frac{\partial^2\left(w_b^1+w_s^1\right)}{\partial x^2}+N_y^0\frac{\partial^2\left(w_b^1+w_s^1\right)}{\partial y^2}+2N_{xy}^0\frac{\partial^2\left(w_b^1+w_s^1\right)}{\partial x\partial y}=0\\
&\frac{\partial Q_x^1}{\partial x}+\frac{\partial Q_y^1}{\partial y}+N_x^0\frac{\partial^2\left(w_b^1+w_s^1\right)}{\partial x^2}+N_y^0\frac{\partial^2\left(w_b^1+w_s^1\right)}{\partial y^2}+2N_{xy}^0\frac{\partial^2\left(w_b^1+w_s^1\right)}{\partial x\partial y}=0
\end{aligned}
\tag{16}
$$

where N_x^0 , N_{xy}^0 and N_y^0 are the pre-buckling forces. Eq. (16) can be written in terms of displacements (($u_0^1, v_0^1, w_b^1, w_s^1$) by substituting for the stress resultants from Eq. (13). For FG sandwich plate, the stability equations Eq. (16) take the form

$$A_{11}d_{11}u_0^1+A_{66}d_{22}u_0^1+\left(A_{12}+A_{66}\right)d_{12}v_0^1-B_{11}d_{111}w_b^1-\left(B_{12}+2B_{66}\right)d_{122}w_b^1=0, \tag{17a}$$

$$A_{22}d_{22}v_0^1+A_{66}d_{11}v_0^1+\left(A_{12}+A_{66}\right)d_{12}u_0^1-B_{22}d_{222}w_b^1-\left(B_{12}+2B_{66}\right)d_{112}w_b^1=0, \tag{17b}$$

$$
\begin{aligned}
&B_{11}d_{111}u_0^1+\left(B_{12}+2B_{66}\right)d_{122}u_0^1+\left(B_{12}+2B_{66}\right)d_{112}v_0^1+B_{22}d_{222}v_0^1-D_{11}d_{1111}w_b^1-2\left(D_{12}+2D_{66}\right)d_{1122}w_b^1\\
&\quad -D_{22}d_{2222}w_b^1+N_x^0 d_{11}\left(w_b^1+w_s^1\right)+N_y^0 d_{22}\left(w_b^1+w_s^1\right)=0
\end{aligned}
\tag{17c}
$$

$$H_{55}d_{11}w_s^1+H_{44}d_{22}w_s^1+N_x^0 d_{11}\left(w_b^1+w_s^1\right)+N_y^0 d_{22}\left(w_b^1+w_s^1\right)=0 \tag{17d}$$

3. Thermal buckling solution

The exact solution of Eq. (17) for the FGMs sandwich plate under various boundary conditions can be constructed. The boundary conditions for an arbitrary edge with simply supported and clamped edge conditions are:

• Clamped (C)

$$u_0=v_0=w_b=\partial w_b/\partial x=\partial w_b/\partial y=w_s=\partial w_s/\partial x=\partial w_s/\partial y=0 \text{ at } x=0,a \text{ and } y=0,b \tag{18}$$

• Simply supported (S)

$$v_0=w_b=\partial w_b/\partial y=w_s=\partial w_s/\partial y=0 \text{ at } x=0,a \tag{19a}$$

$$u_0=w_b=\partial w_b/\partial x=w_s=\partial w_s/\partial x=0 \text{ at } y=0,b \tag{19b}$$

The following representation for the displacement quantities, that satisfy the above boundary conditions, is appropriate in the case of our problem

Table 1 The admissible functions $\boldsymbol{X}_m(x)$ and $\boldsymbol{Y}_n(y)$

	Boundary conditions		The functions $\boldsymbol{X}_m$ and $\boldsymbol{Y}_n$	
	At x=0, a	At y=0, b	$X_m(x)$	$Y_n(y)$
SSSS	$X_m(0)=X''_m(0)=0$ $X_m(a)=X''_m(a)=0$	$Y_n(0)=Y''_n(0)=0$ $Y_n(b)=Y''_n(b)=0$	$\sin(\lambda x)$	$\sin(\mu y)$
CSSS	$X_m(0)=X'_m(0)=0$ $X_m(a)=X''_m(a)=0$	$Y_n(0)=Y''_n(0)=0$ $Y_n(b)=Y''_n(b)=0$	$\sin(\lambda x)[\cos(\lambda x)-1]$	$\sin(\mu y)$
CSCS	$X_m(0)=X'_m(0)=0$ $X_m(a)=X''_m(a)=0$	$Y_n(0)=Y'_n(0)=0$ $Y_n(b)=Y''_n(b)=0$	$\sin(\lambda x)[\cos(\lambda x)-1]$	$\sin(\mu x)[\cos(\mu x)-1]$
CCSS	$X_m(0)=X'_m(0)=0$ $X_m(a)=X'_m(a)=0$	$Y_n(b)=Y''_n(b)=0$ $Y_n(b)=Y''_n(b)=0$	$\sin^2(\lambda x)$	$\sin(\mu y)$
CCCC	$X_m(0)=X'_m(0)=0$ $X_m(a)=X'_m(a)=0$	$Y_n(0)=Y''_n(0)=0$ $Y_n(b)=Y'_n(b)=0$	$\sin^2(\lambda x)$	$\sin^2(\mu y)$
FFCC	$X''_m(0)=X'''_m(0)=0$ $X''_m(a)=X'''_m(a)=0$	$Y_n(0)=Y'_n(0)=0$ $Y_n(b)=Y'_n(b)=0$	$\cos^2(\lambda x)[\sin^2(\lambda x)+1]$	$\sin^2(\mu y)$

()' Denotes the derivative with respect to the corresponding coordinates.

$$\begin{Bmatrix} u_0^1 \\ v_0^1 \\ w_{b0}^1 \\ w_{s0}^1 \end{Bmatrix} = \begin{Bmatrix} U_{mn} \dfrac{\partial X_m(x)}{\partial x} Y_n(y) \\ V_{mn} X_m(x) \dfrac{\partial Y_n(y)}{\partial y} \\ W_{bmn} X_m(x) Y_n(y) \\ W_{smn} X_m(x) Y_n(y) \end{Bmatrix} \tag{20}$$

where U_{mn}, V_{mn}, W_{bmn}, and W_{smn} are arbitrary parameters.

The functions $X_m(x)$ and $Y_n(y)$ are suggested by Sobhy (2013) to satisfy at least the geometric boundary conditions given in Eqs. (18) and (19), and represent approximate shapes of the deflected surface of the plate. These functions, for the different cases of boundary conditions, are listed in Table 1 noting that $\lambda=m\pi/a$ and $\mu=n\pi/b$.

Substituting expressions (20) into the governing Eq. (17), we can obtain, after some mathematical manipulations, the following equations

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33}-\beta\overline{P} & S_{34}-\beta\overline{P} \\ S_{41} & S_{42} & S_{43}-\beta\overline{P} & S_{44}-\beta\overline{P} \end{bmatrix} \begin{Bmatrix} U_{mn} \\ V_{mn} \\ W_{bmn} \\ W_{smn} \end{Bmatrix} = 0 \tag{21}$$

in which

$$S_{11} = A_{11}\alpha_{12} + A_{66}\alpha_8$$
$$S_{12} = (A_{12} + A_{66})\alpha_8$$

$$
\begin{aligned}
&S_{13} = -B_{11}\alpha_{12} - (B_{12} + 2B_{66})\alpha_8 \\
&S_{14} = 0 \\
&S_{21} = (A_{12} + A_{66})\alpha_{10} \\
&S_{22} = A_{22}\alpha_4 + A_{66}\alpha_{10} \\
&S_{23} = -B_{22}\alpha_4 - (B_{12} + 2B_{66})\alpha_{10} \\
&S_{24} = 0 \\
&S_{31} = B_{11}\alpha_{13} + (B_{12} + 2B_{66})\alpha_{11} \\
&S_{32} = (B_{12} + 2B_{66})\alpha_{11} + B_{22}\alpha_5 \\
&S_{33} = -D_{11}\alpha_{13} - 2(D_{12} + 2D_{66})\alpha_{11} - D_{22}\alpha_5 \\
&S_{34} = -D^s_{11}\alpha_{13} - 2(D^s_{12} + 2D^s_{66})\alpha_{11} - D^s_{22}\alpha_5 \\
&S_{41} = 0 \\
&S_{42} = 0 \\
&S_{43} = -D^s_{11}\alpha_{13} - 2(D^s_{12} + 2D^s_{66})\alpha_{11} - D^s_{22}\alpha_5 \\
&S_{44} = -H^s_{11}\alpha_{13} - 2(H^s_{12} + 2H^s_{66})\alpha_{11} - H^s_{22}\alpha_5 + A^s_{44}\alpha_9 + A^s_{55}\alpha_3 \\
&\overline{P} = N^0_x \\
&\xi = N^0_y / N^0_x
\end{aligned}
\tag{22a}
$$

$$
\begin{aligned}
&\beta = \xi\alpha_3 + \alpha_9 \\
&(\alpha_1, \quad \alpha_3, \quad \alpha_5) = \int_0^b\int_0^a \left(X_m Y_n, \quad X_m Y_n'', \quad X_m Y_n''''\right) X_m Y_n \, dx\, dy \\
&(\alpha_2, \quad \alpha_4, \quad \alpha_{10}) = \int_0^b\int_0^a \left(X_m Y_n', \quad X_m Y_n''', \quad X_m'' Y_n'\right) X_m Y_n' \, dx\, dy \\
&(\alpha_6, \quad \alpha_8, \quad \alpha_{12}) = \int_0^b\int_0^a \left(X_m' Y_n, \quad X_m' Y_n'', \quad X_m''' Y_n\right) X_m' Y_n \, dx\, dy \\
&(\alpha_7, \quad \alpha_9, \quad \alpha_{11}, \quad \alpha_{13}) = \int_0^b\int_0^a \left(X_m' Y_n', \quad X_m'' Y_n, \quad X_m'' Y_n'', \quad X_m'''' Y_n\right) X_m Y_n \, dx\, dy
\end{aligned}
\tag{22b}
$$

The non-trivial solution is obtained when the determinant of Eq. (21) equals zero.

3.1 Buckling of FGM plates under uniform temperature rise

The plate initial temperature is assumed to be T_i. The temperature is uniformly raised to a final value T_f in which the plate buckles. The temperature change is $\Delta T = T_f - T_i$. The thermal force resultant is evaluated as

$$\overline{P} = \sum_{n=1}^{3} \int_{h_{n-1}}^{h_n} \frac{E^{(n)}(z)\alpha^{(n)}(z)\Delta T}{1-\nu} dz \tag{23}$$

3.2 Buckling of FGM plates subjected to graded temperature change across the thickness

We assume that the temperature of the top surface is T_t and the temperature varies from T_t, according to the power law variation through-the-thickness, to the bottom surface temperature T_b in which the plate buckles. In this case, the temperature through-the-thickness is given by

$$T(z) = \Delta T\left(\frac{z}{h} + \frac{1}{2}\right)^{\gamma} + T_t \tag{24}$$

where the buckling temperature difference $\Delta T = T_b - T_t$ and γ is the temperature exponent ($0<\gamma<\infty$).

Note that the value of γ equal to unity represents a linear temperature change across the thickness. While the value of γ excluding unity represents a non-linear temperature change through-the-thickness. Similar to the previous loading case, the critical buckling temperature change ΔT_{cr} is obtained by using Eqs. (24) and (23).

4. Numerical results

In this section, numerical examples are proposed and discussed for checking the accuracy of the present formulation in predicting the thermal buckling temperatures. Critical buckling temperatures are obtained and the comparison is carried out with the existing results.

The first comparative study for evaluation of the critical buckling temperature difference T_{cr} between the proposed theory and the solution developed by Zhao *et al*. (2009) based on FSDT, in conjunction with the element- free kp Ritz method, results of Kiani *et al*. (2011) based on the combined Galerkin-power series solution, results of Nguyen-Xuan *et al*. (2011) based on the smoothed finite elements method, results of Bateni *et al*. (2013) based on the multi-term Galerkin solution and those of Bouhadra *et al*. (2015) is performed in Table 2. The plate here is subjected to a uniform temperature rise across the thickness and with clamped boundary conditions. From the results presented in Table 2, it is observed that our results have a good agreement with the

Table 2 Critical buckling temperature difference T_{cr} of a clamped square Al/Al_2O_3 FGM plate under uniform temperature rise for different values of power law index p and side-to-thickness ratio

h/a	Theory	*p*=0	*p*=0.5	*p*=1	*p*=2	*p*=5
0.01	Present	181.299	102.787	84.306	74.738	77.025
	Bouhadra *et al.* (2015)	181.300	102.795	84.307	74.715	76.934
	Zhao *et al*. (2009)	175.817	99.162	82.357	71.013	74.591
	Kiani *et al*. (2011)	182.06	103.15	84.58	74.99	77.36
	Nguyen-Xuan *et al*. (2011)	188.28	105.27	86.07	76.07	78.06
	Bateni *et al.* (2013)	180.30	102.23	83.84	74.30	76.50
0.02	Present	45.529	25.800	21.157	18.756	19.345
	Bouhadra *et al.* (2015)	45.529	25.800	21.156	18.754	19.339
	Zhao *et al*. (2009)	44.171	24.899	20.771	18.489	19.150
	Kiani *et al*. (2011)	45.51	25.79	21.15	18.75	19.34
	Nguyen-Xuan *et al*. (2011)	47.50	26.54	21.70	19.18	19.70
	Bateni *et al.* (2013)	45.28	25.65	21.04	18.65	19.23

Table 3 Material properties used in the FG sandwich plate

Properties	Metal: Ti-6A1-4V	Ceramic: ZrO_2
E (GPa)	66.2	244.27
v	0.3	0.3
α (10^{-6}/K)	10.3	12.766

Table 4 Minimum critical temperature parameter αT_{cr} of the simply supported isotropic plate (a/b=1, α_0=1.0×10^{-6} K, E=1.0×10^{-6} N/m^2, v=0.3)

a/h	Present	Kettaf *et al.* (2013)	Matsunaga (2005)
10	0.1198 10^{-1}	0.1198 10^{-1}	0.1183 10^{-1}
20	0.3119 10^{-2}	0.3119 10^{-2}	0.3109 10^{-2}
100	0.1265 10^{-3}	0.1265 10^{-3}	0.1264 10^{-3}

Table 5 Critical buckling temperature ($10^3\alpha_0\Delta T_{cr}$) of a homogeneous isotropic plate under uniform temperature rise

b/a	Theory	a/h=5	a/h=10	a/h=15	a/h=25	a/h=50
0.5	Present	80.90487	27.72437	13.23020	4.94967	1.25824
	FSDPT(a)	80.90487	27.72437	13.23021	4.94968	1.25824
	HSDPT(a)	81.15170	27.73347	13.23144	4.94979	1.25825
	SSDPT(a)	81.18685	27.73638	13.23205	4.94987	1.25825
	TSDPT(a)	81.09991	27.73011	13.23079	4.94970	1.25824
	CPT	126.5334	31.63335	14.05927	5.06134	1.26533
1	Present	41.29710	11.97782	5.48619	2.00643	0.50500
	FSDPT(a)	41.29710	11.97782	5.48619	2.00643	0.50499
	HSDPT(a)	41.32613	11.97877	5.48633	2.00644	0.50500
	SSDPT(a)	41.33313	11.97927	5.48643	2.00646	0.50500
	TSDPT(a)	41.31747	11.97825	5.48623	2.00643	0.50499
	CPT	50.61336	12.65334	5.62371	2.02453	0.50613
2	Present	27.72437	7.63907	3.46060	1.25824	0.31589
	FSDPT(a)	27.72437	7.63907	3.46060	1.25824	0.31589
	HSDPT(a)	27.73347	7.63938	3.46065	1.25824	0.31589
	SSDPT(a)	27.73638	7.63958	3.46069	1.25825	0.31589
	TSDPT(a)	27.73011	7.63918	3.46061	1.25824	0.31589
	CPT	31.63335	7.90834	3.51482	1.26533	0.31633
5	Present	23.55569	6.39227	2.88670	1.04784	0.26288
	FSDPT(a)	23.55569	6.39227	2.88670	1.04784	0.26288
	HSDPT(a)	23.56145	6.39248	2.88674	1.04785	0.26288
	SSDPT(a)	23.56351	6.39261	2.88676	1.04785	0.26288
	TSDPT(a)	23.55914	6.39233	2.88671	1.04784	0.26288
	CPT	26.31895	6.57974	2.92433	1.05276	0.26319

(a) Taken from Kettaf *et al.* (2013)

available data in (Bouhadra *et al.* 2015, Zhao *et al.* 2009, Kiani *et al.* 2011, Nguyen-Xuan *et al.* 2011, Bateni *et al*. 2013). Good agreement is demonstrate between our results and the available data (Zhao *et al*. 2009, Kiani *et al*. 2011, Nguyen-Xuan *et al*. *et al*. 2011, Bateni *et al*. 2013, Bouhadra *et al*. 2015).

For numerical results, the combination of materials consists of Titanium and Zirconia. The Young's modulus and the coefficient of thermal expansion for Titanium and Zirconia are given in Table 3.

In order to prove the accuracy of the present method, a comparison study is made with the results obtained by both Matsunaga (2005) based on two-dimensional global higher-order deformation theory and Kettaf *et al.* (2013) based on hyperbolic shear deformation plate theory (HSDPT) for simply supported homogeneous isotropic plates under uniform temperature rise. The critical buckling temperature difference is listed in Table 4. As this table shows, the present results have a good agreement with those reported in Ref. (Matsunaga 2005). Excellent agreement can be observed for different values of thickness ratio *a/h*.

Non-dimensional critical buckling temperatures ($10^3\alpha_0\Delta T_{cr}$) of homogeneous isotropic plate (p=0, $E(z)=E_0$, $\alpha(z)=\alpha_0$, ν=0.3) for different values of the side-to-thickness ratio *a/h* and aspect ratio *b/a* are listed in Table 5. The calculated non-dimensional critical buckling temperatures are compared with those reported by Kettaf *et al.* (2013). It should be noted that the results reported by Kettaf *et al* (2013) were based on hyperbolic shear deformation plate theory (HSDPT), sinusoidal shear deformation plate theory (SSDPT), third shear deformation plate theory (TSDPT), and the conventional first shear deformation plate theory (FSDPT) with five independent variables. An excellent agreement between the results is obtained for all values of geometric ratios *a/h* and *b/a*. The difference between the shear deformation plate theories and the CPT decreases as the ratios *a/h* or *b/a* increase because the plate becomes thin or long. It should be recalled that the present theory contains only four unknowns and four governing equations, while the number of unknowns and governing equations of the SSDPT, TSDPT and FSDPT is five. Thus, it can be stated that the present model is not only accurate but also simple in predicting the critical buckling temperature of FG sandwich plates.

For verification of the thermal buckling solutions obtained in this work, the critical buckling temperature difference ($T_{cr}=10^{-3}\Delta T_{cr}$), for FG sandwich plates for the uniform, linear and nonlinear cases of temperature distribution through the thickness are shown in Tables 6-8, respectively. The comparison between the present simple first-order shear deformation theory and different CPT, FSDPT, SSDPT, TSDPT and HSDPT is established. A good agreement between the results is seen for all values of volume fraction index p and thickness of the core $\boldsymbol{t}_C$ of FG sandwich plates. In general, the present FSDPT and existing FSDT gives almost identical results. It should be noted that the proposed FSDPT contains less number of unknowns than the existing FSDPT. It can be concluded that the present FSDPT not only gives comparable results with the existing FSDPT, but also is simpler than the existing FSDPT due to having less number of unknowns, i.e., four as against five.

Tables 6-8 indicate also the effect of the layer thickness of the core $\boldsymbol{t}_C$ (ceramic layer) on the thermal buckling response of the FG sandwich plates. As can be observed from Tables 6 and 7, the thermal buckling temperatures increase with the decrease in volume fraction index p. Thus, the increase in thermal buckling temperature of an FG sandwich plate could be attributed to the ceramic property. Indeed, this remark is also proved when a small volume fraction index is considered ($p\leq 2$) for all values of $\boldsymbol{t}_C$. A small volume fraction index k indicates that the ceramic is the dominant constituent in FG sandwich plates. However, Table 8 shows that the thermal buckling temperatures decrease with the decrease in volume fraction index p when the plate is under non-linear temperature rise with γ=5. It can be seen that the thermal buckling temperature increases with decreasing thickness of the thickness of the core layer ($\boldsymbol{t}_C$) for all considered volume fraction index.

Table 6 Critical buckling temperature T_{cr} of FG sandwich square plates under uniform temperature rise versus volume fraction index p and t_C/h (a/h=5)

t_C/h	Theory	p						
		0	0.2	0.5	1	2	5	10
0	Present	3.23493	3.04858	2.83507	2.64222	2.57355	2.86226	3.23289
	FSDPT[a]	3.23493	3.04858	2.83507	2.64222	2.57355	2.86226	3.23289
	HSDPT[a]	3.23720	3.07138	2.87207	2.68975	2.63325	2.93978	3.30959
	SSDPT[a]	3.23775	3.07197	2.87277	2.69065	2.63460	2.94205	3.31226
	TSDPT[a]	3.23652	3.07042	2.87074	2.68781	2.63018	2.93446	3.30340
	CPT	3.96470	3.66606	3.34559	3.06734	2.96200	3.32950	3.82441
0.2	Present	3.23493	3.03394	2.79675	2.55053	2.34734	2.28926	2.35538
	FSDPT[a]	3.23493	3.03394	2.79675	2.55053	2.34734	2.28926	2.35538
	HSDPT[a]	3.23720	3.05543	2.83135	2.59388	2.39856	2.35252	2.42641
	SSDPT[a]	3.23775	3.05598	2.83194	2.59458	2.39953	2.35401	2.42827
	TSDPT[a]	3.23652	3.05461	2.83030	2.59241	2.39637	2.34898	2.42195
	CPT	3.96470	3.64978	3.30066	2.95538	2.68016	2.59922	2.68195
0.4	Present	3.23493	3.04170	2.81495	2.57037	2.33409	2.15296	2.12571
	FSDPT[a]	3.23493	3.04171	2.81495	2.57038	2.33409	2.15296	2.12571
	HSDPT[a]	3.23720	3.05915	2.84285	2.60512	2.37406	2.19921	2.17624
	SSDPT[a]	3.23775	3.05956	2.84318	2.60545	2.37450	2.19992	2.17714
	TSDPT[a]	3.23652	3.05867	2.84246	2.60462	2.37320	2.19763	2.17417
	CPT	3.96470	3.66567	3.33354	2.99117	2.67295	2.43609	2.39804
0.5	Present	3.23493	3.05527	2.84659	2.62069	2.39542	2.20129	2.13606
	FSDPT[a]	3.23493	3.05527	2.84659	2.62069	2.39542	2.20130	2.13606
	HSDPT[a]	3.23720	3.06980	2.86974	2.64965	2.42885	2.23972	2.17737
	SSDPT[a]	3.23775	3.07014	2.86992	2.64976	2.42900	2.24005	2.17784
	TSDPT[a]	3.23652	3.06952	2.86972	2.64970	2.42873	2.23910	2.17640
	CPT	3.96470	3.68764	3.38155	3.06366	2.75801	2.50252	2.41816
0.6	Present	3.23493	3.07586	2.89364	2.69680	2.49698	2.31286	2.24191
	FSDPT[a]	3.23493	3.07586	2.89364	2.69680	2.49698	2.31286	2.24190
	HSDPT[a]	3.23720	3.08713	2.91139	2.71917	2.52309	2.34313	2.27452
	SSDPT[a]	3.23775	3.08741	2.91146	2.71909	2.52297	2.34310	2.27458
	TSDPT[a]	3.23652	3.08699	2.91168	2.71971	2.52367	2.34345	2.27461
	CPT	3.96470	3.71993	3.45164	3.17226	2.89771	2.65182	2.55878
0.8	Present	3.23493	3.13952	3.03406	2.92193	2.80661	2.72895	2.64315
	FSDPT[a]	3.23493	3.13952	3.03406	2.92193	2.80661	2.72895	2.64315
	HSDPT[a]	3.23720	3.14445	3.04101	2.93052	2.81681	2.74134	2.65659
	SSDPT[a]	3.23775	3.14474	3.04107	2.93038	2.81650	2.74092	2.65609
	TSDPT[a]	3.23652	3.14431	3.04137	2.93131	2.81794	2.74272	2.65798
	CPT	3.96470	3.81800	3.66058	3.49712	3.33246	3.21552	3.10423
1	Present	3.23493	3.23493	3.23493	3.23493	3.23493	3.23493	3.23493
	FSDPT[a]	3.23493	3.23493	3.23493	3.23493	3.23493	3.23493	3.23493
	HSDPT[a]	3.23720	3.23720	3.23720	3.23720	3.23720	3.23720	3.23720
	SSDPT[a]	3.23775	3.23775	3.23775	3.23775	3.23775	3.23775	3.23775
	TSDPT[a]	3.23652	3.23652	3.23652	3.23652	3.23652	3.23652	3.23652
	CPT	3.96470	3.96470	3.96470	3.96470	3.96470	3.96470	3.96470

[a] Taken from Kettaf *et al*. (2013)

Table 7 Critical buckling temperature T_{cr} of FG sandwich square plates under linear temperature rise versus volume fraction index p and t_C/h (a/h=5)

t_C/h	Theory	p						
		0	0.2	0.5	1	2	5	10
0	Present	6.41986	6.04716	5.62014	5.23443	5.09711	5.67452	6.41578
	FSDPT[(a)]	6.41986	6.04716	5.62014	5.23443	5.09711	5.67452	6.41578
	HSDPT[(a)]	6.42441	6.09275	5.69414	5.32949	5.21651	5.82957	6.56918
	SSDPT[(a)]	6.42550	6.09396	5.69554	5.33130	5.21920	5.83411	6.57458
	TSDPT[(a)]	6.42305	6.09084	5.69148	5.32562	5.21036	5.81891	6.55680
	CPT	7.87940	7.28211	6.64118	6.08468	5.87400	6.60901	7.59882
0.2	Present	6.41986	6.01788	5.54350	5.05105	4.64468	4.52851	4.66058
	FSDPT[(a)]	6.41986	6.01789	5.54350	5.05105	4.64468	4.52851	4.66058
	HSDPT[(a)]	6.42441	6.06087	5.61271	5.13775	4.74712	4.65504	4.80264
	SSDPT[(a)]	6.42550	6.06197	5.61388	5.13917	4.74907	4.65803	4.80632
	TSDPT[(a)]	6.42305	6.05922	5.61059	5.13482	4.74275	4.64797	4.79372
	CPT	7.87940	7.24955	6.55131	5.86076	5.31032	5.14843	5.31369
0.4	Present	6.41986	6.03341	5.57990	5.09075	4.61818	4.25591	4.16712
	FSDPT[(a)]	6.41986	6.03341	5.57990	5.09075	4.61818	4.25591	4.16712
	HSDPT[(a)]	6.42441	6.06830	5.63571	5.16024	4.69812	4.34842	4.26735
	SSDPT[(a)]	6.42550	6.06913	5.63636	5.16089	4.69900	4.34984	4.24818
	TSDPT[(a)]	6.42305	6.06734	5.63491	5.15923	4.69640	4.34526	4.26325
	CPT	7.87940	7.28133	6.61708	5.93233	5.29588	4.82217	4.70737
0.5	Present	6.41986	6.06053	5.64319	5.19137	4.74084	4.35259	4.22211
	FSDPT[(a)]	6.41986	6.06053	5.64319	5.19137	4.74084	4.35259	4.22211
	HSDPT[(a)]	6.42441	6.08961	5.68948	5.24929	4.80770	4.42943	4.30474
	SSDPT[(a)]	6.42550	6.09029	5.68986	5.24952	4.80800	4.43011	4.30569
	TSDPT[(a)]	6.42305	6.08903	5.68943	5.24940	4.80746	4.42821	4.30281
	CPT	7.87940	7.32529	6.71310	6.07732	5.46601	4.95505	4.78633
0.6	Present	6.41986	6.10171	5.73727	5.34360	4.94396	4.57561	4.43382
	FSDPT[(a)]	6.41986	6.10171	5.73728	5.34361	4.94396	4.57561	4.43382
	HSDPT[(a)]	6.42441	6.12425	5.77278	5.38833	4.99619	4.63616	4.49905
	SSDPT[(a)]	6.42550	6.12482	5.77291	5.38818	4.99595	4.63609	4.84881
	TSDPT[(a)]	6.42305	6.12398	5.77335	5.38942	4.99734	4.63680	4.49922
	CPT	7.87940	7.38985	6.85328	6.29453	5.74542	5.25352	5.06756
0.8	Present	6.41986	6.22905	6.01812	5.79385	5.56322	5.33541	5.23630
	FSDPT[(a)]	6.41986	6.22905	6.01812	5.79385	5.56322	5.33541	5.23630
	HSDPT[(a)]	6.42441	6.23889	6.03202	5.81104	5.58362	5.35987	5.26317
	SSDPT[(a)]	6.42550	6.23949	6.03215	5.81076	5.58301	5.35923	5.26229
	TSDPT[(a)]	6.42305	6.23862	6.03273	5.81262	5.58589	5.36259	5.26598
	CPT	7.87940	7.58600	7.27115	6.94424	6.61492	6.29563	6.15846
1	Present	6.41986	6.41986	6.41986	6.41986	6.41986	6.41986	6.41986
	FSDPT[(a)]	6.41986	6.41986	6.41986	6.41986	6.41986	6.41986	6.41986
	HSDPT[(a)]	6.42441	6.42441	6.42441	6.42441	6.42441	6.42441	6.42441
	SSDPT[(a)]	6.42550	6.42550	6.42550	6.42550	6.42550	6.42550	6.42550
	TSDPT[(a)]	6.42305	6.42305	6.42305	6.42305	6.42305	6.42305	6.42305
	CPT	6.42363	6.12545	5.77544	5.39175	4.99944	4.63813	4.49981

[(a)] Taken from Kettaf *et al*. (2013)

Table 8 Critical buckling temperature T_{cr} of FG sandwich square plates under non-linear temperature rise versus volume fraction index p and t_C/h (a/h=5 and γ=5)

t_C/h	Theory	p						
		0	0.2	0.5	1	2	5	10
0	Present	19.25957	20.41729	21.34246	22.02700	22.52869	23.12129	23.49484
	FSDPT[(a)]	19.25957	20.41729	21.34246	22.02700	22.52869	23.12129	23.49484
	HSDPT[(a)]	19.27322	20.57122	21.62347	22.42701	23.05643	23.75304	24.05661
	SSDPT[(a)]	19.27655	20.57531	21.62882	22.43468	23.06838	23.77163	24.07624
	TSDPT[(a)]	19.26915	20.56479	21.61337	22.41074	23.02926	23.70963	24.01127
	CPT	23.63820	24.58692	25.21986	25.60494	25.96247	26.92893	27.82720
0.2	Present	19.25957	20.28528	21.08307	21.62417	21.89055	22.03367	22.17958
	FSDPT[(a)]	19.25957	20.28528	21.08307	21.62417	21.89055	22.03367	22.17958
	HSDPT[(a)]	19.27322	20.43016	21.34626	21.99533	22.37338	22.64929	22.85562
	SSDPT[(a)]	19.27655	20.43388	21.35077	22.00145	22.38259	22.66392	22.87344
	TSDPT[(a)]	19.26915	20.42463	21.33822	21.98279	22.35275	22.61489	22.81317
	CPT	23.63820	24.43703	24.91598	25.09061	25.02775	25.04991	25.28770
0.4	Present	19.25957	20.12913	20.79943	21.25144	21.44811	21.40709	21.36153
	FSDPT[(a)]	19.25957	20.12913	20.79943	21.25144	21.44811	21.40709	21.36153
	HSDPT[(a)]	19.27322	20.24553	21.00745	21.54152	21.81937	21.87237	21.87534
	SSDPT[(a)]	19.27655	20.24830	21.00993	21.54429	21.82352	21.87961	21.88463
	TSDPT[(a)]	19.26915	20.24234	21.00447	21.53734	21.81141	21.85652	21.85429
	CPT	23.63820	24.29255	24.66557	24.76464	24.59556	24.25535	24.13098
0.5	Present	19.25957	20.03597	20.63466	21.04804	21.24818	21.22586	21.16437
	FSDPT[(a)]	19.25957	20.03597	20.63466	21.04804	21.24818	21.22586	21.16437
	HSDPT[(a)]	19.27322	20.13209	20.80394	21.28287	21.54783	21.60059	21.57856
	SSDPT[(a)]	19.27655	20.13435	20.80531	21.28380	21.54921	21.60389	21.58333
	TSDPT[(a)]	19.26915	20.13019	20.80375	21.28330	21.54679	21.59462	21.56887
	CPT	23.63820	24.21722	24.54686	24.64006	24.49836	24.16380	23.99263
0.6	Present	19.25957	19.92815	20.44424	20.81202	21.01752	21.04701	21.00808
	FSDPT[(a)]	19.25957	19.92815	20.44424	20.81202	21.01752	21.04701	21.00808
	HSDPT[(a)]	19.27322	20.00176	20.57076	20.98623	21.23955	21.32555	21.31715
	SSDPT[(a)]	19.27655	20.00362	20.57125	20.98565	21.23856	21.32526	21.31771
	TSDPT[(a)]	19.26915	20.00087	20.57280	20.99045	21.24446	21.32848	21.31794
	CPT	23.63820	24.13520	24.42100	24.51562	24.42463	24.16529	24.01085
0.8	Present	19.25957	19.65105	19.95177	20.17887	20.33926	20.43371	20.45455
	FSDPT[(a)]	19.25957	19.65105	19.95177	20.17887	20.33926	20.43371	20.45455
	HSDPT[(a)]	19.27322	19.68210	19.99784	20.23872	20.41383	20.52740	20.55953
	SSDPT[(a)]	19.27655	19.68400	19.99828	20.23774	20.41159	20.52420	20.55608
	TSDPT[(a)]	19.26915	19.68124	20.00022	20.24422	20.42213	20.53780	20.57050
	CPT	23.63820	23.93190	24.10594	24.18546	24.18431	24.11121	24.05679
1	Present	19.25957	19.25957	19.25957	19.25957	19.25957	19.25957	19.25957
	FSDPT[(a)]	19.25957	19.25957	19.25957	19.25957	19.25957	19.25957	19.25957
	HSDPT[(a)]	19.27322	19.27322	19.27322	19.27322	19.27322	19.27322	19.27322
	SSDPT[(a)]	19.27655	19.27655	19.27655	19.27655	19.27655	19.27655	19.27655
	TSDPT[(a)]	19.26915	19.26915	19.26915	19.26915	19.26915	19.26915	19.26915
	CPT	23.63820	23.63820	23.63820	23.63820	23.63820	23.63820	23.63820

[(a)] Taken from Kettaf *et al*. (2013)

Table 9 Critical buckling temperature T_{cr} of FG sandwich square plates under uniform, linear and non-linear temperature rise versus volume fraction index p and t_C/h (a/h=5)

t_C/h	Temperature	p	Boundary condition					
			SSSS	CSSS	CSCS	CCSS	CCCC	FFCC
0	U	0.5	2.83507	4.09320	5.00928	4.66511	6.02729	6.71974
		2	2.57355	3.76007	4.63724	4.32208	5.63185	6.32716
	L	0.5	5.62014	8.13640	9.96856	9.28023	12.0046	13.3895
		2	5.09711	7.47013	9.22448	8.59416	11.2137	12.6043
	NL	0.5	21.3425	30.8980	37.8556	35.2416	45.5874	50.8465
		2	22.5287	33.0172	40.7712	37.9853	49.5634	55.7098
0.4	U	0.5	2.81495	4.05752	4.96039	4.61914	5.96099	6.63909
		2	2.33408	3.41853	4.22279	3.93652	5.13854	5.78242
	L	0.5	5.57990	8.06503	9.87079	9.18828	11.8720	13.2282
		2	4.61818	6.78707	8.39557	7.82304	10.2271	11.5148
	NL	0.5	20.7994	30.0629	36.7940	34.2499	44.2535	49.3088
		2	21.4481	31.5211	38.9914	36.3324	47.4974	53.4781
0.8	U	0.5	3.03406	4.33623	5.27233	4.90741	6.29511	6.97530
		2	2.80661	4.04054	4.93576	4.59588	5.92583	6.59496
	L	0.5	6.01812	8.62246	10.4947	9.76483	12.5402	13.9006
		2	5.56322	7.73077	9.82152	9.14176	11.8017	13.1399
	NL	0.5	19.9518	28.5859	34.7928	32.3732	41.5744	46.0844
		2	20.3393	29.6075	35.9077	33.4225	43.1471	48.0399
1	U	0.5	3.23493	4.59106	5.55750	5.17128	6.60133	7.28509
		2	3.23493	4.59106	5.55750	5.17128	6.60133	7.28509
	L	0.5	6.41986	9.13213	11.0650	10.2926	13.1527	14.5202
		2	6.41986	9.13213	11.0650	10.2926	13.1527	14.5202
	NL	0.5	19.2596	27.3964	33.1950	30.8777	39.4580	43.5606
		2	19.2596	27.3964	33.1950	30.8777	39.4580	43.5606

The effect of the volume fraction index p and thickness of the core $\boldsymbol{t}_C$ on the critical buckling temperature difference of FG sandwich plate under uniform temperature rise (U), linear (L) and non-linear (NL) temperature distributions is shown in Table 9 for various boundary conditions. It can be seen from results presented in Table 9 that the SSSS plate is the softer structure, whereas the FFCC plate is the stiffer structure.

For more clarity, the influence of the volume fraction index p on the non-dimensional critical buckling temperature T_{cr} of a square FG under uniform, linear and non-linear temperature change through-the-thickness is plotted in Fig. 2 using the simple first-order shear deformation theory. It is interesting to note from this figure that the critical buckling temperature T_{cr} for the plates under uniform temperature change is smaller than that for the plates under a non-linear temperature change. While T_{cr} for the plates under linear temperature change is intermediate to the two previous thermal loading cases. It can be concluded also that the non-dimensional critical buckling temperature initially decreases, and then the change of curves are not significant by increasing in the value of the volume fraction index.

Further verification of critical buckling temperature is displayed in Fig. 3 for thick plates. In

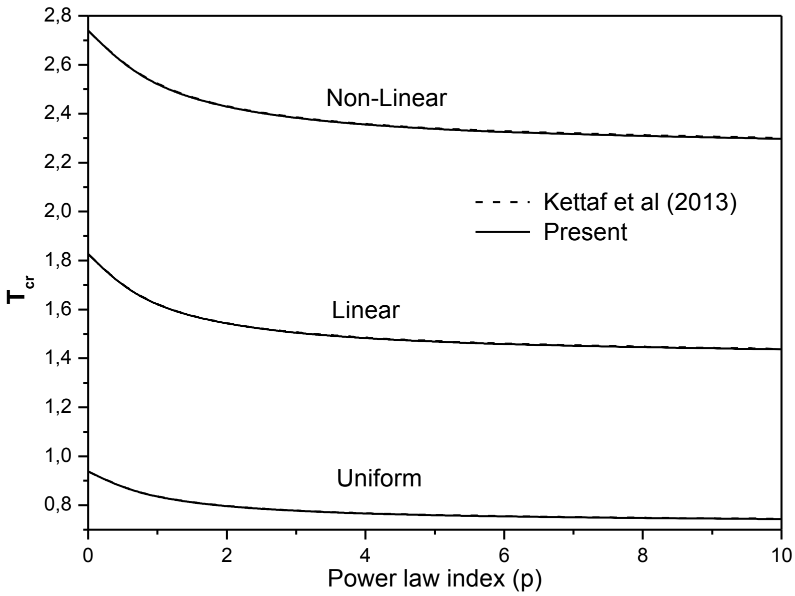

Fig. 2 Critical buckling temperature difference T_{cr} versus the power law index p (t_C=0.8h, a/h=10, a/b=1)

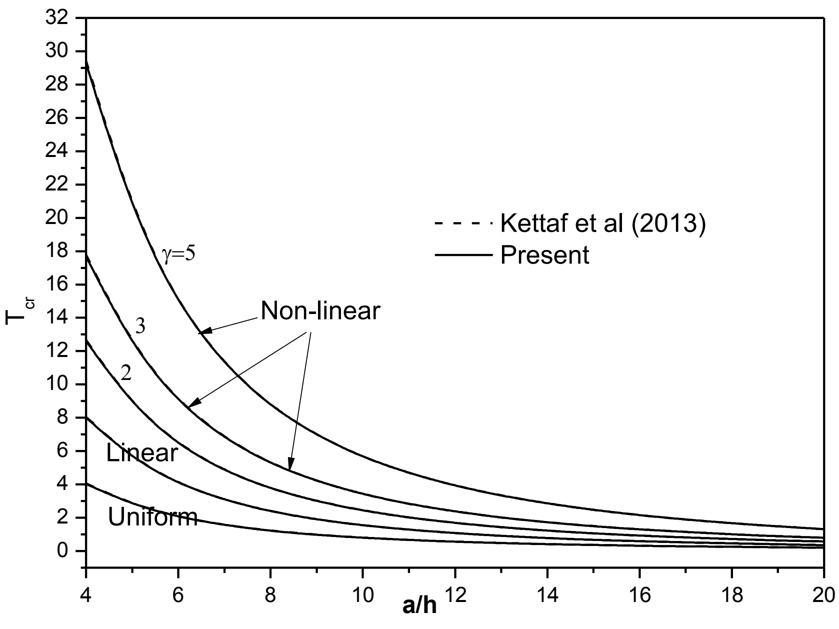

Fig. 3 Critical buckling temperature difference T_{cr} versus the side-to-thickness ratio a/h (t_C=0.8h, a/b=1, k=1)

this figure, the variations of dimensionless critical buckling temperatures versus thickness ratio *a/h* are compared for square FG sandwich plates under uniform, linear and non-linear temperature change through-the-thickness. It is observed that the critical temperature difference decreases monotonically as the thickness ratio *a/h* increases. Also, it is seen that T_{cr} increases as the nonlinearity parameter γ increases.

Fig. 4 indicates the influences of the aspect ratio *b/a* on the critical buckling temperature change T_{cr} of FG sandwich plates under various thermal loading types. It is observed that,

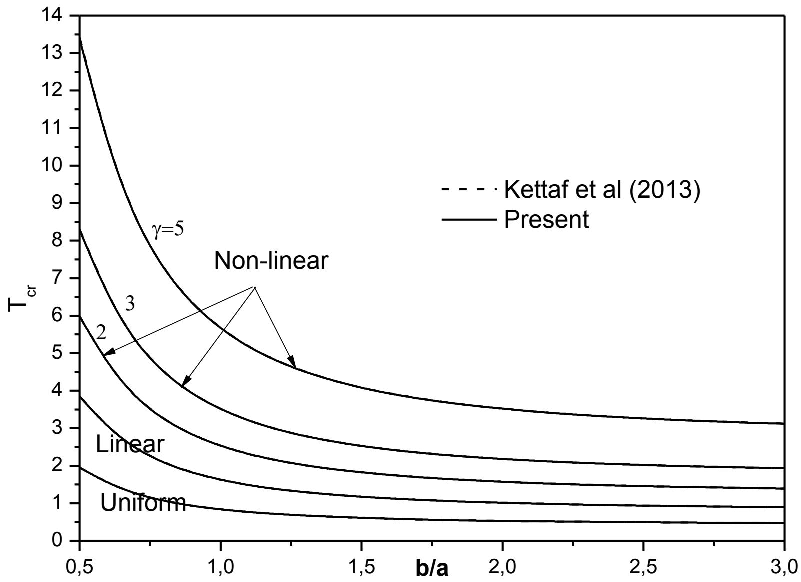

Fig. 4 Critical buckling temperature difference T_{cr} versus the plate aspect ratio *b/a* (t_C=0.8*h*, *a/h*=10, *p*=1)

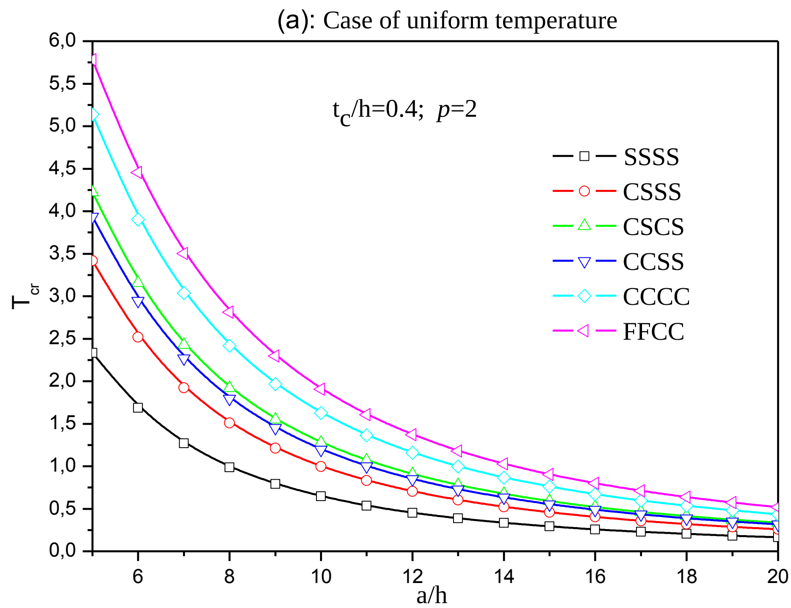

Fig. 5 Critical buckling temperature difference T_{cr} versus the plate side-to-thickness ratio *a/h* with various boundary conditions. (a) uniform temperature; (b) linear temperature; (c) non-linear temperature (γ=5)

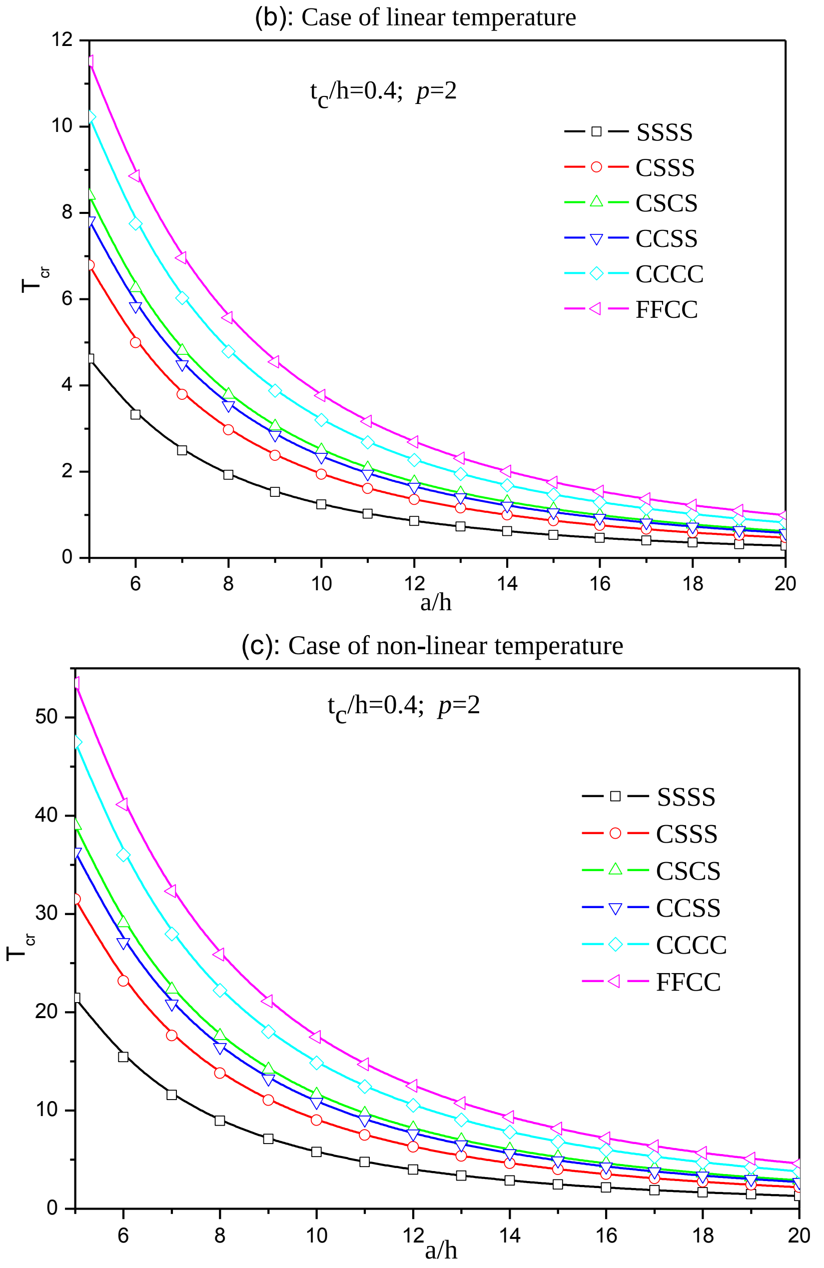

Fig. 5 Continued

regardless of the sandwich plate types, the critical buckling T_{cr} decreases gradually with the increase of the plate aspect ratio *b/a* wherever the loading type is. It is also noticed from Fig. 4 that the $\boldsymbol{T}_{cr}$ increases with the increase of the non-linearity parameter γ. It can be concluded from Figs. 2, 3 and 4, that the obtained results are in excellent agreement with those generated by Kettaf *et al.* (2013) based on the HSDPT. Thus, it can be stated that the present model is not only accurate but also simple in predicting the critical buckling temperature of FG sandwich plates.

Fig. 5 presents the critical buckling temperatures of FG sandwich square plate with various boundary conditions and under various thermal loading types. It can be seen that the critical buckling temperature decreases gradually with the side-to thickness ratio a/h. The results of the simply supported sandwich plate are less than that of the clamped-clamped and free-clamped sandwich plate. For the FG sandwich plate with intermediate boundary conditions, the results take the corresponding intermediate values.

5. Conclusions

A simple and accurate FSDT is presented and implemented in the present study for the thermal buckling analysis of FG sandwich plates with various boundary conditions. By dividing the deflection into bending, and shear components, the number of unknowns and governing equations of the present theory is reduced to four as against five or more unknown in the corresponding theories. Thus, a considerably lower computational effort is reached. The governing differential equations are obtained using the principle of minimum total potential energy. Verification studies prove that the present FSDT is not only more accurate than the conventional one, but also comparable with existing higher-order shear deformation theories which have a greater number of unknowns. The formulation lends itself particularly well to nanostructures (Besseghier *et al*. 2015, Chemi *et al*. 2015, Tagrara *et al*. 2015, Ould Youcef *et al*. 2015, Chakraverty and Behera 2015, Larbi Chaht *et al*. 2015, Bessaim *et al*. 2015, Zemri *et al*. 2015, Belkorissat *et al*. 2015, Rahimi Pour *et al*. 2015, Bounouara *et al*. 2016, Moradi-Dastjerdi 2016), which will be considered in the near future.

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References

Ahmed, A. (2014), "Post buckling analysis of sandwich beams with functionally graded faces using a consistent higher order theory", *Int. J. Civil Struct. Envir.*, **4**(2), 59-64.

Ait Amar Meziane, M., Abdelaziz, H.H. and Tounsi, A. (2014), "An efficient and simple refined theory for buckling and free vibration of exponentially graded sandwich plates under various boundary conditions", *J. Sandw. Struct. Mater.*, **16**(3), 293-318.

Ait Atmane, H., Tounsi, A., Bernard, F. and Mahmoud, S.R. (2015), "A computational shear displacement model for vibrational analysis of functionally graded beams with porosities", *Steel Compos. Struct.*, **19**(2), 369-384.

Ait Atmane, H., Tounsi, A. and Bernard, F. (2016), "Effect of thickness stretching and porosity on mechanical response of a functionally graded beams resting on elastic foundations", *Int. J. Mech. Mater. Des.* (In press)

Ait Yahia, S., Ait Atmane, H., Houari, M.S.A. and Tounsi, A. (2015), "Wave propagation in functionally graded plates with porosities using various higher-order shear deformation plate theories", *Struct. Eng.*

Mech., **53**(6), 1143 - 1165.

Akbaş, Ş.D. (2015), "Wave propagation of a functionally graded beam in thermal environments", *Steel Compos. Struct.*, **19**(6), 1421-1447.

Al-Basyouni, K.S., Tounsi, A. and Mahmoud, S.R. (2015), "Size dependent bending and vibration analysis of functionally graded micro beams based on modified couple stress theory and neutral surface position", *Compos. Struct.*, **125**, 621-630.

Arefi, M. (2015), "Elastic solution of a curved beam made of functionally graded materials with different cross sections", *Steel Compos. Struct.*, **18**(3), 569 - 672.

Attia, A., Tounsi, A., Adda Bedia, E.A. and Mahmoud, S.R. (2015), "Free vibration analysis of functionally graded plates with temperature-dependent properties using various four variable refined plate theories", *Steel Compos. Struct.*, **18**(1), 187-212.

Bakora, A. and Tounsi, A. (2015)," Thermo-mechanical post-buckling behavior of thick functionally graded plates resting on elastic foundations", *Struct. Eng. Mech.*, **56**(1), 85 - 106.

Bateni, M., Kiani, Y. and Eslami, M.R. (2013), "A comprehensive study on stability of FGM plates", *Int. J. Mech. Sci.*, **75**, 134-144.

Belabed, Z., Houari, M.S.A., Tounsi, A., Mahmoud, S.R. and Anwar Bég, O. (2014), "An efficient and simple higher order shear and normal deformation theory for functionally graded material (FGM) plates", *Compos. Part B*, **60**, 274-283.

Bellifa, H., Benrahou, K.H., Hadji, L., Houari, M.S.A. and Tounsi, A. (2016), "Bending and free vibration analysis of functionally graded plates using a simple shear deformation theory and the concept the neutral surface position", *J Braz. Soc. Mech. Sci. Eng.*, **38**, 265-275.

Belkorissat, I., Houari, M.S.A., Tounsi, A., Adda Bedia, E.A. and Mahmoud, S.R. (2015), "On vibration properties of functionally graded nano-plate using a new nonlocal refined four variable model", *Steel Compos. Struct.*, **18**(4), 1063-1081.

Benachour, A., Tahar, H.D., Atmane, H.A., Tounsi, A. and Ahmed, M.S. (2011), "A four variable refined plate theory for free vibrations of functionally graded plates with arbitrary gradient", *Compos. Part B*, **42**, 1386-1394.

Bennai, R., Ait Atmane, H. and Tounsi, A. (2015), "A new higher-order shear and normal deformation theory for functionally graded sandwich beams", *Steel Compos. Struct.*, **19**(3), 521-546.

Bennoun, M., Houari, M.S.A. and Tounsi, A. (2016), "A novel five variable refined plate theory for vibration analysis of functionally graded sandwich plates", *Mech. Adv. Mater. Struct.*, **23**(4), 423-431.

Bessaim, A., Houari, M.S.A., Tounsi, A., Mahmoud, S.R. and Adda Bedia, E.A. (2013), "A new higher-order shear and normal deformation theory for the static and free vibration analysis of sandwich plates with functionally graded isotropic face sheets", *J. Sandw. Struct. Mater.*, **15**, 671-703.

Bessaim, A., Houari, M.S.A., Bernard, F. and Tounsi, A. (2015), "A nonlocal quasi-3D trigonometric plate model for free vibration behaviour of micro/nanoscale plates", *Struct. Eng. Mech.*, **56**(2), 223-240.

Besseghier, A., Heireche, H., Bousahla, A.A., Tounsi, A. and Benzair, A. (2015), "Nonlinear vibration properties of a zigzag single-walled carbon nanotube embedded in a polymer matrix", *Adv. Nano Res.*, **3**(1), 29-37.

Bouchafa, A., Bachir Bouiadjra, M., Houari, M.S.A. and Tounsi, A. (2015), "Thermal stresses and deflections of functionally graded sandwich plates using a new refined hyperbolic shear deformation theory", *Steel Compos. Struct.*, **18**(6), 1493-1515.

Bouderba, B., Houari, M.S.A. and Tounsi, A. (2013), "Thermomechanical bending response of FGM thick plates resting on Winkler-Pasternak elastic foundations", *Steel Compos. Struct.*, **14**(1), 85-104.

Bouguenina, O., Belakhdar, K., Tounsi, A. and Adda Bedia, E.A. (2015), "Numerical analysis of FGM plates with variable thickness subjected to thermal buckling", *Steel Compos. Struct.*, **19**(3), 679-695.

Bouhadra, A., Benyoucef, S., Tounsi, A., Bernard, F., Bachir Bouiadjra, R. and Houari, M.S.A. (2015), "Thermal buckling response of functionally graded plates with clamped boundary conditions", *J. Therm. Stress.*, **38**, 630-650.

Bounouara, F., Benrahou, K.H., Belkorissat, I. and Tounsi, A. (2016), "A nonlocal zeroth-order shear deformation theory for free vibration of functionally graded nanoscale plates resting on elastic

foundation", *Steel Compos. Struct.*, **20**(2), 227 - 249.
Bourada, M., Tounsi, A., Houari, M.S.A. and Adda Bedia, E.A. (2012), "A new four-variable refined plate theory for thermal buckling analysis of functionally graded sandwich plates", *J. Sandw. Struct. Mater.*, **14**, 5 - 33.
Bourada, M., Kaci, A., Houari, M.S.A. and Tounsi, A. (2015), "A new simple shear and normal deformations theory for functionally graded beams", *Steel Compos. Struct.*, **18**(2), 409-423.
Bouremana, M, Houari, M.S.A, Tounsi, A, Kaci, A. and Adda Bedia, E.A. (2013), "A new first shear deformation beam theory based on neutral surface position for functionally graded beams", *Steel Compos. Struct.*, **15**(5), 467-479.
Bousahla, A.A., Houari, M.S.A., Tounsi, A. and Adda Bedia, E.A., (2014), "A novel higher order shear and normal deformation theory based on neutral surface position for bending analysis of advanced composite plates", *Int. J. Comput. Meth.*, **11**(6), 1350082.
Brush, D.O. and Almroth, B.O. (1975), *Buckling of bars, plates, and shells*, McGraw-Hill, New York.
Chakraverty, S. and Pradhan, K.K. (2014), "Free vibration of exponential functionally graded rectangular plates in thermal environment with general boundary conditions", *Aerosp. Sci. Technol.*, **36**, 132-156.
Chakraverty, S. and Behera, L. (2015), "Small scale effect on the vibration of non-uniform nanoplates", *Struct. Eng. Mech.*, **55**(3), 495 - 510.
Chattibi, F., Benrahou, K.H., Benachour, A., Nedri, K. and Tounsi, A. (2015), "Thermomechanical effects on the bending of antisymmetric cross-ply composite plates using a four variable sinusoidal theory", *Steel Compos. Struct.*, **19**(1), 93-110.
Chemi, A., Heireche, H., Zidour, M., Rakrak, K. and Bousahla, A.A. (2015), "Critical buckling load of chiral double-walled carbon nanotube using non-local theory elasticity", *Adv. Nano Res.*, **3**(4), 193-206.
Cunedioglu, Y. (2015), "Free vibration analysis of edge cracked symmetric functionally graded sandwich beams", *Struct. Eng. Mech.*, **56**(6), 1003-1020.
Darılmaz, K., (2015), "Vibration analysis of functionally graded material (FGM) grid systems", *Steel Compos. Struct.*, **18**(2), 395-408.
Draiche, K., Tounsi, A. and Khalfi, Y. (2014), "A trigonometric four variable plate theory for free vibration of rectangular composite plates with patch mass", *Steel Compos. Struct.*, **17**(1), 69-81.
Ebrahimi, F. and Dashti, S. (2015)," Free vibration analysis of a rotating non-uniform functionally graded beam", *Steel Compos. Struct.*, **19**(5), 1279-1298.
Ebrahimi, F. and Habibi, S. (2016), "Deflection and vibration analysis of higher-order shear deformable compositionally graded porous plate", *Steel Compos. Struct.*, **20**(1), 205-225.
El Meiche, N., Tounsi, A., Ziane, N., Mechab, I. and Adda Bedia, E.A. (2011), "A new hyperbolic shear deformation theory for buckling and vibration of functionally graded sandwich plate", *Int. J. Mech. Sci.*, **53**(4), 237-247.
Etemadi, E, Khatibi, AA. and Takaffoli, M. (2009), "3D finite element simulation of sandwich panels with a functionally graded core subjected to low velocity impact", *Compos. Struct.*, **89**, 28-34.
Fekrar, A., Houari, M.S.A., Tounsi, A. and Mahmoud, S.R. (2014), "A new five-unknown refined theory based on neutral surface position for bending analysis of exponential graded plates", *Meccanica*, **49**, 795-810.
Hadji, L., Daouadji, T.H., Tounsi, A. and Adda Bedia, E.A. (2014), "A higher order shear deformation theory for static and free vibration of FGM beam", *Steel Compos. Struct.*, **16**(5), 507-519.
Hadji, L. and Adda Bedia, E.A. (2015a), "Influence of the porosities on the free vibration of FGM beams", *Wind Struct.*, **21(3)**, 273-287.
Hadji, L. and Adda Bedia, E.A. (2015b), "Analyse of the behavior of Functionally graded beams based on neutral surface position", *Struct. Eng. Mech.*, **55**(4), 703-717.
Hadji, L., Hassaine Daouadji, T., Ait Amar Meziane, M., Tlidji, Y. and Adda Bedia, E.A. (2016), "Analysis of functionally graded beam using a new first-order shear deformation theory", *Struct. Eng. Mech.*, **57**(2), 315-325.
Hamidi, A., Houari, M.S.A., Mahmoud, S.R. and Tounsi, A. (2015), "A sinusoidal plate theory with 5-unknowns and stretching effect for thermomechanical bending of functionally graded sandwich plates",

Steel Compos. Struct., **18**(1), 235-253.
Hebali, H., Tounsi, A., Houari, M.S.A., Bessaim, A. and Adda Bedia, E.A. (2014), "A new quasi-3D hyperbolic shear deformation theory for the static and free vibration analysis of functionally graded plates", *ASCE J. Eng. Mech.*, **140**, 374-383.
Houari, M.S.A., Tounsi, A. and Anwar Bég, O. (2013), "Thermoelastic bending analysis of functionally graded sandwich plates using a new higher order shear and normal deformation theory", *Int. J. Mech. Sci.*, **76**, 467-479.
Jha, D.K., Kant, T. and Singh, R.K. (2013), "A critical review of recent research on functionally graded plates", *Compos. Struct*, **96**, 833-849.
Jiang, H.J., Dai, H.L. and Li, S.Z. (2015), "Refined plate theory for bending analysis of a HSLA steel plate under 3D temperature field", *Appl. Math. Comput.*, **250**, 497-513.
Kar, V.R. and Panda, S.K. (2015), "Nonlinear flexural vibration of shear deformable functionally graded spherical shell panel", *Steel Compos. Struct.*, **18**(3), 693-709.
Kettaf, F.Z., Houari, M.S.A., Benguediab, M. and Tounsi, A. (2013), "Thermal buckling of functionally graded sandwich plates using a new hyperbolic shear displacement model", *Steel Compos. Struct.*, **15**(4), 399-423.
Khalfi, Y., Houari, M.S.A. and Tounsi, A. (2014), "A refined and simple shear deformation theory for thermal buckling of solar functionally graded plates on elastic foundation", *Int. J. Comput. Meth.*, **11**(5), 135007.
Kiani, Y., Bagherizadeh, E. and Eslami, M.R. (2011), "Thermal buckling of clamped thin rectangular FGM plates resting on Pasternak elastic foundation (Three Approximate Analytical Solutions)", *J. Appl. Math. Mech.*, **91**(7), 581-593.
Kirkland, B. and Uy, B. (2015), "Behaviour and design of composite beams subjected to flexure and axial load", *Steel Compos. Struct.*, **19**(3), 615-633.
Larbi Chaht, F., Kaci, A., Houari, M.S.A., Tounsi, A., Anwar Bég, O. and Mahmoud, S.R. (2015), "Bending and buckling analyses of functionally graded material (FGM) size-dependent nanoscale beams including the thickness stretching effect", *Steel Compos. Struct.*, **18**(2), 425 - 442.
Liang, X., Wang, Z., Wang, L. and Liu, G. (2014), "Semi-analytical solution for three-dimensional transient response of functionally graded annular plate on a two parameter viscoelastic foundation", *J. Sound Vib.*, **333**(12), 2649-2663.
Liang, X., Wu, Z., Wang, L., Liu, G., Wang, Z. and Zhang, W. (2015), "Semi-analytical three-dimensional solutions for the transient response of functionally graded material", *ASCE J. Eng. Mech.*, **141**(9), 1943-7889.
Lu, C.F., Lim, C.W. and Chen, W.Q. (2009), "Semi-analytical analysis for multi-directional functionally graded plates: 3-d elasticity solutions", *Int. J. Numer. Meth. Eng.*, **79**(1), 25-44.
Mahi, A., Adda Bedia, E.A. and Tounsi, A. (2015), "A new hyperbolic shear deformation theory for bending and free vibration analysis of isotropic, functionally graded, sandwich and laminated composite plates", *Appl. Math. Model.*, **39**, 2489-2508.
Mantari, J.L. and Granados, E.V. (2015), "Thermoelastic behavior of advanced composite sandwich plates by using a new 6 unknown quasi-3D hybrid type HSDT", *Compos. Struct.*, **126**, 132 - 144.
Mantari, J.L. and Guedes Soares, C. (2014). "Four-unknown quasi-3D shear deformation theory for advanced composite plates", *Compos. Struct.*, **109**, 231-239.
Matsunaga, H. (2005), "Thermal buckling of cross-ply laminated composite and sandwich plates according to a global higher-order deformation theory", *Compos. Struct.*, **68**, 439-454.
Meksi, A., Benyoucef, S., Houari, M.S.A. and Tounsi, A. (2015), "A simple shear deformation theory based on neutral surface position for functionally graded plates resting on Pasternak elastic foundations", *Struct. Eng. Mech.*, **53**(6), 1215-1240.
Meradjah, M., Kaci, A., Houari, M.S.A., Tounsi, A. and Mahmoud, S.R. (2015), "A new higher order shear and normal deformation theory for functionally graded beams", *Steel Compos. Struct.*, **18**(3), 793-809.
Miamoto, Y., Kaysser, W.A., Rabin, B.H., Kawasaki, A. and Ford, R. G., (1999), *Functionally graded materials: design, processing and applications*, Kluwer Academic Publishers, Boston.

Moradi-Dastjerdi, R. (2016), "Wave propagation in functionally graded composite cylinders reinforced by aggregated carbon nanotube", *Struct. Eng. Mech.*, **57**(3), 441-456.

Natarajan, S. and Manickam, G. (2012), "Bending and vibration of functionally graded material sandwich plates using an accurate theory", *Finite Elem. Anal. Des.*, **57**, 32-42.

Nedri, K., El Meiche, N. and Tounsi, A. (2014), "Free vibration analysis of laminated composite plates resting on elastic foundations by using a refined hyperbolic shear deformation theory", *Mech. Compos. Mater.*, **49**(6), 641-650.

Nguyen, K.T., Thai, T.H. and Vo, T.P. (2015), "A refined higher-order shear deformation theory for bending, vibration and buckling analysis of functionally graded sandwich plates", *Steel Compos. Struct.*, **18**(1), 91-120.

Nguyen-Xuan, H., Tran, L.V., Nguyen-Thoi, T. and Vu-Do, H.C. (2011), "Analysis of functionally graded plates using an edge-based smoothed finite element method", *Compos. Struct.*, **93**(11), 3019-2039.

Ould Larbi, L, Kaci, A., Houari, M.S.A. and Tounsi, A. (2013), "An efficient shear deformation beam theory based on neutral surface position for bending and free vibration of functionally graded beams", *Mech. Bas. Des. Struct. Mach.*, **41**, 421-433.

Ould Youcef, D., Kaci, A., Houari, M.S.A., Tounsi, A., Benzair, A. and Heireche, H. (2015), "On the bending and stability of nanowire using various HSDTs", *Adv. Nano Res.*, **3**(4), 177-191.

Reddy, J.N. (1984), *Energy principles and variational methods in applied mechanics*, John Wiley, New York.

Pradhan, K.K. and Chakraverty, S. (2015), "Free vibration of functionally graded thin elliptic plates with various edge supports", *Struct. Eng. Mech.*, **53**(2), 337-354.

Rahimi Pour, H., Vossough, H., Heydari, M.M., Beygipoor, G. and Azimzadeh, A. (2015), "Nonlinear vibration analysis of a nonlocal sinusoidal shear deformation carbon nanotube using differential quadrature method", *Struct. Eng. Mech.*, **54**(6), 1061-1073.

Sallai, B., Hadji, L., Hassaine Daouadji, T. and Adda Bedia, E.A. (2015), "Analytical solution for bending analysis of functionally graded beam", *Steel Compos. Struct.*, **19**(4), 829-841.

Shahrjerdi, A., Mustapha, F., Bayat, M. and Majid, D.L.A. (2011), "Free vibration analysis of solar functionally graded plates with temperature-dependent material properties using second order shear deformation theory", *J. Mech. Sci. Tech.*, **25**(9), 2195-2209.

Sobhy, M. (2013), "Buckling and free vibration of exponentially graded sandwich plates resting on elastic foundations under various boundary conditions", *Compos. Struct.*, **99**, 76-87.

Sobhy, M. (2015), "A comprehensive study on FGM nanoplates embedded in an elastic medium", *Compos. Struct.*, **134**, 966-980.

Sofiyev, A.H. and Kuruoglu, N. (2015), "Buckling of non-homogeneous orthotropic conical shells subjected to combined load", *Steel Compos. Struct.*, **19**(1), 1 - 19.

Swaminathan, K. and Naveenkumar, D.T. (2014), "Higher order refined computational models for the stability analysis of FGM plates: Analytical solutions", *Eur. J. Mech. A/Solid.*, **47**, 349 - 361.

Tagrara, S.H., Benachour, A., Bachir Bouiadjra, M. and Tounsi, A. (2015), "On bending, buckling and vibration responses of functionally graded carbon nanotube-reinforced composite beams", *Steel Compos. Struct.*, **19**(5), 1259-1277.

Tebboune, W., Benrahou, K.H., Houari, M.S.A. and Tounsi, A. (2015), "Thermal buckling analysis of FG plates resting on elastic foundation based on an efficient and simple trigonometric shear deformation theory", *Steel Compos. Struct.*, **18**(2), 443 - 465.

Thai, H.T., Vo, T.P., Bui, T.Q. and Nguyen, T.K. (2014), "A quasi-3D hyperbolic shear deformation theory for functionally graded plates", *Acta Mech*, **225**(3), 951-964.

Thai, HT and Kim, SE. (2013), "A simple quasi-3D sinusoidal shear deformation theory for functionally graded plates", *Compos. Struct.*, **99**,172-180.

Thai, HT and Choi, DH. (2013), "Efficient higher-order shear deformation theories for bending and free vibration analyses of functionally graded plates", *Arch. Appl. Mech.*, **83**(12), 1755-1771.

Thai, HT and Choi, DH. (2011), "A refined plate theory for functionally graded plates resting on elastic foundation", *Compos. Sci. Technol.*, **71**(16), 1850-1858.

Talha, M. and Singh, B.N. (2010), "Static response and free vibration analysis of FGM plates using higher order shear deformation theory", *Appl. Math. Model.*, **34**(12), 3991-4011.

Tounsi, A., Houari, M.S.A., Benyoucef, S. and Adda Bedia, E.A. (2013), A refined trigonometric shear deformation theory for thermoelastic bending of functionally graded sandwich plates", *Aerosp. Sci. Tech.*, **24**, 209-220.

Wen, P.H., Sladek, J. and Sladek, V. (2011), "Three-dimensional analysis of functionally graded plates", *Int. J. Numer. Meth. Eng.*, **87**(10), 923-942.

Xiang, S., Kang, G.W., Yang, M.S. and Zhao, Y. (2013), "Natural frequencies of sandwich plate with functionally graded face and homogeneous core", *Compos. Struct.*, **96**, 226 - 231.

Yaghoobi, H. and Yaghoobi, P. (2013), "Buckling analysis of sandwich plates with FGM face sheets resting on elastic foundation with various boundary conditions: an analytical approach", *Meccanica*, **48**, 2019-2035.

Yaghoobi, H. and Fereidoon, A. (2014), "Mechanical and thermal buckling analysis of functionally graded plates resting on elastic foundations: An assessment of a simple refined nth-order shear deformation theory", *Compos. Part B*, **62**, 54-64.

Yaghoobi, H., Valipour, M.S., Fereidoon, A. and Khoshnevisrad, P. (2014), "Analytical study on post-buckling and nonlinear free vibration analysis of FG beams resting on nonlinear elastic foundation under thermo-mechanical loading using VIM", *Steel Compos. Struct.*, **17**(5), 753-776.

Zemri, A., Houari, M.S.A., Bousahla, A.A. and Tounsi, A. (2015), "A mechanical response of functionally graded nanoscale beam: an assessment of a refined nonlocal shear deformation theory beam theory", *Struct. Eng. Mech.*, **54**(4), 693-710.

Zhao, X., Lee, Y.Y. and Liew, K.M. (2009), "Mechanical and thermal buckling analysis of functionally graded plates", *Compos. Struct.*, **90**, 161-171.

Zidi, M., Tounsi, A., Houari M.S.A., Adda Bedia, E. A. and Anwar Bég, O. (2014), "Bending analysis of FGM plates under hygro-thermo-mechanical loading using a four variable refined plate theory", *Aerosp. Sci. Technol.*, **34**, 24-34.

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