

Evaluation of numerical procedures to determine seismic response of structures under influence of soil-structure interaction

Hamid Reza Tabatabaiefar^{*1}, Behzad Fatahi², Kazem Ghabraie³
and Wan-Huan Zhou⁴

¹*School of Engineering and Information Technology, Federation University Australia, Australia*

²*Centre for Built Infrastructure Research, School of Civil and Environmental Engineering,
University of Technology Sydney (UTS), Australia*

³*School of Civil Engineering and Surveying, University of Southern Queensland, Australia*

⁴*Department of Civil and Environmental Engineering, University of Macau, Macau, China*

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Abstract. In this study, the accuracy and reliability of fully nonlinear method against equivalent linear method for dynamic analysis of soil-structure interaction is investigated comparing the predicted results of both numerical procedures with the results of experimental shaking table tests. An enhanced numerical soil-structure model has been developed which treats the behaviour of the soil and the structure with equal rigour. The soil-structural model comprises a 15 storey structural model resting on a soft soil inside a laminar soil container. The structural model was analysed under three different conditions: (i) fixed base model performing conventional time history dynamic analysis, (ii) flexible base model (considering full soil-structure interaction) conducting equivalent linear dynamic analysis, and (iii) flexible base model performing fully nonlinear dynamic analysis. The results of the above mentioned three cases in terms of lateral storey deflections and inter-storey drifts are determined and compared with the experimental results of shaking table tests. Comparing the experimental results with the numerical analysis predictions, it is noted that equivalent linear method of dynamic analysis underestimates the inelastic seismic response of mid-rise moment resisting building frames resting on soft soils in comparison to the fully nonlinear dynamic analysis method. Thus, inelastic design procedure, using equivalent linear method, cannot adequately guarantee the structural safety for mid-rise building frames resting on soft soils. However, results obtained from the fully nonlinear method of analysis fit the experimental results reasonably well. Therefore, this method is recommended to be used by practicing engineers.

Keywords: soil-structure interaction; shaking table test; inelastic seismic response; equivalent linear method; fully nonlinear method; inelastic design procedure

1. Introduction

Soil-Structure Interaction (SSI) includes a set of mechanisms accounting for the flexibility of

^{*}Corresponding author, Lecturer, E-mail: h.tabatabaiefar@federation.edu.au

the foundation support beneath a given structure resulting in altering the ground motion in the vicinity of the foundation compared to the free-field. It determines the actual loading experienced by the soil-structure system resulting from the free-field seismic ground motions. The seismic excitation experienced by structures is a function of the earthquake characteristics, travel path effects, local site effects, and soil-structure interaction effects. The result of the first three of these factors can be summarised as the free-field ground motion. In addition, structural response to the free-field motion is influenced by SSI. In particular, accelerations within the structure are affected by the flexibility of the foundation and the difference between the foundation and the free-field motions (Fatahi *et al.* 2011, Tabatabaiefar *et al.* 2012, Hokmabadi *et al.* 2014).

The importance of the soil-structure interaction both for static and dynamic loads has been well established and the related literature covers at least 30 years of computational and analytical approaches for solving soil-structure interaction problems. Since 1990s, great effort has been made for substituting the classical methods of design by the new ones based on the concept of performance-based seismic design. Performance-based engineering (PBE) is a technique for seismic evaluation and design using performance level prediction for safety and risk assessment. Soil-structure interaction particularly for un-braced structures resting on relatively soft soils may significantly amplify the lateral displacements and inter-storey drifts (Tabatabaiefar *et al.* 2014a, Hokmabadi *et al.* 2016). This amplification of lateral deformations may change the performance level of the building frames. Thus, a comprehensive dynamic analysis to evaluate the realistic performance level of a structure should consider effects of SSI in the model (Tabatabaiefar *et al.* 2014b). In addition, the necessity of estimating the vulnerability of existing structures and assessing reliable methods for their retrofit have greatly attracted the attention of engineering community in most seismic zones throughout the world.

Over the past few years, application of performance-based seismic design concept has been promoted and developed rapidly. The development of this approach has been a natural outgrowth of the evaluation and upgrade process for existing buildings. Performance objectives are expressed as an acceptable level of damage, typically categorised as one of several performance levels. Performance levels describe the state of structures after being subjected to a certain hazard level and are classified as: fully operational, operational, life safe, near collapse, or collapse (FEMA 273 1997). Overall lateral deflection, ductility demand, and inter-storey drifts are the most commonly used damage parameters. The above mentioned five qualitative performance levels are related to the corresponding quantitative maximum inter-storey drifts of: 0.2%, 0.5%, 1.5%, 2.5%, and 2.5%, respectively.

Several researchers (e.g., Spyrakos *et al.* 1989, Veletsos and Prasad 1989, Safak 1995, Krawinkler *et al.* 2003, Galal and Naimi 2008, El Ganainy and El Naggar 2009, Tabatabaiefar *et al.* 2013a, Fatahi *et al.* 2014, Zhang *et al.* 2014) studied structural behaviour of un-braced structures subjected to earthquake under the influence of soil-structure interaction. In addition, during the recent decades, the importance of dynamic soil-structure interaction for several structures founded on soft soils has been well recognised. Examples are given by Gazetas and Mylonakis (1998) including evidence that some structures founded on soft soils are vulnerable to the soil-structure interaction.

Several efforts have been made in recent years in the development of numerical methods for assessing the response of structures and supporting soil media under seismic loading conditions. Successful application of these methods for determining ground seismic response is vitally dependent on the incorporation of the soil properties in the analyses. As a result, substantial effort

has also been made toward the determination of soil attributes for using in these analytical procedures. There are two main numerical procedures for computation of seismic response of structures under the influence soil-structure interaction; (i) equivalent linear method, and (ii) fully nonlinear method. The traditional standard practice for dynamic analysis of soil-structure systems has been based on equivalent linear method which is mainly based on try and error. The try and error process would be continued until approaching to a certain value. Various analytical studies have been carried out to compute the seismic response of structures adopting equivalent linear dynamic analysis for soil-structure interaction (e.g., Stewart *et al.* 1999, Dutta *et al.* 2004, Maheshwari and Sarkar 2011, Turan *et al.* 2013) due to its simplicity and adoptability to the most structural software around the globe. However, effects of soil nonlinearity of the supporting soil on the seismic response of structures have not been fully addressed in the literature adopting fully nonlinear dynamic analysis method. The fully nonlinear analysis has not been applied as often in practical design due to its complexity and requirement to advanced computer programmes. However, practical applications of fully nonlinear analysis have increased in the last decade, as more emphasis is placed on reliable predictions in dynamic analysis of complex soil-structure systems (Byrne *et al.* 2006). Fatahi and Tabatabaiefar (2014) have studied and investigated the accuracy of nonlinear method against equivalent linear method for dynamic analysis of soil-structure interaction using numerical procedures. However, the main shortcoming of the study was lack of verification of the outcomes with experimental results. As a result, in this study, the accuracy of fully nonlinear method against equivalent linear method for dynamic analysis of soil-structure interaction is investigated comparing the predicted results of both numerical procedures with the measured results of shaking table tests. The main goal of this comparison is to pinpoint whether the simplified equivalent linear method of analysis is adequately accurate to determine reliable inelastic seismic response of mid-rise building frames or it is necessary to employ fully nonlinear method in order to attain rigorous and reliable results.

2. Numerical procedures for computation of seismic response

The equivalent linear method has been in use for many years to compute the seismic response of the structures at sites subjected to seismic excitation. In equivalent linear method, a linear analysis is carried out with some assumed initial values for damping and shear modulus ratios of the soil, often referred to as equivalent linear material parameters. Then, the maximum cyclic shear strain of the soil is recorded for each element and used to determine the new values for damping and modulus, utilising the backbone curves relating damping ratio and secant modulus to the amplitude of the shear strain. The new values of damping ratio and shear modulus are then used in the next stage of the numerical analysis. The whole process is repeated several times, until there is no further change in the properties and the structural response. At this stage, “strain-compatible” values of damping and modulus are recorded, and the simulation using these values is deemed to be the best possible prediction of the real behaviour. Rayleigh damping may be used in this method to simulate energy losses in the soil-structure system when subjected to a dynamic loading. Seed and Idriss (1969) described that equivalent linear method employs linear properties for each element, which remain constant under the influence of seismic excitations. Those values, as explained, are estimated from the mean level of the dynamic motion. Other characteristics of the equivalent linear method are as follows (Seed and Idriss 1969):

- The interference and mixing phenomena taking place between different frequency components in a nonlinear material are missing from an equivalent linear analysis;
- The method does not directly provide information on irreversible displacements and the permanent changes; and
- In the case where both shear and compression waves are propagated through a site, the equivalent linear method typically treats these motions independently.

Despite the above mentioned draw backs of the method, as the most structural software can only adopt equivalent linear method, it has been considered as a simple and popular approach for the dynamic analysis considering the soil-structure interaction effects (Maheshwari and Sarkar 2011).

Fully nonlinear method is capable to model nonlinearity in dynamic analysis of soil-structure systems precisely and follows any prescribed nonlinear constitutive relation. In addition, structural geometric nonlinearities (large displacements) can be accommodated precisely in this method. During the solution process, structural materials could behave as isotropic, linearly elastic materials with no failure limit for elastic analysis, or as elasto-plastic materials with specified limiting plastic moment for inelastic structural analysis to simulate elastic-perfectly plastic behaviour. For the dynamic analysis, the damping of the system in the numerical simulation should be reproduced in magnitude and form, simulating the energy losses in the natural system subjected to the dynamic loading. In soil and rock, natural damping is mainly hysteretic (Gemant and Jackson 1937). Hysteretic damping algorithm which is incorporated in this solution method enables the strain-dependent modulus and damping functions to be incorporated directly into the numerical simulation. Other characteristics of fully nonlinear method are as follows (Byrne *et al.* 2006):

- Nonlinear material law, interference and combination of different frequency components can be considered simultaneously;
- Irreversible displacements and other permanent changes can be modelled as required; and
- Both shear and compression waves are propagated together in a single simulation, and the material responds to the combined effect of both components.

In order to perform fully nonlinear dynamic analysis, a computer program treating both soil nonlinearity and structural inelastic behaviour rigorously is required. Thus, structural engineers prefer to use equivalent linear approach using trial and error process based on the available seismic codes recommendations.

Byrne *et al.* (2006), Beaty and Byrne (2001) reviewed the above mentioned methods and discussed the benefits of the fully nonlinear numerical method over the equivalent linear method for different practical applications. The equivalent linear method does not directly capture any nonlinearity effects due to linear solution process. In addition, strain-dependent modulus and damping functions are only taken into account in an average sense, in order to approximate some effects of nonlinearity, while fully nonlinear method correctly represents the physics associated with the problem and follows any stress-strain relation in a realistic way. In this method, small strain shear modulus and damping degradation of soil with strain level can be captured precisely in the modelling. However, the remaining question is that “does the equivalent linear approach result in conservative thus satisfactory design of soil-structure systems or not?” In addition, in this research, it has been tried to figure out the range of error for the mentioned two methods.

3. Developed numerical soil-structure model

The governing equations of motion for the structure incorporating foundation interaction and the method of solving these equations are relatively complex. Therefore, direct method, the method in which the entire soil-structure system is modelled in a single step, is employed in this study. To model soil-structure system in direct method, a novel and enhanced soil-structure model is developed in FLAC2D to simulate various aspects of complex dynamic soil-structure interaction in a realistic and rigorous manner. Rayhani and Naggar (2008), after undertaking comprehensive numerical modelling and centrifuge model tests, concluded that the horizontal distance of the soil lateral boundaries should be at least five times the width of the structure in order to avoid reflection of outward propagating waves back into the model. They also recommended 30 metres as the maximum bedrock depth in the numerical analysis as the most amplification occurs within the first 30 metres of the soil profile, which is in agreement with most of modern seismic codes (e.g., ATC-40 1996, BSSC 2003). Those seismic codes evaluate local site effects just based on the properties of the top 30 meters of the soil profile. Thus, in this study, the horizontal distance of the soil lateral boundaries is assumed to be 60 metres (five times the width of the structure which is 12 metres) and the maximum bedrock depth is 30 metres. The utilised simulation and idealisation procedure to develop numerical soil-structure model as well as the characteristics of the model components and boundary conditions have been explained in detail by Tabatabaiefar *et al.* (2013b).

The soil-structure model employs beam structural elements to model beams, columns and foundation slabs. During analysis process, structural material could behave as an isotropic, linearly elastic material with no failure limit for elastic structural analysis or as an elastic-perfectly plastic material with a specified limiting plastic moment for inelastic structural analysis. Therefore, both elastic and plastic (inelastic) structural behaviour can be captured by the model in dynamic analysis. In addition, structural geometric nonlinearity (large displacements) has been accommodated in dynamic analysis. Two dimensional plane-strain grids composed of quadrilateral elements are utilised to model the soil medium. Nonlinear behaviour of the soil medium has been captured using backbone curves of shear modulus ratio versus shear strain ($G/G_{\max}-\gamma$) and damping ratio versus shear strain ($\xi-\gamma$) while adopting Mohr-Coulomb shear failure model. Employing the backbone curves for simulating nonlinear behaviour of the soil, in this study, equivalent linear and fully nonlinear methods for analysis of dynamic soil-structure interaction have been employed. Fully nonlinear method is capable to precisely model nonlinearity in dynamic analysis of soil-structure systems and follow any prescribed nonlinear constitutive relation.

The new developed model is a novel and enhanced numerical soil-structure model as it is capable of capturing structural plasticity (by introducing the plastic moments, M^p , for the structural sections) and soil nonlinearity, treating the behaviour of both soil and structure with equal rigor simultaneously. Besides, adopting direct method, which perfectly simulates complex geometries and material properties in numerical methods, the model can perform fully nonlinear time history dynamic analysis to simulate realistic dynamic behaviour of soil and structure under seismic excitations as accurate and realistic as possible. In addition, as the model employs a Multi Degree of Freedom (MDOF) structure, inter-storey drifts can be determined and utilised for investigating the performance levels of the building structures under the influence of dynamic soil-structure interaction.

4. Shaking table tests

In this study, the developed numerical soil-structure model has been validated and verified by performing shaking table tests to the scale soil-structure model. The dynamic simulation has been carried out on the shaking table located in the structures laboratory of the University of Technology, Sydney (UTS). It should be noted that UTS shaking table has a uni-axial configuration, allowing for one-dimensional input motions. The shaking table is 3 m×3 m table with testing frequency range between 0.1 to 50 HZ, maximum payload of 10 tonnes, and overturning moment of 100 kN-m.

The prototype building frame of the soil-structure system is a fifteen storey concrete moment resisting frame. The building frame height and width are 45 and 12 metres, respectively and spacing between the frames into the page is 4 metres. The building is resting on a footing which is 4 meters wide and 12 meters long. Natural frequency of the prototype building is 0.384 Hz and its total mass is 953 tonnes. Soil medium underneath the structure is a clayey soil with shear wave velocity of 200 m/s and unit weight of 14.40 kN/m³ (soil density of 1470 kg/m³). The horizontal distance of the soil lateral boundaries and bedrock depth are selected to be 60 metres and 30 metres, respectively.

4.1 Scaling factors for shaking table testing

Scale models can be defined as having geometric, kinematic, or dynamic similarities to the prototype (Langhaar 1951, Sulaeman 2010). Geometric similarity defines a model and prototype with homologous physical dimensions. Kinematic similarity refers to a model and prototype with homologous particles at homologous points at homologous times. Dynamic similarity describes a condition where homologous parts of the model and prototype experience homologous net forces. The objective of the scale modelling procedure for this test program is to achieve “dynamic similarity”, where model and prototype experience homologous forces. For this purpose, adopted methodology by Meymand (1998) is the framework for scale model similitude in this study. According to this approach, three principal test conditions establish many of the scaling parameters. The first condition is that testing is conducted in 1-g environment, which defines model and prototype accelerations to be equal. Secondly, a model with similar density to the prototype is desired, fixing another component of the scaling relations. Thirdly, the test medium is primarily composed of saturated clayey soil, whose undrained stress-strain response is independent of confining pressure, thereby simplifying the constitutive scaling requirements. In addition to the three principal test conditions, Meymand (1998) pointed out that the natural frequency of the prototype should be scaled by an appropriate scaling relation. By defining scaling conditions for density and acceleration, the mass, length, and time scale factors can all be expressed in terms of the geometric scaling factor (λ), and a complete set of dimensionally correct scaling relations (ratio of prototype to model) can be derived for all variables being studied. The scaling relations for the variables contributing to the primary modes of system response, adopted in this study, are shown in Table 1. The mentioned scaling relations have been utilised by many researchers (e.g., Meymand 1998, Turan *et al.* 2009, Moss *et al.* 2010, Sulaeman 2010, Zhu *et al.* 2010, Lee *et al.* 2012, Hokmabadi *et al.* 2015) in soil-structure interaction shaking table test experiments.

Adopting an appropriate geometric scaling factor (λ) is one of the important steps in scale modelling on shaking table. Although small scale models could save cost, the precision of the results could be substantially reduced. Considering the specifications of UTS shaking table,



Fig. 1 Structural model

scaling factor of 1:30 provides the largest achievable scale model with rational scales, maximum payload, and overturning moment which meet the facility limitations. Thus, geometric scaling factor (λ) of 1:30 is adopted for experimental shaking table tests on the scale model in this study.

4.2 Soil-structure model components

In this study, soil-structure model possesses three main components including the structural model, the laminar soil container, and the soil mix. Employing geometric scaling factor of 1:30, height, length, and width of the structural model are determined to be, 1.50 m, 0.40 m, and 0.40 m, respectively. The finalised base plate is a 500×500×10 mm steel plate while the floors consist of 400×400×5 mm plates and four 500×40×2 mm steel plates are used for the columns. The connections between the columns and floors are provided using stainless steel metal screws with 2.5 mm diameter and 15 mm length. After the numerical modelling and design, the structural model was constructed in house. The completed structural model is shown in Fig. 1. The mass of the model (m_m), without the base plate, was measured to be 104 kg which matches the required structural mass. Total measured mass of the structural model considering the mass of the base plate is 115kg. Numerical modelling and design as well as testing and construction procedure of the structural model have been explained by Tabatabaiefar (2012).

The geotechnical model cannot be directly mounted on shake table because of the requirements of confinement. To model the soil in shaking table tests, a container is required to hold the soil in place. During the past few decades, several studies have been conducted on soil-structure systems using various types of soil containers. Many researchers (e.g., Taylor *et al.* 1995, Pitilakis *et al.*

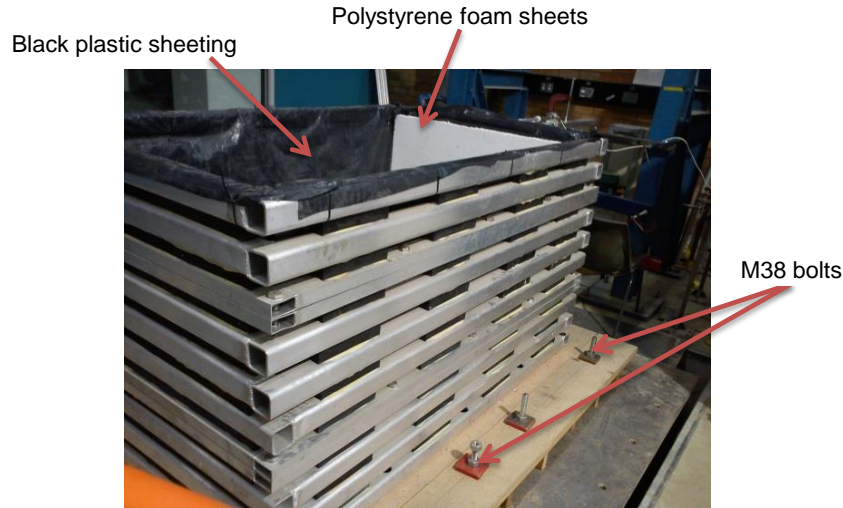


Fig. 2 Laminar soil container view constructed in the UTS civil laboratory

2008, Tang *et al.* 2009, Lu *et al.* 2012) concluded that laminar soil containers are the most appropriate and efficient type of the soil containers. Based on the conclusions made by the above mentioned researchers, well designed laminar soil containers can better model the free field boundary conditions in comparison with rigid and flexible containers as the lateral deformations in laminar soil containers are almost identical to the free field movements. By selecting 1:30 as the geometric scaling factor, the container should have minimum length, width, and depth of 2.0 m, 1.20 m, and 1.0 m, respectively. Allowing a further 10 mm on each side for construction purposes similar to Prasad *et al.* (2004), the final length, width, and depth of the laminar soil container are estimated to be 2.10 m, 1.30 m, and 1.10 m, respectively. In terms of choosing the materials to build the soil container, according to the previous conducted research works (e.g., Taylor 1997, Jakrapiyanun 2002, Pitilakis *et al.* 2008), aluminium frames and rubber layers were employed in an alternating pattern. Therefore, the laminar soil container consists of a rectangular laminar box made of aluminium rectangular hollow section frames separated by rubber layers. The aluminium frames provide lateral confinement of the soil, while the rubber layers allow the container to deform in a shear beam manner. The employed laminar soil container in this study, constructed in house, is shown in Fig. 2. The natural frequency of the laminar soil container was measured to be 10 Hz in the laboratory and it was noted that it fits the required natural frequency. Detailed explanation of this experimental setup can be found in Tabatabaiefar *et al.* (2014c).

In this study, a synthetic clay mixture was adopted as the soil medium for the shaking table testing process. In order to develop the synthetic clay mixture, Q38 kaolinite clay, ActiveBond 23 bentonite, class F fly ash, lime, and water were used as the components of the soil mixture. The proposed mix was prepared three times to control repeatability of the test and each time three cylindrical test specimens of size $D=50$ mm and $h=100$ mm were taken. To measure shear wave velocity of the mix over the cure age, bender element tests were performed. The soil specimens were placed between bender elements, and shear wave velocity of each soil specimen was obtained at different cure ages adopting the approach explained by Fatahi *et al.* (2013). Based on the laboratory measurements, it is understood that the soil mix produces the required shear wave velocity of 36 m/s (based on the scaling factor in Table 1) on the second day of its cure age.

Table 1 Scaling relations in terms of geometric scaling factor

| | | | |
|--------------|------------------|--------------|-----------|
| Mass Density | 1 | Length | λ |
| Force | λ^3 | Stress | λ |
| Stiffness | λ^2 | Strain | 1 |
| Frequency | $\lambda^{-1/2}$ | Acceleration | 1 |

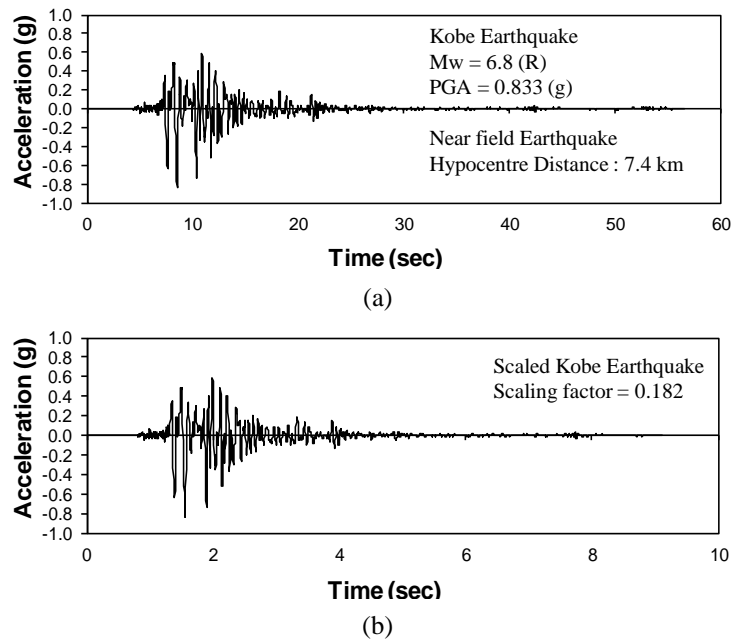


Fig. 3 Kobe earthquake (1995), (a) original record (b) scaled record

Afterwards, the standard method of soil density determination was performed on the second day of the cure age according to AS1289.3.5.1-2006 (Methods of testing soils for engineering purposes). Accordingly, soil density in the second day of the cure age (ρ_s) was determined to be 1450 kg/m³ which is almost equal to the prototype soil density (1470 kg/m³). Thus, shear wave velocity and soil density values of produced soil mix on the second day of the cure age satisfy the dynamic similarity requirements, explained in Section 4.2.

4.3 Scaling of adopted earthquake acceleration records

Four earthquake acceleration records including Kobe, 1995 (Fig. 3(a)), Northridge, 1994 (Fig. 4(a)), El Centro, 1940 (Fig. 5(a)), and Hachinohe, 1968 (Fig. 6(a)) have been adopted for the shaking table tests. The first two earthquakes are near field ground motions and the latter two are far field motions. These earthquakes have been chosen by the International Association for Structural Control and Monitoring for benchmark seismic studies (Karamodin and Kazemi, 2008). Characteristics of the mentioned earthquake ground motions and scaling procedure of the earthquake records are explained by Tabatabaiefar *et al.* (2014c). Scaled earthquake acceleration records are illustrated in Figs. 3(b)-6(b).

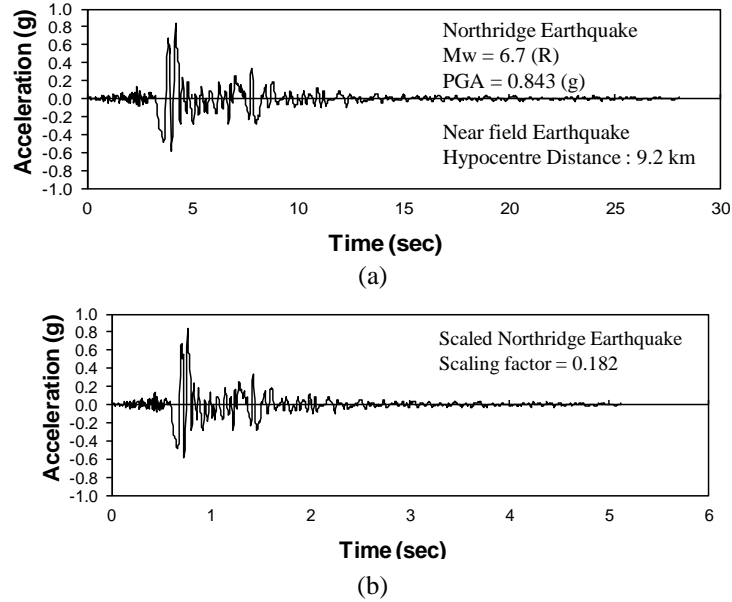


Fig. 4 Northridge earthquake (1994), (a) original record (b) scaled record

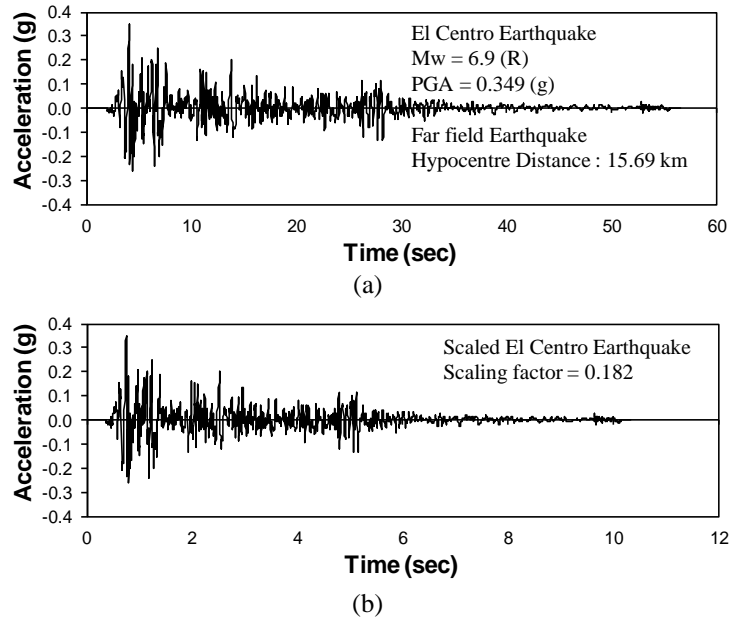


Fig. 5 El Centro earthquake (1940), (a) original record (b) scaled record

4.4 Shaking table tests on fixed base structural model

Tests were carried out on the constructed structural model, described in Section 4.2, as a *fixed base model* (structure directly fixed on top of the shaking table) in order to ensure the

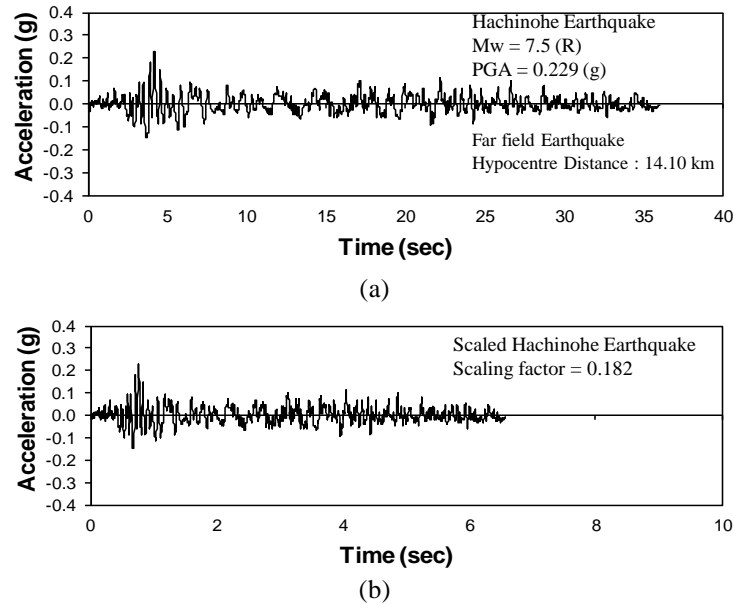


Fig. 6 Hachinohe earthquake (1968), (a) original record (b) scaled record

structural model possesses the targeted natural frequency and determine the damping ratio of the structural model. In addition, to verify the numerical model, seismic response of the fixed base model under the influence of the four scaled earthquake records were obtained.

After ensuring adequacy of the structural model characteristics, shaking table tests were performed by applying scaled earthquake acceleration records of Kobe, 1995 (Fig. 3(b)), Northridge, 1994 (Fig. 4(b)), El Centro, 1940 (Fig. 5(b)), and Hachinohe, 1968 (Fig. 6(b)) to the fixed base structural model. The results of the performed shaking table tests under the influence of four scaled earthquake acceleration records in terms of maximum lateral deflections are determined and presented in Fig. 9. In determination of the lateral deflections, the movement of the shaking table has been subtracted from storey movements. Therefore, all the records are in comparison to the base movements. It should be noted that for the sake of accuracy and consistency, the recorded displacements using displacement transducers, verified against the calculated displacements from accelerometer records, have been presented.

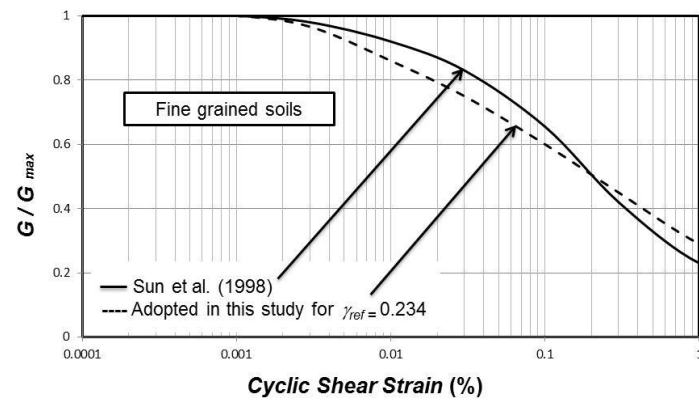
4.5 Shaking table tests on soil-structure model

Fig. 7 shows the final setup of the displacement transducers and accelerometers at different levels of the structural model for the soil-structure system on the shaking table. Details of the tests preparations and various components are explained by Tabatabaiefar *et al.* (2014c).

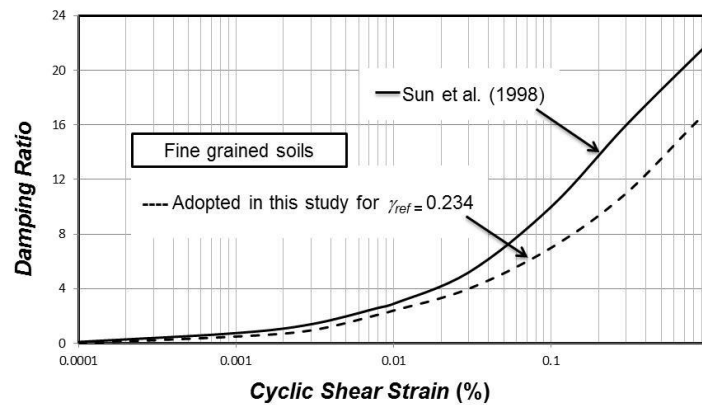
Before applying the scaled earthquake acceleration records to the flexible base model (soil-structure model), Sine Sweep test was carried out in order to estimate the natural frequency of the flexible base model. During the Sin Sweep test, frequency of the shaking table was raised from 0.1 Hz to 50 Hz to obtain the natural frequency of the soil-structure model. The obtained natural frequency of the soil-structure model from the performed Sin Sweep test was estimated to be 1.60 Hz. It can be noted that as expected, natural frequency of the soil-structure model is considerably



Fig. 7 Final setup of the measuring instruments of the soil-structure model



(a)



(b)

Fig. 8 Adopted fitting curves for clay in this study, (a) Relations between G/G_{max} versus cyclic shear strain (b) Relations between material damping ratio versus cyclic shear strain

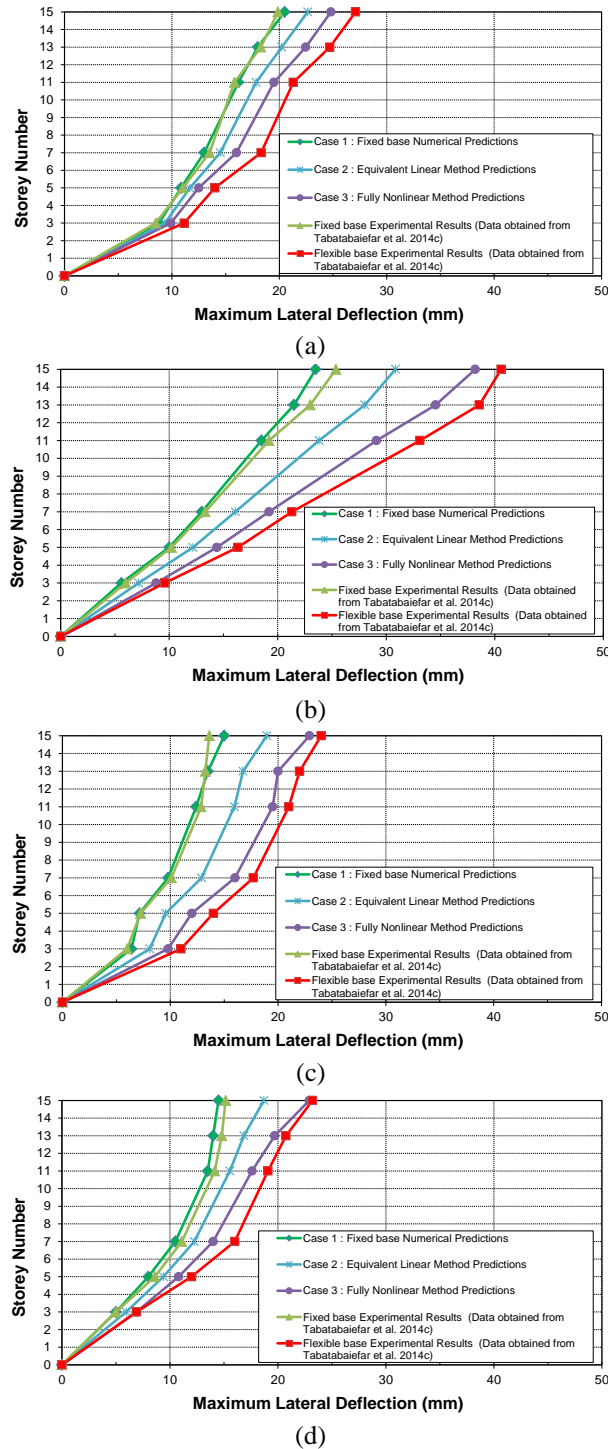


Fig. 9 Numerical and experimental maximum lateral displacements of fixed base and flexible base models under the influence four different scaled earthquake records (a) Kobe (1995) earthquake, (b) Northridge (1994) earthquake, (c) El Centro (1940) earthquake, (d) Hachinohe (1968) earthquake

smaller than the natural frequency of the fixed base structural model, previously determined to be 2.19 Hz. Afterwards, shaking table tests were undertaken by applying scaled earthquake acceleration records of Kobe, 1995 (Fig. 3(b)), Northridge, 1994 (Fig. 4(b)), El Centro, 1940 (Fig. 5(b)), and Hachinohe, 1968 (Fig. 6(b)) to the flexible base model, with the final setup as shown in Fig. 7. The results of the carried out shaking table tests under the influence of four scaled earthquake acceleration records in terms of the maximum lateral deflections of various storey of the structure are illustrated in Fig. 9.

5. Numerical simulation and analysis

In this study, in order to determine the accuracy of the equivalent linear method versus fully nonlinear method, the lateral deflections of the structural model, predicted by both numerical procedures, are compared with the measured experimental shaking table results, illustrated in Fig. 9. Inelastic structural analysis was performed in this study. In addition, geometric nonlinearity of the structures, capturing P-Delta effects, has been accommodated by specifying large-strain solution mode in FLAC2D software in the structural analyses of fixed base and flexible base models.

For numerical simulation and analysis, the structural model with the properties summarised in Section 4.2, was analysed under three different conditions as follows:

Case 1: *Fixed base columns on rigid ground, called fixed base model.* In this case, the numerical model of the constructed structural model, shown in Fig. 1, was built in FLAC2D using dimensions of the physical model. After building the geometry of the structural model, the required structural parameters including cross-sectional area of the beams (A_b), moment of inertia of the beams (I_b), cross-sectional area of the columns (A_c), moment of inertia of the columns (I_c), cross-sectional area of the foundation slab (A_s), moment of inertia of the foundation slab (I_s), modulus of elasticity of steel (E), density (ρ), and structural damping ratio (ξ) were extracted from the construction detail drawings and specifications, and adopted in the numerical simulation of the structure in FLAC2D. It should be noted that for the structural analysis of the fixed base model, constant damping value has been adopted. Inelastic time history dynamic analysis under the influence of scaled earthquake acceleration records of Kobe, 1995 (Fig. 3(b)), Northridge, 1994 (Fig. 4(b)), El Centro, 1940 (Fig. 5(b)), and Hachinohe, 1968 (Fig. 6(b)) earthquake records are performed. The earthquake records are applied to the fixed base of the structural models. The results of inelastic dynamic analyses of Case 1 are considered as pure structural analyses without considering soil-structure interaction effects in the analyses. Finally, the results of inelastic time history dynamic analyses in terms of lateral deflections under the influence of the four earthquake ground motions were derived from FLAC2D history records for fixed base model.

Case 2: *Flexible base model considering soil medium underneath the structure, called soil-structure model, employing direct method, to model and analyse dynamic soil-structure interaction using equivalent linear method of analysis.* In order to simulate flexible base (soil-structure) model in FLAC2D, the proposed soil-structure model, explained in Section 3, has been employed. The numerical soil-structure model (flexible base model) in FLAC2D is illustrated in Fig. 9(b). As reported in Section 4.6, ten cylindrical soil specimens of size $D=50$ mm and $h=100$ mm were successively taken from the soil mix, during the soil mixing process. In order to adopt the most accurate soil parameters in simulation of the physical soil-structure model, shear wave velocity (V_s) and soil density (ρ) of the samples in the second day of curing were determined by

Table 2 Adopted soil parameters in numerical simulation of soil-structure model

| Parameters | S_u (kPa) | V_s (m/s) | G_{max} (kPa) | K (kPa) | ρ (kg/m ³) | Reference |
|------------|-------------|-------------|-----------------|-----------|-----------------------------|-------------------------------------|
| Values | 1.57 | 35.5 | 1830 | 90760 | 1450 | Tabatabaiefar <i>et al.</i> (2014c) |

performing bender element and density tests in the UTS soils laboratory. The average results of the ten specimens indicated that the values of shear wave velocity (V_s) and soil density (ρ) were 35.5 m/s and 1450 kg/m³, respectively. These results have been in very good agreement and conformity to the initial laboratory test results. The adopted soil properties in the numerical simulation of the flexible base model consist of shear strength (S_u), shear wave velocity (V_s), low strain shear modulus (G_{max}), bulk modulus (K), and density (ρ), summarised in Table 2.

In order to perform equivalent linear analysis, a linear analysis is carried out with assumed initial values for equivalent linear material parameters including damping ratio (ξ) and shear modulus ratio (G/G_{max}) of the model. For this purpose, with respect to Peak Ground Accelerations (PGA) of the employed scaled earthquake records the initial value of shear modulus ratio (G/G_{max}), equal to 0.42, was derived from Table 10-5 of ATC-40 (1996). Having the initial value of (G/G_{max}) = 0.42, the initial value of cyclic shear strain was extracted, equal to 0.3%, from backbone curve of G/G_{max} versus cyclic shear strain for cohesive soils, presented by Sun *et al.* (1998), shown in Fig. 8(a). Afterwards, the initial value of soil damping ratio (ξ) = 16% was found from backbone curve of damping versus cyclic shear strain for cohesive soils (Fig. 8(b), presented by Sun *et al.* (1998).

Employing the extracted initial values for equivalent linear material parameters, inelastic time history dynamic analysis under the influence of the four mentioned scaled earthquake records are performed on flexible base models. The earthquake records were applied to the combination of soil and structure directly at the bedrock level. Then, the maximum soil cyclic shear strain values are determined from FLAC2D outputs. With respect to the extracted maximum soil cyclic shear strain values and using backbone curves of G/G_{max} versus cyclic shear strain and damping versus cyclic shear strain for cohesive soils (presented in Fig. 8) the new values of G/G_{max} and damping ratio are determined. The whole process has been repeated several times, until achieving the strain-compatible values of damping ratio (ξ) and shear modulus ratio (G/G_{max}). It should be noted that acceptable error for reaching the strain-compatible values of damping ratio (ξ) and shear modulus ratio (G/G_{max}) is 5% according to ATC-40 (1996) which has been adopted in this study. As mentioned earlier, the simulation using these values is deemed to be the best possible prediction of the real soil-structure system seismic behaviour while adopting linearity during the analysis. Eventually, the results of the equivalent linear dynamic analyses, employing the strain compatible values of damping ratio (ξ) and shear modulus ratio (G/G_{max}), in terms of lateral deflections under the influence of four scaled earthquake acceleration records including Kobe, 1995 (Fig. 3(b)), Northridge, 1994 (Fig. 4(b)), El-Centro, 1940 (Fig. 5(b)), and Hachinohe, 1968 (Fig. 6(b)) were derived from FLAC2D history records for the flexible model.

Case 3: *Flexible base model in order to model and analyse dynamic soil-structure interaction using fully nonlinear method of analysis.* The soil-structure model in this case is similar to Case 2 in which the developed soil-structure models has been utilised. For this case, adopting fully nonlinear method in dynamic analysis, nonlinear behaviour of the soil medium has been captured using backbone curves of shear modulus ratio versus shear strain (G/G_{max} - γ) and damping ratio versus shear strain (ξ - γ) adopting Mohr-Coulomb failure model. Fully nonlinear method is capable to model nonlinearity in dynamic analysis of soil-structure systems precisely and follow any

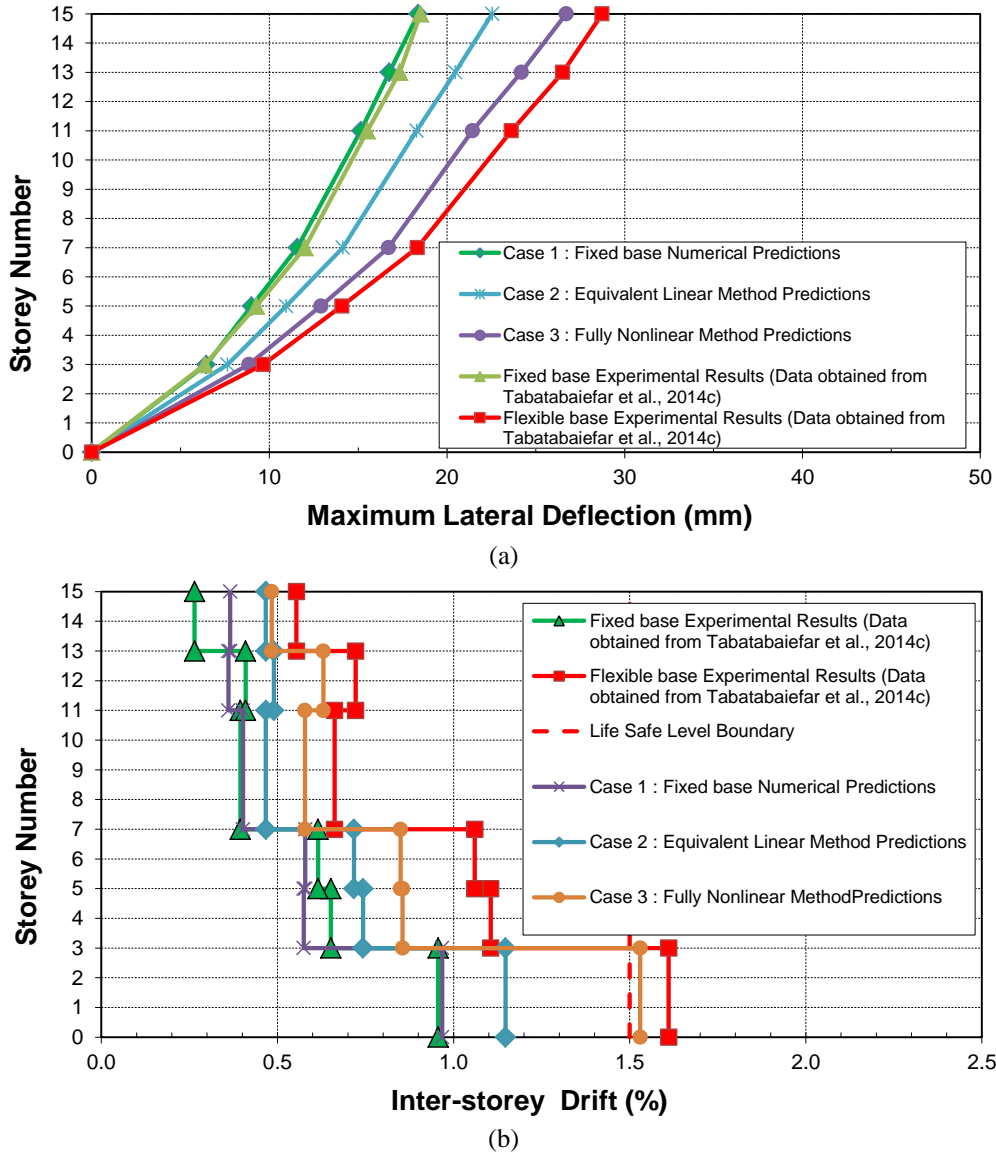


Fig. 10 (a) Average values of the numerical predictions and experimental values of the maximum lateral displacements of fixed base and flexible base models (b) average experimental inter-storey drifts of fixed base and flexible base models

prescribed nonlinear constitutive relation. Fully nonlinear method adopts hysteretic damping algorithm which captures the hysteresis curves and energy absorbing characteristics of the real soil. Small strain shear modulus and damping degradation of the soil with strain level can be considered in the modelling accurately. It should be noted, in the soil-structure model, the built-in tangent modulus function presented by Hardin and Drnevich (1972), known as Hardin model, is employed as the model provides reliable fits to backbone curves represented by Sun *et al.* (1998) in order to implement hysteretic damping to the model. Adopted model in FLAC2D generates

backbone curves representing Sun *et al.* (1998) curves for clay, adopting $\gamma_{ref}=0.234$ (Fig. 8) as the numerical fitting parameter with acceptable accuracy. In this way, nonlinear behaviour of the subsoil has been considered in the dynamic analysis. Afterwards, the numerical results of the inelastic time history dynamic analyses are obtained in terms of the lateral deflections from FLAC2D displacement history records for each scaled earthquake.

6. Results and discussions

The results of the numerical analyses for Cases 1, 2, and, 3 under the influence of four scaled earthquake acceleration records including Kobe, 1995 (Fig. 3(b)), Northridge, 1994 (Fig. 4(b0)), El-Centro, 1940 (Fig. 5(b)), and Hachinohe, 1968 (Fig. 6(b)) are presented and compared with the experimental results of the shaking table tests in Fig. 9. Average values of the numerical predictions and experimental values of the lateral deflections of the fixed base and the flexible base models were determined and compared in Fig. 10(a), while their corresponding inter-storey drifts have been calculated using the following equation based on AS 1170.4-2007 (Earthquake Actions in Australia)

$$drift = (d_{i+1} - d_i) / h \quad (1)$$

where, d_{i+1} is deflection at $(i+1)$ level, d_i is deflection at (i) level, and h is the storey height. The average values of numerical and experimental inter-storey drifts, determined by Eq. (1), are illustrated in Fig. 10(b).

Reviewing the numerical results for the three cases, it is observed that lateral deflections of flexible base model, predicted by the equivalent linear method (Case 2), have increased by 25% in comparison to the fixed base model (Case 1), while the lateral deflections and inter-storey drifts of flexible base model, determined by fully nonlinear method (Case 3) have enlarged by 49%, comparatively. For example, maximum lateral deflection of the fixed base model (Case 1) under the influence of Hachinohe, 1968 is found to be 14.5 mm. This value is determined equal to 18.4 mm for Case 2 (equivalent linear method), while the calculated value for the maximum lateral deflection of Case 3 (fully nonlinear method) is equal to 21.8 mm. Consequently, it can be observed that lateral deflection amplification due to SSI obtained through the equivalent linear method are almost half of the deflection amplification determined by the fully nonlinear method for the studied structural model. Thus, it can be noted that the discrepancies between the lateral deflections, predicted by equivalent linear method and fully nonlinear method, can be up to 50% for the examined structural model in this study.

Comparing the predicted and observed values of the maximum lateral displacements of the three cases, the accuracy of the numerical predictions are examined against the experimental measurements. Accordingly, the trend and the values of the numerical seismic response, predicted by Case 1 (fixed base numerical model) as well as the new developed numerical soil-structure model using the fully nonlinear method of seismic analysis (Case 3) are in a good agreement with the experimental results. However, the numerical predications by Case 2 (equivalent linear method) show some disparities in comparison to the experimental results, shown in Fig. 9. Reviewing the average maximum lateral deflections (Fig. 10(a)), it is noted that the numerical predictions and laboratory measurements for Cases 1 and 3 are in a good agreement (less than 10% difference). Therefore, the numerical soil-structure model using fully nonlinear method (Case 3) can replicate the behaviour of the real soil-structure system with acceptable accuracy. However,

the numerical predictions adopting equivalent linear analysis approach (Case 2) are almost 30% less than the experimental values.

As explained earlier, the equivalent linear method does not directly capture any nonlinearity effects due to linear solution process, and takes the strain-dependent modulus and damping functions into account only in an average sense in order to approximate some effects of nonlinearity. However, the fully nonlinear method correctly represents the physics associated with the problem and follows realistic stress-strain relationships which enable the small strain shear modulus and damping degradation of the soil with strain level to be captured in the modelling precisely.

It should be mentioned that soil-structure interaction increases the lateral deflections and corresponding inter-story drifts in both numerical and experimental procedures. However, based on the determined results, linear solution process and approximations utilised in the equivalent linear method result in significant inaccuracies in lateral deflection prediction of mid-rise building frames resting on soft soil deposits. This lack of accuracy may potentially underestimate the performance level of the building frames. For instance, based on the average inter-storey results, predicted by equivalent linear method (Fig. 10(b)), the performance level of the model remains in life safe level. However, realistic predictions (i.e., fully nonlinear method) suggest that the performance level of the models have been shifted from life safe to near collapse level due to soil-structure interaction effects. Such a considerable change in the performance level of the models is extremely dangerous and safety threatening.

Thus, it can be observed that the equivalent linear method for dynamic analysis may underestimate the lateral deflections and corresponding inter-storey drifts of mid-rise building frames in comparison to fully nonlinear dynamic analysis. Therefore, the dangerous and safety threatening effects of soil-structure interaction could be overlooked by using this simplified analytical method in the seismic design procedure while adopting performance base design. Evidently, in order to guarantee the safety and integrity of the seismic design of mid-rise building frames under the influence of soil-structure interaction, fully nonlinear method for dynamic analysis is recommended to be employed.

7. Conclusions

In this study, in order to examine the accuracy of the equivalent linear method versus fully nonlinear method, the lateral deflections of the structural model, predicted by both numerical procedures, are compared with the measured experimental shaking table test results. According to the numerical results, it is observed that the discrepancies between the lateral deflections, predicted by equivalent linear method and fully nonlinear method, can be up to 50% for the examined structural model in this study.

In addition, it is noted that the numerical predictions and laboratory measurements for the soil-structure model using fully nonlinear method are in a good agreement (less than 10% difference). Therefore, the numerical soil-structure model using fully nonlinear method can replicate the behaviour of the real soil-structure system with acceptable accuracy. However, the numerical predictions of lateral deformations adopting the equivalent linear method are almost 30% less than the experimental results obtained from the scale model in the laboratory. Thus, adopting the equivalent linear method, results in under-prediction of the lateral deflections of mid-rise building frames resting on soft soils. This lack of accuracy may potentially underestimate the performance

level of the building frames. As a result, extremely dangerous and safety threatening effects of the soil-structure interaction could be overlooked and misinterpreted employing the equivalent linear method in seismic design of mid-rise building frames resting on soft soils.

It can be concluded that, while adopting performance base design, the equivalent linear method of dynamic analysis may not be an accurate and qualified method for seismic design and cannot adequately guarantee the structural safety of the mid-rise building frames resting on soft soil deposits.

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