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Correlation between chloride-induced corrosion initiation and time to cover cracking in RC Structures

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Abstract. Numerical value of correlation between effective parameters in the strength of a structure is as important as its stochastic properties in determining the safety of the structure. In this article investigation is made about the variation of coefficient of correlation between effective parameters in corrosion initiation time of reinforcement and the time of concrete cover cracking in reinforced concrete (RC) structures. Presence of many parameters and also error in measurement of these parameters results in uncertainty in determination of corrosion initiation and the time to crack initiation. In this paper, assuming diffusion process as chloride ingress mechanism in RC structures and considering random properties of effective parameters in this model, correlation between input parameters and predicted time to corrosion is calculated using the Monte Carlo (MC) random sampling. Results show the linear correlation between corrosion initiation time and effective input parameters increases with increasing uncertainty in the input parameters. Diffusion coefficient, concrete cover, surface chloride concentration and threshold chloride concentration have the highest correlation coefficient respectively. Also the uncertainty in the concrete cover has the greatest impact on the coefficient of correlation of corrosion initiation time and the time of crack initiation due to the corrosion phenomenon

Keywords: corrosion initiation time; time to cracking; correlation coefficient; lifetime; structural safety

1. Introduction

Calculation of structural safety and also prediction of residual lifetime of them, based upon their situation, is of great importance. For this purpose the residual strength of the structure is calculated, considering the environmental degrading effects and using reliability methods, the probability of right functioning of the structure according to the limit-state conditions, is calculated. In all the analytical and or numerical methods for calculation of reliability, consideration of stochastic properties of the parameters and also correlation between these parameters is of great importance. For example in Hasofer-Lind (HL) method in which the factor

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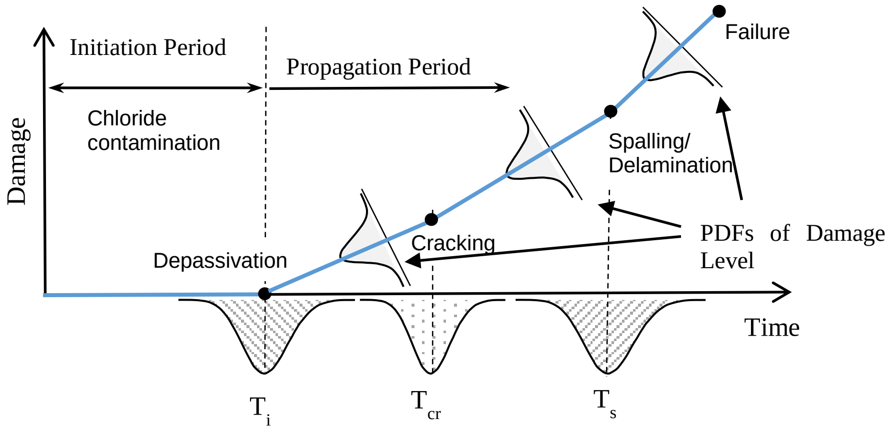


Fig. 1 Main events related to the service life of concrete structures exposed to chloride ion

of safety for the structure is calculated on the basis of reliability index: the first step is to transfer the variables to the standard normal space (Hasofer and Lind 1974). Transfer to the standard normal space should be accomplished in such a way that no correlation exist between the variables, for this purpose the NATAF and Rosenblatt transformations are used to transfer the correlated variables to the non-correlated standard normal space (DerKiureghian and Liu 1986, Nataf 1962). In order to obtain the correlation of effective parameters in the limit-state function of a structure, first all the effective factors in the strength of the limit-state function should be investigated. In a concrete structure which is under destructive environmental factors, such as Chloride ion, the damage propagation model will appear schematically like shown in Fig. 1. As shown in Fig. 1, the lifetime of the structure consists of initiation and propagation phases. The initiation period is a period during which chloride ingress occurs into the concrete cover until its concentration on the reinforcement bar reaches the critical value. The second phase starts from the corrosion initiation of the bar and includes reduction in reinforcement cross-sectional area, occurrence of the surface cracks and peeling of the concrete cover until, eventually, the structure loses its functionality. Corrosion-induced cover cracking is an important criterion for evaluating the service life of RC structures. When these cracks occur on the concrete cover, a path for quicker ingress of aggressive elements to the steel bars may be provided, and the corrosion-induced deterioration processes will be accelerated (Lu *et al.* 2011). Therefore it is widely accepted that the time to corrosion induced crack initiation (T_{cr}), is identified as serviceability limit state of RC structures (Vu *et al.* 2005, Stewart and Mullard 2007).

Initiation of rebar corrosion itself does not necessarily represent an undesirable state but in many service life approaches, initiation is taken as an indicator of the need to carry out maintenance. For presentation of a lifetime model for a structure and calculations of structural reliability, considering the correlation between events due to deterioration processes is of great importance.

Existence of various models for the prediction of the time to corrosion-induced crack initiation and uncertainty in evaluation of effective parameters in these models, cause increase in uncertainty of the time to crack initiation. A lot of studies have been made on the impact of effective parameters within the diffusion model on the corrosion initiation time and structural safety (Lu *et* *al.* 2011, Moodi *et al.* 2014). Enright and Frangopol (1998) have investigated the uncertainty effect in the calculation of corrosion initiation time. According to their calculations, the increase in coefficient of variation of input parameters in diffusion model, both extends the mean value of corrosion initiation time and uncertainty related to it. But they have not presented any analysis on the correlation between these parameters. Bhargava *et al.* (2011) assuming no correlation between the parameters have investigated the impact of uncertainties present in the diffusion model and the corrosion initiation time, on the probability of failure of concrete structures. Li *et al.* (2003) considering spatial variability have calculated reliability of reinforced concrete buildings for the limit-state corrosion initiation and appearance of surface cracks and have neglected the correlation between T_i and T_{cr}. Although many studies have been made on the impact of several factors on the corrosion initiation time and the time of surface cracks initiation due to the corrosion, but up to now, there have been no investigations on the correlation between these two times and its impact on the probability of non-efficiency of the structure.

In this paper, corrosion initiation time has been calculated by using diffusion model for different scenarios, and then by calculation of time to crack initiation implementing experimental and analytical models, the correlation between these two times has been calculated. In this research, the coefficient of correlation ($\rho_{Ti,Tcr}$), is used to state the relationship between variables and by implementing this coefficient, the correlation between predicted time for corrosion initiation and parameters of diffusion model, also the correlation between the time of corrosion initiation and the time to crack initiation, are calculated. In calculation of this coefficient, the impact of uncertainties present in the parameters is considered. In this article to study the impact of uncertainties present in the parameters, the effective parameters in the model are taken as random variables. In calculation of results, the Monte Carlo random sampling method is implemented.

2. Corrosion initiation time

When a concrete structure is exposed to chloride ions, the ions gradually penetrated into the concrete cover. Penetration of chloride ions over time leads to destruction of alkaline protection of reinforcements and initiation of corrosion. Ions generally penetrate concrete through diffusion. The type of concrete, the water to cement ratio, curing time, surface chlorides concentration, temperature and thickness of concrete cover over reinforcements are among the most important factors influencing the time required for initiation of reinforcement corrosion (Duffo *et al*. 2004, Hosseini *et al*. 2014, Oh and Jang 2007). The concentration of chloride ions is expressed as follow using the Fick's second law (Collepardi *et al*. 1972).

$$C(x,t) = C_0 + (C_S - C_0)\left[1 - erf\left(\frac{x}{\sqrt{4Dt}}\right)\right] \tag{1}$$

where, $C(x, t)$ denotes concentration of chlorides at a depth of x and time of t. D is also the diffusion coefficient. C_0 and C_s are the initial concentration of chlorides in the concrete mortar and concentration of chlorides on the concrete surface respectively. Corrosion of reinforcement starts when the concentration of ions reaches the critical concentration (C_{cr}) level. The time of reinforcement corrosion initiation (T_i) is obtained as follows by setting the value of Eq. (1) to C_{cr} for the reinforcement surface (x=c). Considering diffusion coefficient as a time dependent parameter and using Fick's second law of diffusion gives (Tang and Gulikers 2007)

$$T_i = \left[\frac{(1-n)}{4D_0 t_0^n} \left(erf^{-1} \left(\frac{C_s - C_{cr}}{C_s - C_0} \right) \right)^{-2} \right]^{\frac{1}{1-n}} \tag{2}$$

where, D_0 is the diffusion coefficient for the initial time, t_0, and n is the lifetime coefficient ($0<n<1$). The complete derivation of Eq. (2) from Fick's second law of diffusion is given in Appendix A. The lifetime coefficient falls in the 0.2-0.3 range for concretes with typical Portland cement and falls in the 0.5-0.7 range for cements containing fly-ash or slag (Bamforth 2004). The critical concentration for initiation of reinforcement corrosion is influenced by many factors including concrete type, water to cement ratio, temperature and oxygen (Duffo *et al*. 2004). The values vary between 0.2% and 3% of cement content (Yu *et al*. 2012, Alonso *et al*. 2000). According to the American Concrete Regulations, introduces the conservative value of 0.15% for the cement content of concrete components exposed to moisture and chlorides at time of operation (ACI 2011). In regions where conditions for reinforcement corrosion are met, the average value of 0.5 is obtained as the critical value (Polder and Rooij 2002).

3. Corrosion induced cracking

The increase in volume causes cracking of the concrete cover and finally leading to its eventual spalling and/or delamination. Vu *et al.* (2005) by simulating the reinforcement corrosion for a typical RC bridge deck and proposed an empirical model to predict the time to excessive cracking for RC structures for cracks up to 1 mm in width by considering concrete quality and cover as influencing variables. Jamali *et al.* (2013) have been reviewed a number of empirical, analytical and numerical models to predict the time to cracking, they observed that the majority of the investigated models were only capable of adequately predicting the time-to-cracking for the experiments to which they were fitted. Some researchers like Real and O'Connor (2012) have done some modification in existing models. These modified models were in compliance with previous experimental results and so is ultimately has been proposed as a means of predicting time to crack initiation when the use of more advanced, computationally intensive FE models is inappropriate. Firouzi and Rahai (2012) evaluated the width of cracks in bridge decks caused by corrosion and when the width of these cracks reaches the critical level, and accordingly they offered a program for optimizing the inspection of structures. Corrosion-induced cracking models might be classified into three main categories: empirical, analytical, and numerical. In principal for all models the time of the propagation period until cracking, t_{cr} is proportional to the critical amount of the corrosion products W_{cr} (e.g., g/mm^2 reinforcing bar surface) and inversely proportional to the corrosion current density, i_{cor} as Eq. (3)

$$t_{cr} \propto W_{crit} / i_{cor} \tag{3}$$

The experimental data forming the basis for deriving the empirical models and cracking has been marked by the appearance of <0.05 mm cracks. Critical attack penetration, x_0 is main issue in all empirical models. Analytical models are primarily based on the concepts of solid mechanics and almost they are only capable of describing uniform reinforcement corrosions. Some of empirical and analytical Corrosion-induced cracking models shown in Table. 1 Analytical models in Table 1 are based on thick-walled uniform cylinder approach (TWUC) whereby the concrete surrounding the reinforcing bars can be considered as a thick-walled cylinder which becomes

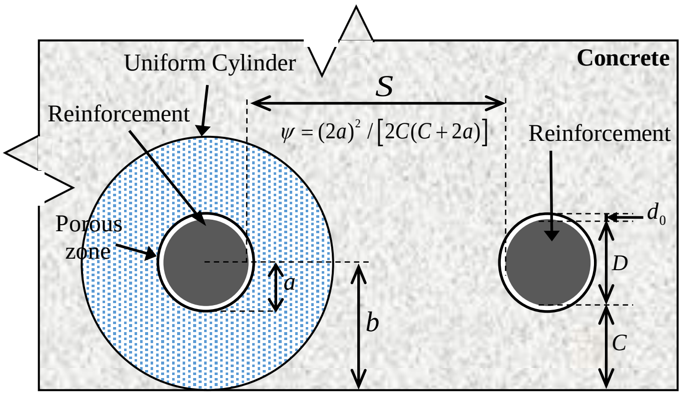


Fig. 2 Thick-walled uniform cylinder model

Table 1 Empirical and analytical models of corrosion-induced cracking

Reference	T_{cr}	Type
Rodriguez *et al.*(1996)	$[83.8+7.4(C/D)-22.6f_t]\times 10^{-3}/0.0116i_{cor}$	Empirical
Alonso *et al.* (1998)	$[7.53+9.32(C/D)]\times 10^{-3}/0.0116i_{cor}$	Empirical
Webster (2000)	$1.25C\times 10^{-3}/0.0116i_{cor}$	Empirical
Liu and Weyers (1998)	$\dfrac{\pi D\left\{\left(\frac{1}{\rho_r}-\frac{\alpha_m}{\rho_s}\right)^{-1}\left[\frac{Cf_t}{E_{eff}}\left(\frac{b^2+a^2}{b^2-a^2}+\nu_c\right)+d_0\right]\times 1000\right\}^2-1}{2\times 0.098\times 1.07\pi D i_{cor}/\alpha_m}$	Analytical
El Maaddawy And Soudki (2007)	$\left[\dfrac{7117.5(D+2d_0)(1+\upsilon_c+\psi)}{i_{cor}E_{eff}}\right]\left[\dfrac{2Cf_t}{E_{eff}}+\dfrac{2d_0E_{eff}}{(D+2d_0)(1+\upsilon_c+\psi)}\right]/365$	Analytical
Lu *et al.* (2011)	$234762\times(D+KC)\dfrac{\left\{\frac{f_t}{E_{eff}}\left(0.3+\frac{0.6C}{D}\right)\left(\frac{b^2+a^2}{b^2-a^2}+\nu_c\right)+1+\frac{2d_0}{D}\right\}^2-1}{24\times 365(\gamma-1)i_{cor}}$	Analytical

Notations: c: clear cover (mm); D: Reinforcement Diameter (mm); L: length of corroding (mm); i_{cor}: corrosion current density (μA/cm^2); ρ_r: Density of rust (gr/mm^3); ρ_s: Density of iron (0.00785gr/mm^3); α_m: Molar mass of corrosion products; f_t: Tensile strength of concrete (Mpa); j_r: Rate of rust production per unit area (g/mm^2/year); k: Hole flexibility (mm(Mpa)$^{-1}$); K: An empirical coefficient that depends on the corrosion conditions; E_{eff}: Effective modulus of elasticity (Mpa); υ_c: Poisson's ratio of the concrete; γ: Volumetric ratio of a rust product; n: Moles of electrons released per mole of iron (n=2 for iron)

perfectly plastic when in tension (Fig. 2). Some useful parameters of these models shown in Fig. 2.

4. Coefficient of correlation

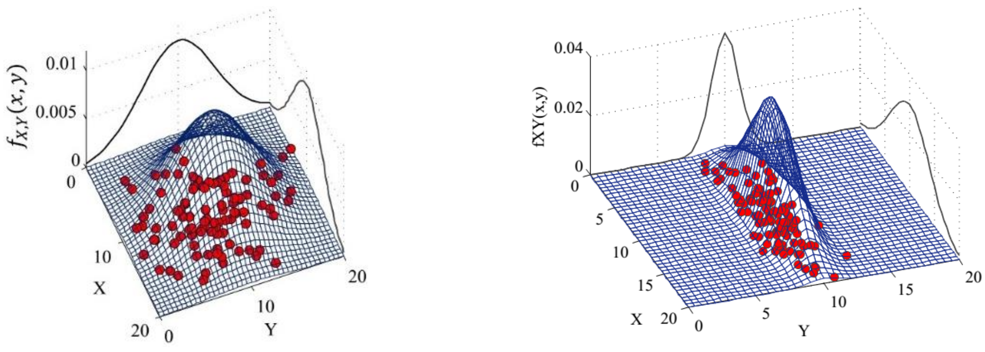
In the theory of probability, when occurrence of an event does not have any effect on the occurrence of another event, the two events are called, independent. From theory point of view when two events are independent, then there is no correlation between them. On the other hand, if two variables possess no correlation then they are not independent necessarily. For example for two random variables *x* and *y*, by pair wise plotting of these two variables data, one can investigate their situation and define a probability density function of two variables in the form of $f_{x,y}(x,y)$. To understand the correlation between the two variables numerically, different parameters are implemented, among them Co-variance and correlation coefficient are the most important ones. The co-variance of two continuous variables *x* and *y*, having mean values μ_x and μ_y and conjunct probability density function $f_{x,y}(x,y)$, is calculated by integration over the space of variables as indicated by Eq. (4).

$$CoV\left(X,Y\right)=E\left[\left(X-\mu_X\right)\left(Y-\mu_Y\right)\right]=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\left(x-\mu_x\right)\left(y-\mu_y\right)f_{XY}(x,y)dxdy \tag{4}$$

where *E* [.], is the (mathematical) expectation of the variable within accolade, and is calculated by integration over the space of variables. The coefficient of correlation between two variables *x* and *y* is calculated using Eq. (5), on the basis of co-variance and values of standard deviation of the two variables.

$$\rho_{xy}=\frac{CoV\left(X,Y\right)}{\sigma_X\sigma_Y} \tag{5}$$

In this equation, *Cov* (.), σ_x and σ_x are covariance and standard deviation of the two variables, respectively. This coefficient also called the Pearson product moment correlation coefficient and indicates the degree of linear dependence between the two random variables *X* and *Y*. The coefficient of correlation is limited to values between -1 and 1 inclusive, that is $-1\leq\rho_{xy}\leq1$. If $|\rho_{xy}|$ is closed to 1, then *X* and *Y* are linearly correlated. If ρ_{xy} is close to zero, then the two variables are nor linearly related to each other. When ρ_{xy} is close to zero, it does not mean that there is no dependence at all; there may be some nonlinear relationship between two variables (Nowak and Collins 2012). The concept of this kind of correlation, is shown in Fig. 3. The distribution of variables *x* and *y* shown in Fig. 3(a), indicates low correlation between these two variables, while in part (b) of this figure, these two variables are correlated.



(a) (b)

Fig. 3 Distribution of variables that randomly simulated; (a) uncorrelated variables. (b) linearly correlated variables

Coefficient of correlation has an important role in calculations related to the evaluation of damage probability of a structure. A kind of correlation used in the evaluation of reliability of structures, is implemented to consider random spatial variation (dependent to location) of a parameter within a structure or an element (Stewart and Mullard 2007). The most important kind of correlation used in the calculation of reliability, is the correlation between components of a system and its limit state functions (Stuart 1958).

For a structure susceptible to Chloride ion, as a limit state function, the non-initiation of corrosion could be taken as the limit state function in the following expression.

$$G(X) = T_i(X) - t_a \tag{6}$$

In which t_a, is the lifetime considered for the structure, T_i is the corrosion initiation time, which is dependent on the parameter X, and is calculated using Eq. (2), based on the increasing concentration of Chloride ion on the reinforcement bar surface to a critical value. As stated before, in concrete structures after initiation of corrosion, due to increase of pressure of corrosion products, cracks appear in the concrete cover. For some structures, the nonappearance of corrosion induced cracks is defined as the limit state function in the following equation:

$$G(Y) = T_{cr}(Y) - t_a \tag{7}$$

Where T_{cr}, is the time to crack initiation that is dependent on the Y parameters. For calculation of reliability or its complement that is the probability of noncompliance of limit state functions considered , use is made of reliability methods such as FORM , SORM and … . If for a structure, both limit sate conditions presented in Eqs. (6)-(7) are applied simultaneously, the correlation between the corrosion initiation time and the time to crack initiation, due to the common parameters present in their descriptive model, should be considered.

5. Calculation of correlation

5.1 Numerical simulation procedure

In this section, first by taking the effective parameters in the corrosion initiation of reinforcement and using coefficient of correlation concept, relationship of input parameters in the diffusion model and corrosion initiation is calculated. Then the coefficient of correlation between corrosion initiation time and time to surface crack initiation ($\rho_{Ti,Tcr}$), is calculated. In calculation of $\rho_{Ti,Tcr}$, the time to crack initiation, obtained from each of the models is taken. For this purpose the stochastic properties of effective parameters in the models are taken. Considering Eq. (2) and also models presented for crack initiation, it is obvious that concrete cover as a common parameter in all the models is influential. The increase in concrete cover, causes increase of corrosion initiation time and on the other hand, in models describing crack initiation, the increase in the ratio of concrete cover to the bar diameter ,causes delay in crack initiation. For investigation of effect of concrete cover on the correlation of T_i and T_{cr}, the minimum concrete cover for various environmental conditions and for a reinforced concrete beam based on the ACI Code, are taken. As, in customary reinforced concrete structures, bar diameters ranging from 10 to 36 mm are used as longitudinal reinforcement bars for beams, then based on the minimum concrete cover, the least ratio *C/D* for any diameter of the bar and for various conditions of bar placing, are indicated in

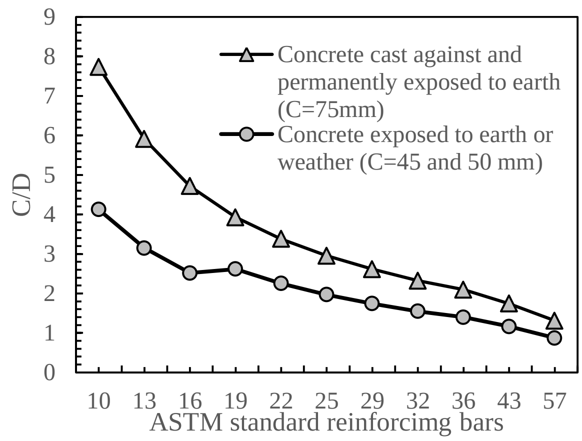


Fig. 4 Minimum ratio of concrete cover thickness to bar diameter (*C/D*) for different environmental condition

Table 2 Main random variables affecting the corrosion initiation time

Variables	Mean	Coefficient of variation	Distribution
Surface concentration, C_s (kg/m^3)	2.5, 5.0 , 8.0	0.05, 0.10, 0.15	Lognormal
Critical concentration, C_{cr} (kg/m^3)	0.8, 1.1, 1.4	0.05, 0.10, 0.15	Lognormal
Diffusion coefficient, D_o (10^{-12} m2/s)	1.0, 3.0, 5.0	0.2, 0.35, 0.50	Lognormal
Clear cover, C (cm)	4.0,5.0,7.5	0.10, 0.15, 0.20	Normal

Fig. 4. As is obvious from this figure, by increasing of the bar diameter, the minimum value of *C/D* ratio decreases, so that for bar no. 10 this ratio has values of 4.12 and 7.73 and for bar no. 57 it amounts to 0.87 and 1.31.

In addition to the concrete cover, the most important effective parameters in the model of corrosion initiation time are Diffusion coefficient, critical chloride concentration, surface Chloride concentration and initial present Chloride in the concrete. Assumed values for these parameters together with stochastic properties are shown in Table 2. These values are based on the ranges reported by other researchers.

For investigation of uncertainties effect on the time of corrosion initiation, use is made of mean values of every parameter together with the range of their coefficient of variation. And by Monte Carlo random sampling, the corrosion initiation time is calculated. For the concrete cover, values of 4 cm, 5 cm and 7.5 cm are used, which are the minimum values for concrete cover for various conditions (ACI 2011).

For investigation of correlation of corrosion initiation to present parameters in Eq. (2), use has been made of the Monte Carlo random sampling. For this purpose, first by generation of 100 000 random samples from every effective parameter in Eq. (2), the corrosion initiation time is calculated. The distribution of samples generated by Monte Carlo method is shown in Fig. 5. The corrosion initiation time is calculated from Eq. (2). As Monte Carlo method is applied in this modeling, Eq. (5) is used for calculation of coefficient of correlation in the form Eq. (8)

$$\bar{\rho}_{xy} = \frac{\sum_{i=1}^{n}\left(X_i - \bar{X}\right)\left(Y_i - \bar{Y}\right)}{\sqrt{\sum_{i=1}^{n}\left(X_i - \bar{X}\right)^2}\sqrt{\sum_{i=1}^{n}\left(Y_i - \bar{Y}\right)^2}} \tag{8}$$

where $\bar{X}$ and $\bar{Y}$ are the mean values of two variables *X* and *Y*.

5.2 Correlation between effective parameters in the diffusion model and T_i

The effect of cover and uncertainty present in it, on the coefficient of correlation are shown in Fig. 6. As is evident from this figure, the coefficient of correlation between corrosion initiation time and concrete cover $\rho_{Ti,C}$ is a positive value. The increase in mean concrete cover has no effect on the value of $\rho_{Ti,C}$, while by increasing of uncertainty in concrete cover, the coefficient of uncertainty also increases. $\rho_{Ti,C}$ Values are 0.48 for minimum uncertainty in concrete cover up to 0.7 for maximum uncertainty defined for concrete cover. As the value and the sign of this coefficient in this case indicates the way the input parameter impacts the output value of the model (T_i), then it is clear from the results, that increase of concrete cover, causes increase in the time of

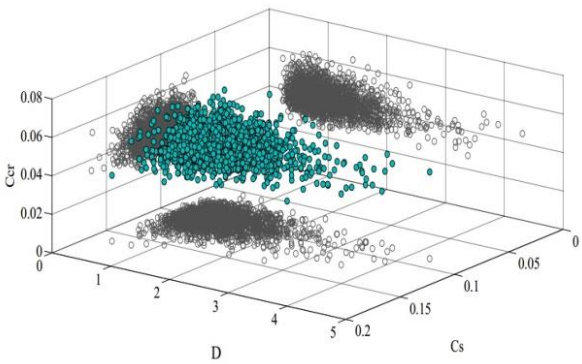
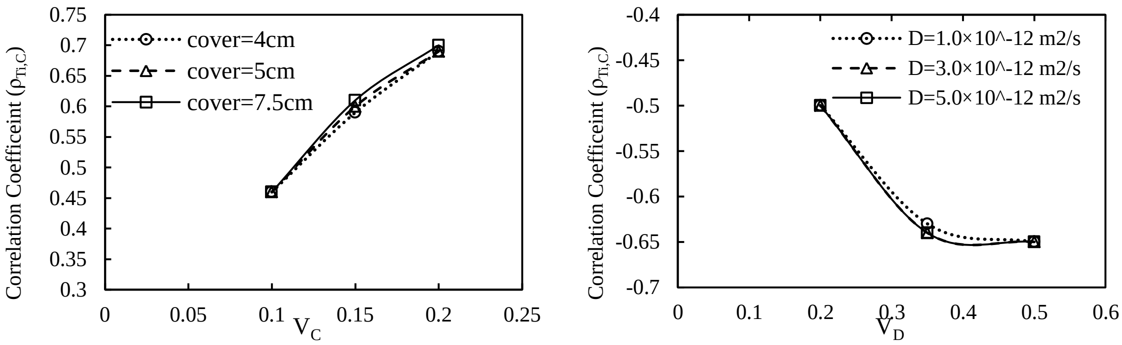
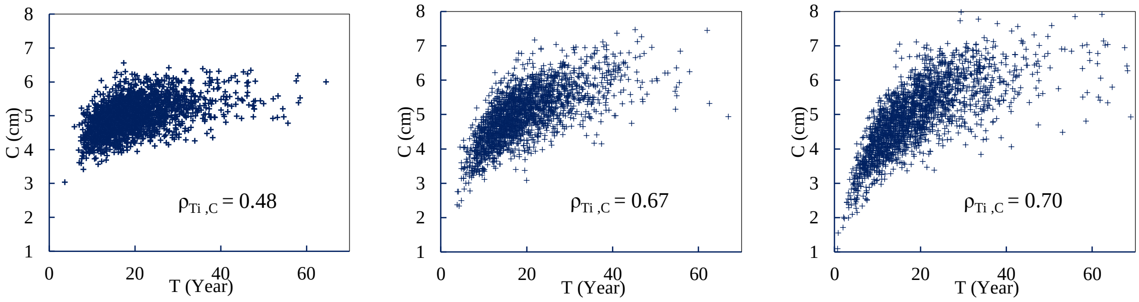


Fig. 5 Random variables simulated by MC sampling method (V_c=0.1,V_{Cs}=0.05,V_{Ccr}=0.05,V_D=0.2)



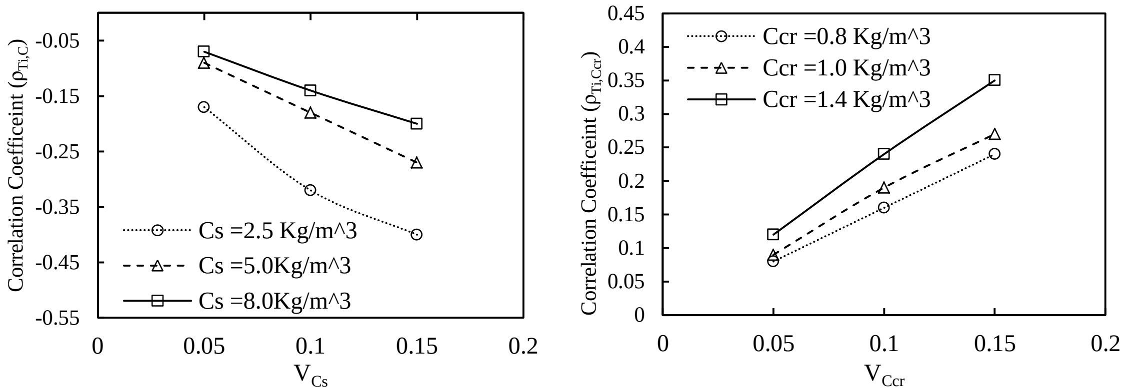
(a) (b)

Fig. 6 Effect of uncertainty of: (a) concrete cover; (b) diffusion coefficient on correlation coefficient



(a) V_C=0.10 (b) V_C=0.15 (c) V_C=0.20

Fig. 7 Distribution of concrete cover and corrosion initiation time based on MC method



(a) (b)

Fig. 8 Effect of uncertainty of: (a) surface chloride concentration; (b) critical chloride concentration on correlation coefficient

corrosion initiation. And the coefficient of increase, based on the uncertainty present in concrete cover, is proportional to the coefficient of correlation between concrete cover and the time of corrosion initiation.

The coefficient of correlation between chloride diffusion coefficient and the corrosion initiation time ($\rho_{Ti,D}$) is shown in Fig. 6(b). The results show linear correlation with negative value. Increase or decrease of the mean value of diffusion coefficient has no effect on the $\rho_{Ti,D}$ While increase of uncertainty in diffusion coefficient up to V_D=0.35 causes increase in linear correlation, but after it increase of V_D had no effect on $\rho_{Ti,D}$. The negative sign of $\rho_{Ti,D}$ indicates the inverse effect of diffusion coefficient on the time of corrosion initiation.

The effect of Chloride ion in two forms of surface Chloride and critical Chloride is influential on the time of corrosion initiation. As is seen in Fig. 8, contrary to the two previous parameters, variation in mean values of critical Chloride and surface Chloride can have significant impact on the coefficient of correlation between the two parameters and the corrosion initiation time ($\rho_{Ti,Ccr}$, $\rho_{Ti,Cs}$). With reference to this figure, by increase of C_s, the value of coefficient of correlation reduces significantly. The effect of increase in uncertainty of surface Chloride is similar to the other two parameters, such that by increase of V_{Cs}, linear correlation increases. From Fig. 8(a) also is obvious that when uncertainty in C_s increases, the effect of C_s value on the coefficient of linear correlation increases. The coefficient of correlation between surface Chloride and the corrosion initiation time is a negative value which indicates the inverse effect of increase of surface Chloride concentration on the corrosion initiation time of reinforcement bar. The variation of critical Chloride concentration, similar to the surface Chloride concentration, impacts the coefficient of correlation but with this difference that the increase of critical Chloride concentration causes increase in the coefficient of correlation related to this parameter. As is apparent from Fig. 8(b) the increase of critical Chloride concentration value and also increase of uncertainty of this parameter both cause increase in the coefficient of correlation related to this parameter.

In all the parameters influential in corrosion initiation it is observed that by reduction of uncertainty that is applied in the form of reduction in the coefficient of variation between parameters, The correlation between these parameters and the corrosion initiation time decreases.

5.3 Coefficient of correlation between T_i and T_{cr}

In this section the coefficient of correlation between T_i and T_{cr} is calculated. For this purpose the effects of concrete cover thickness as a common parameter in the diffusion model and the crack initiation models and also the effects of uncertainties present in the corrosion intensity are considered. By combining the least required concrete cover for various environmental conditions for a beam and also all diameters of standard reinforcement bars. According to the ACI Code, the least *C/D* ratio for various practical conditions was obtained as shown in Fig. 2. In diagrams, label A indicates conditions under soil proximity and label B indicates conditions under weather proximity. By utilizing values of the coefficient of variation for the concrete cover (V_c), 0.10 and 0.2 as the upper and lower limits of uncertainty present in this parameter and also coefficient of variation 0.10 and 0.3 for the corrosion current density (V_i), the effect of the stochastic properties of these two parameters in the coefficient of correlation, is calculated. Utilizing the MC method and simultaneous use of Eq. (2) and relationships in Table 1, the value of $\rho_{Ti,Tcr}$ for the environmental conditions A and B, and various bar diameters are calculated. In Fig. 9, $\rho_{Ti,Tcr}$ is calculated based on the *Rodriguez et al*. model, as is evident from this figure, for condition A, the linear correlation between the corrosion initiation time and the time to crack initiation is greater, such that for example for bar No. 10 and values V_i=0.1 and V_c=0.2 it is observed that $\rho_{Ti,Tcr}$ for condition A is 0.31 and for condition B equals 0.19. Therefore for a specific bar diameter, the increase of concrete cover results in the increase of linear correlation of the time to crack initiation, and the corrosion initiation time. By increase in the bar diameter and consequently reduction of *C/D* ratio, it is observed that $\rho_{Ti,Tcr}$ declines. So both for this model and for a constant concrete cover increase of bar diameter causes decline in the coefficient of correlation. In both conditions A and B, increase of uncertainty in the concrete cover causes increase of the coefficient of correlation, while increase of uncertainty in the corrosion current density, has resulted in the decline of $\rho_{Ti,Tcr}$. So that the maximum correlation for conditions A and B is for the case V_i=0.1 and V_c=0.2 and the minimum value of correlation is obtained for the case V_i=0.1 and V_c=0.2. Fig. 9(b), the state of distribution of the corrosion initiation time and crack initiation time are shown, based on the samples produced by the MC method for the maximum and minimum values of $\rho_{Ti,Tcr}$, obtained from Rodriguez et al model.

Correlation coefficients based on the *Alonso et al.* model are shown in Fig. 10. From this figure it is clear that $\rho_{Ti,Tcr}$ obtained from this model are larger than the results obtained from the *Rodriguez et al.* model. So that for example, for condition A and the case of V_i=0.1 and V_c=0.2 the value of $\rho_{Ti,Tcr}$ is equal to 54%. For this model values for both conditions are similar to each other,

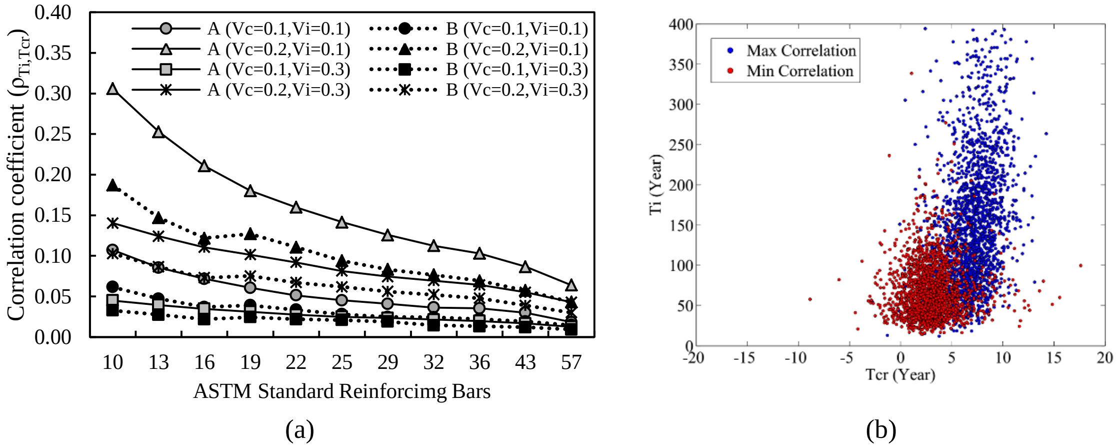


Fig. 9 Correlation coefficient ($\rho_{Ti,Tcr}$) based on *Rodriguez et al.* model; (a) Effect of environmental condition and bar diameter on correlation coefficient; (b) distribution of T_i and T_{cr} for max and min correlation coefficient

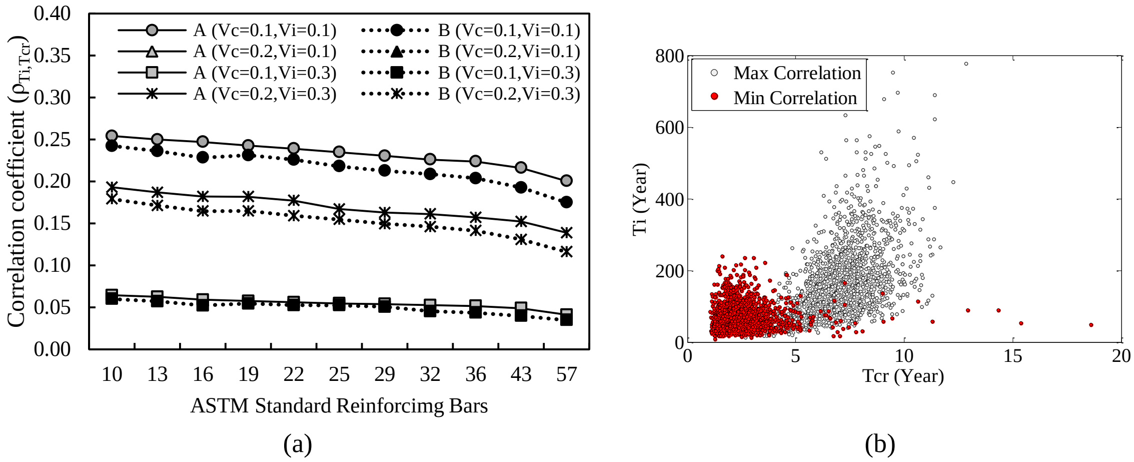
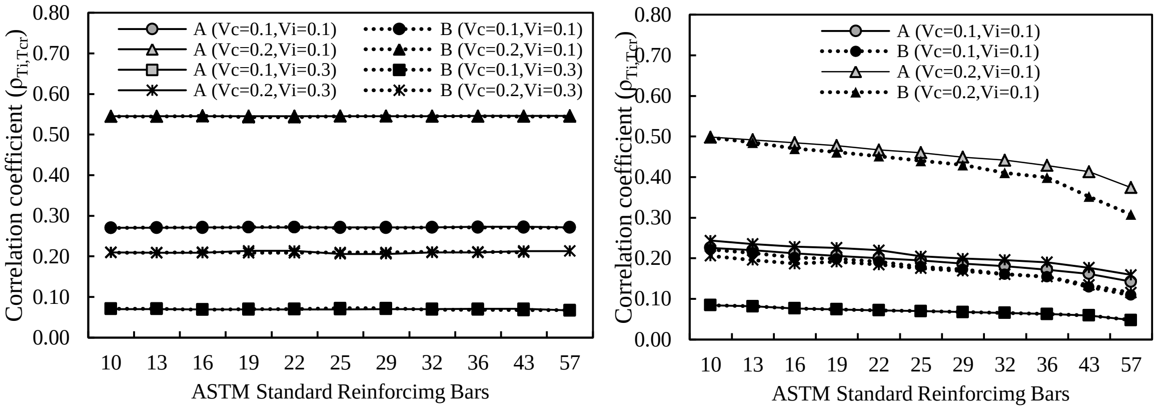


Fig. 10 Correlation coefficient ($\rho_{Ti,Tcr}$) based on *Alonso et al.* model; (a) Effect of environmental condition and bar diameter on correlation coefficient; (b) distribution of T_i and T_{cr} for max and min correlation coefficient

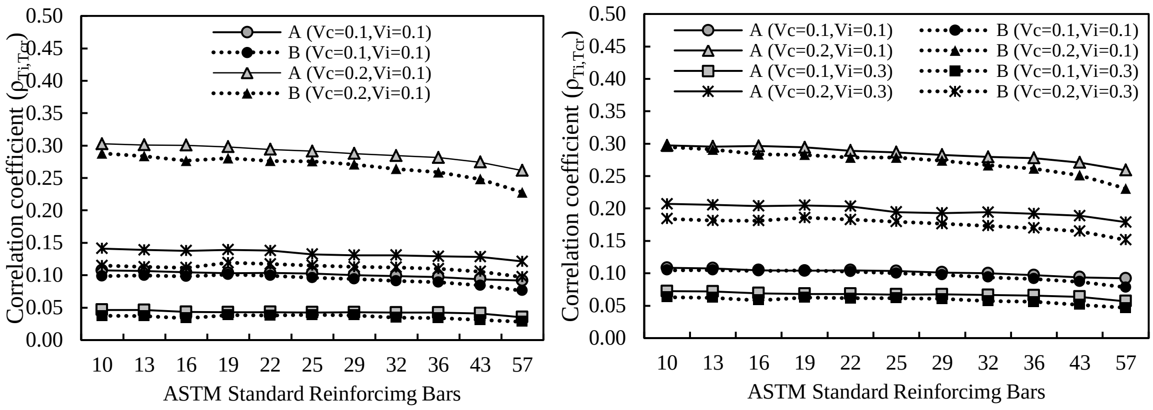
and contrary to the *Rodriguez et al.* model, don't differ much. On the other hand the increase of bar diameter also does not have much impact on the values of $\rho_{Ti,Tcr}$. For the *Alonso et al.* model as *Rodriguez et al.*, the increase of uncertainty in concrete cover, increases $\rho_{Ti,Tcr}$ and increase of uncertainty in the corrosion current density, causes decline in $\rho_{Ti,Tcr}$. The greatest coefficient of correlation is 0.53 for the case A and bar diameter 10 mm and V_i =0.1 and V_c=0.2. And the least value of $\rho_{Ti,Tcr}$ is 0.04 for the case B and bar diameter no. 57 and V_i =0.3 and V_c=0.2 .The state of distribution of both T_i and T_{cr}, are shown in Fig. 10(b).

The calculated $\rho_{Ti,Tcr}$ from the *Webster* and *Lu et al.* models are shown in Fig. 11. Results of these models are similar to each other, so that for models, variation of reinforcement diameter and also variation of concrete cover had no impact on the results and only variation of uncertainty



(a) (b)

Fig. 11 Correlation coefficient ($\rho_{Ti,Tcr}$) based on; (a) Webster model; (b) Lu *et al*. model



(a) (b)

Fig. 12 Correlation coefficient ($\rho_{Ti,Tcr}$) based on: (a) El Maaddawy and Soudki model; (b) on Liu and Weyers model

present in the corrosion current density and concrete cover have influenced the results. In these models, similar to the two previous models, increase of the coefficient of variation of the concrete cover and corrosion current density, causes increase and decrease of the linear correlation of T_i and T_{cr}, respectively. Increase of V_c from 0.1 to 0.2 in the two models for V_i equal to 0.1 and 0.3, causes increase of $\rho_{Ti,Tcr}$ from 0.27 to 0.55 and from 0.07 to 0.21, respectively.

In Fig. 12, calculated $\rho_{Ti,Tcr}$ from the *El Maaddawy & Soudki* and on *Liu & Weyers* which are analytical models are shown. In these models the variation of the concrete cover and the bar diameter had insignificant impact on $\rho_{Ti,Tcr}$. The effect of uncertainty in the concrete cover and corrosion current density on $\rho_{Ti,Tcr}$ was similar to the experimental models.

6. Conclusions

In this paper, the correlation coefficient between corrosion initiation time and the effective parameters in the diffusion model and also the correlation coefficient between the corrosion initiation time and the time to crack initiation due to corrosion, are investigated. The four main parameters such as concrete cover, coefficient of diffusion, surface and critical Chloride concentration, are considered as random variables by appropriate probability distribution function. Using the Monte Carlo sampling method, sampling is made and for various cases, the coefficient of correlation between these parameters and the time of corrosion initiation are calculated. Based on the results for all the parameters, with increase in uncertainty, the correlation is also increased. Amongst all parameters discussed, the diffusion coefficient and the surface Chloride concentration have negative correlation coefficient and the concrete cover and the critical Chloride concentration have positive correlation coefficient. Furthermore, in the range of considered uncertainties, the correlation values of surface Chloride and critical Chloride is smaller than the correlations obtained for concrete cover and the diffusion coefficient. The maximum correlation for surface Chloride and critical Chloride are limited to 0.32 and 0.35 respectively. While the maximum values of correlation for concrete cover and diffusion coefficient are 0.7 and 0.65 respectively. The variation of the mean value of the coefficient of diffusion and concrete cover has no impact on the coefficient of correlation. While the variation of mean value of the concentration for surface Chloride and critical Chloride causes variation in the coefficient of correlation. Increase in the mean value of surface Chloride causes decline in the coefficient of correlation, but increase in the mean Critical Chloride causes increase in the linear correlation between this parameter and the corrosion initiation time. The coefficient of correlation between the corrosion initiation time and the time to crack initiation, considering environmental conditions, bar diameter and uncertainty in corrosion current density and concrete cover, is calculated. Results show that increase in the coefficient of correlation is with increase in uncertainty in the concrete cover, while increase in uncertainty in the corrosion current density had the inverse impact. Increase in the bar diameter in most of models had no impact on the coefficient of correlation but in some models like *Rodriguez et al.* with increase in the bar diameter, the coefficient of correlation has declined.

References

ACI Building Code Requirements for Structural Concrete (ACI 318M-11) (2011), American Concrete Institute, Farmington Hills.

Alonso, C., Andrade, C., Castellote, M. and Castro, P. (2000), “Chloride threshold values to depassivate reinforcing bars embedded in a standardized OPC mortar”, *Cement Concrete Res.*, **30**(7), 1047-1055.

Alonso, C., Andrade, C., Rodriguez, J. and Diez, J.M. (1998), “Factors controlling cracking of concrete affected by reinforcement corrosion”, *Mater. Struct.*, **31**, 435-441.

Amey, S.L., Johnson, D.A., Miltenberger, M.A. and Farzam, H. (1998), “Predicting the service life of concrete marine structures: an environmental methodology”, *ACI Struct. J.*, **95**(2), 205-214.

Bamforth, P.B. (1999), “The derivation of input data for modeling chloride ingress from eight year UK coastal exposure trials”, *Mag. Concrete Res.*, **51**(2), 87-96.

Bamforth, P.B. and Price, W.F. (1997), *An International Review of Chloride Ingress Into Structural Concrete*, Contractor report 359, Taywood engineering Ltd and Emerson M.

Bamforth, P.B. (2004), *Technical Report No. 61: Enhancing Reinforced Concrete Durability- Guidance on Selecting Measures for Minimizing the Risk of Reinforcement in Concrete*, Concrete Society.

Bhargava, K., Mori, Y. and Ghosh, A.K. (2011), "Time-dependent reliability of corrosion-affected RC beams. Part 3: Effect of corrosion initiation time and its variability on time-dependent failure probability", *Nucl. Eng. Des.*, **241**, 1395-1402.

Collepardi, M., Marcialis, A. and Turriziani, R. (1972), "Penetration of chloride ions into cement pastes and concretes", *J. Am. Ceram. Soc.*, **55**(10), 534-535.

Der Kiureghian, A. and Liu, P.L. (1986), "Structural reliability under incomplete probability information" *J. Eng. Mech.*, **112**(1), 85-104.

Duffo, G.S., Morris, W., Raspini, I. and Saragovi, C. (2004), "A study of steel rebars embedded in concrete during 65 years", *Corros Sci.*, **46**, 2143-2157.

DuraCrete-Statistical Quantification of the Variables in the Limit State Functions (R9) (2000), Civieltechnisch Centrum Uitvoering Research en Regelgeving, Gouda.

El Maaddawy, T. and Soudki, K. (2007), "A model for prediction of time from corrosion initiation to corrosion cracking", *Cement Concrete Compos.*, **29**, 168-175.

Enright, M.P. and Frangopol, D.M. (1998), "Probabilistic analysis of resistance degradation of Reinforced Concrete Bridge beams under corrosion", *Eng. Struct.*, **20**(11), 960-971.

Firouzi, A. and Rahai, A. (2012), "An integrated ANN-GA for reliability based inspection of concrete bridge decks considering extent of corrosion-induced cracks and life cycle costs", *Scientia Iranica*, **19**(4), 974-981.

Jamali, A., Angst, U., Adey, B. and Elsener, B. (2013), "Modeling of corrosion-induced concrete cover cracking: A critical analysis", *Constr. Build. Mater.*, **42**, 225-237.

Hasofer, A.M and Lind, N.C. (1974), "Exact and invariant second moment code format", *J. Eng. Mech.*, **100**, 111-121.

Hosseini, S.A., Shabakhty, N. and Mahini, S.S. (2014), "The effect of uncertainties on calculation of initiation of corrosion of reinforcement for assessment of reliability of concrete structures", *J. Civil Eng. Urban*, **4**(4), 364-369.

Li, Y., Vrouwenvelder, T. and Wijnants, G. (2003), "Spatial variability of concrete degradation", *Life-Cycle Performance of Deteriorating Structures*, 49-58.

Lin, S.H. (1990), "Chloride diffusion in a porous concrete slab", *Corrosion*, **46**(12), 964-967.

Liu, Y. and Weyers, R.E. (1998), "Modeling the time-to-corrosion cracking in chloride contaminated reinforced concrete structures", *ACI Mater. J.*, **95**(6), 675-681.

Lu, C., Jin, W. and Liu, R. (2011), "Reinforcement corrosion-induced cover cracking and its time prediction for reinforced concrete structures", *Corros Sci.*, **53**, 1337-1347.

Lu, Z.H., Zhao, Y.G., Yu, Z.W. and Ding, F.X. (2011), "Probabilistic evaluation of initiation time in RC bridge beams with load-induced cracks exposed to de-icing salts", *Cement Concrete Res.*, **41**(3), 365-372.

Moodi, F., Ramezanianpour, L. and Jahangiri, E. (2014), "Assessment of some parameters of corrosion initiation prediction on reinforced concrete in marine environments", *Comput. Concrete*, **13**(1), 71-82.

Nataf, A. (1962), "Determination des distributions dont les marges sont données", *Comptes Rendus Acad´emie des Sciences*, **225**, 42-43.

Nowak, A.S. and Collins, K.R. (2012), *Reliability of Structures*, 2th Edition, McGraw-Hill, Singapore.

Oh, B.H. and Jang, S.Y. (2007), "Effects of material and environmental parameters on chloride penetration profile in concrete structures", *Cement Concrete Res.*, **37**, 37-53.

Polder, R.B. and Rooij, M.R. (2002), "*Investigation of the concrete structure of the Pier of Scheveningen after 40 years of exposure to marine environment*", TNO Building and Construction Research, Delft.

Reale, T. and O'Connor, A. (2012), "A review and comparative analysis of corrosion-induced time to first crack models", *Constr. Build. Mater.*, **36**, 475-483.

Rodriguez, J., Ortega, L., Casal, J. and Diez, J. (1996), "Corrosion of reinforcement and service life of concrete structures", *Durab. Build. Mater. Compon.*, **1**(1), 117-126.

Stewart, M.G. and Mullard, J.A. (2007), "Spatial time-dependent reliability analysis of corrosion damage and the timing of first repair for RC structures", *Eng. Struct.*, **29**, 1457-1464.

Stuart, A.J. (1958), "Equally correlated variates and multinormal integral", *J. Roy. Statis. Soc.*, **B20**, 373-378.

Tang, L. and Gulikers, J. (2007), "On the mathematics of time-dependent chloride coefficient in concrete", *Cement Concrete Res.*, **37**, 589-595.

Vu, K.A.T., Stewart, M.G. and Mullard, J.A. (2005), "Corrosion-induced cracking: experimental data and predictive models", *ACI Struct. J.*, **102**(5), 719-726.

Webster, M.P. (2000), "The assessment of corrosion-damaged concrete structures", Ph.D. Dissertation, The University of Birmingham, Birmingham.

Yu, H., Chiang, K.K. and Yang, L. (2012), "Threshold chloride level and characteristics of reinforcement corrosion initiation in simulated concrete pore solutions", *Constr. Build. Mater.*, **26**, 723-729.

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Appendix A: Derivation of Eq. (2)

The derivation of the solution with the time dependent diffusion coefficient from the partial differential equation:

Combining the equation of the time dependent diffusion coefficient ($D(t)=D_0(t_0/t)^n$) with Fick's second law of diffusion gives

$$\frac{\partial C}{\partial t}=D\frac{\partial^2 C}{\partial x^2} \quad \rightarrow \quad \frac{\partial C}{\partial t}=D_0\left(\frac{t_0}{t}\right)^n\frac{\partial^2 C}{\partial x^2} \quad \rightarrow \quad \frac{1}{D_0\left(\frac{t_0}{t}\right)^n}\frac{\partial C}{\partial t}=\frac{\partial^2 C}{\partial x^2} \tag{a}$$

Assuming $\partial Y = D_0\left(\frac{t_0}{t}\right)^n \partial t$ therefore Eq. (a) change to $\frac{\partial C}{\partial Y}=\frac{\partial^2 C}{\partial x^2}$ the standard solution of this equation is given in Eq. (b)

$$C(x,t)=C_s\left(1-erf\left(\frac{x}{\sqrt{4Y}}\right)\right) \tag{b}$$

Integrating Eq. (2) results in Eq. (c)

$$\int_0^t D(t)dt=\frac{D_0}{1-n}\left(\frac{t_0}{t}\right)^n t \tag{c}$$

Finally Eq. (d) results from the substitution of Eq. (c) into Eq. (b) by considering initial chloride concentration

$$C(x,t)=C_0+\left(C_s-C_0\right)\left(1\text{-erf}\left(\frac{x}{\sqrt{4\frac{D_0}{1\text{-n}}\left(\frac{t_0}{t}\right)^n t}}\right)\right)=C_0+\left(C_s-C_0\right)\left(1\text{-erf}\left(\frac{x}{\sqrt{4\frac{D_0}{1\text{-n}}t_0{}^n t^{1-n}}}\right)\right) \tag{d}$$

The time of reinforcement corrosion initiation (T_i) is obtained as Eq. (2) by setting the value of Eq. (d) to C_{cr} for the reinforcement surface ($x=c$).