Structural Engineering and Mechanics, *Vol. 58, No. 1 (2016) 1-19*
DOI: http://dx.doi.org/10.12989/sem.2016.58.1.001

Curvature ductility of high strength concrete beams according to Eurocode 2

Haytham Bouzid* and Amar Kassoul[a]

Department of Civil Engineering, Laboratory of Structures, Geotechnics and Risks (LSGR), Université of Hassiba Benbouali of Chlef, Algeria BP 151, Hay Essalam, UHB Chlef, 02000, Algeria

(Received December 5, 2014, Revised October 6, 2015, Accepted December 23, 2015)

Abstract. Recently, the high-strength concrete is increasingly used in the construction of reinforced concrete structures due to its benefits, but this use is influenced negatively on the local ductility of structural elements. The objective of this study is the prediction of a new approach to evaluate the curvature ductility factor of high strength concrete beams according to Eurocode 2. After the presentation of the Constitutive laws of materials and the evaluation method of curvature ductility according to the Eurocode 2, we conduct a parametric study on the factors influencing the curvature ductility of inflected sections. The calibrating of the obtained results allows predicting a very simple approach for estimating the curvature ductility factor. The proposed formula allows to calculate the curvature ductility factor of high strength concrete beams directly according to the concrete strength f_{ck}, the yield strength of steel f_{yk} and the ratio of tension and compression reinforcements ρ and ρ' respectively, this proposed formula is validated by theoretical and experimental results of different researchers.

Keywords: Eurocode 2; beam; reinforcement; curvature ductility; high strength concrete

1. Introduction

The known progress in the construction of reinforced concrete structures and buildings towers in the recent years requires very high strength materials, to reduce the vertical element sections and ameliorate their strength and local ductility. For this reason the use of a concrete with high compressive strength is inevitable, the increasing of this strength is frequently accompanied by improvement of other properties such as tensile strength, stiffness and durability, as it is characterized by a strong adhesion between aggregates and cement matrix. Furthermore, several concrete codes tolerate the use of strength up to 90 MPa and more.

Currently, several numerical and experimental investigations have been conducted on the nonlinear behavior study of reinforced concrete structures with high strength materials, where the local ductility study of structural elements columns and beams took à large occupation of researchers. One of the structure characteristics dissipative the energy in high seismicity zone is to promote the apparition of plastic hinges in the beams rather than columns. From here comes the

*Corresponding author, Ph.D. Candidate, E-mail: haytambouzid@hotmail.com
[a]Professor, E-mail: amarkassoul@gmail.com

http://www.techno-press.org/?journal=sem&subpage=8 ISSN: 1225-4568 (Print), 1598-6217 (Online)

importance of study the local ductility of concrete beams where the seismic codes require their verification during the design.

Among these studies cited Pam *et al.* (2001a, b). In these both studies two formulas are proposed for estimating the curvature ductility factor of reinforced concrete beams. The first study (Pam *et al.* 2001a) is based on the experimental results and the proposed formula depended on the cube compressive strength of concrete f_{cu}, the percentage of tensile reinforcements ρ and a very important parameter considered by the ACI code (ACI 318M-14), which is the balanced reinforcement ratio ρ_b. This formula is valid for beams reinforced with ordinary and high strength concrete and it is given as follows

$$\mu_{\varphi} = 9.5\left(f_{cu}\right)^{-0.3}\left(\frac{\rho}{\rho_b}\right)^{-0.75} \tag{1}$$

The second study (Pam *et al.* 2001b) is based on the numerical analysis and the proposed formula depended on the same parameters as (Pam *et al.* 2001a) but this time the ratio of compression reinforcement ρ' is taken into consideration and concrete cylinder strength f_{ck} is used, this formula is written as follows

$$\mu_{\varphi} = 10.7\left(f_{ck}\right)^{-0.45}\left(\frac{\rho-\rho'}{\rho_b}\right)^{-1.25}\left(1+95.2\left(f_{ck}\right)^{-01.1}\left(\frac{\rho'}{\rho}\right)^{3}\right) \tag{2}$$

In other numerical investigations, Kwan *et al.* (2002) studied the formula proposed by Pam *et al.* (2001b) and they are eliminated its last term, so the formula Eq. () became as follows

$$\mu_{\varphi} = 10.7\left(f_{ck}\right)^{-0.45}\left(\frac{\rho-\rho'}{\rho_b}\right)^{-1.25} \tag{3}$$

Arslan and Cihanli (2010) developed another simplified approach based on the concrete strength variation up to 110 MPa, the proposed formula is given by

$$\mu_{\varphi} = 40\left(f_{ck}\right)^{-0.17}\left(f_{y}\right)^{-0.42}\left(\frac{\rho}{\rho_b}\right)^{-1.18} \tag{4}$$

Recently, Lee (2013a, b) conducted two researches in order to predict the curvature ductility factor of reinforced High Strength concrete beams. In the first research a new approach has been proposed based on the numerical analysis and taking into account the same parameters used by different researchers mentioned previously, the obtained formula is given as follows

$$\mu_{\varphi} = \left[\left(\rho-\rho'\frac{f_{sc}}{f_{yc}}\right)/\rho_b\right]^{-1.279} * \left(f_y\right)^{-0.215} * \left[-0.6\left(f_{ck}\right)^2 + 88f_{ck} + 2.285\right] * 10^{-3} \tag{5}$$

In which, f_y and f_{yc} are the yield strength of tension and compression reinforcement respectively and f_{sc} is the stress of the compressive reinforcement in the ultimate state.

The second research is also based on the numerical analysis and the obtained results are compared with the experimental results of (Jang *et al.* 2008, Hong 2011, Rashid and Mansur 2005).

On the other hand, many researches have been conducted in the laboratories to study and calculate the curvature ductility of beams**,** among these researches cited Maghsoudi and Bengar (2006), Maghsoudi and Sharifi (2009), they are conducted an experimental tests on a series of beams, where the validity of the obtained results is made with the ACI and CSA theory. In other research Shohana *et al.* (2012) have realized an experimental program on singly reinforced beams with tensile reinforcement only using 500 grade steel, in order to classify the performance of the specimens according to the ductility three different reinforcement ratios and three different concrete strengths have been tested. In conclusion, it has confirmed that the use of higher strength steel would allow a higher flexural strength, and the use of a higher strength concrete would not allow a higher flexural strength. Also, Mohammad et *al.* (2013) are tested nine rectangular-sections of high strength concrete beams reinforced with tensile reinforcement only, the tested beams are designed and casted based on the American Concrete Institute (ACI) code. The comparison between the obtained results and the theoretical ductility coefficient from CSA94, NZS95 and ACI showed that the three mentioned codes exhibit conservative values for low reinforced HSC beams.

Although the mentioned studies have been used different Constitutive laws of materials, the basic consideration in these researches is to take into account the balanced reinforcement ratio adopted by the ACI code as a basic element, but the researchers who use the Eurocode 2 (EN 1992, 2004) are not allowed to use this ratio. Seen the importance accorded by the Eurocode 8 (prEN 1998-1, 2003) to take into account the curvature ductility factor during the design of structural elements (beams, columns and …), and this by requirement of admissible curvature ductility factor. Accordingly, it is necessary to have a simplified relation allow verifying the local ductility condition according to Eurocode 8 (prEN 1998-1, 2003) and takes into account parameters in Accordance with Eurocode 2, in particular the Constitutive laws of materials.

The main objective of this work is the development of simplified relationship to estimate the curvature ductility factor taking into account the main characteristics of the Eurocode 2 (EN 1992, 2004) in particular the Constitutive laws of materials steel and concrete.

2. Constitutive laws of materials

2.1 Concrete

The study of the reinforced concrete structures behavior according to Eurocode 2 (EN 1992, 2004) uses the characteristic compressive strength of concrete f_{ck}. For the high-performance concrete, the maximum value of this strength at 28 days is limited to 90 MPa for a cylindrical concrete Specimens and 105 MPa for a cubic specimens. The design value of the compressive strength of a cylindrical concrete Specimens f_{cd} is defined by

$$f_{cd} = \frac{\alpha_{cc}\ f_{ck}}{\gamma_c} \tag{6}$$

Where, γ_c is the partial safety factor for concrete, equal to 1.5 for durable situations and 1.2 for accident situations. α_{cc} is the coefficient taking account of long term effects on the compressive strength and of unfavorable effects resulting from the way the load is applied, its value varies between 0.8 and 1.

In the following, the accident situation is fully considered.

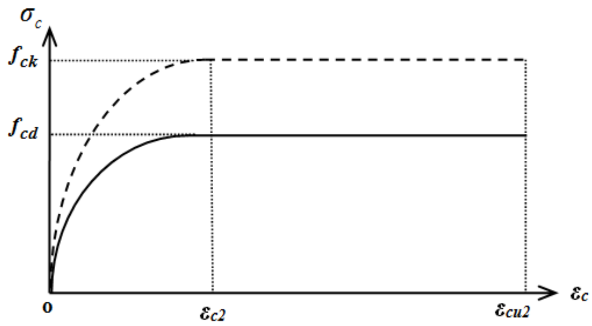


Fig. 1 Parabola-rectangle diagram for unconfined concrete under compression after the Eurocode 2 (EN 1992 2004)

$$\sigma_c = \begin{cases} f_{cd}\left[1-\left(1-\dfrac{\varepsilon_c}{\varepsilon_{c2}}\right)^n\right] & \text{for } 0 \le \varepsilon_c \le \varepsilon_{c2} \\ f_{cd} & \text{for } \varepsilon_{c2} \le \varepsilon_c \le \varepsilon_{cu2} \end{cases} \tag{7}$$

Where ε_c is the compressive strain in the concrete and ε_{c2} is the strain at reaching the maximum strength f_{cd}, and is expressed by

$$\varepsilon_{c2}(‰) = \begin{cases} 2 & \text{for } f_{ck} \le 50\,MPa \\ 2.0+0.085(f_{ck}-50)^{0.53} & \text{for } f_{ck} > 50\,MPa \end{cases} \tag{8}$$

And, ε_{cu2} is ultimate compressive strain in the concrete, defined as

$$\varepsilon_{cu2}(‰) = \begin{cases} 3.5 & \text{for } f_{ck} \le 50\,MPa \\ 2.6+35\left(\dfrac{90-f_{ck}}{100}\right)^4 & \text{for } f_{ck} > 50\,MPa \end{cases} \tag{9}$$

The exponent n takes the following values

$$n = \begin{cases} 2 & \text{for } f_{ck} \le 50\,MPa \\ 1.4+23.4\left(\dfrac{90-f_{ck}}{100}\right)^4 & \text{for } f_{ck} > 50\,MPa \end{cases} \tag{10}$$

2.2 Steel

According to Eurocode 2 (EN 1992 2004) the design of reinforced concrete section is performed from a specified class of frames represented by the characteristic value of yield strength f_{yk}. This value of f_{yk} varies from 400 up to 600 MPa.

The stress-strain steel diagram shown in Fig. 2, is distinguished by the bilinear elasto-plastic

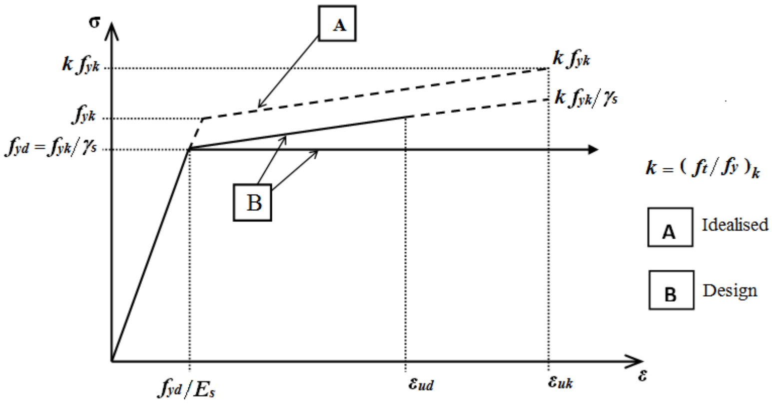


Fig. 2 Idealized and design stress-strain diagrams for reinforcing steel (for tension and compression) after the Eurocode 2 (EN 1992 2004)

curve, characterized by a inclined branch up to a value of deformation equal to $\varepsilon_{sy,d}$ and a stress in steel equal to f_{yd}, and a top branch supposed horizontal corresponding to a maximum deformation ε_{uk} and a stress in steel equal to f_{yd}, where

$$f_{yd}=\frac{f_{yk}}{\gamma_s} \tag{11}$$

Where, γ_s is the partial safety factor for steel, equal to 1.15 for durable situations and 1 for accident situations.

$\varepsilon_{sy,d}=f_{yd}/E_s$: Elastic elongation of steel at maximum load.

E_s: Modulus of elasticity of the steel, equal to 200000 MPa.

$k=(f_t/f_y)_k$: ratio of tensile strength to the yield stress its recommended value is 10%.

ε_{uk}: Characteristic strain of reinforcement or prestressing steel at maximum load, this ultimate strain is limited to 5% for class B and 7.5% for class C. The recommended value of ε_{ud} is $0.9\varepsilon_{uk}$.

3. Evaluation method of curvature ductility factor

The nonlinear behavior analysis of a doubly reinforced beam cross section in simple flexure usually requires a study in the limit states (Park and Ruittong 1988). The evaluation procedure of the curvature ductility factor is adapted according to the Eurocode 2 recommendations (Kassoul and Bougara 2010).

3.1 Curvature at first yield

The use of the serviceability limit state in reinforced concrete beams is conditioned mostly by the limit stresses in the concrete and reinforcements. To avoid the longitudinal cracks and micro cracks, the compressive stress in the concrete is limited to $k_1 f_{ck}$ ($f_{cd} \leq k_1\, f_{ck}$), which generally $k_1=0.6$. To avoid the inelastic deformations, cracks and the unacceptable deformations the tensile stresses in the reinforcement will be limited to $k_3\, f_{yk}$ ($f_{yd} \leq k_3\, f_{yk}$), in our case $k_3=0.8$.

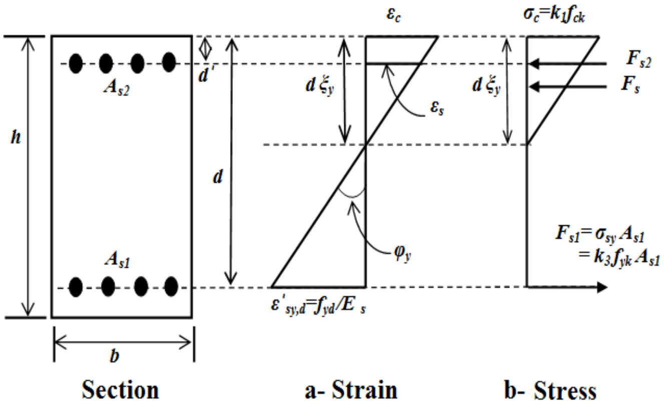


Fig. 3 Behavior of reinforced concrete beam section in flexure at the serviceability limit state (the end of the elastic phase)

Fig. 3 shows a cross section of a doubly reinforced concrete beam in serviceability limit state, where ξ_y represents the height factor of the compressed zone in the elastic state, d is the effective depth of the section and d' is the distance from extreme compression fiber to centroid of compression reinforcements. From Fig. 3(a), the curvature at first yield is expressed by

$$\varphi_y = \frac{\varepsilon_{sy,d}}{d\left(1-\xi_y\right)} \tag{12}$$

And, the strain in the compressed reinforcement ε_{s2}, is written

$$\varepsilon_{s2} = \frac{\left(\xi_y d - d'\right)}{d\left(1-\xi_y\right)} \frac{k_3 f_{yk}}{E_s} \tag{13}$$

Knowing that the stress in the compression reinforcement $\sigma_{s2}=\varepsilon_{s2}E_s$, the static equilibrium equation of the internal forces acting section in the Fig. 3(b) is written

$$\frac{1}{2}\xi_y d\, b\, k_1 f_{ck} + \frac{\left(\xi_y d - d'\right)}{d\left(1-\xi_y\right)} k_3\ f_{yk}\ A_{s2} = \sigma_{sy}\ A_{s1} \tag{14}$$

The solution of this equation leads to a second order polynomial function with the variable ξ_y, and the acceptable solution chosen is

$$\xi_y = \left(\frac{1}{2} + \frac{k_3\ f_{yk}}{k_1 f_{ck}}\left(\rho + \rho'\right)\right) - \sqrt{\left(\frac{1}{2} + \frac{k_3\ f_{yk}}{k_1 f_{ck}}\left(\rho + \rho'\right)\right)^2 - \frac{2 k_3\ f_{yk}}{k_1 f_{ck}}\left(\rho + \frac{d'}{d}\rho'\right)} \tag{15}$$

Where $\rho=A_{s1}/bd$ is the ratio of tension reinforcement, and $\rho'=A_{s2}/bd$ is the ratio of compression reinforcement.

After determining ε_{s2} expressed by Eq. (13), if $\varepsilon_{s2}\leq f_{yk}/E_s$, we retain the value of ξ_y obtained by Eq. (15). Otherwise, the compression frames A_{s2} are yielding in compression, in this case the Eq. (14) becomes

$$\frac{1}{2}\xi_y d\, b\, k_1 f_{ck} + k_3\ f_{yk}\ A_{s2} = k_3\ f_{yk}\ A_{s1} \tag{16}$$

So, it is clear that

$$\xi_y = \frac{2k_3\ f_{yk}}{k_1 f_{ck}}\left(\rho - \rho'\right) \tag{17}$$

3.2 Curvature at the ultimate limit state

At the ultimate state, several considerations recommended by Eurocode 2 (EN 1992 2004) are taken into account i.e.: section remains plane after deformation, the concrete in tension is neglected, the stresses in concrete and reinforcement are determined by Eqs. (6)-(11) and the strains are limited to ε_{cu2} in compressed concrete and ε_{ud} in tension reinforcements. Fig. 4 shows the behavior of unconfined cross section, considered as the worst critical region of a reinforced concrete beam where ξ_u represents the height factor of the compressed zone in the ultimate state.

The curvature at the ultimate state, from Fig. 4(a), is expressed by

$$\varphi_u = \frac{\varepsilon_{cu2}}{\xi_u d} \tag{18}$$

Suppose the compression reinforcement A_{s2} remain in the elastic state, its deformation ε_{s2}, is obtained

$$\varepsilon_{s2} = \frac{\left(\xi_u d - d'\right)}{\xi_u d}\varepsilon_{cu2} \tag{19}$$

Knowing that the stress in the compression reinforcement $\sigma_{s2}=\varepsilon_{s2}E_s$, the equilibrium equation of compression and tension internal forces, is written

$$\lambda\xi_u db\,\eta f_{cd} + \frac{\left(\xi_u d - d'\right)}{\xi_u d}\varepsilon_{cu2}E_s\ A_{s2} = f_{yd}\ A_{s1} \tag{20}$$

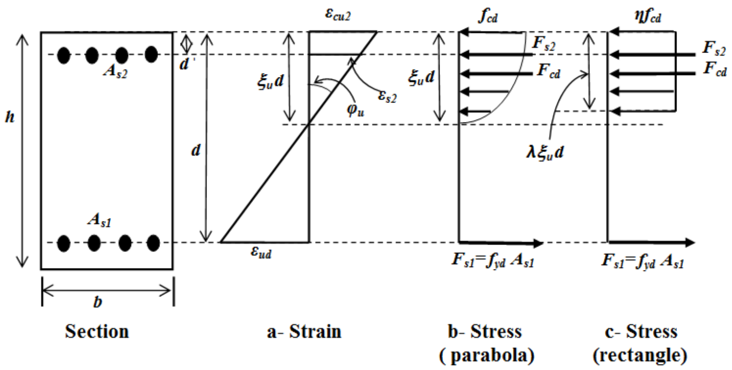


Fig. 4 Behavior of reinforced concrete beam section in flexure at the ultimate limit state.

Where, the factor λ defining the effective height of the compression zone (Fig. 4(c)). According to Eurocode 2, it is expressed by

$$\lambda = \begin{cases} 0.8 & for \quad f_{ck} \le 50\,MPa \\ 0.8 - \dfrac{f_{ck} - 50}{400} & for \quad 50\,MPa < f_{ck} \le 90\,MPa \end{cases} \tag{21}$$

And η define the effective strength (Fig. 4(c)). According to Eurocode 2, it is expressed by

$$\eta = \begin{cases} 1.0 & for \quad f_{ck} \le 50\,MPa \\ 1.0 - \dfrac{f_{ck} - 50}{200} & for \quad 50\,MPa < f_{ck} \le 90\,MPa \end{cases} \tag{22}$$

The solution of Eq. (20) leads to a second order expression with the variable ξ_u, its acceptable solution can be written as follows

$$\xi_u = \frac{\left(f_{yd}\,\rho - \varepsilon_{cu2} E_s \rho'\right)}{2\lambda\eta\, f_{cd}} + \frac{\sqrt{\left(f_{yd}\,\rho - \varepsilon_{cu2} E_s \rho'\right)^2 + 4\lambda\eta\, f_{cd}\,\varepsilon_{cu2}\,E_s \rho' \dfrac{d'}{d}}}{2\lambda\eta\, f_{cd}} \tag{23}$$

3.3 Conventional curvature ductility factor

The curvature ductility factor is obtained by the ratio between the curvature determined at the ultimate limit state Eq. (18) and the curvature determined at the first yield Eq. (12)

$$\mu_\varphi = \frac{\varepsilon_{cu2}}{\varepsilon_{sy,d}} \frac{\left(1 - \xi_y\right)}{\xi_u} \tag{24}$$

4. Parametric study

This study will be organized around parameters influencing the curvature ductility a reinforced concrete beams. These parameters are divided according to their influences into two categories. The first affects the element of overall beam, namely: the compressive strength of concrete f_{ck} and the yield strength of reinforcement f_{yk}. The second group affects the neutral axis for each section of the reinforced concrete beam, these parameters are particularly relevant the ratios of tension and compression reinforcements, and finally we see the effect of geometrical ratio (d'/d) on the curvature ductility.

4.1 Influence of the concrete compressive strength f_{ck} on curvature ductility

In order to examine the effect of the concrete compressive strength on the curvature ductility, we try to use a concrete strength f_{ck} up to 50 MPa for ordinary concrete and up to 90 MPa for high-strength concrete, as exercised within Eurocode 2 (EN 1992, 2004). Fig. 5(a) shows the curvature ductility factor corresponding to different values of strength f_{ck}. From this histogram, it is observed

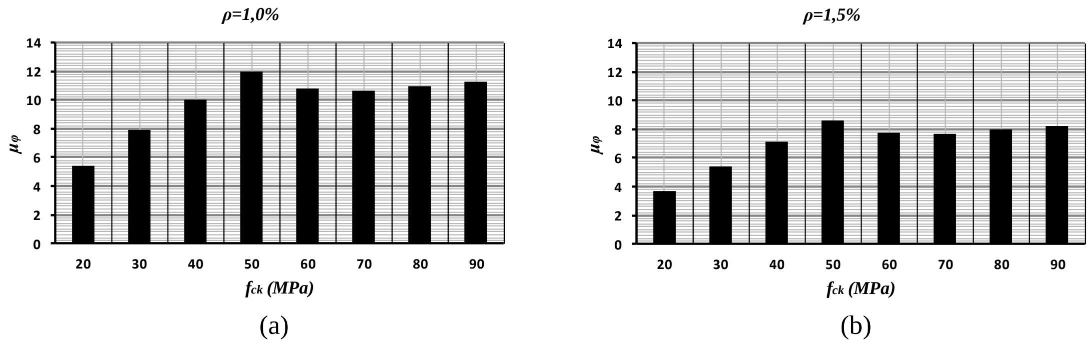


Fig. 5 Effect of the concrete compressive strength f_{ck} for (f_{yk}=400 MPa; ρ'/ρ = ½ .)

that the factor μ_φ increases with the increase of the strength f_{ck} up to value of 50 MPa, where the curvature ductility factor reaches its maximum value. After that, the ductility factor μ_φ decreases up to a strength equal to 70 MPa. Beyond this point, μ_φ returns to increase but with small variation. This observation continues in Fig. 5(b) when ρ=1.5%. Accordingly, we can deduce that the curvature ductility factor continue to improve with the increase of the concrete compressive strength until strength equal to 50 MPa.

4.2 Influence of the yield strength of steel f_{yk} on curvature ductility

Respecting the application domain of the Eurocode 2 (EN 1992, 2004), three values of yield strength f_{yk} are considered 400, 500 and 600 MPa. The obtained results are illustrated in Fig. 6. This figure shows the curvature ductility factor for each value of the yield strength of steel f_{yk}. According to the histogram of Fig. 6(a), it is observed that the factor μ_φ decreases with the increasing of the yield strength f_{yk}, this observation is clearly seen in Figs. 6(b) when the concrete strength increases 90 MPa. Contrary to the finding deducted in (4.1), the curvature ductility factor increases when the yield strength of steel f_{yk} decreases.

4.3 Influence of tension reinforcement ratio ρ on curvature ductility

The effect of tension reinforcement ratio on curvature ductility is treated in accordance with the

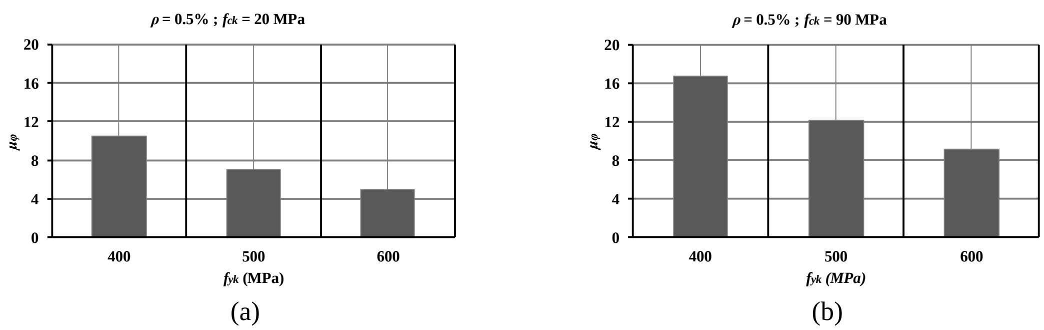
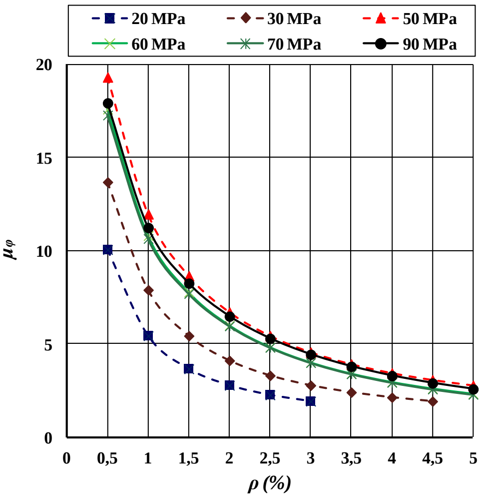
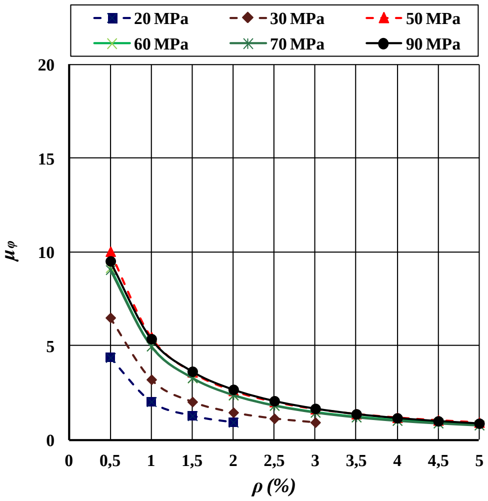


Fig. 6 Effect of the yield strength of steel f_{yk} for (ρ'/ρ = 3/4)



(a) f_{yk}=400 MPa; ρ'/ρ=½



(b) f_{yk}=600 MPa; ρ'/ρ=½

Fig. 7 Effect of the ratio of tension reinforcement ρ on the curvature ductility

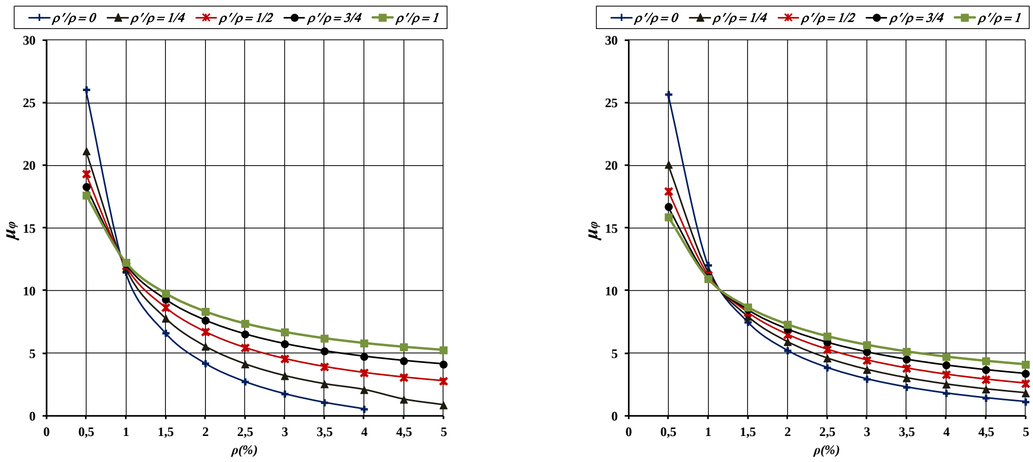
increase of the compressive strength f_{ck}, this ratio will be varied from 0.005 to 0.05 with a step equal to 0.005, in percentage from 0.5 to 5 % with a step of 0.5 %. Fig. 7(a) shows the curves $\mu_\varphi(\rho)$ for various concrete strengths. From this figure, it is observed that each curve $\mu_\varphi(\rho)$ decreases proportional inversely with the increase of the percentage ρ (%). In the same figure, we also observed that the curves $\mu_\varphi(\rho)$ increase with the increasing of the concrete strength f_{ck} up to the curve corresponding to f_{ck}=50 MPa, beyond this strength, the curves stop to increase and coincide together. Fig. 7(b) shows the same curves plotted in Fig. 7(a) with yield strength of steel f_{yk}=600 MPa, the same appearances as Fig. 7(a) are observed in this figure. Accordingly, we can say that the factor μ_φ decreases proportionally with the increasing of tension reinforcement ratio ρ. The overall comparison between the six curves of Fig. 7(a) and others of Fig. 7(b) shows the decrease of the envelope of curves when the yield strength f_{yk} increases from 400 to 600 MPa.

4.4 Influence of compression reinforcement ratio ρ' on curvature ductility

The effect of compression reinforcement ratio ρ' on curvature ductility is expressed by the ratio of compression to tension reinforcement ($\rho' \rho$). The compression reinforcement ratio will be varied from zero to the value of tension reinforcement ratio ρ, so (ρ'/ρ) varies from 0 to 1. Fig. 8 illustrates the influence of this ratio on the curvature ductility. Fig. 8(a) shows three intervals of variation of curves μ_φ (ρ):

- For ρ <0.8%, the curves decrease with increasing of the ratio ρ'/ρ.
- For ρ=0.8%, the influence of compression reinforcement is negligible, all curves coincide for any value of ratio ρ'/ρ.
- For ρ>0.8%, the effect of the ratio ρ'/ρ becomes very advantageous. Here, the curves μ_φ (ρ) increase with increasing of the ratio ρ'/ρ.

The same observation is illustrated in Fig. 8(b) when f_{ck}=90 MPa, but with a slight modification in the coincidence point.



(a) f_{ck}=50 MPa (b) f_{ck}=90 MPa

Fig. 8 Effect of the ratio of compression reinforcement ρ' on the curvature ductility for f_{yk}=400 MPa

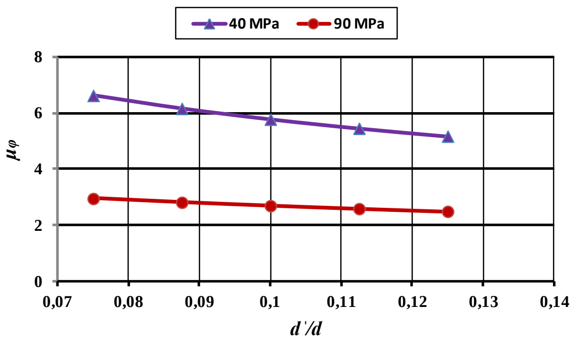


Fig. 9 Effect of the ratio d'/d (f_{yk}=400 MPa)

4.5 Influence of the ratio (d ′/ d) on curvature ductility

Fig. 9 shows the effect of the geometrical ratio on the curvature ductility. From this figure, in the case where the concrete strength equal to 40 MPa and the tensile reinforcement ratio equal to 2 %, we can observe that the curvature ductility factor is reduced bit by bit when the ratio d'/d increases. The same observation is noted when the concrete strength and the ratio of tensile reinforcement increase to 90 MPa and 5% respectively, but in this case the effect of this ratio is almost nil. Finally, we can say that the ratio d'/d has a slight effect on the curvature ductility.

5. Proposed formula

The parametric study of the influence of different parameters affecting the curvature ductility factor of unconfined concrete beam sections showed that this factor can be represented based on the parameters studied; it can be expressed as the following function:

$$\mu_\varphi = f(f_{ck}, \rho, (\rho'/\rho), f_{yk}) \tag{25}$$

Where, the effect of the geometrical ratio (d'/d) is neglected, due to its little effect on the curvature ductility.

According to the various curves of curvature ductility factor μ_φ (ρ) plotted based on the tension reinforcement ratio ρ, as shown in Figs. 7 and 8, the ductility factor is inversely proportional with the ratio of tensile reinforcement, and they form hyperbolic shape. So the curvature ductility factor μ_φ can be expressed in the following form

$$\mu_\varphi = A\rho^B \tag{26}$$

Where A and B are coefficients can be determined based on the parameters previously studied (f_{ck}, ρ, ρ'/ρ and f_{yk}).

The proposed Expressions for determining the available curvature ductility factor in the beam sections vary according to the effect of the concrete compressive strength f_{ck}. According to Eurocode 2, the first proposition is for ordinary concrete with compressive strength less or equal to 50 MPa, and the second is for beams having strength higher than 50 MPa.

5.1 For $f_{ck} \leq 50$ MPa

In the case of the concrete strength f_{ck} equal to 40 MPa, the yield strength of steel f_{yk} equal to 400 MPa and the ratio (ρ'/ρ) equal to 0.5, the exponential function of the curvature ductility factor μ_φ according to the tensile reinforcement ratio ρ is written as follow

$$\mu_\varphi = 0.1341\rho^{-0.94} \tag{27}$$

To facilitate the determination of a general formula, the coefficient B is fixed by the value -0.94 and the coefficient A is written according to the parameters studied, so

$$A = ?\ f_{ck}, (\rho'/\rho), f_{yk}) \tag{28}$$

Or

$$A = \alpha_1(f_{ck}) * \beta_1(f_{yk}) * \gamma_1(\rho'/\rho) \tag{29}$$

Where: α_1 (f_{ck}), β_1 (f_{yk}) and γ_1 (ρ'/ρ) are functions with the variables f_{ck}, f_{yk} and (ρ'/ρ) respectively.

Based on the parametric study:

The concrete strength f_{ck} has a positive effect on the ductility, the function α_1 (f_{ck}) is obtained as follow

$$\alpha_1(f_{ck}) = 0.00335 \ * f_{ck} \tag{30}$$

The yield strength of steel f_{yk} has a negative effect on the ductility, so the function β_1 (f_{yk}) is obtained as follows

$$\beta_1(f_{yk}) = 620987 * f_{yk}^{-2.226} \tag{31}$$

And the function γ_1 (ρ'/ρ) can be expressed as

$$\gamma_1(\rho'/\rho)=(44\rho(\frac{\rho'}{\rho}-\frac{1}{2})+1) \tag{32}$$

Then, the coefficient *A* is written as follows

$$A=2080*\,f_{ck}*\,f_{yk}^{\;-2.226}*(44\rho(\frac{\rho'}{\rho}-\frac{1}{2})+1) \tag{33}$$

The proposed expression is a simplified formula that meets to the practical requirements for the factors used in beams. In general, it is valid for 30≤ f_{ck}≤50 MPa, 1≤ ρ≤5%, 0.25≤ ρ'/ρ≤1 and $f_{yk} \in$ [400,600] MPa.

5.2 For f_{ck}>50 MPa

In the case of the concrete strength f_{ck} equal to 51 MPa, the yield strength of steel f_{yk} equal to 400 MPa and the ratio (ρ'/ρ) equal to 0.5, the exponential function of the curvature ductility factor μ_φ according to the ratio of tension reinforcements ρ is written as follow

$$\mu_\varphi=0.1667\rho^{-0.93} \tag{34}$$

Fixing the coefficient *B* by the value -0.93, the coefficient *A* will be written according to the parameters studied

$$A=?\;\; f_{ck},(\rho'/\rho),f_{yk}\;) \tag{35}$$

Or

$$A=\alpha_2(\,f_{ck}\,)*\beta_2(\,f_{yk}\,)*\gamma_2(\rho'/\rho) \tag{36}$$

Where $\alpha_2\,(f_{ck})$, $\beta_2\,(f_{yk})$ and $\gamma_2\,(\rho'/\rho)$ are functions with the variables f_{ck}, f_{yk} and ($\rho'\rho$) respectively.

Based on the parametric study:

When the concrete strength f_{ck} exceeds 50 MPa the curvature ductility factor starts to decrease up to strength equal to 70 MPa, beyond this strength the curvature ductility factor returns to increase. The change of the curvature ductility according to the concrete strength is in the form of second degree function, the function $\alpha_2\,(f_{ck})$ obtained as follows

$$\alpha_2(\,f_{ck}\,)=\frac{0.1667}{-0.0003f_{ck}^{\;2}+0.0424f_{ck}-0.367} \tag{37}$$

The effect of the yield strength of steel f_{yk} is also negative in the case of high strength concrete and the function $\beta_2\,(f_{yk})$ is obtained as follows

$$\beta_2(\,f_{yk}\,)=797824*\,f_{yk}^{\;-2.268} \tag{38}$$

And the function $\gamma_2\,(\rho'/\rho)$ can be expressed as

$$\gamma_2(\rho'/\rho)=36\rho(\frac{\rho'}{\rho}-\frac{1}{2})-\frac{1}{3}(\frac{\rho'}{\rho}-\frac{7}{2}) \tag{39}$$

So, the coefficient A is written as follows

$$A=\left(\frac{132997{,}261}{-0.0003f_{ck}^{\ 2}+0.0424f_{ck}-0.367}\right)*\left(36\rho(\frac{\rho'}{\rho}-\frac{1}{2})-\frac{1}{3}(\frac{\rho'}{\rho}-\frac{7}{2})\right)*f_{yk}^{\ -2.268} \tag{40}$$

In general, the proposed expressions is valid for beams have a concrete strength $51\leq f_{ck}\leq 90$ MPa, the yield strength of steel $f_{yk}\in[400,600]$ MPa, a ratio of tension reinforcements $1\leq\rho\leq 5\%$ and a ratio of compression reinforcements $0.25\,\rho\leq\rho'\leq\rho$.

6. Validation of proposed formula

The formula of curvature ductility proposed in Eq. (26) according to the different parameters f_{ck}, ρ, ρ'/ρ and f_{yk} is compared firstly with the numerical results and afterwards with the theoretical results of Lee (2013a, b) and also by the experimental results of (Maghsoudi and Bengar 2006, Maghsoudi and Sharifi 2009).

Tables 1,2,3 and 4 show the mean value (MV) and standard deviation (SD) for the ratio of proposed formula in Eq. (26) ($\mu_{\varphi,\,prop}$) to the numerical results of Eurocode 2 ($\mu_{\varphi,\,num}$) according to the concrete strength f_{ck}, the yield strength of steel f_{yk}, the tensile reinforcement ratio ρ and the ratio of compression to tensile reinforcement ρ'/ρ respectively.

The means values and standard deviations of the errors ($\mu_{\varphi,\,prop}/\mu_{\varphi,\,num}$) shown in Table 1 are calculated when (f_{yk}=400 MPa; ρ=1, 2, 3, 4 and 5%; ρ'/ρ=1/4, 1/2, 3/4 and 1), in the case of ordinary concrete $f_{ck}\leq 50$ MPa, for a group of 89 errors, the mean value is equal to 1.005 and the standard deviation equal to 0.085, in the case of high-strength concrete $f_{ck}>50$ MPa, for a group of 177 errors the mean value is equal to 0.989 and the standard deviation equal to 0.049. According to these results the proposed formula is in good agreement with the numerical results of the Eurocode 2.

On the other hand, Table 2 shows the mean values and standard deviations of the errors ($\mu_{\varphi,prop}/\mu_{\varphi,num}$) according to the change of the yield strength of steel (f_{yk}=400, 500 and 600 MPa). When f_{yk} =400 MPa, we have a group of 74 error values, MV and SD are 0.993 and 0.087 respectively. In the case where f_{yk}=500 MPa, for a group of 70 error, the MV is equal to 0.978 and the SD is equal to 0.091. When the yield strength of steel increases to 600 MPa, MV and. These results confirm

Table 1 Comparison between the proposed formula and the numerical method used according to the concrete strength

	$f_{ck}\leq 50$ MPa		$f_{ck}>50$ MPa	
	MV	SD	MV	SD
($\mu_{\varphi,prop}/\mu_{\varphi,num}$)	1.005	0.085	0.989	0.049

Table 2 Comparison between the proposed formula and the numerical method used according to the yield strength of steel

	f_{yk} =400 MPa		f_{yk} =500 MPa		f_{yk} =600 MPa	
	MV	SD	MV	SD	MV	SD
($\mu_{\varphi,prop}/\mu_{\varphi,num}$)	0.993	0.087	0.978	0.091	0.990	0.118

Table 3 Comparison between the proposed formula and the numerical method used according to tensile reinforcement

	ρ=1%		ρ=2%		ρ=3%		ρ=4%		ρ=5%	
	MV	SD	MV	SD	MV	SD	MV	SD	MV	SD
$(\mu_{\varphi,prop}/\mu_{\varphi,num})$	0.967	0.090	0.965	0.086	0.988	0.094	1.002	0.100	1.026	0.120

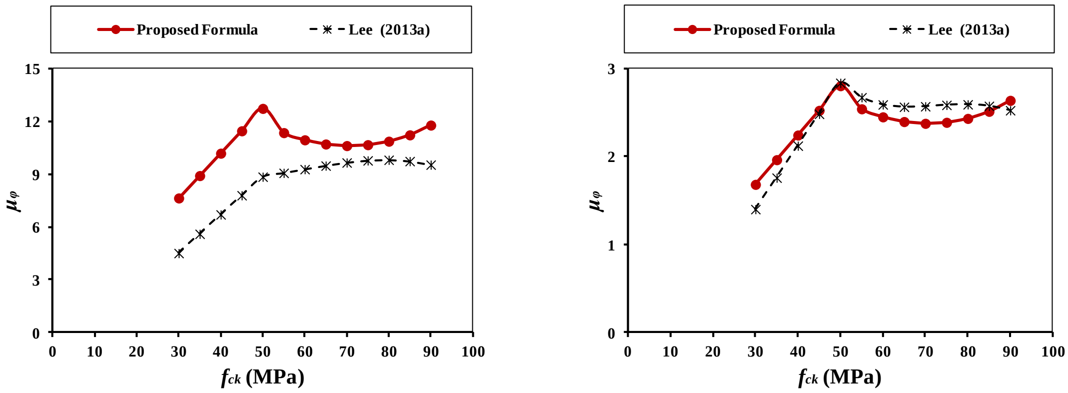
Table 4 Comparison between the proposed formula and the numerical method used according to compression reinforcement

	ρ'/ρ=1/4		ρ'/ρ=1/2		ρ'/ρ=3/4		ρ'/ρ=1	
	MV	SD	MV	SD	MV	SD	MV	SD
$(\mu_{\varphi,prop}/\mu_{\varphi,num})$	1.010	0.083	1.019	0.082	0.975	0.088	0.956	0.120

the reliability between the proposed formula Eq. (26) and the numerical results of the Eurocode 2 according to the change of the yield strength of steel f_{yk}.

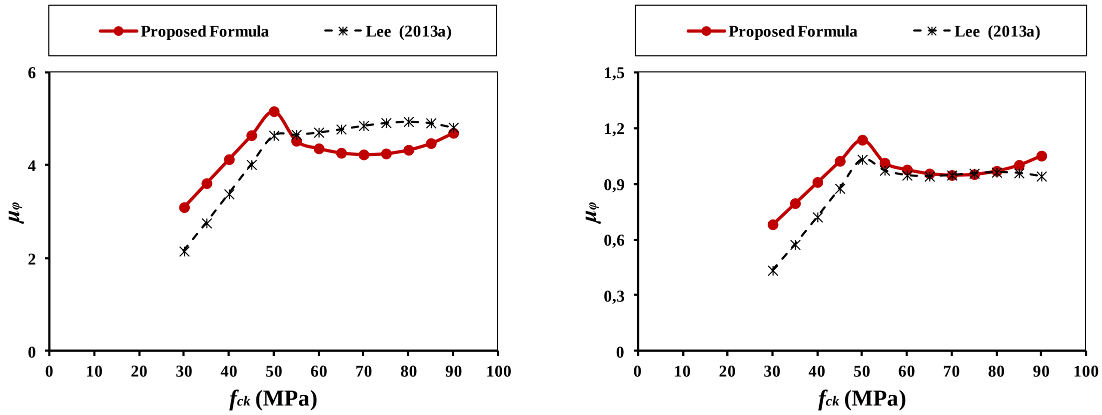
In the same context, MV and SD of Table 3 are calculated with percentage of tension reinforcement equal to 1, 2, 3, 4, and 5%, concrete strength equal to 30, 50, 51 and 90 MPa, and yield strength of steel equal to 400, 500 and 600 MPa. From this table, all MV and SD calculated according to the tensile reinforcements confirm the reliability of the proposed formula according to this parameter. On the other hand, Table 4 shows MV and SD of the errors $(\mu_{\varphi,\ prop}/\mu_{\varphi,\ num})$ according to the ratio between tensile and compression reinforcements (ρ'/ρ) when the ratio ρ'/ρ=1/4, 1/2, 3/4 and 1, and the yield strength of steel equal to 400, 500 and 600 MPa, and the concrete strength equal to 30, 50, 51 and 90 MPa. Also in Table 4, all MV and SD calculated according to the ratio ρ'/ρ confirm the good agreement between the proposed formula Eq. (26) and the numerical results of the Eurocode 2 according to the change of compression reinforcements.

The Figs. 10 and 11 show a comparison between the different curves of curvature ductility factor μ_φ according to the compressive concrete strength f_{ck}. This comparison is between the curvature ductility factor obtained by the proposed formula Eq. (26) and other prediction of Lee (2013a). The curvature ductility factor is calculated with two values of ρ = 1 and 5 % and



(a) ρ=1% (b) ρ=5%

Fig. 10 comparison between the proposed formula Eq. (26) and prediction of Lee (2013a)



(a) ρ=1% (b) ρ=5%

Fig. 11 comparison between the proposed formula Eq. (26) and prediction of Lee (2013a)

Table 5 comparison between the proposed formula Eq. (26) and the results of Lee (2013b)

N° Beam	f_{ck} (MPa)	ρ (%)	ρ' (%)	d (mm)	d' (mm)	f_{yk} (MPa)	Curvature ductility factor μ_φ			$\frac{\mu_{\varphi,prop}}{\mu_{\varphi,exp}}$	$\frac{\mu_{\varphi,prop(Eq.26)}}{\mu_{\varphi,Lee(2013b)}}$
							Proposed formula	Results of Lee (2013b)			
								Exp. Rashid and Mansour (2005)	Numerical results		
1	42.8	2.2	0.3	345	55	460	2.47	2.39	2.69	1.03	0.92
2	73.6	2.2	0.3				3.10	2.82	4.96	1.10	0.62
3	72.8	3.46	0.31				1.52	1.59	2.66	0.96	0.57
4	77.0	3.46	0.62				1.74	2.22	3.08	0.78	0.56
5	72.8	3.46	0.94				1.93	1.64	3.15	1.18	0.61
6	77.0	4.73	0.32				-	1.97	1.84	-	-
Average										**1.01**	**0.66**

intermediate value of ρ'/ρ equal to 1/2 and two values of steel yield strength f_{yk}=400 MPa for the curves of Fig. 10 and 600 MPa for curves shown in Fig. 11. In Fig. 10(a), the percentage of tension reinforcement ρ(%) equal to 1%, we note an harmonization between our formula and other prediction, this notation continues in the Fig. 10(b) when ρ=5%, but in this case the two curves are almost identical. When the yield strength of steel f_{yk} increases to 600 MPa, in Fig. 11(a), the consistency of our curve continues, where there is a clear superposition between our results and other work. Also in Fig. 11(b) when ρ=5% we note a large harmonization between the two curves. Consequently, the proposed formula Eq. (26) shows a broad consistency with the prediction of Lee (2013a), and it expresses all parameters influenced the curvature ductility of high strength unconfined concrete beams.

In the same context, Tables 5 and 6 show a comparison between the results obtained by the proposed formula Eq. (26) and other results of Lee (2013b) and (Maghsoudi and Bengar 2006, Maghsoudi and Sharifi 2009), respectively. From Table 5, the mean of errors calculated between Eq. (26) results and the experimental results of Rashid and Mansour (2005) approximately equal to

Table 6 comparison between the proposed formula Eq. (26) and experimental results

	N° Beam	f_{ck} (MPa)	d (mm)	d' (mm)	ρ (%)	ρ' (%)	ρ'/ρ	f_{yk} (MPa)	Curvature ductility factor μ_φ Experimental results	ACI	CSA	Proposed formula
Maghsoudi and Sharifi (2009)	1	73.65	256	40	4.103	2.0515	0.5	400	4.33	2.75	3.51	**2.86**
	2	66.81	266	40	4.773	2.3865	0.5	400	-	2.07	2.65	**2.49**
	3	77.72	258	42	5.851	2.9255	0.5	400	3.38	1.76	2.18	**2.08**
Maghsoudi and Bengar (2006)	4	56.31	254	42	0.61	0.61	1	398	11.84	9.89	11.91	**16.96**
	5	69.5	254	42	0.61	0.61	1	398	10.25	19.13	23.98	**16.04**
	6	63.48	250	47	1.25	0.61	0.488	401	6.84	6.68	8.13	**8.68**
	7	70.5	250	47	1.25	0.61	0.488	401	5.38	8.22	10.31	**8.57**
	8	63.21	251	42	2.03	1.01	0.4975	373	5.75	5.53	6.87	**6.53**
	9	70.8	251	42	2.03	1.01	0.4975	373	4.52	5.34	6.65	**6.43**
	10	71.45	250	47	2.51	1.24	0.494	401	5.6	4.75	5.87	**4.47**
	11	72.8	250	47	2.51	1.24	0.494	401	2.82	3.64	4.48	**4.48**

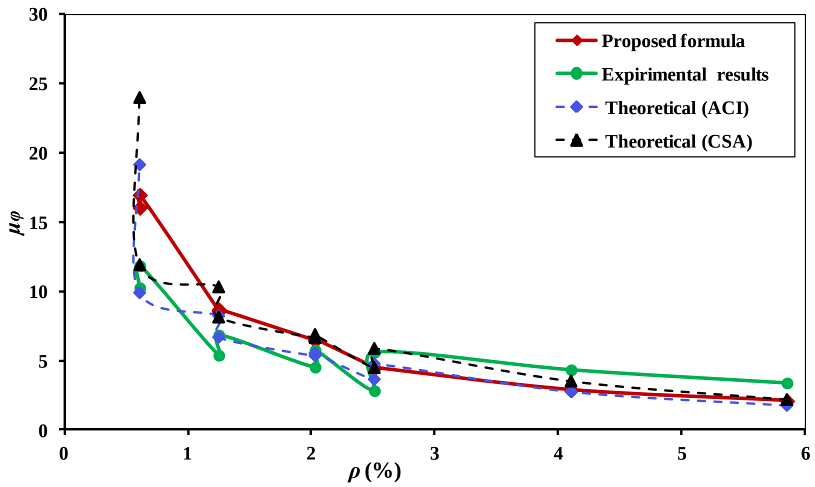


Fig. 12 comparison between the proposed formula Eq. (26) and experimental results

1.01, this finding indicate the affinity of our formula with these experimental results. This affinity remains valid with the errors calculated between the results obtained by Eq. (26) and the numerical results of Lee (2013b). On the other hand, Table 6 shows that the results obtained by our proposal Eq. (26) are acceptable compared with the experimental results and the theoretical results of ACI and CSA, except in the case where the ratio of tensile reinforcement ρ is inferior to 1%, a less agreement is observed between our proposal and the experimental results because the proposed formula is very effective when the percentage ρ is between 1 and 5%. The graphical representation of Table 6 in Fig. 12 confirms all findings obtained from this Table.

7. Conclusions

The work presented in this paper is principally studied the parameters affecting the curvature ductility of high strength unconfined concrete beams according to the Eurocode 2. The results generated in this parametric study, as well as those collected in the bibliography, we have provided the necessary data to propose a new simplified formula according to the concrete strength f_{ck}, the yield strength of steel f_{yk} and the ratio of tension and compression reinforcements ρ and ρ'. The proposed formula is applicable for unconfined reinforced concrete beams having a concrete strength f_{ck} from 30 up to 90 MPa, yield strength of steel f_{yk} from 400 to 600 MPa, a percentage of tension reinforcement $1 \leq \rho \leq 5$ percent and a ratio of compression reinforcement ρ' from 1/4 up to the value of tension reinforcement ρ.

The proposed formula in the practical cases allows designers to choose beam configurations for a curvature ductility factor initially selected, and therefore a good control of beam damage during major earthquakes.

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