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A nonlocal quasi-3D trigonometric plate model for free vibration behaviour of micro/nanoscale plates

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Abstract. In this work, a nonlocal quasi-3D trigonometric plate theory for micro/nanoscale plates is proposed. In order to introduce the size influences, the Eringen's nonlocal elasticity theory is utilized. In addition, the theory considers both shear deformation and thickness stretching effects by a trigonometric variation of all displacements within the thickness, and respects the stress-free boundary conditions on the top and bottom surfaces of the plate without considering the shear correction factor. The advantage of this theory is that, in addition to considering the small scale and thickness stretching effects ($\varepsilon_z \neq 0$), the displacement field is modelled with only 5 unknowns as the first order shear deformation theory (FSDT). Analytical solutions for vibration of simply supported micro/nanoscale plates are illustrated, and the computed results are compared with the available solutions in the literature and finite element model using ABAQUS software package. The influences of the nonlocal parameter, shear deformation and thickness stretching on the vibration behaviors of the micro/nanoscale plates are examined.

Keywords: trigonometric shear deformation theory; nanoplates; nonlocal elasticity theory; navier solution; stretching effect; vibration

1. Introduction

Nanostructures have large kinds of applications due to their high mechanical, thermal, and electrical characteristics. These characteristics allow manufacture of small devices that were, previously impossible to realize. Recently, the nano-structures such as nanoplates, which are made from nano-materials, have shown significant potential applications in various fields of modern

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nanotechnology (Kiani 2013a, Kiani 2013b). Nanoplates are being used in the fields of energy storage and conversion (Ma *et al*. 2008), nano and electromechanical systems (Fritz *et al.* 2000), biological sensors (Yguerabide and Yguerabide 2001), solar cells (Aagesen and Sørensen 2008), sensors, actuators, switchers, ultra thin films (Ma and Clarke 1995), and nanovehicle transporters (Kiani 2011b, Kiani 2011c).

Two approaches are employed for mechanical investigation of nanostructures, the molecular dynamic model and continuum models. It has been confirmed in several works that continuum models can be utilized effectively for investigating of nanostructures (Reddy and Pang 2008, Benzair *et al.* 2008, Heireche *et al.* 2008, Aghababaei and Reddy 2009, Amara *et al.* 2010, Janghorban and Zare 2011, Janghorban (2012), Nami and Janghorban 2013, Nami and Janghorban 2015, Berrabah *et al.* 2013, Benguediab *et al.* 2014, Besseghier *et al.* 2015, Larbi Chaht *et al.* 2015, Zemri *et al*. 2015, Adda Bedia *et al.* 2015, Aissani *et al.* 2015).

All these studies were carried out in the context of the nonlocal continuum theory of Eringen (Eringen and Edelen 1972, Eringen 1983). According to this theory, the state of the stress at a point of a nanostructure does not only depend on the stress state of that point but also on the stresses of other points (Kiani 2015, Tounsi *et al.* 2013abc). Such a dependency is incorporated into the constitutive equations of the nanostructure through a so-called small-scale parameter, commonly presented by e_0a. Choice of e_0a (in dimension of length) is crucial to ensure the validity of nonlocal models. This parameter was determined by matching the dispersion curves based on the atomic models (Eringen and Edelen 1972, Eringen 1983). For a specific material, the corresponding nonlocal parameter can be estimated by fitting the results of atomic lattice dynamic and experiment (Aksencer and Aydogdu 2011). There are several studies that deal with the magnitude of the small-scale parameter for the problems of nanoplates (Malekzadeh *et al.* 2011, Zhang *et al.* 2015, Belkorissat *et al*. 2015).

Recently, considerable interests have been devoted to experimental and theoretical works of the mechanical response of nanoscale structures. Since, controlling the experimental conditions is not evident for nanoscale structures, theoretical models become necessary.

Liu and Rajapakse (2010) presented a general model incorporating surface energy for the investigation of the response of nano-beams with arbitrary cross sections. Based on Timoshenko beam model, Wang and Feng (2009) discussed the stability and vibration of nano-wires. Phadikar and Pradhan (2010) presented finite element formulations for non-local elastic Euler–Bernoulli beams and Kirchoff plates. They employed non-local differential elasticity theory and the Galerkin finite-element method. By using the generalized Kirchhoff and Mindlin plate theory. Nami and Janghorban (2014) presented the strain gradient elasticity formulation and the non local elasticity theory for analysis the resonance behaviors of functionally graded rectangular micro/nano plates using Kirchhoff plate theory. Lu *et al*. (2006) investigated the influence of other surface characteristics and explained the size-dependent mechanical response of nano-plates. Sheng *et al* (2010) studied the 3D elasticity of nano-plates, incorporating their surface characteristics, by employing the theory of laminated structures. By employing the non-local continuum theory, (Murmu and Pradhan 2009, Murmu and Pradhan 2010) analyzed the vibration behavior of nano-plates by use of non-local continuum theory. Murmu and Adhikari (2011) presented an analytical formulation to compute the natural frequencies of the non-local double-nanoplate system. They obtained explicit closed-form expressions for natural frequencies for the case in which all four ends are simply supported. Kiani (2011a) studied the small scale effect on the vibration response of elastic thin nano-plates subjected to a moving nano-particle. Mohammadi *et al*. (2013) analyzed the free vibration response of circular and annular graphene sheet by employing the non-local elasticity theory. Huang (2008) investigated size-dependent bending, buckling, and vibration of nano-plates by employing the non-linear Kirchhoff plate theory and Von-Karman non-linearity assumptions. By using the non-local plate model, Babaei and Shahidi (2010) discussed the buckling behavior of the quadrilateral nano-plates. Pradhan and Murmu (2009) studied the stability of rectangular single-layered graphene sheets (SLGSs) under biaxial compression by use of the non-local elasticity. Sobhy (2014) investigated the free vibration, mechanical buckling and thermal buckling analyses of multi-layered graphene sheets (MLGSs) by the use of new non-local two-variable plate theories.

Due to the widespread employ of micro/nano-plates in micro/nano-electro-mechanical systems (MEMS/NEMS) components, mechanical behavior of them is of a considerable interest. Recently, new quasi-3D shear deformations theories were developed (Belabed *et al.* 2014, Bessaim *et al.* 2013, Bourada *et al.* 2015) and to the best of authors' knowledge, most of these theories have not been employed for studying nanostructures yet. So it may be useful to develop a new model based on one of these new theories for micro/nano-plates. In the work discussed in this research, the free vibration behavior of micro/nano-plates is studied on the basis of quasi-3D trigonometric plate theory in conjunction with Eringen's nonlocal elasticity theory. The displacement field is proposed based on a trigonometric variation of in-plane and transverse displacements through the thickness. By partitioning the deflection into the bending, shear and thickness stretching parts, the number of unknowns of the theory is reduced, thus saving computational time. Equations of motion are obtained from Hamilton's principle based on the nonlocal constitutive expressions of Eringen. Analytical solutions for natural frequency are determined for simply supported plates, and the computed results are compared with the existing solutions and finite element model using ABAQUS software package to check the accuracy of the present model.

2. Nonlocal plate model

2.1 Kinematics

A rectangular nano-plate (length *a*, width *b* and thickness *h*) is considered with coordinates *x*, *y* along the in-plane directions and *z* along the thickness direction. The plane *z*=0 coincident with the mid-surface of the nanoplates. The plate is made of isotropic material. The displacement field of the present theory is chosen based on the following assumptions (Hamidi *et al.* 2015, Bennai *et al.* 2015, Bousahla *et al.* 2014, Fekrar *et al.* 2014, Hebali *et al.* 2014, Zidi *et al*. 2014, Tounsi *et al.* 2013d, Houari *et al.* 2013) : (1) The transverse displacement is composed by a three parts namely: the bending, shear and thickness stretching components; (2) the in-plane displacement is divided into extension, bending and shear components; (3) the bending parts of the in-plane displacements are similar to those given by classical plate theory (CPT); and (4) the shear parts of the in-plane displacements give rise to the trigonometric variations of shear strains and hence to shear stresses through the thickness of the plate in such a way that the shear stresses vanish on the top and bottom surfaces of the plate. Based on these assumptions, the following displacement field relations can be obtained

$$u(x,y,z,t) = u_0(x,y,t) - z\frac{\partial w_b}{\partial x} - f(z)\frac{\partial w_s}{\partial x}$$

$$v(x,y,z,t) = v_0(x,y,t) - z\frac{\partial w_b}{\partial y} - f(z)\frac{\partial w_s}{\partial y}$$
$$w(x,y,z,t) = w_b(x,y,t) + w_s(x,y,t) + g(z)\,\varphi(x,y,t) \tag{1}$$

where u_0 and v_0 denote the displacements along the x and y coordinate directions of a point on the mid-plane of the plate; w_b and w_s are the bending and shear components of the transverse displacement, respectively; and the additional component displacement φ due to the normal stress (stretching effect).

In this study, the shape functions $f(z)$ and $g(z)$ are chosen based on the trigonometric function as

$$f(z) = z - \sin\left(\frac{\pi z}{h}\right) \quad \text{and} \quad g(z) = 1 - f'(z) \tag{2}$$

The non-zero strains associated with the displacement field in Eq. (1) are

$$\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix} = \begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{Bmatrix} + z\begin{Bmatrix}k_x^b\\ k_y^b\\ k_{xy}^b\end{Bmatrix} + f(z)\begin{Bmatrix}k_x^s\\ k_y^s\\ k_{xy}^s\end{Bmatrix}, \quad \begin{Bmatrix}\gamma_{yz}\\ \gamma_{xz}\end{Bmatrix} = g(z)\begin{Bmatrix}\gamma_{yz}^0\\ \gamma_{xz}^0\end{Bmatrix}, \quad \varepsilon_z = g'(z)\,\varepsilon_z^0 \tag{3}$$

where

$$\begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{Bmatrix} = \begin{Bmatrix}\dfrac{\partial u_0}{\partial x}\\ \dfrac{\partial v_0}{\partial x}\\ \dfrac{\partial u_0}{\partial y}+\dfrac{\partial v_0}{\partial x}\end{Bmatrix}, \quad \begin{Bmatrix}k_x^b\\ k_y^b\\ k_{xy}^b\end{Bmatrix} = \begin{Bmatrix}-\dfrac{\partial^2 w_b}{\partial x^2}\\ -\dfrac{\partial^2 w_b}{\partial y^2}\\ -2\dfrac{\partial^2 w_b}{\partial x\partial y}\end{Bmatrix}, \quad \begin{Bmatrix}k_x^s\\ k_y^s\\ k_{xy}^s\end{Bmatrix} = \begin{Bmatrix}-\dfrac{\partial^2 w_s}{\partial x^2}\\ -\dfrac{\partial^2 w_s}{\partial y^2}\\ -2\dfrac{\partial^2 w_s}{\partial x\partial y}\end{Bmatrix}, \quad \begin{Bmatrix}\gamma_{yz}^0\\ \gamma_{xz}^0\end{Bmatrix} = \begin{Bmatrix}\dfrac{\partial w_s}{\partial y}+\dfrac{\partial \varphi}{\partial y}\\ \dfrac{\partial w_s}{\partial x}+\dfrac{\partial \varphi}{\partial x}\end{Bmatrix}, \varepsilon_z^0 = \varphi \tag{4a}$$

and

$$g'(z) = \frac{dg(z)}{dz} \tag{4b}$$

2.2 Equations of motion

The Hamilton's principle is utilized for the free vibration problem of nanoscale plate. The Hamilton's principle is expressed as (Mahi *et al.* 2015, Al-Basyouni *et al.* 2015, Ait Yahia *et al.* 2015, Ait Amar Meziane *et al.* 2014, Draiche *et al.* 2014, Ould Larbi *et al.* 2013, Benachour *et al.* 2011, El Meiche *et al.* 2011)

$$0 = \int_0^T (\delta U - \delta K)dt \tag{5}$$

where δU is the virtual strain energy; and δk is the virtual kinetic energy.

The virtual strain energy of the plate is expressed as

$$\begin{aligned}\delta U &= \int_{-h/2}^{h/2}\int_A \left[\sigma_x \delta\,\varepsilon_x + \sigma_y \delta\,\varepsilon_y + \sigma_z \delta\,\varepsilon_z + \tau_{xy}\delta\,\gamma_{xy} + \tau_{yz}\delta\,\gamma_{yz} + \tau_{xz}\delta\,\gamma_{xz}\right] dA\,dz \\ &= \int_A \Big[N_x \delta\,\varepsilon_x^0 + N_y \delta\,\varepsilon_y^0 + N_z \delta\,\varepsilon_z^0 + N_{xy}\delta\,\gamma_{xy}^0 + M_x^b \delta\,k_x^b + M_y^b \delta\,k_y^b + M_{xy}^b \delta\,k_{xy}^b \\ &\quad + M_x^s \delta\,k_x^s + M_y^s \delta\,k_y^s + M_{xy}^s \delta\,k_{xy}^s + S_{yz}^s \delta\,\gamma_{yz} + S_{xz}^s \delta\,\gamma_{xz}\Big] dA = 0\end{aligned} \tag{6}$$

where *AA* is the top surface and the stress resultants *N*, *M*, and *Q* are defined by

$$\begin{Bmatrix} N_x, & N_y, & N_{xy} \\ M_x^b, & M_y^b, & M_{xy}^b \\ M_x^s, & M_y^s, & M_{xy}^s \end{Bmatrix} = \int_{-h/2}^{h/2} \left(\sigma_x, \sigma_y, \tau_{xy}\right) \begin{Bmatrix} 1 \\ z \\ f(z) \end{Bmatrix} dz, \tag{7a}$$

$$N_z = \int_{-h/2}^{h/2} \sigma_z g'(z)dz, \tag{7b}$$

$$\left(S_{xz}^s, S_{yz}^s\right) = \int_{-h/2}^{h/2} \left(\tau_{xz}, \tau_{yz}\right) g(z)dz. \tag{7c}$$

The virtual kinetic energy of the plate can be written as

$$\begin{aligned}&\delta K = \int_{-h/2}^{h/2}\int_A \left[\dot{u}\delta\,\dot{u} + \dot{v}\delta\,\dot{v} + \dot{w}\delta\,\dot{w}\right]\rho(z)\,dA\,dz \\ &= \int_A \Big\{I_0\left[\dot{u}_0\delta\dot{u}_0 + \dot{v}_0\delta\dot{v}_0 + \left(\dot{w}_b + \dot{w}_s\right)\left(\delta\dot{w}_b + \delta\dot{w}_s\right)\right] - I_1\left(\dot{u}_0 \frac{\partial\delta\dot{w}_b}{\partial x} + \frac{\partial\dot{w}_b}{\partial x}\delta\,\dot{u}_0 + \dot{v}_0 \frac{\partial\delta\dot{w}_b}{\partial y} + \frac{\partial\dot{w}_b}{\partial y}\delta\,\dot{v}_0\right) \\ &- J_1\left(\dot{u}_0 \frac{\partial\delta\dot{w}_s}{\partial x} + \frac{\partial\dot{w}_s}{\partial x}\delta\,\dot{u}_0 + \dot{v}_0 \frac{\partial\delta\dot{w}_s}{\partial y} + \frac{\partial\dot{w}_s}{\partial y}\delta\,\dot{v}_0\right) + I_2\left(\frac{\partial\dot{w}_b}{\partial x}\frac{\partial\delta\,\dot{w}_b}{\partial x} + \frac{\partial\dot{w}_b}{\partial y}\frac{\partial\delta\,\dot{w}_b}{\partial y}\right) \\ &+ K_2\left(\frac{\partial\dot{w}_s}{\partial x}\frac{\partial\delta\,\dot{w}_s}{\partial x} + \frac{\partial\dot{w}_s}{\partial y}\frac{\partial\delta\,\dot{w}_s}{\partial y}\right) \\ &+ J_2\left(\frac{\partial\dot{w}_b}{\partial x}\frac{\partial\delta\,\dot{w}_s}{\partial x} + \frac{\partial\dot{w}_s}{\partial x}\frac{\partial\delta\,\dot{w}_b}{\partial x} + \frac{\partial\dot{w}_b}{\partial y}\frac{\partial\delta\,\dot{w}_s}{\partial y} + \frac{\partial\dot{w}_s}{\partial y}\frac{\partial\delta\,\dot{w}_b}{\partial y}\right) + J_1^s\left(\left(\dot{w}_b + \dot{w}_s\right)\delta\,\dot{\varphi} + \dot{\varphi}\,\delta\left(\dot{w}_b + \dot{w}_s\right)\right) + K_2^s\dot{\varphi}\,\delta\,\dot{\varphi}\Big\} dA\end{aligned} \tag{8}$$

where dot-superscript convention indicates the differentiation with respect to the time variable *t*; and (I_0, I_1, J_1, I_2, J_2, K_2) are mass inertias defined as

$$\left(I_0, I_1, J_1, J_1^s, I_2, J_2, K_2, K_2^s\right) = \int_{-h/2}^{h/2} \left(1, z, f, g, z^2, z\ f, f^2, g^2\right)\rho(z)dz \tag{9}$$

Substituting the Eqs. (6), and (8) into Eq. (5) and integrating by parts, and collecting the coefficients of δu_0, δv_0, δw_b, δw_s and $\delta\varphi$, the following equations of motion of the plate are obtained

$$
\begin{aligned}
&\delta u_0: \ \frac{\partial N_x}{\partial x}+\frac{\partial N_{xy}}{\partial y}=I_0\ddot{u}_0-I_1\frac{\partial \ddot{w}_b}{\partial x}-J_1\frac{\partial \ddot{w}_s}{\partial x}\\
&\delta v_0: \ \frac{\partial N_{xy}}{\partial x}+\frac{\partial N_y}{\partial y}=I_0\ddot{v}_0-I_1\frac{\partial \ddot{w}_b}{\partial y}-J_1\frac{\partial \ddot{w}_s}{\partial y}\\
&\delta w_b: \ \frac{\partial^2 M_x^b}{\partial x^2}+2\frac{\partial^2 M_{xy}^b}{\partial x\partial y}+\frac{\partial^2 M_y^b}{\partial y^2}=I_0\left(\ddot{w}_b+\ddot{w}_s\right)+I_1\left(\frac{\partial \ddot{u}_0}{\partial x}+\frac{\partial \ddot{v}_0}{\partial y}\right)-I_2\nabla^2\ddot{w}_b-J_2\nabla^2\ddot{w}_s+J_1^s\ddot{\varphi}\\
&\delta w_s: \ \frac{\partial^2 M_x^s}{\partial x^2}+2\frac{\partial^2 M_{xy}^s}{\partial x\partial y}+\frac{\partial^2 M_y^s}{\partial y^2}+\frac{\partial S_{xz}^s}{\partial x}+\frac{\partial S_{yz}^s}{\partial y}=I_0\left(\ddot{w}_b+\ddot{w}_s\right)+J_1\left(\frac{\partial \ddot{u}_0}{\partial x}+\frac{\partial \ddot{v}_0}{\partial y}\right)-J_2\nabla^2\ddot{w}_b-K_2\nabla^2\ddot{w}_s+J_1^s\ddot{\varphi}\\
&\delta \varphi: \ \frac{\partial S_{xz}^s}{\partial x}+\frac{\partial S_{yz}^s}{\partial y}-N_z=J_1^s\left(\ddot{w}_b+\ddot{w}_s\right)+K_2^s\ddot{\varphi}
\end{aligned}
\tag{10}
$$

2.3 Constitutive relations

The nonlocal theory considers that the stress at a point is related not only on the strain at that point but also on strains at all other points of the body. Such dependencies are related to the inter-atomic bonds between an atom and its neighboring atoms (Kiani 2013b). According to the nonlocal continuum theory (Eringen and Edelen 1972, Eringen 1983), the nonlocal stress tensor σ at a point is expressed as

$$\left(1-\mu\nabla^2\right)\sigma=\tau \tag{11}$$

where ∇^2 is the Laplacian operator in two-dimensional Cartesian coordinate system; τ is the classical stress tensor at a point related to the strain by the Hooke's law; and $\mu=(e_0a)^2$ is the nonlocal parameter which includes the small scale effect, a is the internal characteristic length and e_0 is a constant appropriate to each material.

For an isotropic micro/nanoscale plate, the nonlocal constitutive relation in Eq. (11) takes the following forms.

$$
\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \sigma_z\\ \tau_{yz}\\ \tau_{xz}\\ \tau_{xy}\end{Bmatrix}-\mu\nabla^2\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \sigma_z\\ \tau_{yz}\\ \tau_{xz}\\ \tau_{xy}\end{Bmatrix}=\begin{bmatrix}C_{11}&C_{12}&C_{13}&0&0&0\\ C_{12}&C_{22}&C_{23}&0&0&0\\ C_{13}&C_{23}&C_{33}&0&0&0\\ 0&0&0&C_{44}&0&0\\ 0&0&0&0&C_{55}&0\\ 0&0&0&0&0&C_{66}\end{bmatrix}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_z\\ \gamma_{yz}\\ \gamma_{xz}\\ \gamma_{xy}\end{Bmatrix} \tag{12}
$$

where C_{ij} are the three-dimensional elastic constants given by

$$C_{11}=C_{22}=C_{33}=\frac{(1-\nu)E}{(1-2\nu)(1+\nu)}$$

$$C_{12} = C_{13} = C_{23} = \frac{\nu\left(1-\nu\right)E}{\left(1-2\nu\right)\left(1+\nu\right)}$$
$$C_{44} = C_{55} = C_{66} = \frac{E}{2\left(1+\nu\right)} \tag{13}$$

E and ν are the elastic modulus and Poisson's ratio, respectively. By utilizing Eqs. (3), (13) and (7), the stress resultants can be written in terms of displacements as

$$\begin{Bmatrix} N \\ M^b \\ M^s \end{Bmatrix} - \mu\nabla^2 \begin{Bmatrix} N \\ M^b \\ M^s \end{Bmatrix} = \begin{bmatrix} A & B & B^s \\ B & D & D^s \\ B^s & D^s & H^s \end{bmatrix} \begin{Bmatrix} \varepsilon \\ k^b \\ k^s \end{Bmatrix} + \begin{bmatrix} L \\ L^a \\ R \end{bmatrix} \varepsilon_z^0, \quad S - \mu\nabla^2 S = A^s\gamma, \tag{14a}$$

$$N_z - \mu\nabla^2 N_z = R^a\varphi + L\left(\varepsilon_x^0 + \varepsilon_y^0\right) + L^a\left(k_x^b + k_y^b\right) + R\left(k_x^s + k_y^s\right), \tag{14b}$$

where

$$N = \left\{N_x, N_y, N_{xy}\right\}, \quad M^b = \left\{M_x^b, M_y^b, M_{xy}^b\right\}, \quad M^s = \left\{M_x^s, M_y^s, M_{xy}^s\right\}, \tag{15a}$$

$$\varepsilon = \left\{\varepsilon_x^0, \varepsilon_y^0, \gamma_{xy}^0\right\}, \quad k^b = \left\{k_x^b, k_y^b, k_{xy}^b\right\}, \quad k^s = \left\{k_x^s, k_y^s, k_{xy}^s\right\}, \tag{15b}$$

$$A = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix}, \quad B = \begin{bmatrix} B_{11} & B_{12} & 0 \\ B_{12} & B_{22} & 0 \\ 0 & 0 & B_{66} \end{bmatrix}, \quad D = \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{12} & D_{22} & 0 \\ 0 & 0 & D_{66} \end{bmatrix}, \tag{15c}$$

$$B^s = \begin{bmatrix} B_{11}^s & B_{12}^s & 0 \\ B_{12}^s & B_{22}^s & 0 \\ 0 & 0 & B_{66}^s \end{bmatrix}, \quad D^s = \begin{bmatrix} D_{11}^s & D_{12}^s & 0 \\ D_{12}^s & D_{22}^s & 0 \\ 0 & 0 & D_{66}^s \end{bmatrix}, \quad H^s = \begin{bmatrix} H_{11}^s & H_{12}^s & 0 \\ H_{12}^s & H_{22}^s & 0 \\ 0 & 0 & H_{66}^s \end{bmatrix}, \tag{15d}$$

$$S = \left\{S_{xz}^s, S_{yz}^s\right\}, \quad \gamma = \left\{\gamma_{xz}, \gamma_{yz}\right\}, \quad A^s = \begin{bmatrix} A_{44}^s & 0 \\ 0 & A_{55}^s \end{bmatrix}, \quad \begin{Bmatrix} L \\ L^a \\ R \\ R^a \end{Bmatrix} = \int_{-h/2}^{h/2} \lambda(z) \begin{Bmatrix} 1 \\ z \\ f(z) \\ g'(z)\dfrac{1-\nu}{\nu} \end{Bmatrix} g'(z)dz \tag{15b}$$

where the stiffness coefficients A_{ij} and B_{ij},… etc., are defined as

$$\begin{Bmatrix} A_{11} & B_{11} & D_{11} & B_{11}^s & D_{11}^s & H_{11}^s \\ A_{12} & B_{12} & D_{12} & B_{12}^s & D_{12}^s & H_{12}^s \\ A_{66} & B_{66} & D_{66} & B_{66}^s & D_{66}^s & H_{66}^s \end{Bmatrix} = \int_{-h/2}^{h/2} \lambda(z)\left(1, z, z^2, f(z), z\, f(z), f^2(z)\right) \begin{Bmatrix} \dfrac{1-\nu}{\nu} \\ 1 \\ \dfrac{1-2\nu}{2\nu} \end{Bmatrix} dz, \tag{16a}$$

and

$$\left(A_{22}, B_{22}, D_{22}, B_{22}^s, D_{22}^s, H_{22}^s\right) = \left(A_{11}, B_{11}, D_{11}, B_{11}^s, D_{11}^s, H_{11}^s\right), \tag{16b}$$

$$A_{44}^s = A_{55}^s = \int_{-h/2}^{h/2} \mu(z)[g(z)]^2 dz, \tag{16c}$$

2.4 Equations of motion in terms of displacements

The nonlocal equations of motion of the present formulation can be written in terms of generalized displacements (u_0, v_0, w_b, w_s and φ) by using the linear differential operator ($1-\mu\nabla^2$) on Eq. (10)

$$A_{11}d_{11}u_0 + A_{66}d_{22}u_0 + (A_{12}+A_{66})d_{12}v_0 = (1-\mu\nabla^2)I_0\ddot{u}_0, \tag{17a}$$

$$A_{22}d_{22}v_0 + A_{66}d_{11}v_0 + (A_{12}+A_{66})d_{12}u_0 = (1-\mu\nabla^2)I_0\ddot{v}_0 \tag{17b}$$

$$\begin{aligned} &- D_{11}d_{1111}w_b - 2(D_{12}+2D_{66})d_{1122}w_b - D_{22}d_{2222}w_b \\ &+ L^a(d_{11}\varphi + d_{22}\varphi) = (1-\mu\nabla^2)\left[I_0(\ddot{w}_b+\ddot{w}_s) - J_2(d_{11}\ddot{w}_b + d_{22}\ddot{w}_b) - K_2(d_{11}\ddot{w}_s + d_{22}\ddot{w}_s) + J_1^s\ddot{\varphi}\right] \end{aligned} \tag{17c}$$

$$\begin{aligned} &- D_{11}^s d_{1111}w_b - 2(D_{12}^s + 2D_{66}^s)d_{1122}w_b - D_{22}^s d_{2222}w_b - H_{11}^s d_{1111}w_s - 2(H_{12}^s + 2H_{66}^s)d_{1122}w_s - H_{22}^s d_{2222}w_s \\ &+ A_{44}^s d_{11}w_s + A_{55}^s d_{22}w_s + R(d_{11}\varphi + d_{22}\varphi) \\ &+ A_{44}^s d_{11}\varphi + A_{55}^s d_{22}\varphi = (1-\mu\nabla^2)\left[I_0(\ddot{w}_b+\ddot{w}_s) - J_2(d_{11}\ddot{w}_b + d_{22}\ddot{w}_b) - K_2(d_{11}\ddot{w}_s + d_{22}\ddot{w}_s) + J_1^s\ddot{\varphi}\right] \end{aligned} \tag{17d}$$

$$\begin{aligned} &- L^a(d_{11}w_b + d_{22}w_b) - (R + A_{44}^s)d_{11}w_s - (R + A_{55}^s)d_{22}w_s \\ &+ R^a\varphi - A_{44}^s d_{11}\varphi - A_{55}^s d_{22}\varphi = (1-\mu\nabla^2)\left[J_1^s(\ddot{w}_b+\ddot{w}_s) + K_2^s\ddot{\varphi}\right] \end{aligned} \tag{17e}$$

It is observed from Eq. (17) that the in-plane displacements (u_0, v_0) are uncoupled from the transverse displacements (w_b, w_s and φ). Thus, the equations of motion for the transverse response of the plate are reduced to Eqs. (17c)-(17e).

3. Analytical solution of simply supported nanoplate

In this work, we are concerned with the exact solutions of Eq. (17) for a simply supported nanoplate. The following boundary conditions are imposed at the side edges

$$w_b = w_s = \frac{\partial w_s}{\partial y} = \varphi = M_x^b = M_x^s = 0 \quad \text{at} \quad x = 0,\ a \tag{18a}$$

$$w_b = w_s = \frac{\partial w_s}{\partial x} = \varphi = M_y^b = M_y^s = 0 \quad \text{at} \quad y = 0,\ b \tag{18b}$$

Following the Navier solution procedure, we assume the following solution form for w_b, w_s and φ that satisfies the boundary conditions given in Eq. (18)

$$\begin{Bmatrix} w_b \\ w_s \\ \varphi \end{Bmatrix} = \begin{Bmatrix} W_{bmn}e^{i\omega t}\sin(\alpha\, x)\sin(\beta\, y) \\ W_{smn}e^{i\omega t}\sin(\alpha\, x)\sin(\beta\, y) \\ \Phi_{mn}e^{i\omega t}\sin(\alpha\, x)\sin(\beta\, y) \end{Bmatrix} \tag{19}$$

where W_{bmn}, W_{smn} and Φ_{mn} are arbitrary parameters to be determined, ω is the eigenfrequency associated with (m, n)th eigenmode, and $\alpha=m\pi/a$ and $\beta=n\pi/b$.

Substituting Eq. (19) into Eq. (17), the analytical solutions can be obtained from

$$\left(\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{12} & a_{22} & a_{23} \\ a_{13} & a_{23} & a_{33} \end{bmatrix} - \lambda\omega^2 \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{12} & m_{22} & m_{23} \\ m_{13} & m_{23} & m_{33} \end{bmatrix}\right)\begin{Bmatrix} W_{bmn} \\ W_{smn} \\ \Phi_{mn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \tag{20}$$

where

$$a_{11} = -\left(D_{11}\alpha^4 + 2(D_{12} + 2D_{66})\alpha^2\beta^2 + D_{22}\beta^4\right)$$
$$a_{12} = -\left(D_{11}^s\alpha^4 + 2(D_{12}^s + 2D_{66}^s)\alpha^2\beta^2 + D_{22}^s\beta^4\right)$$
$$a_{13} = -L^a\left(\alpha^2 + \beta^2\right)$$
$$a_{22} = -\left(H_{11}^s\alpha^4 + 2(H_{11}^s + 2H_{66}^s)\alpha^2\beta^2 + H_{22}^s\beta^4 + A_{55}^s\lambda^2 + A_{44}^s\beta^2\right)$$
$$a_{23} = -\left(A_{44}^s\alpha^2 + A_{55}^s\beta^2 + R\left(\alpha^2 + \beta^2\right)\right)$$
$$a_{33} = -\left(A_{44}^s\alpha^2 + A_{55}^s\beta^2 + R^a\right)$$
$$m_{11} = -\left(I_0 + I_2\left(\alpha^2 + \beta^2\right)\right), \quad m_{12} = -\left(I_0 + J_2\left(\alpha^2 + \beta^2\right)\right), m_{22} = -\left(I_0 + K_2\left(\alpha^2 + \beta^2\right)\right),$$
$$m_{13} = m_{23} = -J_1^s, \quad m_{33} = -K_2^s, \lambda = 1 + \mu\left(\alpha^2 + \beta^2\right) \tag{21}$$

4. Validation and comparison of results

In this section, the accuracy of the presented quasi-3D trigonometric plate theory for the free vibration of the nanoplates is demonstrated by comparing the analytical solution with those of other available results in the literature. In addition, the influences of the nonlocal parameter, shear deformation and thickness stretching on the vibration behaviors of the micro/nanoscale plates are investigated.

As a first example, in Table 1, a comparison of the first non-dimensional natural frequency parameters $\hat{\omega}$ is carried out for the isotropic rectangular plate (μ=0), with the solution of Liew *et al*. (1993) and Alibeigloo (2011). It can be observed that the results of the three-dimensional Ritz method developed by Liew *et al*. (1993) and the three-dimensional elasticity solutions developed by Alibeigloo (2011) are in a good agreement with the present results of quasi-3D trigonometric plate theory.

In the second example, the validation of the solution of the proposed quasi-3D trigonometric plate model is carried out by comparing the obtained results with those computed via finite element model using ABAQUS software package with considering the mesh convergence study to optimize the results. The FEM solution of abaqus software is obtained by using "S4R" shell elements.

The analysis of a FEM using ABAQUS software package was performed and results are tabulated in Table 2 and plotted in Fig. 1. The following parameters are used for numerical computations: a=10h, E=1, v=0.3, ρ=1. As clearly shown in Table 2 and Fig. 1, the convergence

Table 1 Comparison of dimensionless first natural frequency $\hat{\omega} = \left(\omega a^2/\pi^2\right)\sqrt{\rho h/D}$ of the isotropic rectangular plate, μ=0, ($D=Eh^3/12(1-\nu^2)$), (a/b=1.5)

Theory	*a/h*				
	5/2	10/3	5	10	100
3D Ritz method Liew *et al*. (1993)	1.0954	1.2088	1.3209	1.4096	1.444
Elasticity 3D Alibeigloo (2011)	1.0940	1.2075	1.3200	1.4096	1.444
Present ($\varepsilon_z \neq 0$)	1.0996	1.2122	1.3237	1.4120	1.446

Table 2 Number of elements used to achieve optimum mesh for isotropic plates "S4R"

Approximate Global Size	Number of Mesh	Frequency $\bar{\omega}_{11} = \omega_{11} h\sqrt{\rho/G}$
0.2	50×50	0.089425
0.1	100×100	0.089390
0.05	200×200	0.089381
0.04	250×250	0.089380
0.025	400×400	0.089380

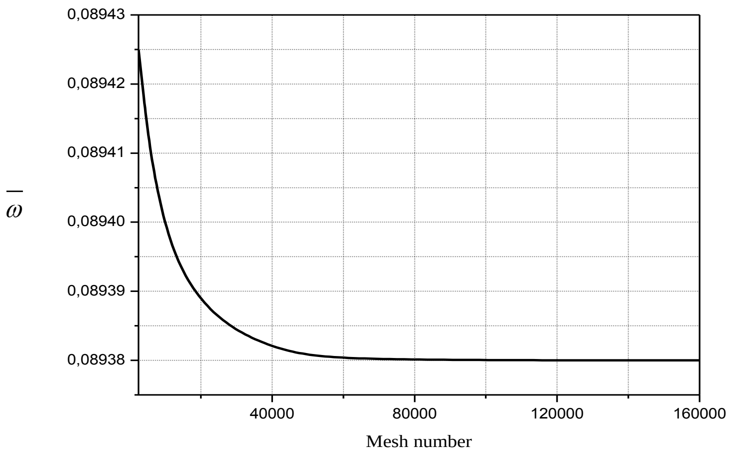

Fig. 1 The natural frequency against mesh number «S4R»

begins from 50×50 mesh number with highest non-dimensional frequency 0.089425 and fully converges at 400×400 with lowest non-dimensional frequency 0.089380, but 250×250 mesh number with lowest non-dimensional frequency 0.089380 was chosen for the comparison, in order to reduce the number of nodes and elements in the analyses.

The dimensionless natural frequencies corresponding to different quasi-3D vibration modes are provided for isotropic square plates. For the modes (m, n), the integers m and n denote the number of half-waves in the x and y directions, respectively. The comparison of the dimensionless frequencies of isotropic square plates for three different planar half-wave numbers (i.e., m and n) are presented in Tables 3. The dimensionless natural frequencies $\bar{\omega}$ obtained by the finite element model are approximately equal to those of the present quasi-3D trigonometric plate model

Table 3 Natural frequencies $\bar{\omega}=\omega h\sqrt{\rho/G}$ of an isotropic plate with ν=0.3, a/h=10 and a/b=1

(m, n)	Present ε_z=0	Present $\varepsilon_z\neq$0	Abaqus FEM	Error (%)
(1,1)	0.0930	0.0931	0.08938	4,16
(1,2) or (2,1)	0.2220	0.2226	0.21678	2,68
(2,2)	0.3406	0.3421	0.32894	4,00

Table 4 Non-dimensional first mode frequency $\bar{\omega}=\omega h\sqrt{\rho/G}$ of simply supported square plate (a=10, E=30×10^6, ν=0.3, ρ=1)

μ (nm^2)	Aghababaei and Reddy (2009): TSDT	Lee *et al*. (2012): TSDT	Present without Stretching effect (ε_z=0)	Present Stretching effect ($\varepsilon_z\neq$0)
0	0.0935	0.0930	0.0930	0.0933
1	0.0854	0.0850	0.0850	0.0853
2	0.0791	0.0788	0.0788	0.0790
3	0.0741	0.0737	0.0737	0.0740
4	0.0699	0.0695	0.0695	0.0698
5	0.0663	0.0660	0.0660	0.0662

(Table 3).

The measurement of a "relative error" is defined by the relationship

$$error\,(\%)=\frac{\left(M_c-M_m\right)}{M_m}\times 100\% \tag{22}$$

Where M_m is the natural frequencies given by the discrete finite element model, and M_c that of present quasi-3D trigonometric plate theory. The errors can be seen in Table 3. It is observed that the present quasi-3D trigonometric plate theory is in a good agreement with the finite element solution.

To illustrate the influence of nonlocal parameter μ on the vibration response of the nanoplates, the non-dimensional natural frequency parameters are computed and compared with the solution of Aghababaei and Reddy (2009) and Lee *et al.* (2012) using the third shear deformation plates theory (TSDT) with respect to various non-local parameters. The following parameters are employed to obtain the numerical values: a=10h, E=30×10^6, ν=0.3, ρ=1. The non-dimensional frequencies $\bar{\omega}$ of a simply supported nanoplates are presented in Tables 4 for various values of the small scale parameter μ. A good agreement is demonstrated between the present results and those of Aghababaei and Reddy (2009) and those of Lee *et al.* (2012) and this whatever the value of the nonlocal parameter μ. Since the quasi-3D model of the present formulation includes the thickness stretching effect, the non-dimensional natural frequency parameters are slightly raised with respect to other frequencies documented in Table 4. Thus, the inclusion of thickness stretching effect makes the nanoplate stiffer.

In the next example, the nondimensional natural frequency parameters $\hat{\omega}$ of a simply supported nanoplate are presented in Table 5 for various values of thickness ratio a/h and scale parameter μ. For this end, the geometric and the material properties of nanoplates are taken as E=1.02 TPa, ν=0.16 and ρ=2.250 kg/m^3. A value of h=0.34 nm is assumed for the thickness of SLGS (Kitipornchai *et al.* 2005). For the FSDT solutions the shear correction factor k=5/6 is

Table 5 Nondimensional fundamental frequency $\hat{\omega}$ of simply supported square plates (h=0.34 nm, E=1.02 TPa, ρ=2.250 kg/m^3, v=0.16)

(a/h)	$\sqrt{\mu}(nm)$	Frequency $\hat{\omega}=\left(\omega a^2/h\right)\sqrt{\rho/E}$			
		FSDT	TSDT	Present (ε_z=0)	Present ($\varepsilon_z\neq$0)
5	0	5.1759	5.1767	5.1774	5.1897
	0.5	3.1456	3.1461	3.1465	3.1539
	1	1.8497	1.8500	1.8502	1.8546
	1.5	1.2794	1.2795	1.2797	1.2828
	2	0.9726	0.9726	0.9729	0.9752
10	0	5.5997	5.599	5.5999	5.6050
	0.5	4.6878	4.6878	4.6880	4.6922
	1	3.4031	3.4031	3.4032	3.4063
	1.5	2.5448	2.5448	2.5449	2.5472
	2	2.0011	2.0011	2.0012	2.0030
20	0	5.7275	5.7275	5.7275	5.7300
	0.5	5.4444	5.4443	5.4444	5.4468
	1	4.7948	4.7948	4.7948	4.7969
	1.5	4.0906	4.0905	4.0906	4.0924
	2	3.4808	3.4808	3.4808	3.4823

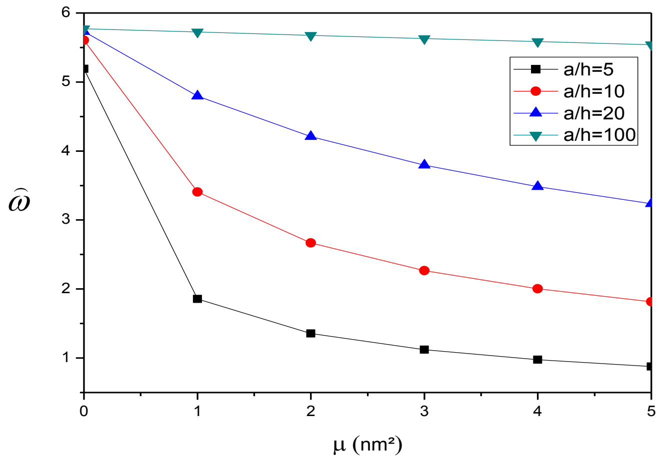

Fig. 2 The variation of non-dimensional natural frequency parameters $\hat{\omega}$ against the nonlocal parameter with different the aspect of thickness ratio (a/h) of the simply supported square nanoplates

adopted. It can be seen that the results of present theory without the thickness stretching (i.e., ε_z=0) are in excellent agreement with those predicted by (TSDT) and (FSDT) for all values of scale parameter even, for of nanoplates having different thickness ratios a/h. The inclusion of thickness stretching effect (i.e., $\varepsilon_z\neq$0) leads to an increase in frequency.

In general, it can be concluded from Tables 4-5 that the inclusion of thickness stretching effect (i.e., $\varepsilon_z\neq$0) makes a beam stiffer, and hence, leads to increase of frequency. This effect is considerable for deep nanoplates. Furthermore, the local theory overestimates the natural

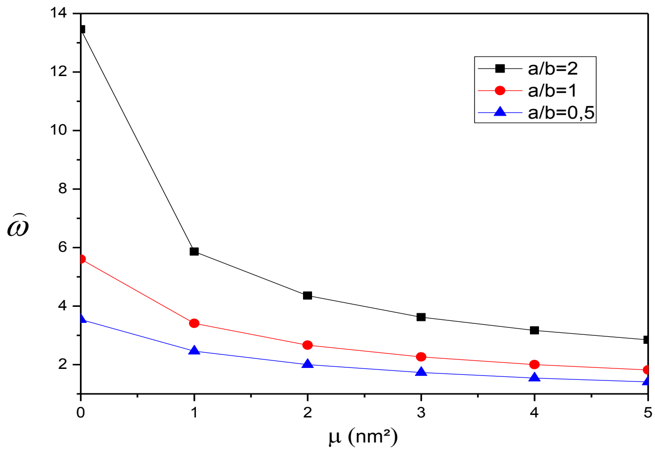

Fig. 3 The variation of non-dimensional natural frequency parameters $\hat{\omega}$ against the nonlocal parameterwith different the aspect of ratio (*a/b*) of the simply supported square nanoplates.

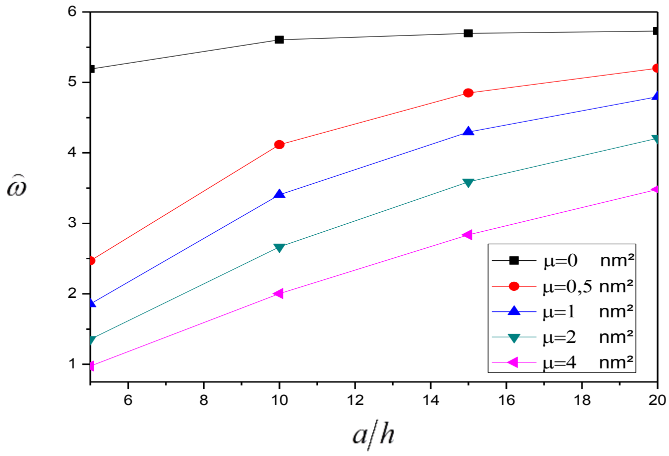

Fig. 4 The variation of non-dimensional natural frequency parameters $\hat{\omega}$ against the aspect of thickness ratio (*a/h*) with different nonlocal parameter of the simply supported square nanoplates

frequencies of the nanoplates compared to the nonlocal one, and the difference between local and nonlocal theories is significant for high value of the scale parameter. This is due to the fact that the local theory is unable to capture the small scale effect of the nanoplates.

The effects of the nonlocal parameter together with the influence of aspect thickness ratio on the first nondimensional natural frequency parameters $\hat{\omega}$ of the simply supported square nanoplates are shown in Fig. 2. It can be seen that the first nondimensional natural frequency parameters decrease monotonically by increasing the nonlocal parameter.

The impacts of aspect ratio (*a/b*) in conjunction with the nonlocal parameter on non-dimensional natural frequency parameters of the nanoplates are displayed in Fig. 3. The results show that increasing the aspect ratio, the non-dimensional natural frequency parameters of the nanoplates reduces. This is due to the reduction in the stiffness of the nanoplates. In addition, it is found that the non-dimensional natural frequency parameters decrease with increase of the nonlocal parameter for all aspect ratio considered. For smaller aspect ratios of the nanoplates, the non-dimensional natural frequency parameters decrease in a linear way with nonlocal parameter. However, for higher aspect ratios of the nanoplates, the variation of the non-dimensional natural frequency parameters is smoother and follows a slightly nonlinear way with nonlocal parameter.

The influence of the aspect thickness ratio (*a/h*) and the nonlocal parameter on the non-dimensional natural frequency parameters is shown in Fig. 4 for a square plate. It can be seen that as the aspect of thickness ratio (*a/h*) of the plate increases, the fundamental frequency tends to increase. And as nonlocal parameter decreases, the fundamental frequency increases.

5. Conclusions

A quasi-3D trigonometric plate theory for micro/nanoscale plates is developed for the vibration of nanoplates. The present model is capable of capturing small scale, shear deformation and thickness stretching effects of nanoplates, and satisfies the zero traction boundary conditions on the top and bottom surfaces of the nanoplates without considering the shear correction factor. Based on the nonlocal differential constitutive relation of Eringen, the nonlocal equations of motion of the proposed theory are derived from Hamilton's principle. Results show that the inclusion of thickness stretching effect ($\varepsilon_z \neq 0$) makes a nanoplates stiffer, and hence, leads to increase of the natural frequency. However, it is observed that the inclusion of the small scale and shear deformation effects lead to a reduction of the natural frequencies of nanoplates. This work is expected to be useful to design and analyze the vibration responses of nanoscale physical devices. The formulation lends itself particularly well to functionally graded plates (Khalfi *et al.* 2014), free vibration response of a single-layer graphene sheet embedded in an elastic medium (Samaei *et al.* 2014), and also combining surface effects and non-local plate theories on the buckling and vibration of nanoplates (Karimi *et al.* 2015), which will be considered in the near future.

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