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Effects of Hall current in a transversely isotropic magnetothermoelastic with and without energy dissipation due to normal force

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Abstract. This investigation is concerned with the disturbances in a homogeneous transversely isotropic thermoelastic rotating medium with two temperature, in the presence of the combined effects of Hall currents and magnetic field due to normal force of ramp type. The formulation is applied to the thermoelasticity theories developed by Green-Naghdi Theories of Type-II and Type-III. Laplace and Fourier transform technique is applied to solve the problem. The analytical expressions of displacements, stress components, temperature change and current density components are obtained in the transformed domain. Numerical inversion technique has been applied to obtain the results in the physical domain. Numerically simulated results are depicted graphically to show the effects of Hall current and anisotropy on the resulting quantities. Some special cases are also deduced from the present investigation.

Keywords: transversely isotropic; thermoelastic; laplace transform; fourier transform; normal force; hall current

1. Introduction

As the importance of anisotropic devices has increased in many fields of optics and microwaves, a wide research in anisotropic media has been widely done over in the last decades. The anisotropic nature basically stems from the polarization or magnetization that can occur in materials when external fields pass by. During the past few decades, widespread attention has been given to thermoelasticity theories that admit a finite speed for the propagation of thermal signals. In contrast to the conventional theories based on parabolic-type heat equation, these theories are referred to as generalized theories. Because of the experimental evidence in support of the finiteness of the speed of propagation of a heat wave, generalized thermoelasticity theories are more realistic than conventional thermoelasticity theories in dealing with practical problems involving very short time intervals and high heat fluxes such as those occurring in laser units,

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energy channels, nuclear reactors, etc. The phenomenon of coupling between the thermomechanical behavior of materials and magnetic behavior of materials have been studied since the 19th century.

Chen and Gurtin (1968), Chen *et al*. (1968), Chen *et al*. (1969), have formulated a theory of heat conduction in deformable bodies which depends upon two distinct temperatures, the conductive temperature □ and the thermo dynamical temperature *T*. For time independent situations, the difference between these two temperatures is proportional to the heat supply, and in absence of heat supply, the two temperatures are identical. For time dependent problems, the two temperatures are different, regardless of the presence of heat supply. The two temperatures, and the strain are found to have representations in the form of a travelling wave plus a response, which occurs instantaneously throughout the body (Boley and Tolins 1962). The wave propagation in two temperature theory of thermoelasticity was investigated by Warren and Chen (1973). Green and Naghdi (1991), postulated a new concept in thermoelasticity theories and proposed three models which are subsequently referred to as GN-I, II, and III models. The linearised version of model-I corresponds to classical thermoelastic model (based on Fourier’s law). The linearised version of model-II and III permit propagation of thermal waves at finite speed. Green-Naghdi’s second model (GN-II) (1993), in particular exhibits a feature that is not present in other established thermoelastic models as it does not sustain dissipation of thermal energy .In this model the constitutive equations are derived by starting with the reduced energy equation and by including the thermal displacement gradient among other constitutive variables. Green-Naghdi’s third model (GN-III) admits dissipation of energy. In this model the constitutive equations are derived by starting with the reduced energy equation, where the thermal displacement gradient in addition to the temperature gradient, are among the constitutive variables. Green and Naghdi (1992), included the derivation of a complete set of governing equations of a linearised version of the theory for homogeneous and isotropic materials in terms of the displacement and temperature fields and a proof of the uniqueness of the solution for the corresponding initial boundary value problem.

A comprehensive work has been done in thermoelasticity theory with and without energy dissipation and thermoelasticity with two temperature. Youssef (2011), constructed a new theory of generalized thermoelasticity by taking into account two-temperature generalized thermoelasticity theory for a homogeneous and isotropic body without energy dissipation. Youssef *et al*. (2007), investigated State space approach of two temperature generalized thermoelasticity of infinite body with a spherical cavity subjected to different types of thermal loading. Abbas (2011), discussed two dimensional problem with energy dissipation. Quintanilla (2002), investigated thermoelasticity without energy dissipation of materials with microstructure. Abbas, Kumar and Reen (2014), discussed response of thermal source in transversely isotropic thermoelastic materials without energy dissipation and with two temperature. Several researchers studied various problems involving two temperature e.g., (Youssef and AI-Lehaibi 2007, Youssef 2006, Youssef 2013, Kumar *et al*. 2014, Kaushal *et al*. 2011, Kaushal Sharma and Kumar 2010, Kumar and Mukhopdhyay 2010, Ezzat and Awad 2010, Sharma and Marin 2013, Sharma and Bhargav 2014, Sharma *et al*. 2013, Sharma and Kumar 2013, Sharma and Kumar 2012, Sharma *et al*. 2012). In view of the fact that most of the large bodies like the earth, the moon and other planets have an angular velocity, as well as earth itself behaves like a huge magnet, it is important to study the propagation of thermoelastic waves in a rotating medium under the influence of magnetic field. So, the attempts are being made to study the propagation of finite thermoelastic waves in an infinite elastic medium rotating with angular velocity. Several authors (Das and Kanoria 2014, Kumar and Kansal 2010, Kumar and Rupender 2009, Kumar and Devi 2010, Atwa and Jahangir

2014, Mahmoud 2013) have studied various problems in generalized thermoelasticity to study the effect of rotation.

When the magnetic field is very strong, the conductivity will be a tensor and the effect of Hall current cannot be neglected. The conductivity normal to the magnetic field is reduced due to the free spiraling of electrons and ions about the magnetic lines of force before suffering collisions and a current is induced in a direction normal to both the electric and magnetic fields. This phenomenon is called the Hall effect. Authors like (Zakaria 2011, 2012, Salem 2007, Attia 2009, Sarkar and Lahiri 2021) have considered the effect of Hall current for two dimensional problems ins micropolar thermoelasticity. Inspite of these, not much work has been done in thermoelastic solid with the combined effects of Hall current, rotation and two temperature. Keeping these considerations in mind, we formulated a two dimensional problem in transversely isotropic thermoelastic solid with and without energy dissipation in the presence of magnetic field, two temperature and rotation taking into consideration the effect of Hall current. The components of normal displacement, normal stress, tangential stress, conductive temperature and current density are obtained by using Laplace and Fourier transforms. Numerical computation is performed by using a numerical inversion technique and the resulting quantities are shown graphically.

2. Basic equations

The constitutive relations for anisotropic thermoelastic medium are given by

$$t_{ij} = C_{ijkl}e_{kl} - \beta_{ij}T \tag{1}$$

Equation of motion for anisotropic thermoelastic medium rotating uniformly with an angular velocity $\boldsymbol{\Omega} = \Omega n$, where n is a unit vector representing the direction of axis of rotation and taking into account Lorentz force

$$t_{ij,j} + F_i \ = \ \rho\{\ddot{u}_i + \left(\boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times u)\right)_i + (2\boldsymbol{\Omega} \times \dot{u})_i \ \} \tag{2}$$

Following Chandrasekharaiah (1998) and Youssef (2013), the heat conduction equation with two temperature and with and without energy dissipation is given by

$$K_{IJ}^{*} \varphi_{,ij} + K_{ij}\dot{\varphi}_{,ij} = \beta_{ij}T_0\ddot{e_{ij}} + \rho C_E\ddot{T} \tag{3}$$

The above equations are supplemented by generalized Ohm's law for media with finite conductivity and including the Hall current effect

$$\boldsymbol{J} = \frac{\sigma_0}{1+m^2}\left(\boldsymbol{E} + \mu_0\left(\dot{\boldsymbol{u}} \times \boldsymbol{H} - \frac{1}{en_e}\boldsymbol{J} \times \boldsymbol{H}_0\right)\right) \tag{4}$$

and the strain displacement relations are

$$e_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right) i,j = 1,2,3 \tag{5}$$

Here

$F_i = \mu_0(\boldsymbol{J} \times \boldsymbol{H}_0)_i$ are the components of Lorentz force.

$\beta_{ij} = C_{ijkl}\alpha_{ij}$ and $T = \varphi - a_{ij}\varphi_{,ij}$

$\beta_{ij} = \beta_i\delta_{ij}$, $K_{ij} = K_i\delta_{ij}$, $K_{ij}{}^{*} = K_i{}^{*}\delta_{ij}$, i is not summed

$C_{ijkl}(C_{ijkl} = C_{klij} = C_{jikl} = C_{ijlk})$ are elastic parameters, β_{ij} is the thermal tensor, T is the temperature, T_0 is the reference temperature, t_{ij} are the components of stress tensor, e_{kl} are the components of strain tensor, u_i are the displacement components, ρ is the density, C_E is the specific heat, K_{ij} is the thermal conductivity, K_{IJ}^* is the materialistic constant, a_{ij} are the two temperature parameters, α_{ij} is the coefficient of linear thermal expansion, Ω is the angular velocity of the solid, H is the magnetic strength, $\dot{\boldsymbol{u}}$ is the velocity vector , **E** is the intensity vector of the electric field, $\boldsymbol{J}$ is the current density vector, $m(= \omega_e t_e = \frac{\sigma_0 \mu_0 H_0}{e n_e})$ is the Hall parameter, t_e is the electron collision time, $\omega_e = \frac{e \mu_0 H_0}{m_e}$ is the electronic frequency, e is the charge of an electron, m_e is the mass of the electron, $\sigma_0 = \frac{e^2 t_e n_e}{m_e}$, is the electrical conductivity and n_e is the number of density of electrons.

3. Formulation and solution of the problem

We consider a homogeneous perfectly conducting transversely isotropic magnetothermoelastic medium which is rotating uniformly with an angular velocity $\boldsymbol{\Omega}$ initially at uniform temperature T_0. The rectangular Cartesian co-ordinate system (u_1, u_2, u_3) having origin on the surface (x_3=0) with x_3-axis pointing vertically downwards into the medium is introduced. The surface of the half-space is subjected to normal force. For two dimensional problem in x_1x_3-plane, we take

$$\boldsymbol{u} = (u_1, 0, u_3). \tag{6}$$

We also assume that

$$\boldsymbol{E}=0 \text{ , } \boldsymbol{\Omega} = (0, \Omega, 0). \tag{7}$$

The generalized Ohm's law

$$J_2 = 0 \tag{8}$$

the current density components J_1 and J_3 using (4) are given as

$$J_1 = \frac{\sigma_0 \mu_0 H_0}{1+m^2} \left(m \frac{\partial u_1}{\partial t} - \frac{\partial u_3}{\partial t} \right) \tag{9}$$

$$J_3 = \frac{\sigma_0 \mu_0 H_0}{1+m^2} \left(\frac{\partial u_1}{\partial t} + m \frac{\partial u_3}{\partial t} \right) \tag{10}$$

Following Slaughter (2002), using appropriate transformations, on the set of Eqs. (2) and (3) and with the aid of (6)-(10), we obtain the equations for transversely isotropic thermoelastic solid as

$$c_{11} \frac{\partial^2 u_1}{\partial x^2} + c_{13} \frac{\partial^2 u_3}{\partial x_1 \partial x_3} + c_{44} \left(\frac{\partial^2 u_1}{\partial {x_3}^2} + \frac{\partial^2 u_3}{\partial x_1 \partial x_3} \right) - \beta_1 \frac{\partial}{\partial x_1} \left\{ \varphi - \left(a_1 \frac{\partial^2 \varphi}{\partial {x_1}^2} + a_3 \frac{\partial^2 \varphi}{\partial {x_3}^2} \right) \right\} - \mu_0 J_z H_0 = \rho \left(\frac{\partial^2 u_1}{\partial t^2} - \Omega^2 u_1 + 2\Omega \frac{\partial u_3}{\partial t} \right) \tag{11}$$

$$(c_{13} + c_{44}) \frac{\partial^2 u_1}{\partial x_1 \partial x_3} + c_{44} \frac{\partial^2 u_3}{\partial {x_1}^2} + c_{33} \frac{\partial^2 u_3}{\partial {x_3}^2} - \beta_3 \frac{\partial}{\partial x_3} \left\{ \varphi - \left(a_1 \frac{\partial^2 \varphi}{\partial {x_1}^2} + a_3 \frac{\partial^2 \varphi}{\partial {x_3}^2} \right) \right\} + \mu_0 J_x H_0 = \rho \left(\frac{\partial^2 u_3}{\partial t^2} - \Omega^2 u_3 - 2\Omega \frac{\partial u_1}{\partial t} \right) \tag{12}$$

$$\left(k_1 + {k_1}^* \frac{\partial}{\partial t}\right)\frac{\partial^2 \varphi}{\partial {x_1}^2} + \left(k_3 + {k_3}^* \frac{\partial}{\partial t}\right)\frac{\partial^2 \varphi}{\partial {x_3}^2} = T_0 \frac{\partial^2}{\partial t^2}\left\{\beta_1 \frac{\partial u_1}{\partial x_1} + \beta_3 \frac{\partial u_3}{\partial x_3}\right\} + \rho C_E \ddot{T} \tag{13}$$

and

$$t_{11} = c_{11}e_{11} + c_{13}e_{33} - \beta_1 T \tag{14}$$

$$t_{33} = c_{13}e_{11} + c_{33}e_{33} - \beta_3 T \tag{15}$$

$$t_{13} = 2c_{44}e_{13} \tag{16}$$

where

$$T = \varphi - \left(a_1 \frac{\partial^2 \varphi}{\partial x_1^2} + a_3 \frac{\partial^2 \varphi}{\partial x_3^2}\right)$$

$$\beta_1 = (c_{11}+c_{12})\alpha_1 + c_{13}\alpha_3 \text{ , } \beta_3 = 2c_{13}\alpha_1 + c_{33}\alpha_3$$

In the above equations we use the contracting subscript notations ($11 \to 1, 22 \to 2, 33 \to 3, 23 \to 4, 31 \to 5, 12 \to 6$) to relate c_{ijkl} to c_{mn}

We assume that medium is initially at rest. The undisturbed state is maintained at reference temperature. Then we have the initial and regularity conditions are given by

$$u_1(x_1, x_3, 0) = 0 = \dot{u_1}(x_1, x_3, 0)$$

$$u_3(x_1, x_3, 0) = 0 = \dot{u_3}(x_1, x_3, 0) \; \varphi(x_1, x_3, 0) = 0 = \dot{\varphi}(x_1, x_3, 0) \text{ For } x_3 \geq 0, -\infty < x_1 < \infty$$

$$u_1(x_1, x_3, t) = u_3(x_1, x_3, t) = \varphi(x_1, x_3, t) = 0 \; for \; t > 0 \; when \; x_3 \to \infty \tag{17}$$

To facilitate the solution, following dimensionless quantities are introduced

$$x_1{}' = \frac{x_1}{L}, \, x_3{}' = \frac{x_3}{L}, \, u_1{}' = \frac{\rho c_1^2}{L\beta_1 T_0}u_1, \, u_3' = \frac{\rho c_1^2}{L\beta_1 T_0}u_3, \, T' = \frac{T}{T_0}, \, t' = \frac{c_1}{L}t, \, t'_{11} = \frac{t_{11}}{\beta_1 T_0}, \, J' = \frac{\rho c_1^2}{\beta_1 T_0}J$$

$$t'_{33} = \frac{t_{33}}{\beta_1 T_0}, \, t'_{31} = \frac{t_{31}}{\beta_1 T_0}, \, \varphi' = \frac{\varphi}{T_0}, \, a'_1 = \frac{a_1}{L}, \, a'_3 = \frac{a_3}{L}, \, h' = \frac{h}{H_0}, M = \frac{\sigma_0 \mu_0 H_0}{\rho c_1 L}, \, \Omega' = \frac{L}{c_1}\Omega \tag{18}$$

Making use of (18) in Eqs. (11)-(13) and suppressing the primes, and applying Laplace and Fourier transforms defined by

$$\bar{f}(x_1, x_3, s) = \int_0^\infty f(x_1, x_3, t)e^{-st}dt \text{ (19) } \hat{f}(\xi, x_3, s) = \int_{-\infty}^\infty \bar{f}(x_1, x_3, s)e^{i\xi x_1}dx_1 \tag{20}$$

on the resulting equations, we obtain a system of homogeneous equations in terms of $\widetilde{u_1}$, $\widetilde{u_3}$ and $\tilde{\varphi}$ which yield a non trivial solution if determinant of coefficient $\{\widetilde{u_1}, \widetilde{u_3}, \tilde{\varphi}\}^T$ vanishes and we obtain the following characteristic equation

$$(PD^6 + QD^4 + RD^2 + S)(\widetilde{u_1}, \widetilde{u_3}, \tilde{\varphi}) = 0 \tag{21}$$

where P, Q , R and S are given in appendix A.

The solution of the Eq. (21) satisfying the radiation condition that $\widetilde{u_1}, \widetilde{u_3}, \tilde{\varphi} \to 0$ as $x_3 \to \infty$, can be written as

$$\widetilde{u_1} = A_1 e^{-\lambda_1 x_3} + A_2 e^{-\lambda_2 x_3} + A_3 e^{-\lambda_3 x_3} \tag{22}$$

$$\widetilde{u_3} = d_1 A_1 e^{-\lambda_1 x_3} + d_2 A_2 e^{-\lambda_2 x_3} + d_3 A_3 e^{-\lambda_3 x_3} \tag{23}$$

$$\tilde{\varphi} = l_1 A_1 e^{-\lambda_1 x_3} + l_2 A_2 e^{-\lambda_2 x_3} + l_3 A_3 e^{-\lambda_3 x_3} \tag{24}$$

where $\pm\lambda_i, (i = 1,2,3)$, are the roots of (21) and d_i and l_i are given in appendix B.

4. Applications

The boundary conditions are taken as

$$(1)\ t_{33}(x_1, x_3, t) = G(t)\delta(x) \tag{25}$$

$$(2)\ t_{31}(x_1, x_3, t) = 0 \tag{26}$$

$$(3)\ \varphi(x_1, x_3, t) = 0 \tag{27}$$

where $\delta\ (x)$ is an arbitrary function of x and $G(t)$ is a function defined as

$$G(t) = \begin{cases} 0 & t \leq 0 \\ T_1 \frac{t}{t_0} & 0 < t \leq t_0 \\ T_1 & t > t_0 \end{cases} \tag{28}$$

where t_0 indicates the length of time to rise the heat and T_1 is a constant, this means that the boundary of the half-space, which is initially at rest and has a fixed temperature t_0, is suddenly raised to a temperature equal to the function $G(t)\delta(x)$ and maintained at this temperature afterwards.

Applying the Laplace and Fourier transforms to both sides of (28), we obtain

$$\widetilde{t_{33}} = \bar{G}(s)$$

where $\bar{G}(s) = T_1 \frac{(1-e^{-st_0})}{t_0 s^2}$

Applying the Laplace and Fourier transform defined by (19)-(20) on the boundary conditions (25)-(27) and with the aid of Eqs. (14)-(16), and (18) we obtain the components of displacement ,normal stress, tangential stress, conductive temperature and the components of current density as given by

$$\widetilde{u_1} = \frac{\bar{G}(s)}{\Delta}\left(\Delta_1 e^{-\lambda_1 x_3} + \Delta_2 e^{-\lambda_2 x_3} + \Delta_3 e^{-\lambda_3 x_3}\right) \tag{29}$$

$$\widetilde{u_3} = \frac{\bar{G}(s)}{\Delta}\left(d_1\Delta_1 e^{-\lambda_1 x_3} + d_2\Delta_2 e^{-\lambda_2 x_3} + d_3\Delta_3 e^{-\lambda_3 x_3}\right) \tag{30}$$

$$\widetilde{t_{33}} = \frac{\bar{G}(s)}{\Delta}\left(\Delta_{11}\Delta_1 e^{-\lambda_1 x_3} + \Delta_{12}\Delta_2 e^{-\lambda_2 x_3} + \Delta_{13}\Delta_3 e^{-\lambda_3 x_3}\right) \tag{31}$$

$$\widetilde{t_{31}} = \frac{\bar{G}(s)}{\Delta}\left(\Delta_{21}\Delta_1 e^{-\lambda_1 x_3} + \Delta_{22}\Delta_2 e^{-\lambda_2 x_3} + \Delta_{23}\Delta_3 e^{-\lambda_3 x_3}\right) \tag{32}$$

$$\tilde{\varphi} = \frac{\bar{G}(s)}{\Delta}\left(\Delta_{31}\Delta_1 e^{-\lambda_1 x_3} + \Delta_{32}\Delta_2 e^{-\lambda_2 x_3} + \Delta_{33}\Delta_3 e^{-\lambda_3 x_3}\right) \tag{33}$$

$$\widetilde{J_1} = \frac{c_1\sigma_0 H_0 \mu_0}{1+m^2}\frac{\bar{G}(s)}{\Delta}\left((m-d_1)\Delta_1 e^{-\lambda_1 x_3} + (m-d_2)\Delta_2 e^{-\lambda_2 x_3} + (m-d_3)\Delta_3 e^{-\lambda_3 x_3}\right) \tag{34}$$

$$\widetilde{J_3} = \frac{c_1\sigma_0 H_0 \mu_0}{1+m^2}\ \frac{\bar{G}(s)}{\Delta}\left((1+md_1)\Delta_1 e^{-\lambda_1 x_3} + (1+md_2)\Delta_2 e^{-\lambda_2 x_3} + (1+md_3)\Delta_3 e^{-\lambda_3 x_3}\right) \tag{35}$$

where Δ_1, Δ_2 , Δ_3, Δ_{11}, Δ_{12}, Δ_{13}, Δ_{21}, Δ_{22}, Δ_{23}, Δ_{31} , Δ_{32}, and Δ_{33} are given in appendix C.

5. Particular cases

(i) If $k_1{}^* = k_3{}^* = 0$, then from (29)-(35) ,we obtain the corresponding expressions for displacements, and stresses, conductive temperature and components of current density for transversely isotropic magnetothermoelastic solid without energy dissipation and with two temperature with Hall current effect and rotation.

(ii) If $a_1 = a_3 = 0$, then from (29)-(35),we obtain the corresponding expressions for displacements, stresses, conductive temperature and components of current density for transversely isotropic magnetothermoelastic solid with and without energy dissipation alongwith with Hall current effect and rotation.

(iii) If we take $c_{11} = \lambda + 2\mu = c_{33}$, $c_{12} = c_{13} = \lambda$, $c_{44} = \mu$, $\beta_1 = \beta_3 = \beta$, $\alpha_1 = \alpha_3 = \alpha$, $K_1 = K_3 = K$ and $a_1 = a_3 = a$ in Eqs. (29)-(35), we obtain the corresponding expressions for displacements, stresses , conductive temperature components of current density in isotropic magnetothermoelastic solid with two temperature and with and without energy dissipation alongwith combined effects of Hall current and rotation.

(iv) If m=0, in Eqs. (29)-(35) ,we obtain the components of displacements, stresses, conductive temperature and components of current density for transversely isotropic magnetothermoelastic solid and with and without energy dissipation and with two temperature alongwith rotation.

6. Inversion of the transformation

To obtain the solution of the problem in physical domain, we must invert the transforms in Eqs. (29)-(35). Here the displacement components, normal and tangential stresses and conductive temperature are functions of x_3 , the parameters of Laplace and Fourier transforms s and ξ respectively and hence are of the form f (ξ, x_3, s). To obtain the function $f(x_1, x_3, t)$ in the physical domain, we first invert the Fourier transform using

$$\overline{f}(x_1, x_3, s)=\frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-i\xi x_1}\,\hat{f}(\xi, x_3, s)d\xi = \frac{1}{2\pi}\int_{-\infty}^{\infty}|\cos(\xi x)\,f_e - isin(\xi x)\mathrm{f_o}|\,d\xi \qquad (36)$$

Where $f_e\ and\ f_o$ are respectively the odd and even parts of $\hat{f}$ (ξ, x_3, s). Thus the expression (36) gives the Laplace transform $\overline{f}(x_1, x_3, s)$ of the function $f(x_1, x_3,, t)$. Following Honig and Hirdes (1984), the Laplace transform function $\overline{f}(x_1, x_3, s)$ can be inverted to $f(x_1, x_3, t)$.

The last step is to calculate the integral in Eq. (36). The method for evaluating this integral is described in Press *et al*. (1986). It involves the use of Romberg's integration with adaptive step size. This also uses the results from successive refinements of the extended trapezoidal rule followed by extrapolation of the results to the limit when the step size tends to zero.

7. Numerical results and discussion

For the purpose of numerical evaluation, cobalt material has been chosen following Dhaliwal and Singh (1980) as

Table Cobalt material

$c_{11} = 3.071 \times 10^{11} Nm^{-2}$	$c_{33} = 3.581 \times 10^{11} Nm^{-2}$	$c_{13} = 1.027 \times 10^{11} Nm^{-2}$,	$c_{44} = 1.510 \times 10^{11} Nm^{-2}$	$\rho = 8.836 \times 10^{3} Kgm^{-3}$
$T_0 = 298°K$	$C_E = 4.27 \times 10^{2} JKg^{-1}deg^{-1}$	$K_1 = .690 \times 10^{2} wm^{-1}deg^{-1}$	$K_3 = .690 \times 10^{2} wm^{-1}deg^{-1}$	$\beta_1 = 7.04 \times 10^{6} Nm^{-2}deg^{-1}$
$\beta_3 = 6.90 \times 10^{6} Nm^{-2}deg^{-1}$	$K_1^* = 0.02 \times 10^{2} Nsec^{-2}deg^{-1}$	$K_3^* = 0.04 \times 10^{2} Nsec^{-2}deg^{-1}$	$\mu_0 = 1.2571 \times 10^{-6} Hm^{-1}$	$H_0 = 1Jm^{-1}nb^{-1}$
$\varepsilon_0 = 8.838 \times 10^{-12} Fm^{-1}$	$\sigma_0 = 9.36 \times 10^{5} col^2/Cal.cm$.sec	$t_0 = 0.02$		

with non-dimensional parameter L=1 and , Ω=2, , M=3 and two temperature parameters is taken as a_1=0.03 and a_3=0.06

Using the above values a comparison has been made of values of transverse displacement u, normal displacement w, normal stress t_{33}, tangential stress t_{31}, conductive temperature φ, transverse conduction current density J_1 and normal conduction current density J_3 for a transversely isotropic magneto-thermoelastic to show the effect of Hall current and isotropy and are presented in the Figs. 1-7.The computations are carried out in the range $0 \le x \le 10$.

1) The solid line, small dashed line corresponds to transversely isotropic thermoelastic solid (TIS) with Hall current parameter $m = 0.6\ and\ m = 0$ respectively.

2) The solid line with centre symbol circle, the small dashed line with centre symbol diamond corresponds to isotropic thermoelastic solid (IS) with hall parameter as $m = 0.6\ and\ m = 0$ respectively.

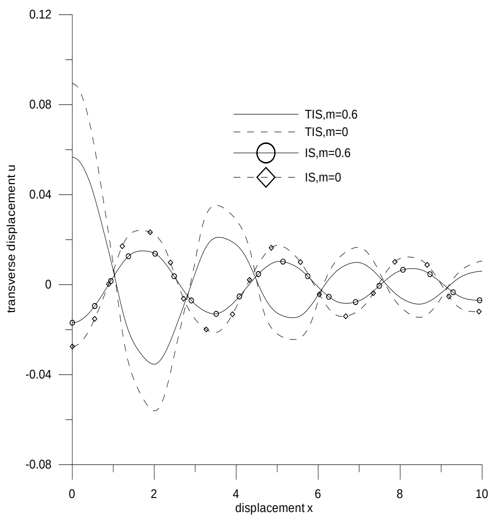

Fig. 1 Variation of transverse displacement u_1 with distance x

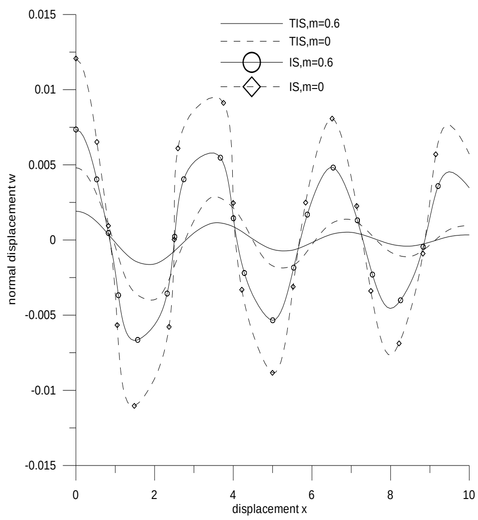

Fig. 2 Variation of normal displacement u_3 with distance x

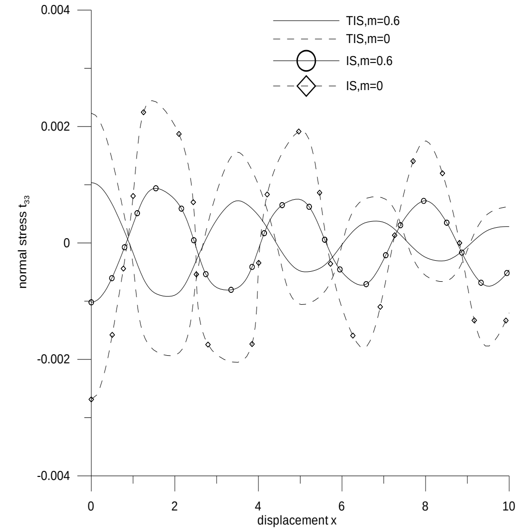

Fig. 3 Variation of normal stress t_{33} with distance x (mechanical force)

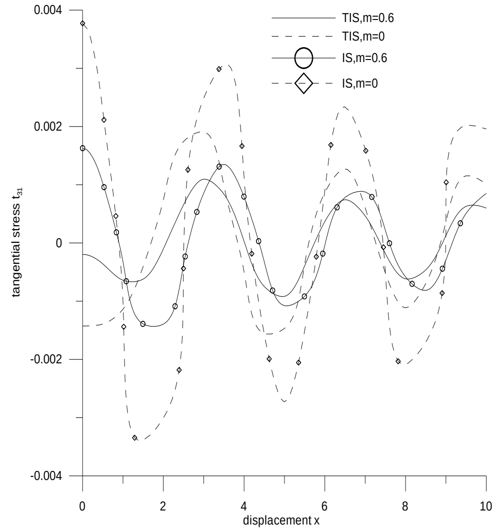

Fig. 4 Variation of tangential stress t_{31} with distance x

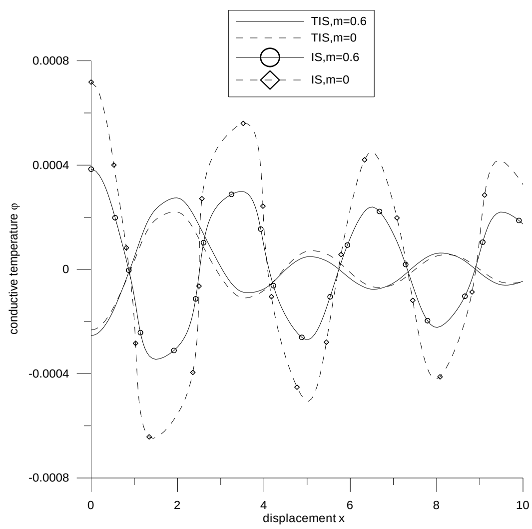

Fig. 5 Variation of conductive temperature φ with distance x

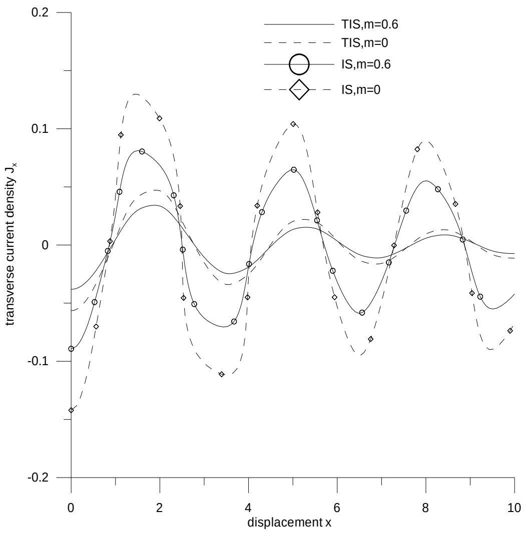

Fig. 6 Variation of transverse current density J_1 with distance x

Fig.1 depicts transverse displacement u_1 with distance x. We notice that the variations corresponding to (TIS) and (IS) follow opposite oscillatory pattern in case of both the values of Hall parameter for the whole range. For m=0.6, the amplitude of oscillation is smaller as compared to m=0. Fig. 2 presents the normal displacement u_3 with distance x. We notice that, the values of normal displacement corresponding to (TIS) and (IS) follow similar oscillatory behaviour with change in magnitude for the whole range. Also as m increases, the amplitude of oscillation decreases. Fig. 3 exhibits the trends of normal stress t_{33} with distance x. Here opposite oscillatory

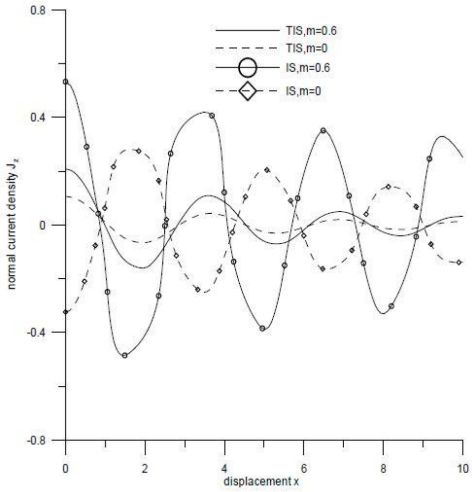

Fig. 7 Variation of normal current density J_3 with distance x

pattern in the variations is noticed corresponding to the two solids. Also, we notice that presence of Hall current parameter decreases the magnitude of oscillation. Fig. 4 shows variations of tangential stress t_{31} with distance x. Here, in all the cases similar oscillatory pattern with difference in magnitudes is noticed. Amplitude of variations corresponding to (TIS) is smaller as compared to (IS). The behaviour of conductive temperature φ is depicted in Fig. 5. Here opposite oscillatory behaviour is noticed in the values of conductive temperature corresponding to the two solids with difference in magnitudes. As x moves away from the point of application of the source, amplitude of oscillation decreases. Fig. 6 and Fig. 7 describe the trends of current density components J_1 and J_3 respectively with respect to distance x. We notice that the pattern is oscillatory in all the cases. In Fig. 6, we notice that, with the absence of Hall current, the amplitude of oscillation is greater than as compared to the presence of Hall current. Also we notice maximum variations corresponding to (IS). In Fig. 7, we notice opposite pattern in the values of J_3 as compared to J_1.

8. Conclusions

The analysis of graphs permits us some concluding remarks

1. Hall current parameter and isotropy has great impact in transversely isotropic magneto-thermoelastic solid.
2. The Hall current plays a significant role in the distribution of all the physical quantities. The amplitude of all the physical quantities vary (increase or decrease) as Hall parameter increases.
3. Presence of Hall current restricts the quantities to oscillate near the point of application of source as well as away from the source.
4. The effect of isotropy tends the variations to move in opposite oscillatory pattern in case of transverse displacement, normal stress and conductive temperature whereas in similar oscillatory pattern in the rest of cases.
5. Behaviour of both the components of transverse current density is just the reflection of each

other in presence of hall current.

6. The resulting quantities with and without Hall current show opposite oscillatory pattern in the form of waves. These trends obey elastic and thermoelastic properties of a solid under investigation. The research work is useful in further studies, for both theoretical and observational in more realistic models of transversely isotropic thermoelastic solid present in the earth's interior.

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CC

Appendix A

$$P = \delta_2\delta_3\zeta_7 + \varepsilon_5\zeta_5\delta_2$$

$$Q = \zeta_{10}\zeta_7\delta_3 + \delta_2\zeta_7\zeta_{11} - \delta_3\delta_2\zeta_6 - \varepsilon_5{\beta_3}^2\zeta_4\delta_2 - \zeta_3^2\zeta_7 - i\xi\zeta_3 p_3\zeta_5\varepsilon_5{\beta_1}^2 + \xi^2\varepsilon_5\delta_3\zeta_5{\beta_1}^2 - i\xi\varepsilon_5\zeta_5\beta_1\beta_3\zeta_3 - \xi^2\varepsilon_5{\beta_1}^2\delta_3\zeta_5$$

$$R = \zeta_{10}\zeta_{11}\zeta_7 - \zeta_{10}\delta_3\zeta_6 - \varepsilon_5{\beta_3}^2\zeta_{10}\zeta_4 - \delta_2\zeta_6\zeta_{11} + \zeta_3^2\zeta_6 + \zeta_2^2\zeta_7 + p_3\zeta_3\zeta_4\varepsilon_5{\beta_1}^2 i\xi + \mathrm{i}\xi\zeta_3\zeta_4\beta_1\beta_3\varepsilon_5 - \xi^2\zeta_5\zeta_{11}{\beta_1}^2\varepsilon_5$$

$$S = -\zeta_{10}\zeta_{11}\zeta_6 - \zeta_2^2\zeta_6 + \xi^2\zeta_{11}{\beta_1}^2\zeta_4\varepsilon_5$$

$$\zeta_1 = -\left(\frac{M}{1+m^2}\mu_0 H_0 s + s^2\right) + \Omega^2\ ,\ \zeta_2 = -\frac{M}{1+m^2}\mu_0 H_0 m s - 2\Omega s\ ,\ \zeta_3 = (\delta_4 + \delta_2)i\xi\ ,\ \zeta_4 = 1 + \frac{a_1}{L}\xi^2,\ \zeta_5 = \frac{a_3}{L}$$

$$\zeta_6 = (\varepsilon_1 + \varepsilon_3 s)\xi^2 + s^2\left(1 + \frac{a_1}{L}\xi^2\right)\ ,\ \zeta_7 = \varepsilon_2 + \varepsilon_4 s - \frac{a_3}{L}s^2\ ,\ \zeta_{10} = \zeta_1 - \xi^2,$$

$$\zeta_{11} = \zeta_1 - \delta_2\xi^2\ ,\varepsilon_5 = \frac{T_0 s^2}{\rho^2 C_E c_1^2}\ ,p_3 = \frac{\beta_3}{\beta_1},\ \delta_1 = \frac{(c_{13}+c_{44})}{c_{11}}\ ,\ \delta_2 = \frac{c_{44}}{c_{11}}\ ,\ \delta_3 = \frac{c_{33}}{c_{11}}\ ,\ \delta_4 = \frac{c_{13}}{c_{11}}\ ,\ \varepsilon_1 = \frac{k_1}{\rho C_E c_1^2}\ ,\ \varepsilon_2 = \frac{k_3}{\rho C_E c_1^2}\ ,\ \varepsilon_3 = \frac{{k_1}^*}{L\rho C_E c_1}$$

$$\varepsilon_4 = \frac{{k_3}^*}{L\rho C_E c_1}\ \ {\varepsilon_5}' = \frac{T_0}{\rho^2 C_E c_1^2}$$

Appendix B

$$d_i = \frac{\lambda_i^2(\varepsilon_5 i\xi\delta_3 - \zeta_3\beta_3\beta_1\xi\,) - \lambda_i(\xi\beta_1\beta_3\zeta_2) + \varepsilon_5 i\xi\zeta_{11}}{\lambda_1^4(\delta_3\zeta_7 + \varepsilon_5\zeta_5{\beta_3}^2) + \lambda_1^2(\zeta_7\zeta_{11} - \delta_3\zeta_6 - {\beta_3}^2\varepsilon_5\zeta_4) - \zeta_6\zeta_{11}}\ \ i\ = 1,2,3$$

$$l_i = \frac{-\lambda_i^3(\zeta_3\zeta_7 + \varepsilon_5 i\xi\zeta_5 \mathrm{p}_3) - \zeta_2\zeta_7\lambda_i^2 + (\zeta_3\zeta_6 + \varepsilon_5 i\xi \mathrm{p}_3\zeta_4)\lambda_i + \zeta_6\zeta_2}{\lambda_1^4(\delta_3\zeta_7 + \varepsilon_5\zeta_5{\beta_3}^2) + \lambda_1^2(\zeta_7\zeta_{11} - \delta_3\zeta_6 - {\beta_3}^2\varepsilon_5\zeta_4) - \zeta_6\zeta_{11}}\ \ i = 1,2,3$$

Appendix C

$$(\Delta_{22}\Delta_{33} - \Delta_{23}\Delta_{32}) = \Delta_1,\ (\Delta_{23}\Delta_{31} - \Delta_{21}\Delta_{33}) = \Delta_2,\ (\Delta_{21}\Delta_{32} - \Delta_{22}\Delta_{31}) = \Delta_3$$

$$\Delta_{1j} = \frac{c_{13}}{\rho c_1^2}i\xi - \frac{c_{33}}{\rho c_1^2}d_j\lambda_j - \frac{\beta_3}{\beta_1}l_j + \frac{\beta_3}{\beta_1}a_3 l_j{\lambda_j}^2 - \frac{\beta_3}{\beta_1}l_j a_1\xi^2\ j = 1,2,3$$

$$\Delta_{2j} = -\lambda_j + i\xi d_j\ j = 1,2,3$$

$$\Delta_{3j} = l_j,\ j = 1,2,3$$