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An efficient shear deformation theory for wave propagation of functionally graded material plates

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Abstract. An efficient shear deformation theory is developed for wave propagation analysis of an infinite functionally graded plate in the presence of thermal environments. By dividing the transverse displacement into bending and shear parts, the number of unknowns and governing equations of the present theory is reduced, and hence, makes it simple to use. The thermal effects and temperature-dependent material properties are both taken into account. The temperature field is assumed to be a uniform distribution over the plate surface and varied in the thickness direction only. Material properties are assumed to be temperature-dependent, and graded in the thickness direction according to a simple power law distribution in terms of the volume fractions of the constituents. The governing equations of the wave propagation in the functionally graded plate are derived by employing the Hamilton's principle and the physical neutral surface concept. There is no stretching–bending coupling effect in the neutral surface-based formulation, and consequently, the governing equations and boundary conditions of functionally graded plates based on neutral surface have the simple forms as those of isotropic plates. The analytic dispersion relation of the functionally graded plate is obtained by solving an eigenvalue problem. The effects of the volume fraction distributions and temperature on wave propagation of functionally graded plate are discussed in detail. It can be concluded that the present theory is not only accurate but also simple in predicting the wave propagation characteristics in the functionally graded plate. The results carried out can be used in the ultrasonic inspection techniques and structural health monitoring.

Keywords: wave propagation; functionally graded plate; thermal effects; efficient shear deformation theory; neutral surface position

1. Introduction

Functionally graded materials (FGMs) are new materials which are designed to achieve a functional performance with gradually variable properties in one or more directions (Koizumi

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1992). This continuity prevents the material from having disadvantages of composites such as delamination due to large interlaminar stresses, initiation and propagation of cracks because of large plastic deformation at the interfaces and so on. Typically, FGMs are made of a mixture of ceramics and a combination of different metals (Bennoun *et al.* 2016, Ebrahimi and Dashti 2015, Sallai *et al.* 2015, Meradjah *et al.* 2015, Kar and Panda 2015, Pradhan and Chakraverty 2015, Bakora and Tounsi 2015, Bouchafa *et al.* 2015, Arefi 2015, Akbaş 2015, Mansouri and Shariyat 2015, Belabed *et al.* 2014, Khalfi *et al.* 2014, Mansouri and Shariyat 2014, Hadji *et al.* 2014, Fekrar *et al.* 2014, Tounsi *et al.* 2013a, Bouderba *et al.* 2013, Bourada *et al.* 2012, Benachour *et al.* 2011). So the key point is an accurate description of the variables and the material properties in the thickness direction, to perform a satisfactory analysis of the mechanical behavior of FGM plates. Many works on FGM structures have been studied in literature. For example, Reddy (2000) has analyzed the static behavior of FGM rectangular plates based on his third-order shear deformation plate theory. Reddy and Cheng (2001) have presented a three-dimensional model for an FGM plate subjected to mechanical and thermal loads, both applied at the top of the plate. Vel and Batra (2004) have proposed a three-dimensional solution for free vibration of FGM rectangular plates. Zenkour (2006) presented a generalized shear deformation theory in which the in-plane displacements are expanded as sinusoidal types across the thickness. Woo *et al*. (2006) studied the non-linear free vibration behavior of plates made of FGMs using the Von Karman theory for large transverse deflection. Also, Park and Kim (2006) investigated the thermal postbuckling and vibration analyses of FG plates. Kim (2005) discussed the temperature dependent vibration analysis of FGM rectangular plates. Matsunaga (2008) studied natural frequencies and buckling stresses of FG simply supported rectangular plates based on 2D higher-order approximate plate theory (2D HAPT). Shahrjerdi *et al*. (2011) employed the second-order shear deformation theory to analyze vibration of temperature-dependent solar functionally graded plates. Arefi and Rahimi (2011) investigated the nonlinear response of a FG square plate with two smart layers as a sensor and actuator under pressure. Arefi (2013) analyzed the nonlinear thermo-elastic behavior of thick-walled functionally graded piezoelectric cylinder. Sobhy (2013) studied the vibration and buckling behavior of exponentially graded material sandwich plate resting on elastic foundations under various boundary conditions. The first-order shear deformation theory (FSDT), including the effects of transverse shear deformation, was employed by some researches to analyze buckling behavior of moderately thick FGM plates (Yaghoobi and Yaghoobi 2013, Bouazza *et al.* 2010). By using an efficient and simple refined theory, Ait Amar Meziane *et al*. (2014) studied the buckling and free vibration of exponentially graded sandwich plates under various boundary conditions. Hebali *et al.* (2014) proposed a new quasi-3D hyperbolic shear deformation theory for the static and free vibration analysis of FG plates. Bousahla *et al.* (2014) presented a novel higher order shear and normal deformation theory based on neutral surface position for bending analysis of advanced composite plates. Zidi *et al* (2014) employed a four variable refined plate theory for bending analysis of FG plates under hygro-thermo-mechanical loading. Yaghoobi *et al.* (2014) presented an analytical study on post-buckling and nonlinear free vibration analysis of FG beams resting on nonlinear elastic foundation under thermo-mechanical loading using VIM. Ait Yahia *et al.* (2015) studied the wave propagation in functionally graded plates with porosities using various higher-order shear deformation plate theories. Nguyen *et al*. (2015) proposed a refined higher-order shear deformation theory for bending, vibration and buckling analysis of FG sandwich plates. Bourada *et al*. (2015) discussed the bending and vibration responses of FG thick beams by proposing a novel simple shear and normal deformations theory. Mahi *et al*. (2015) developed a novel hyperbolic shear deformation model for static and dynamic analysis of isotropic, functionally graded, sandwich and laminated composite plates. Ait Atmane *et al*. (2015) used a variationally consistent shear deformation theory for dynamic behavior of thick FG beams with porosities. Attia *et al*. (2015) examined the dynamic response of FG plates with temperature-dependent properties by employing various four variable refined plate models. Larbi Chaht *et al.* (2015) studied the bending and buckling behaviors of FG size-dependent nanoscale beams including the thickness stretching effect. Bouguenina *et al*. (2015) presented a numerical analysis of FGM plates with variable thickness subjected to thermal buckling. Tagrara *et al.* (2015) investigated the bending, buckling and vibration responses of functionally graded carbon nanotube-reinforced composite beams. Belkorissat *et al*. (2015) studied the dynamic properties of FG nanoscale plates using a novel nonlocal refined four variable theory. Bennai *et al*. (2015) proposed a novel higher-order shear and normal deformation theory for FG sandwich beams. Tebboune *et al.* (2015) analyzed the thermal buckling behavior of FG plates resting on elastic foundation based on an efficient and simple trigonometric shear deformation theory. Hamidi *et al.* (2015) presented a sinusoidal plate theory with 5-unknowns and stretching effect for thermomechanical bending of FG sandwich plates. Bennoun *et al*. (2016) proposed a novel five variable refined plate theory for vibration analysis of FG sandwich plates. Ait Atmane *et al*. (2016) studied the effect of thickness stretching and porosity on mechanical response of a FG beams resting on elastic foundations.

The study of the wave propagation in the FGM structures has received also much attention from various researchers. Chen *et al*. (2007) studied the dispersion behavior of waves in a functionally graded plate with material properties varying along the thickness direction. Han and Liu (2002) investigated SH waves in FGM plates, where the material property variation was assumed to be a piecewise quadratic function in the thickness direction. Han *et al*. (2001) proposed an analytical-numerical method for analyzing the wave characteristics in FGM cylinders. Han *et al*. (2002) also proposed a numerical method to study the transient wave in FGM plates excited by impact loads. Sun and Luo (2011a) also studied the wave propagation and dynamic response of rectangular functionally graded material plates with completed clamped supports under impulsive load. Considering the thermal effects and temperature-dependent material properties, Sun and Luo (2011b) investigated the wave propagation of an infinite functionally graded plate using the higher-order shear deformation plate theory.

Among the aforementioned higher-order shear deformation theories (HSDTs), the Reddy’s theory is the most widely used due to its high efficiency and simplicity (Reddy 2000, Sun and Luo 2011b). Since the in-plane displacements of the Reddy’s theory are expanded as cubic function of the thickness coordinate, the equations of motion are more complicated than those of FSDT. Hence, there is a scope to develop an accurate theory, which is simple to use.

The purpose of this study is to develop a shear deformation plate theory for the wave propagation of an infinite functionally graded plate which is simple to use. The theory is based on assumption that the in-plane and transverse displacements consist of bending and shear components, in which the bending components do not contribute toward shear forces and, likewise, the shear components do not contribute toward bending moments. The most interesting feature of this theory is that it accounts for a quadratic variation of the transverse shear strains across the thickness and satisfies the zero traction boundary conditions on the top and bottom surfaces of the plate without using shear correction factors. In addition, it contains four unknowns and has strong similarities with the CPT in some aspects such as equations of motion, boundary conditions, and stress resultant expressions. To simplify the governing equations for the FGM plate, the coordinate system is located at the physical neutral surface of the plate. This is due to the fact that the stretching-bending coupling in the constitutive equations of an FGM plate does not exist when the physical neutral surface is considered as a coordinate system (Bellifa *et al.* 2016, Ould Larbi *et al.* 2013, Yahoobi and Feraidoon 2010). The governing equations of the wave propagation in the functionally graded plate are derived by using the Hamilton's principle, which the effects of shear deformation and the inertia rotation are taken into account. The dispersion, phase velocity and group velocity curves of the wave propagation in the functionally graded plate in thermal environments are plotted. The influences of the volume fraction index and temperature on the dispersion, phase velocity and group velocity of the wave propagation in the functionally graded plate are clearly discussed.

2. Properties of the FGM constituent materials and physical neutral surface

Consider a rectangular plate made of FGMs of thickness h. Since in functionally graded plates the condition of mid-plane symmetry does not exist, the stretching and bending equations are coupled. But, if the origin of the coordinate system is suitably selected in the thickness direction of the FGM plate so as to be the neutral surface, the analysis of the FGM plates can easily be treated with the homogenous isotropic plate theories, because the stretching and bending equations of the plate are not coupled. In order to determine the position of neutral surface of FGM plates, two different datum planes are considered for the measurement of z, namely, z_{ms} and z_{ns} measured from the middle surface and the neutral surface of the plate, respectively, as shown in Fig. 1. The volume fraction of ceramic V_C can be written in terms of z_{ms} and z_{ns} coordinates as

$$V_C = \left(\frac{z_{ms}}{h} + \frac{1}{2}\right)^n = \left(\frac{z_{ns} + C}{h} + \frac{1}{2}\right)^n \tag{1}$$

Material non-homogeneous properties of a functionally graded material plate may be obtained by means of the Voigt rule of mixture (Suresh and Mortensen 1998). Thus, using Eq. (1), the material non-homogeneous properties of FG plate P, as a function of thickness coordinate, become

$$P(z) = P_M + P_{CM}\left(\frac{z_{ns} + C}{h} + \frac{1}{2}\right)^n, \quad P_{CM} = P_C - P_M \tag{2}$$

in which P_M and P_C are the corresponding properties of the metal and ceramic and may be expressed as a function of temperature (Touloukian 1967)

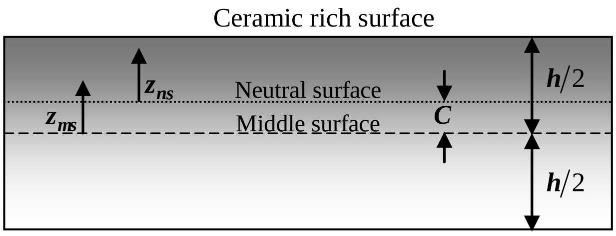

Fig. 1 The position of middle surface and neutral surface for a functionally graded plate

$$P = P_0\left(P_{-1}T^{-1} + 1 + P_1T + P_2T^2 + P_3T^3\right) \tag{3}$$

where P_0, P_{-1}, P_1, P_2 and P_3 are the coefficients showing the temperature-dependency in material properties and are unique to the constituent materials, T (in K) is the environment temperature. n is the material parameter which takes the value greater or equal to zero. Also, the parameter C is the distance of neutral surface from the middle surface. It is assumed that the effective Young's modulus E, Poisson's ratio v and thermal expansion coefficient α of an FGM plate are temperature -dependent, whereas the mass density ρ and thermal conductivity λ of an FGM plate are independent of the temperature (Sun and Luo, 2011b and 2012). The position of the neutral surface of the FGM plate is determined to satisfy the first moment with respect to Young's modulus being zero as follows (Ould Larbi *et al.* 2013)

$$\int_{-h/2}^{h/2} E(z_{ms})(z_{ms} - C)dz_{ms} = 0 \tag{4}$$

Consequently, the position of neutral surface can be obtained as

$$C = \frac{\int_{-h/2}^{h/2} E(z_{ms})z_{ms}dz_{ms}}{\int_{-h/2}^{h/2} E(z_{ms})dz_{ms}} \tag{5}$$

It is clear that the parameter C is zero for homogeneous isotropic plates, as expected. The temperature field assumed to be uniform over the plate surface but varying along the thickness direction due to heat conduction. In such a case, the temperature distribution along the thickness can be obtained by solving the steady-state heat transfer equation as

$$\frac{d}{dz_{ns}}\left(\lambda(z_{ns})\frac{dT}{dz_{ns}}\right) = 0\,, \tag{6}$$

with the boundary conditions $T(h/2-C)=T_C$ and $T(-h/2-C)=T_M$. Substituting Eq. (2) into Eq. (6) yields a second-order differential equation in terms of temperature which can be written as

$$-\frac{d^2T}{dr^2} + \frac{n\lambda_{CM}r^{n-1}}{\lambda_M + \lambda_{CM}r^n}\frac{dT}{dr} = 0 \tag{7a}$$

where

$$r = \left(\frac{z_{ns} + C}{h} + \frac{1}{2}\right) \tag{7b}$$

The differential Eq. (7a) can be easily solved by using the polynomial series. Thus, the temperature distribution across the plate thickness is obtained as

$$T(z_{ns}) = T_M + \Delta T\frac{\theta(z_{ns} + C)}{\eta} \tag{8}$$

where

$$\theta(z_{ns}) = \left[\left(\frac{z_{ns}+C}{h} + \frac{1}{2} \right) - \frac{(\lambda_C - \lambda_M)}{(n+1)\lambda_M} \left(\frac{z_{ns}+C}{h} + \frac{1}{2} \right)^{n+1} \right.$$
$$+ \frac{(\lambda_C - \lambda_M)^2}{(2n+1)\lambda_M^2} \left(\frac{z_{ns}+C}{h} + \frac{1}{2} \right)^{2n+1} - \frac{(\lambda_C - \lambda_M)^3}{(3n+1)\lambda_M^3} \left(\frac{z_{ns}+C}{h} + \frac{1}{2} \right)^{3n+1} \quad (9a)$$
$$\left. + \frac{(\lambda_C - \lambda_M)^4}{(4n+1)\lambda_M^4} \left(\frac{z_{ns}+C}{h} + \frac{1}{2} \right)^{4n+1} - \frac{(\lambda_C - \lambda_M)^5}{(5n+1)\lambda_M^5} \left(\frac{z_{ns}+C}{h} + \frac{1}{2} \right)^{5n+1} \right]$$

$$\eta = 1 - \frac{(\lambda_C - \lambda_M)}{(n+1)\lambda_M} + \frac{(\lambda_C - \lambda_M)^2}{(2n+1)\lambda_M^2} - \frac{(\lambda_C - \lambda_M)^3}{(3n+1)\lambda_M^3} + \frac{(\lambda_C - \lambda_M)^4}{(4n+1)\lambda_M^4} - \frac{(\lambda_C - \lambda_M)^5}{(5n+1)\lambda_M^5} \quad (9b)$$

3. Fundamental equations

It is noted that a two variable refined plate theory (RPT) using only two unknown functions was developed by Shimpi (2002) for isotropic plates, and was extended by Shimpi and Patel (2006ab) for orthotropic plates, by Kim *et al*. (2009) for laminated composite plates and Mechab *et al*. (2010) for FG plates. In this study, RPT is extended for wave propagation analysis of an infinite FG plate.

3.1 Basic assumptions

The assumptions of the present theory are as follows:

(i) The origin of the Cartesian coordinate system is taken at the neutral surface of the FGM plate.

(ii) The displacements are small in comparison with the plate thickness and, therefore, strains involved are infinitesimal.

(iii) The transverse displacement w includes two components of bending w_b and shear w_s. Both these components are functions of coordinates x, y, and time t only.

$$w(x, y, z_{ns}, t) = w_b(x, y, t) + w_s(x, y, t) \quad (10)$$

(iv) The displacements u in x-direction and v in y-direction consist of extension, bending, and shear components.

$$u = u_0 + u_b + u_s, \qquad v = v_0 + v_b + v_s \quad (11)$$

The bending components u_b and v_b are assumed to be similar to the displacements given by the classical plate theory. Therefore, the expression for u_b and v_b can be given as

$$u_b = -z_{ns} \frac{\partial w_b}{\partial x}, \qquad v_b = -z_{ns} \frac{\partial w_b}{\partial y} \quad (12)$$

The shear components u_s and v_s give rise, in conjunction with w_s, to the parabolic variations of shear strains γ_{xz}, γ_{yz} and hence to shear stresses τ_{xz}, τ_{yz} through the thickness of the plate in such a way that shear stresses τ_{xz}, τ_{yz} are zero at the top and bottom faces of the plate. Consequently, the expression for u_s and v_s can be given as

$$u_s = -f(z_{ns})\frac{\partial w_s}{\partial x}, \qquad v_s = -f(z_{ns})\frac{\partial w_s}{\partial y} \tag{13}$$

where the shape function proposed by Shimpi (2002) is modified based on the concept of the physical neutral surface

$$f(z_{ns}) = (z_{ns} + C)\left[-\frac{1}{4} + \frac{5}{3}\left(\frac{z_{ns} + C}{h}\right)^2\right] \tag{14}$$

3.2 Kinematics and constitutive equations

Based on the assumptions made in the preceding section, the displacement field can be obtained using Eqs. (10)-(14) as

$$u(x, y, z_{ns}, t) = u_0(x, y, t) - z_{ns}\frac{\partial w_b}{\partial x} + (z_{ns} + C)\left[\frac{1}{4} - \frac{5}{3}\left(\frac{z_{ns} + C}{h}\right)^2\right]\frac{\partial w_s}{\partial x} \tag{15a}$$

$$v(x, y, z_{ns}, t) = v_0(x, y, t) - z_{ns}\frac{\partial w_b}{\partial y} + (z_{ns} + C)\left[\frac{1}{4} - \frac{5}{3}\left(\frac{z_{ns} + C}{h}\right)^2\right]\frac{\partial w_s}{\partial y} \tag{15b}$$

$$w(x, y, z_{ns}, t) = w_b(x, y, t) + w_s(x, y, t) \tag{15c}$$

The kinematic relations can be obtained as follows

$$\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix} = \begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{Bmatrix} + z_{ns}\begin{Bmatrix}k_x^b\\ k_y^b\\ k_{xy}^b\end{Bmatrix} + f(z_{ns})\begin{Bmatrix}k_x^s\\ k_y^s\\ k_{xy}^s\end{Bmatrix}, \quad \begin{Bmatrix}\gamma_{yz}\\ \gamma_{xz}\end{Bmatrix} = g(z_{ns})\begin{Bmatrix}\gamma_{yz}^0\\ \gamma_{xz}^0\end{Bmatrix}, \tag{16}$$

where

$$\begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{Bmatrix} = \begin{Bmatrix}\dfrac{\partial u_0}{\partial x}\\ \dfrac{\partial v_0}{\partial x}\\ \dfrac{\partial u_0}{\partial y} + \dfrac{\partial v_0}{\partial x}\end{Bmatrix}, \quad \begin{Bmatrix}k_x^b\\ k_y^b\\ k_{xy}^b\end{Bmatrix} = \begin{Bmatrix}-\dfrac{\partial^2 w_b}{\partial x^2}\\ -\dfrac{\partial^2 w_b}{\partial y^2}\\ -2\dfrac{\partial^2 w_b}{\partial x \partial y}\end{Bmatrix}, \quad \begin{Bmatrix}k_x^s\\ k_y^s\\ k_{xy}^s\end{Bmatrix} = \begin{Bmatrix}-\dfrac{\partial^2 w_s}{\partial x^2}\\ -\dfrac{\partial^2 w_s}{\partial y^2}\\ -2\dfrac{\partial^2 w_s}{\partial x \partial y}\end{Bmatrix}, \quad \begin{Bmatrix}\gamma_{yz}^0\\ \gamma_{xz}^0\end{Bmatrix} = \begin{Bmatrix}\dfrac{\partial w_s}{\partial y}\\ \dfrac{\partial w_s}{\partial x}\end{Bmatrix}, \tag{17a}$$

and

$$g(z_{ns}) = 1 - \frac{df(z_{ns})}{dz_{ns}} \tag{17b}$$

For elastic and isotropic FGMs, the constitutive relations can be written as

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x - \alpha\,\Delta T \\ \varepsilon_y - \alpha\,\Delta T \\ \gamma_{xy} \end{Bmatrix} \quad \text{and} \quad \begin{Bmatrix} \tau_{yz} \\ \tau_{zx} \end{Bmatrix} = \begin{bmatrix} Q_{44} & 0 \\ 0 & Q_{55} \end{bmatrix} \begin{Bmatrix} \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix} \tag{18}$$

where (σ_x, σ_y, τ_{xy}, τ_{yy}, τ_{yx}) and (ε_x, ε_y, γ_{xy}, γ_{yz}, γ_{yx}) are the stress and strain components, respectively. Using the material properties defined in Eq. (2), stiffness coefficients, Q_{ij}, can be expressed as

$$Q_{11} = Q_{22} = \frac{E(z_{ns}, T)}{1 - \nu(z_{ns}, T)^2}, \tag{19a}$$

$$Q_{12} = \frac{\nu\, E(z_{ns}, T)}{1 - \nu(z_{ns}, T)^2}, \tag{19b}$$

$$Q_{44} = Q_{55} = Q_{66} = \frac{E(z_{ns}, T)}{2[1 + \nu(z_{ns}, T)]}, \tag{19c}$$

3.3 Governing equations

Hamilton's principle is used herein to derive the equations of motion. The principle can be stated in analytical form as (Reddy 2002)

$$0 = \int_0^t (\delta U - \delta K)\, dt \tag{20}$$

where δU is the variation of strain energy; and δK is the variation of kinetic energy.
The variation of strain energy of the plate stated as

$$\begin{aligned} \delta U &= \int_V \left[\sigma_x\, \delta\varepsilon_x + \sigma_y\, \delta\varepsilon_y + \tau_{xy}\, \delta\gamma_{xy} + \tau_{yz}\, \delta\gamma_{yz} + \tau_{zx}\, \delta\gamma_{zx} \right] dA dz_{ns} \\ &= \int_A \left[N_x\, \delta\varepsilon_x^0 + N_y\, \delta\varepsilon_y^0 + N_{xy}\, \delta\varepsilon_{xy}^0 + M_x^b\, \delta k_x^b + M_y^b\, \delta k_y^b + M_{xy}^b\, \delta k_{xy}^b + M_x^s\, \delta k_x^s \right. \\ &\quad \left. + M_y^s\, \delta k_y^s + M_{xy}^s\, \delta k_{xy}^s + S_{yz}^s\, \delta\gamma_{yz}^s + S_{xz}^s\, \delta\gamma_{xz}^s \right] dA \end{aligned} \tag{21}$$

where the stress resultants N, M, and S are defined by

$$\left(N_i, M_i^b, M_i^s\right) = \int_{-\frac{h}{2}-C}^{\frac{h}{2}-C} (1, z_{ns}, f)\, \sigma_i\, dz_{ns}, \quad (i = x, y, xy) \quad \text{and} \quad S_i = \int_{-\frac{h}{2}-C}^{\frac{h}{2}-C} g\, \sigma_i\, dz_{ns}, \quad (i = xz, yz) \tag{22}$$

The variation of kinetic energy of the plate is expressed as

$$
\begin{aligned}
\delta K &= \int_{-\frac{h}{2}-C}^{\frac{h}{2}-C} \int_A \left[\dot{u}\delta\,\dot{u} + \dot{v}\delta\,\dot{v} + \dot{w}\delta\,\dot{w}\right] \rho(z_{ns})\, dA\, dz_{ns} \\
&= \int_A \Big\{ I_0 \left[\dot{u}_0 \delta \dot{u}_0 + \dot{v}_0 \delta \dot{v}_0 + \left(\dot{w}_b + \dot{w}_s\right)\left(\delta \dot{w}_b + \delta \dot{w}_s\right)\right] \\
&\quad + I_2 \left(\frac{\partial \dot{w}_b}{\partial x} \frac{\partial \delta\,\dot{w}_b}{\partial x} + \frac{\partial \dot{w}_b}{\partial y} \frac{\partial \delta\,\dot{w}_b}{\partial y} \right) + \frac{I_2}{84} \left(\frac{\partial \dot{w}_s}{\partial x} \frac{\partial \delta\,\dot{w}_s}{\partial x} + \frac{\partial \dot{w}_s}{\partial y} \frac{\partial \delta\,\dot{w}_s}{\partial y} \right) \Big\} dA
\end{aligned}
\tag{23}
$$

where dot-superscript convention indicates the differentiation with respect to the time variable t; and (I_0, I_2) are mass inertias defined as

$$
\{I_0, I_2\} = \int_{-\frac{h}{2}-C}^{\frac{h}{2}-C} \left\{1, (z_{ns} + C)^2\right\} \rho(z_{ns})\, dz_{ns} \tag{24}
$$

Substituting the expressions for δU and δK from Eqs. (21) and (23) into Eq. (20) and integrating by parts, and collecting the coefficients of δu_0, δv_0, δw_b and δw_s, the following equations of motion of the plate are obtained

$$
\begin{aligned}
&\delta\, u: \quad \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = I_0 \ddot{u} \\
&\delta\, v: \quad \frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = I_0 \ddot{v} \\
&\delta\, w_b: \quad \frac{\partial^2 M_x^b}{\partial x^2} + 2\frac{\partial^2 M_{xy}^b}{\partial x \partial y} + \frac{\partial^2 M_y^b}{\partial y^2} = I_0(\ddot{w}_b + \ddot{w}_s) - I_2 \left(\frac{\partial^2 \ddot{w}_b}{\partial x^2} + \frac{\partial^2 \ddot{w}_b}{\partial y^2} \right) \\
&\delta\, w_s: \quad \frac{\partial^2 M_x^s}{\partial x^2} + 2\frac{\partial^2 M_{xy}^s}{\partial x \partial y} + \frac{\partial^2 M_y^s}{\partial y^2} + \frac{\partial S_{xz}^s}{\partial x} + \frac{\partial S_{yz}^s}{\partial y} = I_0(\ddot{w}_b + \ddot{w}_s) - \frac{I_2}{84} \left(\frac{\partial^2 \ddot{w}_s}{\partial x^2} + \frac{\partial^2 \ddot{w}_s}{\partial y^2} \right)
\end{aligned}
\tag{25}
$$

By substituting Eq. (16) into Eq. (18) and the subsequent results into Eq. (22), the stress resultants are obtained as

$$
\begin{Bmatrix} N \\ M^b \\ M^s \end{Bmatrix} = \begin{bmatrix} A & 0 & B^s \\ 0 & D & D^s \\ B^s & D^s & H^s \end{bmatrix} \begin{Bmatrix} \varepsilon \\ k^b \\ k^s \end{Bmatrix} - \begin{Bmatrix} N^T \\ M^{bT} \\ M^{sT} \end{Bmatrix}, \quad S = A^s \gamma, \tag{26}
$$

where

$$
N = \left\{N_x, N_y, N_{xy}\right\}^t, \qquad M^b = \left\{M_x^b, M_y^b, M_{xy}^b\right\}^t, \qquad M^s = \left\{M_x^s, M_y^s, M_{xy}^s\right\}^t, \tag{26a}
$$

$$N^T = \left\{N_x^T, N_y^T, 0\right\}^t, \quad M^{bT} = \left\{M_x^{bT}, M_y^{bT}, 0\right\}^t, \quad M^{sT} = \left\{M_x^{sT}, M_y^{sT}, 0\right\}^t, \tag{26b}$$

$$\varepsilon = \left\{\varepsilon_x^0, \varepsilon_y^0, \gamma_{xy}^0\right\}^t, \quad k^b = \left\{k_x^b, k_y^b, k_{xy}^b\right\}^t, \quad k^s = \left\{k_x^s, k_y^s, k_{xy}^s\right\}^t, \tag{26c}$$

$$A = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix}, \quad D = \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{12} & D_{22} & 0 \\ 0 & 0 & D_{66} \end{bmatrix}, \quad B^s = \begin{bmatrix} B_{11}^s & B_{12}^s & 0 \\ B_{12}^s & B_{22}^s & 0 \\ 0 & 0 & B_{66}^s \end{bmatrix} \tag{26d}$$

$$D^s = \begin{bmatrix} D_{11}^s & D_{12}^s & 0 \\ D_{12}^s & D_{22}^s & 0 \\ 0 & 0 & D_{66}^s \end{bmatrix}, \quad H^s = \begin{bmatrix} H_{11}^s & H_{12}^s & 0 \\ H_{12}^s & H_{22}^s & 0 \\ 0 & 0 & H_{66}^s \end{bmatrix}, \tag{26e}$$

$$S = \left\{S_{xz}^s, S_{yz}^s\right\}^t, \quad \gamma = \left\{\gamma_{xz}, \gamma_{yz}\right\}^t, \quad A^s = \begin{bmatrix} A_{44}^s & 0 \\ 0 & A_{55}^s \end{bmatrix}, \tag{26f}$$

where A_{ij}, D_{ij}, etc., are the plate stiffness, defined by

$$\begin{aligned}
&\left\{A_{ij}, D_{ij}, E_{ij}, F_{ij}, H_{ij}\right\} = \int_{-\frac{h}{2}-C}^{\frac{h}{2}-C} \left\{1, (z_{ns}+C)^2, (z_{ns}+C)^3, (z_{ns}+C)^4, (z_{ns}+C)^6\right\} Q_{ij}\, dz_{ns} \quad (i, j = 1,2,6) \\
&B_{ij}^s = \frac{5}{3h^2} E_{ij} \quad (i, j = 1,2,6) \\
&D_{ij}^s = -\frac{1}{4} D_{ij} + \frac{5}{3h^2} F_{ij} \quad (i, j = 1,2,6) \\
&H_{ij}^s = \frac{1}{16} D_{ij} - \frac{5}{6h^2} F_{ij} + \frac{25}{9h^4} H_{ij} \quad (i, j = 1,2,6) \\
&\left\{A_{ij}, D_{ij}, F_{ij}\right\} = \int_{-\frac{h}{2}-C}^{\frac{h}{2}-C} \left\{1, (z_{ns}+C)^2, (z_{ns}+C)^4\right\} Q_{ij}\, dz_{ns} \quad (i, j = 4,5) \\
&A_{ij}^s = \frac{25}{16} A_{ij} - \frac{25}{2h^2} D_{ij} + \frac{25}{h^4} F_{ij} \quad (i, j = 4,5)
\end{aligned} \tag{27}$$

Substituting from Eq. (26) into Eq. (25), we obtain the following equation

$$A_{11} \frac{\partial^2 u_0}{\partial x^2} + A_{66} \frac{\partial^2 u_0}{\partial y^2} + \left(A_{12} + A_{66}\right) \frac{\partial^2 v_0}{\partial x \partial y} - B_{11}^s \frac{\partial^3 w_s}{\partial x^3} - \left(B_{12}^s + 2B_{66}^s\right) \frac{\partial^3 w_s}{\partial x \partial y^2} = I_0 \ddot{u}_0 \tag{28a}$$

$$\left(A_{12} + A_{66}\right) \frac{\partial^2 u_0}{\partial x \partial y} + A_{66} \frac{\partial^2 v_0}{\partial x^2} + A_{22} \frac{\partial^2 v_0}{\partial y^2} - B_{22}^s \frac{\partial^3 w_s}{\partial y^3} - \left(B_{12}^s + 2B_{66}^s\right) \frac{\partial^3 w_s}{\partial x^2 \partial y} = I_0 \ddot{v}_0 \tag{28b}$$

$$-D_{11}\frac{\partial^4 w_b}{\partial x^4}-2\left(D_{12}+2D_{66}\right)\frac{\partial^4 w_b}{\partial x^2\partial y^2}-D_{22}\frac{\partial^4 w_b}{\partial y^4}-D_{11}^s\frac{\partial^4 w_s}{\partial x^4}-2\left(D_{12}^s+2D_{66}^s\right)\frac{\partial^4 w_s}{\partial x^2\partial y^2}$$
$$-D_{22}^s\frac{\partial^4 w_s}{\partial y^4}=I_0(\ddot{w}_b+\ddot{w}_s)-I_2\nabla^2\ddot{w}_b \tag{28c}$$

$$B_{11}^s\frac{\partial^3 u_0}{\partial x^3}+\left(B_{12}^s+2B_{66}^s\right)\frac{\partial^3 u_0}{\partial x\partial y^2}+\left(B_{12}^s+2B_{66}^s\right)\frac{\partial^3 v_0}{\partial x^2\partial y}+B_{22}^s\frac{\partial^3 v_0}{\partial y^3}-D_{11}^s\frac{\partial^4 w_b}{\partial x^4}-2\left(D_{12}^s+2D_{66}^s\right)\frac{\partial^4 w_b}{\partial x^2\partial y^2}$$
$$-D_{22}^s\frac{\partial^4 w_b}{\partial y^4}-H_{11}^s\frac{\partial^4 w_s}{\partial x^4}-2\left(H_{12}^s+2H_{66}^s\right)\frac{\partial^4 w_s}{\partial x^2\partial y^2}-H_{22}^s\frac{\partial^4 w_s}{\partial y^4}$$
$$+A_{55}^s\frac{\partial^2 w_s}{\partial x^2}+A_{44}^s\frac{\partial^2 w_s}{\partial y^2}=I_0(\ddot{w}_b+\ddot{w}_s)-\frac{I_2}{84}\nabla^2\ddot{w}_b \tag{28d}$$

The Eqs. (28) are the governing equations of the FGM plate in thermal environments in terms of the displacements.

4. Dispersion relations

We assume solutions for u_0, v_0, w_b and w_s representing propagating waves in the x-y plane with the form

$$\begin{Bmatrix} u_0(x,y,t) \\ v_0(x,y,t) \\ w_b(x,y,t) \\ w_s(x,y,t) \end{Bmatrix}=\begin{Bmatrix} U\exp\left[i(k_1x+k_2y-\omega t\right] \\ V\exp\left[i(k_1x+k_2y-\omega t\right] \\ W_b\exp\left[i(k_1x+k_2y-\omega t\right] \\ W_s\exp\left[i(k_1x+k_2y-\omega t\right] \end{Bmatrix} \tag{29}$$

where U; V; W_b and W_s are the coefficients of the wave amplitude, k_1 and k_2 are the wave numbers of wave propagation along x-axis and y-axis directions respectively, ω is the frequency.
Substituting Eq. (29) into Eq. (28), we obtain

$$\left(\left[K\right]-\omega^2\left[M\right]\right)\left\{\Delta\right\}=\left\{0\right\} \tag{30}$$

where

$$\left\{\Delta\right\}=\left\{U,V,W_b,W_s\right\}^T, \tag{31a}$$

$$\left[K\right]=\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix},\quad \left[M\right]=\begin{bmatrix} m_{11} & 0 & m_{13} & m_{14} \\ 0 & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{bmatrix} \tag{31b}$$

in which

$$
\begin{aligned}
a_{11} &= -\left(A_{11}k_1^2 + A_{66}k_2^2\right)\\
a_{12} &= -k_1\,k_2\left(A_{12}+A_{66}\right)\\
a_{13} &= 0\\
a_{14} &= i\,k_1\,k_2^2\,B_{12}^s + 2\,i\,k_1\,k_2^2\,B_{66}^s + i\,B_{11}^s\,k_1^3\\
a_{41} &= -i\,k_1\,k_2^2\,B_{12}^s - 2\,i\,k_1\,k_2^2\,B_{66}^s - i\,B_{11}^s\,k_1^3\\
a_{22} &= -\left(A_{22}k_2^2 + A_{66}k_1^2\right)\\
a_{23} &= 0\\
a_{24} &= i\,k_1^2\,k_2\,B_{12}^s + 2\,i\,k_1^2\,k_2\,B_{66}^s + i\,B_{22}^s\,k_2^3\\
a_{33} &= -\left(2k_1^2k_2^2D_{12} + 4k_1^2k_2^2D_{66} + D_{11}k_1^4 + D_{22}k_2^4\right)\\
a_{34} &= -\left(2k_1^2k_2^2D_{12}^s + 4k_1^2k_2^2D_{66}^s + D_{11}^s k_1^4 + D_{22}^s k_2^4\right)\\
a_{44} &= -\left(H_{11}^s k_1^4 + 2(H_{12}^s + 2H_{66}^s)k_1^2k_2^2 + H_{22}^s k_2^4 + A_{55}^s k_1^2 + A_{44}^s k_2^2\right)\\
m_{11} &= m_{22} = m_{34} = m_{43} = -I_0\\
m_{33} &= -I_0 - I_2\left(k_1^2+k_2^2\right)\\
m_{44} &= -I_0 - \frac{I_2}{84}\left(k_1^2+k_2^2\right)
\end{aligned}
\qquad (31c)
$$

The dispersion relations of wave propagation in the functionally graded plate are given by

$$\left|\,[K]-\omega^2[M]\,\right| = 0 \qquad (32)$$

Assuming k_1=k_2=k, the roots of Eq. (32) can be expressed as

$$\omega_1 = W_1(k),\quad \omega_2 = W_2(k),\quad \omega_3 = W_3(k)\ \text{ and }\ \omega_4 = W_4(k) \qquad (33)$$

They correspond with the wave modes M_0, M_1, M_2 and M_3, respectively. The wave modes M_0 and M_3 correspond to the flexural wave, the wave modes M_1 and M_2 correspond to the extensional wave.

The phase velocity of wave propagation in the functionally graded plate can be expressed as

$$C_i = \frac{W_i(k)}{k},\quad (i = 1,2,3,4) \qquad (34)$$

In Eq. (31c), the element of matrix is containing real and imaginary parts and consequently, the solution of characteristic equation for calculation of phase velocity yields to imaginary and real phase velocity. It is noted here, that the presented results are devoted to real part of phase velocity and the imaginary part is a measure of attenuation.

5. Numerical results and discussion

In this section, the eigenvalues problem for a Si_3N_4/SUS304 functionally graded material plate is considered. The thickness of the functionally graded plate is 0.02 m. The Young's modulus E, density ρ, Poisson's ratio v and thermal expansion coefficient α of these materials are listed in

Table 1 Temperature-dependent coefficients for ceramics and metals

Materials	Proprieties	P_{-1}	P_0	P_1	P_2	P_3
Si_3N_4	E (Pa)	0	348.43e+9	–3.070e–4	2.160 e–7	–8.946 e–11
	υ	0	0.24	0	0	0
	ρ (kg/m^3)	0	2370	0	0	0
	α (1/K)	0	5.8723e–6	9.095e–4	0	0
SUS304	E (Pa)	0	201.04e+9	3.079e–4	–6.534 e–7	0
	υ	0	0.3262	–2.002 e–4	3.797e–7	0
	ρ (kg/m^3)	0	8166	0	0	0
	α (1/K)	0	12.330e–6	8.086 e–4	0	0

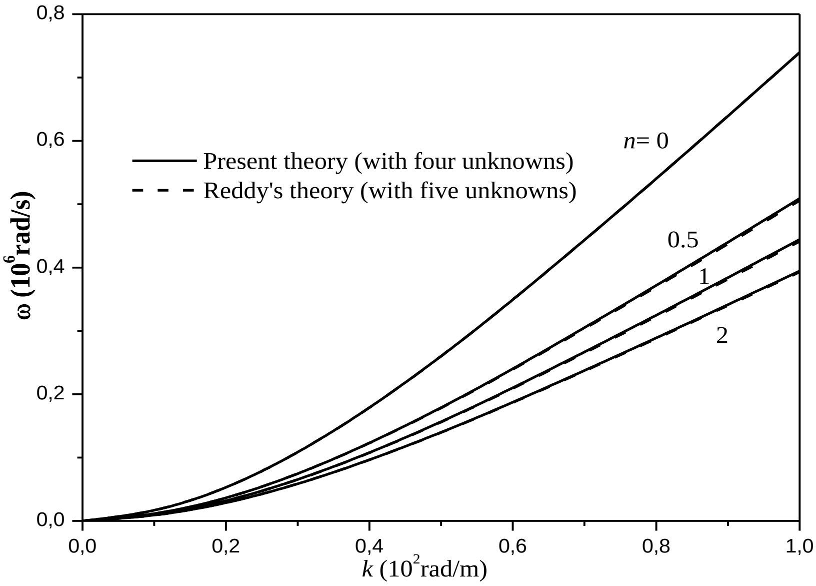

Fig. 2 The dispersion curves of the different functionally graded plates for flexural wave mode M_0 (T_M=T_C=300 K)

Table 1, which are taken from reference (Yang and Shen 2002, Reddy and Chin 1998).

The accuracy of the present neutral surface-based model involving only four unknown displacement functions is verified by comparing the obtained results with those computed using Reddy's theory (Sun and Luo 2011b). Figs. 2 and 3 show, respectively, the dispersion curves and the phase velocity for flexural wave mode M_0 of the different functionally graded plates under thermal environmental condition T_M=T_C=300 K. It can be seen that the results of the present neutral surface-based model (with only four unknown displacement functions) are in excellent agreement with those of Reddy's theory (with five unknown displacement functions) for all values of power law index$n$. This indicates that the partition of the transverse displacement into the bending and shear parts lead not only to accurate results, but it can improve the computational cost due to reducing the number of unknowns as well as governing equations of the wave propagation in the functionally graded plate.

The dispersion curves of functionally graded plate (with T_M=T_C=300 K) are shown in Fig. 4 for different wave modes (M_0, M_1, M_2 and M_3) and different values of power law index n. From these results, it can be concluded that the dispersion curves of the functionally graded plates are

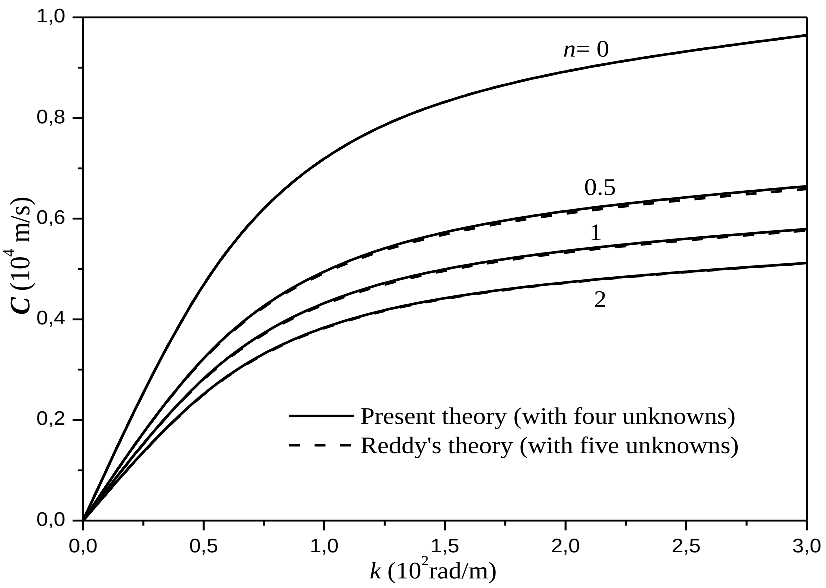

Fig. 3 The phase velocity curves of the different functionally graded plates for flexural wave mode M_0 (T_M=T_C=300 K)

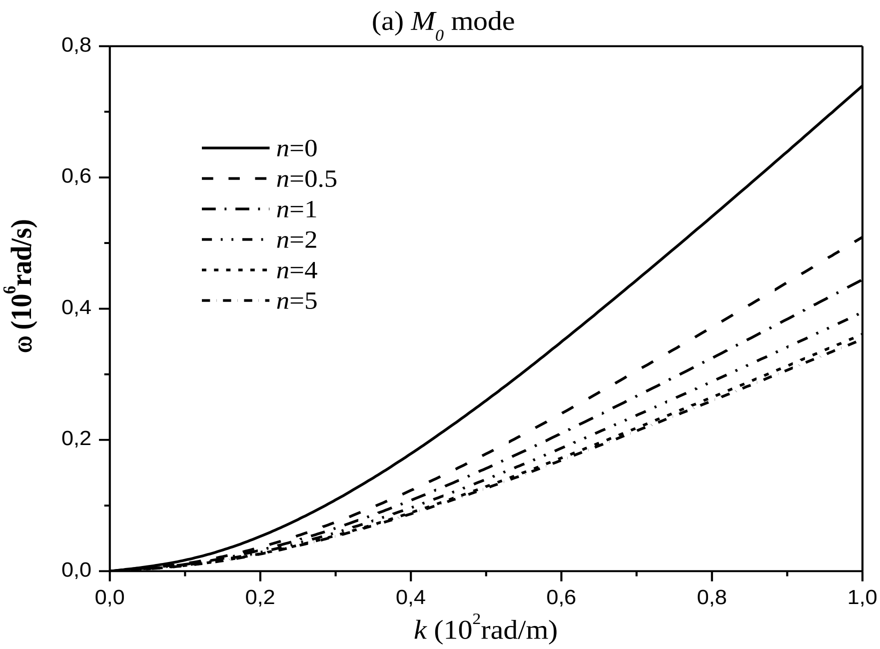

Fig. 4 The dispersion curves of the different functionally graded plates (T_M=T_C=300 K)

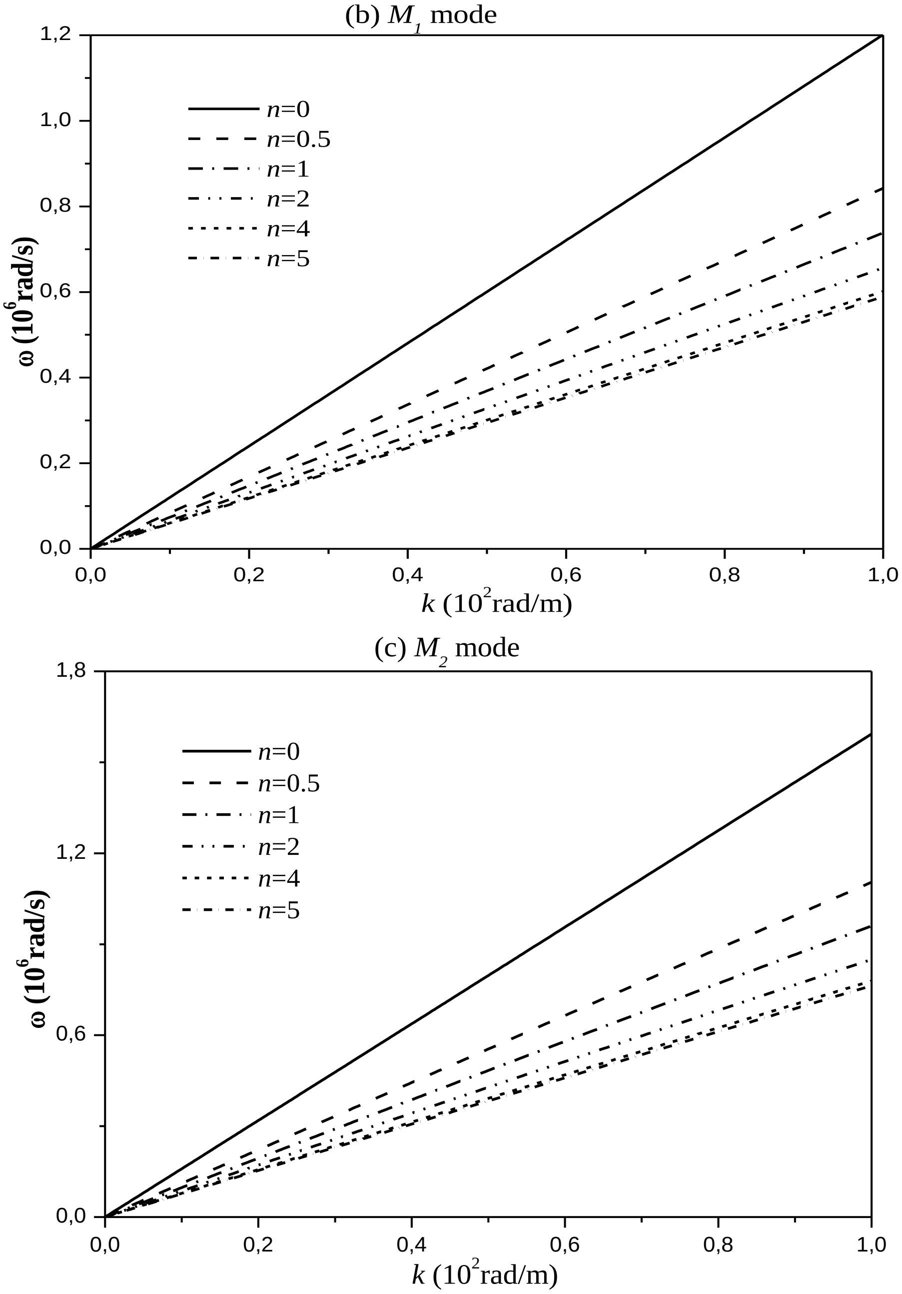

Fig. 4 Continued

considerably influenced by the power law index. Indeed, it can be seen that for the same wave number *k*, the frequency of the wave propagation in the functionally graded plate is decreased with increasing the power law index *n*, and the frequency of the wave propagation in the homogeneous plate (*n*=0) is the maximum among those of all functionally graded plates. Consequently, the

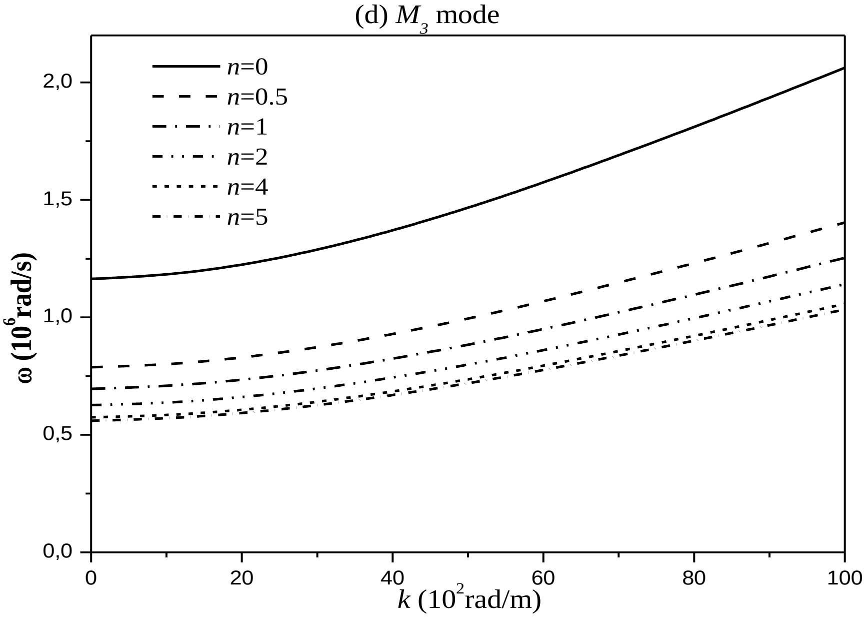

Fig. 4 Continued

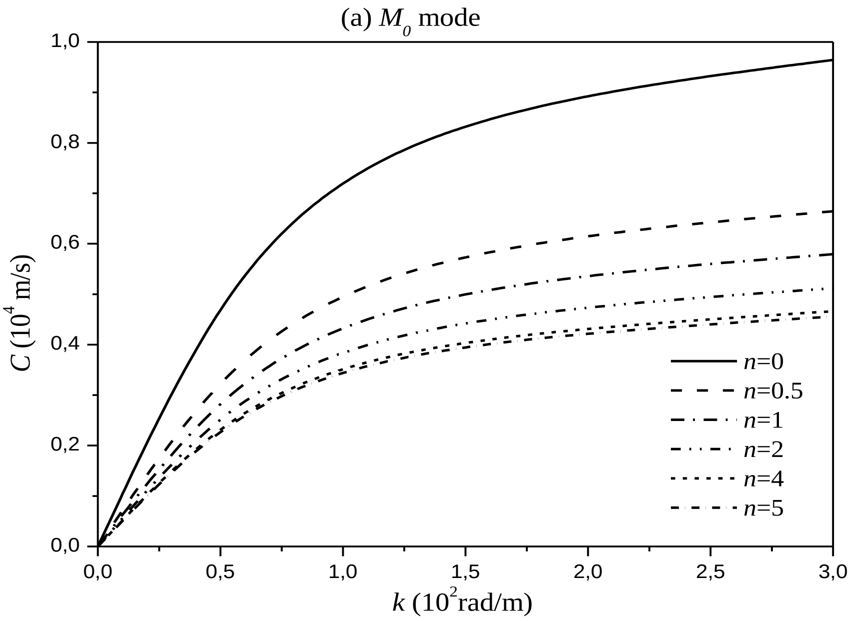

Fig. 5 The phase velocity curves of the different functionally graded plates (T_M=T_C=300 *K*)

increase of the power law index makes a plate flexible, and hence, leads to a reduction of frequency. Furthermore, it is observed that the frequency of the wave propagation for the extensional wave modes M_1 and M_2 of the functionally graded plates, vary linearly with the wave number *k*.

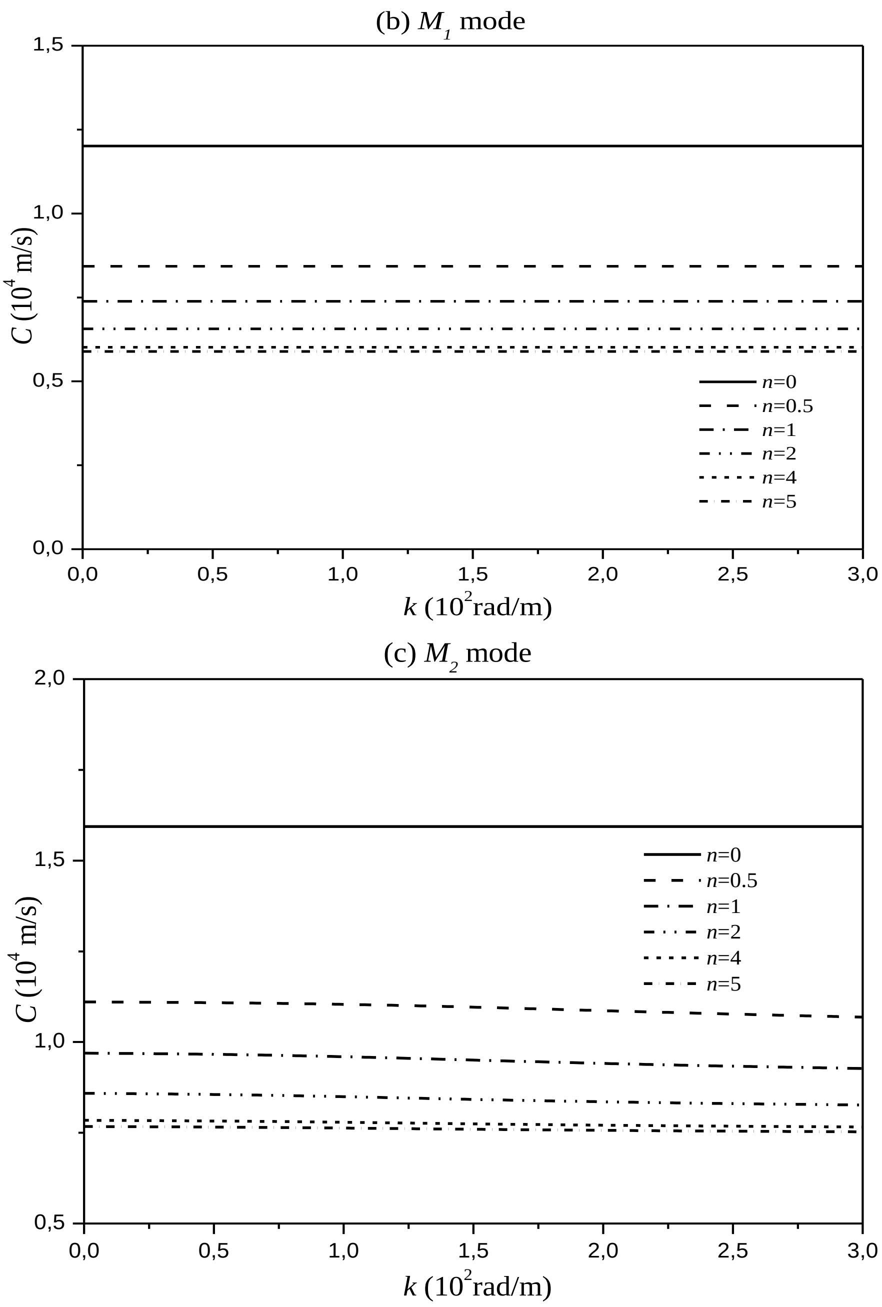

Fig. 5 Continued

The phase velocity curves of the different functionally graded plates under thermal environmental condition T_M=T_C=300 K are illustrated in Fig. 5. It is observed that the decrease of the power law index n leads to the decrease of the phase velocity and group velocity of the wave propagation in the functionally graded plate for the same wave number k. In addition, it can be

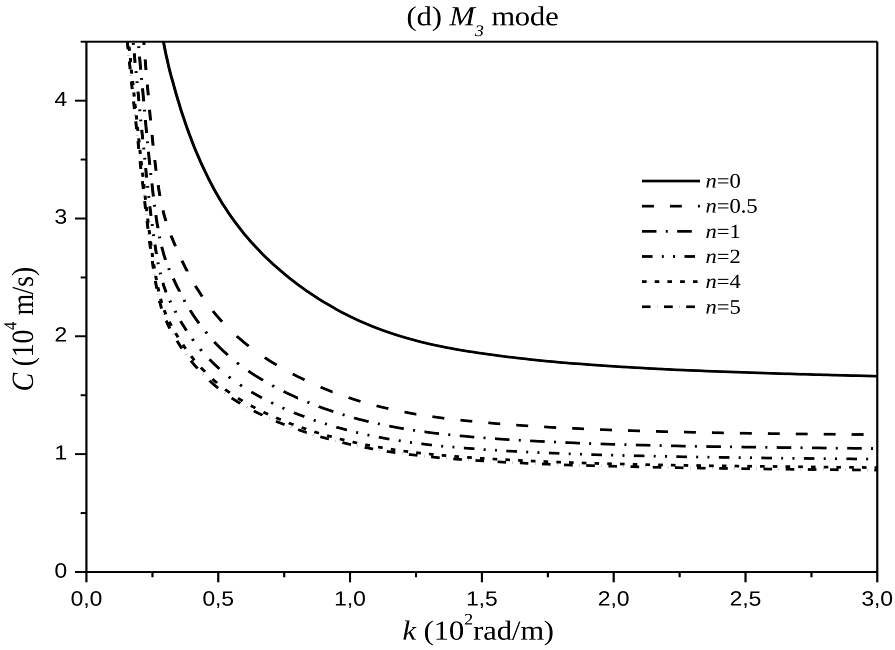

Fig. 5 Continued

seen that the phase velocity for the extensional wave modes M_1 and M_2 of the functionally graded plates is almost a constant. The phase velocity of the wave propagation in the homogeneous plate (*n*=0) is the maximum among those of all FG plates. This is expected because the ceramic plate (*n*=0) is the one with the highest stiffness. So, it is clear that the heterogeneity of FGMs has great influence on the phase velocity of the wave propagation in the FG plate.

6. Conclusions

The wave propagation of an infinite functionally graded plate in thermal environment load is studied based on an efficient shear deformation theory. The proposed theory has an advantage over the existing higher-order shear deformation theories since they involve less unknowns as well as equations of motion. The computational cost can therefore be reduced. In addition, the partition of the transverse displacement of the proposed theory into the bending and shear parts helps one to see the contributions due to shear and bending to the total one. Material properties are assumed to be temperature-dependent, and graded in the thickness direction according to a simple power law distribution in terms of the volume fractions of the constituents. The analytic dispersion relation of the functionally graded plate is obtained by solving an eigenvalue problem. Finally, it can be said that the proposed higher order shear and normal deformation theory is not only accurate but also provides an elegant and easily implementable approach for simulating the characteristics of wave propagation of the functionally graded plate. The formulation lends itself particularly well to composite structures (Kirkland and Uy 2015, Ozturk 2015, Darılmaz 2015, Chattibi *et al*. 2015, Sadoune *et al*. 2014, Draiche *et al*. 2014), micro/nano-structures (Bounouara *et al*. 2016, Al-Basyouni *et al*. 2015, Besseghier *et al*. 2015, Chemi *et al*. 2015, Zemri *et al*. 2015, Tounsi *et al*. 2013b), finite element simulations (Curiel Sosa *et al*. 2013) and also other numerical methods employing symbolic computation for plate bending problems (Rashidi *et al*. 2012), which will be considered in the near future.

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