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Performance study on the whole vibration process of a museum induced by metro

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Abstract. The vibrations caused by metro operation propagate through surrounding soil, further induce secondary vibrations of the nearby underground structures and adjacent buildings. In order to investigate the effects of vibrations caused by metro on use performance of buildings, vibration experiment of Chengdu museum was carried out firstly. Then, the coupling tunnel-soil-structure finite element model was established with software ANSYS detailedly, providing a useful tool for investigating the vibration performances of structures. Furthermore, the dynamic responses and vibration predictions of museum building were obtained respectively by the whole process time-domain analysis and frequency-domain analysis, which were compared with the vibration reference values of museum. Quantitative analyses of the museum building performance were carried out, and the possible tendency and changing laws of vibration level with floors were proposed. Finally, the related vibration isolation measures were compared and discussed. The tests and analysis results show that: The vertical vibration responses almost increased with the increasing of building floors, while weak floors existed for the curve of horizontal vibration; The vertical vibrations were larger than the horizontal vibrations, indicating the vibration performances of building caused by metro were characterized with vertical vibrations; The frequencies of the museum corresponding to the peak vibration levels were around 6~17Hz; The damping effect of structure with 33m-span cantilever on vertical vibration was obvious, however, the damping effect of structure with foundation vibration isolators was not obvious.

Keywords: metro vibration; museum structure vibration; vibration experiment; finite element method; performance analysis

1. Introduction

The metros are commonly located at densely populated areas. The dynamic loads are caused by the coupling effect of wheels and tracks (such as non-uniform geometry of wheel-rail contact, variations in track flexibility (Bruni *et al*. 2009) and resilience of wheels (Kouroussis *et al*. 2012)), which induce vibrations to the structures through the surrounding soils (Gupta *et al.* 2008, 2009).

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buildings (Nejati *et al.* 2012), especially have adverse effects on the buildings sensitive to vibrations, such as museums, music hall, hospital, etc.

At present, theoretical analysis methods (including analytical method and numerical method) combined with actual measurements are usually used for solving the vibration problem of ground and buildings induced by metro (Au *et al.* 2011). Firstly, the theoretical model and the numerical analysis results are verified by the measured data. Then, dynamic analyses of environmental vibration are conducted by establishing theory models, the effects of structural parameters on vibration are studied and the vibration values of the ground and buildings under various operation conditions are analyzed. For instance, Wang *et al.* (1999) carried out the contrastive analysis of vibration distribution characteristics of different floors caused by metro, and the differences of the vibration distribution in multi-storey and high-rise buildings were discussed. Xia *et al.* (2004) pointed out that the vibration intensities of upper floors were higher than lower floors for the multi-storey buildings. Zhou *et al.* (2004) measured the vibration levels and responses of Shanghai music hall structures under the situations with and without metro near Hengshan Road, and predicted the vibration performances of music hall after relocation. According to the measured results, the vibration level was about 84dB when the train passed at the position eight meters away from metro line. The main frequency bands of vibration were around 40-100Hz. Xia *et al.* (2007) proposed a finite element approach to study the ground vibrations induced by metro, and gave a propagation distribution of ground accelerations for prediction. Ma *et al.* (2011) analyzed the vibration influence on a monument caused by Chengdu metro Line 2, and then compared the effect of different kinds of fasteners and train speeds. Galvin *et al.* (2007) presented a general numerical model for analyzing the soil motion caused by high-speed train, and discussed the effects of vibrations on nearby surface and underground structures. Francois *et al.* (2007) studied the dynamic responses of buildings due to the wave field induced by traffic, with a method that fully considering the dynamic interaction between the soil and the structure. Forrest *et al.* (2006) used a three-dimensional tunnel model with track model to assess the efficiency of floating-slab track. Stupazzini *et al.* (2010) proposed the spectral element method to analyze the ground motion and structural responses induced by train, which could improve the numerical simulation efficiency. Vogiatzis *et al.* (2012) carried out the numerical prediction to investigate the ground borne noise and vibration levels to give some suggestions for reducing vibrations. All of them have conducted outstanding works about the effects of metro on surrounding ground and structures.

Chengdu museum is located close to the Tianfu Square Station of Chengdu metro Line 2, Chengdu, China. The vibrations caused by metro operation may be transmitted to the museum floors through the building foundation, which would adversely affect the use performance of buildings. Furthermore, the distance of cultural relic storeroom of museum from the metro is less than 20 meters, so the vibrations induced by metro will cause slight fatigue damage of cultural relics. Therefore, it is necessary to carry out the vibration performance analysis of Chengdu museum induced by metro operation. The effective measures of vibration isolation should be suggested and provided for the exhibits safety and staff comfort in the museum.

In order to investigate the vibration performance of Chengdu museum caused by metro, firstly, vibration experiment of site measurement is carried out, and the vibration behaviors of the structure caused by surroundings are obtained. Then, the coupling finite element model of the tunnel - surrounding soil - building floor - building structure is established with software ANSYS, based on the structural dynamics and wave theory. Elements types, material properties of soils and structures, parameters of vibration isolators, the train moving load and the treatment of boundary

Table 1 Control standard of city area environmental vibration (*Z*-direction vibration level: dB)

Application range	Daytime	Night	Description of areas
Special residential area	65	65	The residential areas with particularly peaceful requirements
Residential, cultural and educational area	70	67	The areas only for residents, culture and education
Mixing district, central business district	75	72	The mixing areas with general industry, commercial center, residents and small amounts of traffic
Industrial concentration district	75	72	The clearly defined industrial areas planned by a city or a region
Both sides of the main traffic road	75	72	The areas on both sides of the roads with traffic of more than 100 per hour
Both sides of the trunk Railway	80	80	The areas 30m away from the railroad with daily traffic of no less than 20m

conditions are described detailedly. Furthermore, based on the finite element model, the dynamic responses of museum building calculated by time-domain analysis and frequency-domain analysis are obtained, comparing with the proposed vibration reference values of museum. Quantitative analyses of the museum building performance are carried out, and the possible tendency of vibration level with the increasing of building floors is proposed. Finally, the related vibration isolation measures are compared and discussed, which provide a useful reference for engineering applications and design proposals of similar buildings.

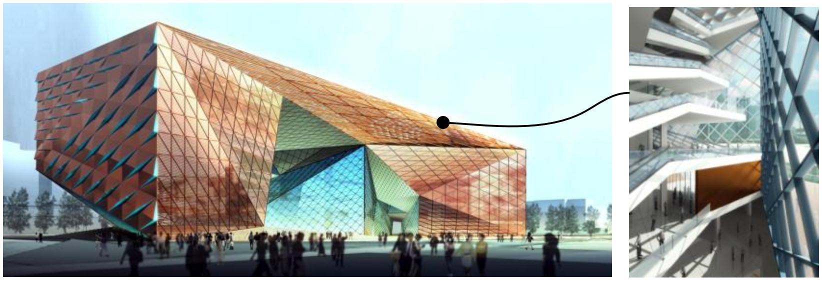
2. The vibration criteria of the museum

In order to limit the effect of environmental vibration on residents sleeping, learning and resting, the state environmental protection agency developed and approved the "*Standard of vibration in urban area environment (GB10070-88)*" (1988), in which, limit values of generalized environmental vibration were given, as shown in Table 1. The vibration level is widely used to describe the intensity of vibration in international community (unit dB). In order to ensure the usability of the museum, the vibration reference value of residential and cultural districts at night is adopted for the museum building on the ground, and the limit value of vibration level is 67dB; the vibration reference value of special residential areas at night (the limit value is 65dB) is used for the underground part of the museum, where precious cultural relics are stored.

In addition, the vibration peak accelerations of the key positions in the museum are also controlled in this paper. According to the design guide of "*Minimizing floor vibration*" published by American Applied Technology Council (Allen *et al.* 1999), 25mm/s^2 of hospital operation room standard is used as the limit value of peak vibration acceleration for the underground part of the museum, and 50mm/s^2 of the residence and office standard is used as the limit value of vibration acceleration of the museum building on the ground.

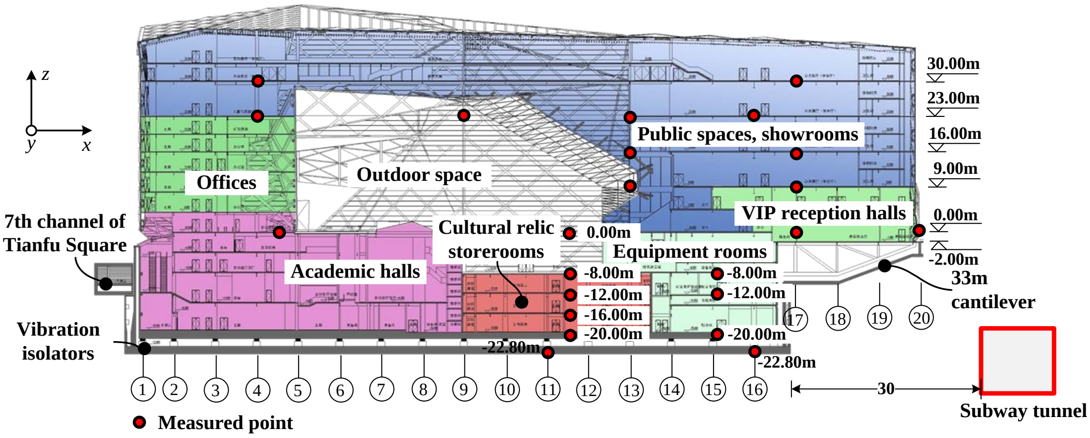
3. Vibration experiment of the museum

Due to the complexity of soil material, transmission path and coupling effect, it is quite difficult



Architectural appearance Inside sight

(a) Building structure of museum



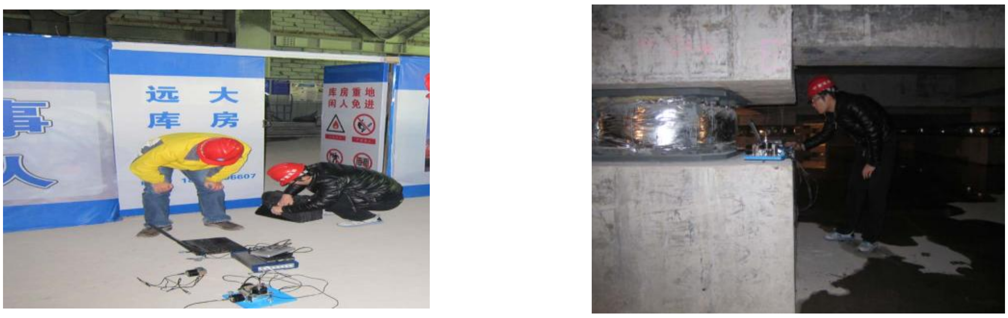
(b) The profile map of the building and arrangement of measured points

Fig.1 Building structure and arrangement of measured points of Chengdu museum

to study the vibration interaction only by theoretical method. So the Site measurement must be carried out simultaneously to investigate the vibrations of structures.

3.1 Engineering situation of Chengdu museum

Chengdu museum is located on the west of Chengdu Tianfu Square, which is a complex structure beyond the specifications, as shown in Fig. 1(a). The total construction area is 64161 m^2.The total area of five floors on the ground is 38239 m^2, and the area of four floors underground is 25921 m^2. The structural system on the ground consists of the multi-story concrete core tube, surrounding steel frames and oblique steel space lattice. The reinforced concrete shear walls are used for underground structures. It is an irregular structure in plane and vertical directions. Considering the flexibility of the museum exhibition spaces, the span of the main hall is 30 m without any column. The 15 m cantilever structure on the southwest corner is designed for space. The 33 m cantilever structure with five stories height is designed for avoiding the metro. The vibration isolators are set at the basement. The position relationship of building and metro is shown in Fig. 1(b). It can be seen that the building is quite near to the metro, so the vibration caused by metro may adversely affect the use performance of the building.



(a) Measuring inside the building (b) Measuring at vibration isolator (-22.8m)

Fig. 2 Image of test site

3.2 Site measurement for vibrations

When the test was carried out, most structures of Chengdu museum had been completed. The test purpose is to measure the vibration responses of ground and internal points of the museum, further to validate the finite element model for predicting the whole process performances of this building (Shi 2014). The typical test sites are in Chengdu museum buildings, as shown in Fig. 2. The measured points were arranged at the key positions of the ground and all the floors, including -22.8 m floor, -20 m floor,-16 m floor,-12 m floor,-8 m floor, 0m floor, 9 m floor, 16 m floor, 23 m floor and 30 m floor. Most of the measured positions are shown in Fig. 1(b) and Table 1. The measurement configurations consist of INV3018-C data acquisition system produced by Beijing Dongfang institute of vibration and noise technology and 941-B vibration sensor developed by institute of engineering mechanics of China Earthquake Administration. The sampling frequency of all test records is 256 Hz, and sampling time is 1000s. A total of 27 points were measured with totally 31 times.

3.3 The results analysis

In order to evaluate the actual vibration behaviors of the structure, the calculation methods of vibration indices should be described, including RMS, RMSA, and V_{L10}. Firstly, the effects of vibration on the building structures and people are actually caused by vibrational energy conversion. The RMS (Root-Mean-Square) acceleration can better reflect this situation, and avoid accidental and interferential data in site measurement. Therefore, in the analysis of environmental vibration, the vibration strength is generally expressed by RMS acceleration a_{rms}, shown as Eq. (1).

$$a_{\mathrm{rms}} = [\frac{1}{T}\int_0^T a^2(t)dt]^{1/2} \tag{1}$$

Where, $a(t)$ is the time history record of vibration acceleration.

Secondly, in order to investigate the effect of metro operation (the records will be amplified due to metro operation), the measurement time of 1000s is divided into 200 sections averagely. In comprehensive analysis of the test results, the average accelerations of 200 sections are in ascending sort. The mean value of 180$^{\mathrm{th}}$ section and 181$^{\mathrm{th}}$ section (considering 10% exceedance probability) is treated as the relative maximum value of periods a_{rmsa}, called RMSA (Relative

Table 2 The feeling correction value of vibration acceleration provided in ISO2631/1-1985

Frequency (Hz)	1	2	4	6.3	8	10	16	20	25	31.5	40	50	63	80
Vertical (dB)	−6	−3	0	0	0	−2	−6	−8	−10	−12	−14	−16	−18	−20
Horizontal (dB)	3	3	-3	−7	−9	−11	−15	−17	−19	−21	−23	−25	−27	−29

Maximum of Section Average).

Finally, the vibration acceleration levels VL_z and $VL_{x\text{-}y}$ (x-y is in plane, and z is vertical direction) is used for describing the intensity of vibration, which should be corrected in various frequencies, according to the “GB10071-88 Measuring method of city area environmental vibration” (1988). The corrected values are listed in Table 2 provided by ISO 2631/1-1985 (1985), which is referred in GB10071-88. The vibration level VL (VL_{z} and $VL_{\mathrm{x\text{-}y}}$) (dB) considering comprehensive weighting can be obtained by Eq. (2).

$$VL\left(VL_{\mathrm{z}}\mathrm{and}VL_{\mathrm{x-y}}\right)=10\lg\sum 10^{(VAL_i+\alpha_i)/10} \tag{2}$$

Where, α_i is the correction value of each frequency (dB) (Table 2). VAL_i is the vibration level of each frequency (dB). The vibration acceleration level VAL is determined by Eq. (3).

$$VAL = 20\lg\frac{a_{\mathrm{rms}}}{a_0} \tag{3}$$

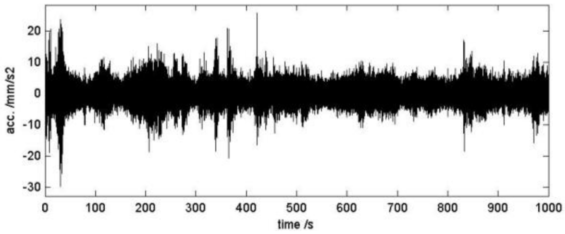
Where, a_{rms} is the effective value of acceleration, which is defined by Eq. (1). a_0 is basic acceleration value as 1×10^{-6} m/s^2.

In this test, VL_{10} is used for vibration level VL (VL_{z} and $VL_{\mathrm{x\text{-}y}}$). The process to obtain the values of VL_{10} is similar to RMSA.

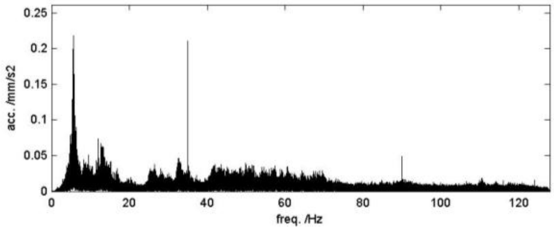
The typical time history curves of vertical vibration acceleration and corresponding frequency spectrums are shown in Fig. 3. The values of RMS, RMSA and V_{L10} of each measured point of the whole period are summarized in Table 3.

Based on the design guide of “Minimizing floor vibration” (Allen *et al.* 1999), interferential and accidental data should be excluded. It is inappropriate to use the maximum values of all-time records to evaluate the vibration behaviors. Therefore, RMSA and VL_{10} value are adopted to compare with vibration reference values in section 2. The vibration levels of most measured points (Table 3) satisfy the requirements. However, the individual measured points exceed the criteria. The probable reason is that some parts of museum project are under construction, the maintenance structure is not completed, and the effective damping measures do not work. Therefore, it can be considered that the actual vibrations will be less than the measured results when the museum is in formal operation.

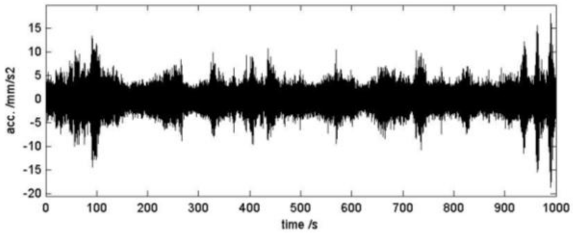
According to the analysis of measurement results, the frequencies with larger responses of all measured points at ground and -22.8 m floor are between 15-18 Hz. Except for this frequency band, the bands of 20-25 Hz, 50-60 Hz and 80-90 Hz are also found for underground part, and the frequency with larger responses is about 20 Hz in the middle of stage at -16 m floor. These frequency bands are the local vibration frequency of the measured points. The vibration frequency bands of measuring points on upper structure contain the floor slab vibration frequencies of 5 Hz, 10 Hz and 55 Hz, and the vibration frequency bands of measured points at cantilever part also contain the cantilever overall vibration frequencies of 2-4 Hz.



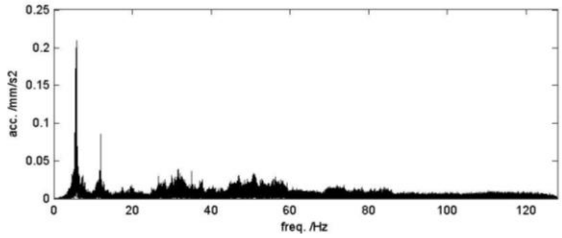
(a) The time history curve of acceleration at 23 m F16



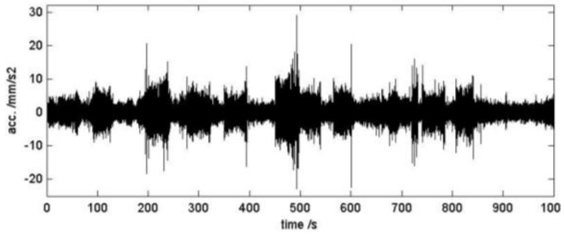
(b) The frequency spectrum at 23 m F16



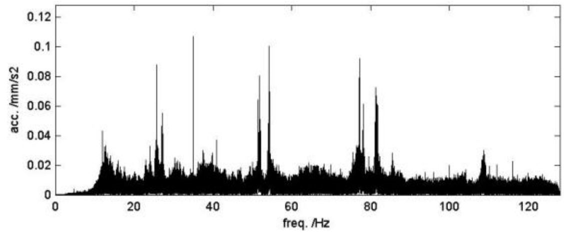
(c) The time history curve of acceleration at 16 m F17



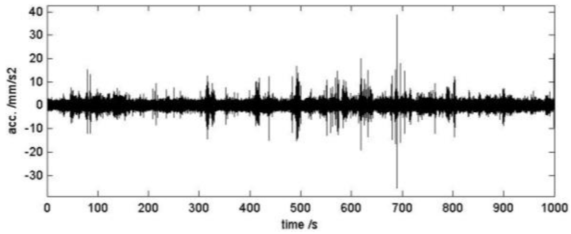
(d) The frequency spectrum at 16 m F17



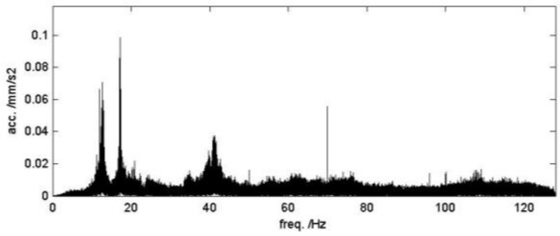
(e) The time history curve of acceleration at -8 m EF-11-12



(f) The frequency spectrum at -8 m EF-11-12



(g) The time history curve of acceleration at -16 m F5



(h) The frequency spectrum at -16 m F5

Fig. 3 The measured results

The vibration level comparison results of typical measured point at daytime (metro and road traffic) and night (no metro) are shown in Table 4, showing the effects of metro and traffic on structure are relatively obvious. The relationships of vertical and horizontal vibration level with floors are shown in Fig. 4. In general, the vibration on the ground is larger than the one underground. From the distribution curves, we can see that the vertical vibration responses almost increase with the increasing of building floors, while weak floors exist for the curve of horizontal vibration during the middle floors. The horizontal vibration levels are mostly between 39-43dB, and vertical vibration levels are mostly between 58-68dB. The vertical vibration is larger than the horizontal vibration, indicating the vertical vibration is in control of vibration evaluation caused by metro.

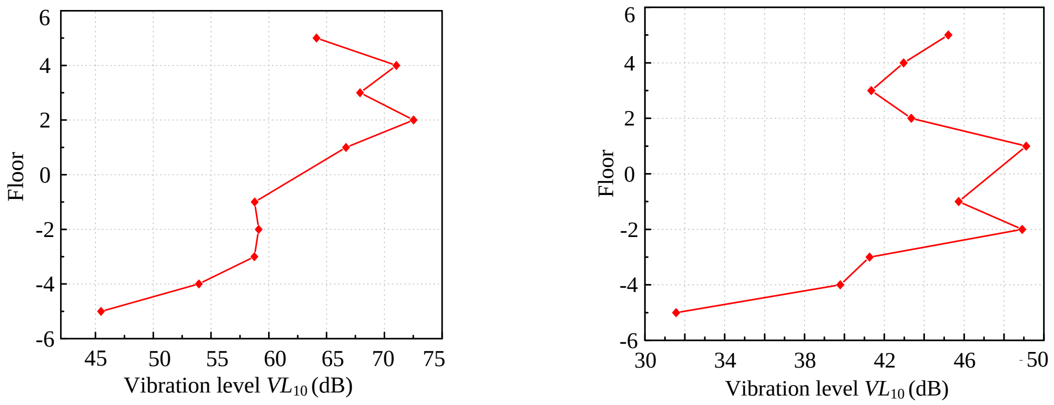
Table 3 The measured results of vertical vibration and horizontal vibration at daytime

Floor	Measured points	Vertical vibration			Horizontal vibration		
		RMS a_{rms} (mm/s^2)	RMSA a_{rmsa} (mm/s^2)	VL_{10} (dB)	RMS a_{rms} (mm/s^2)	RMSA a_{rmsa} (mm/s^2)	VL_{10} (dB)
Ground	Ground	1.86	1.91	62.93	1.38	1.52	48.12
-22.8m	F11 column footing	0.55	0.53	45.48	0.84	0.94	31.56
	F16 column footing	0.55	0.52	45.53	0.86	1.01	32.76
	G16 column footing	0.51	0.46	44.71	0.74	0.79	31.83
-20m	EF, 11-12	1.45	1.83	53.95	2.24	2.75	39.79
	EF, 15-16	1.79	1.62	53.61	3.68	4.03	41.90
-16m	EF, 11-12	2.40	2.64	58.75	3.00	3.53	41.26
	F5 The center of stage	1.34	1.36	58.08	1.36	1.48	39.70
-12m	EF, 11-12	7.11	10.26	65.60	8.12	11.26	48.91
	EF, 15-16	2.69	3.46	59.12	2.91	3.74	41.87
-8m	EF, 11-12	2.00	2.30	56.56	3.73	4.76	43.19
	EF, 15-16	2.51	3.25	58.77	4.83	6.50	45.73
0m	F17 The center of hall	5.80	5.41	66.68	8.79	9.43	46.26
	F4-5 The center of hall	4.89	5.40	68.26	5.42	6.22	45.03
	C20 Cantilever position	3.89	3.61	58.89	1.90	2.01	43.43
9m	F13 The center of hall	1.96	2.23	58.10	2.82	3.34	42.59
	F17 The center of hall	4.16	4.42	72.53	1.97	2.35	43.35
16m	F13 The center of hall	2.86	2.84	67.89	1.71	1.92	40.47
	F17 The center of hall	2.02	2.08	67.37	1.52	1.68	41.35
23m	F4 The center of hall	2.46	2.77	58.64	2.49	2.58	40.50
	F9 The center of hall	3.16	3.27	62.74	2.47	2.50	42.97
	F13 The center of hall	4.33	4.64	66.94	2.85	3.79	42.66
	F16 The center of hall	2.79	3.06	71.04	1.64	1.78	41.53
30m	F17 The center of hall	2.28	2.20	64.13	2.02	2.25	41.65
	C20 Cantilever position	3.00	3.15	57.42	4.95	5.55	49.86

Note: *E* and *F* are the axes in *y*-direction. 1-20 are the axes in *x*-direction. For example, "EF, 11-12" means the measured point is located between *E* axis and *F* axis, and between 11 axis and 12 axis. "F11" means the measured point is located at the intersection point of F axis and 11 axis.

Table 4 The maximum results of vibration level at daytime and night

Floor	Vibration level VL_{10} (dB)			
	Vertical vibration		Horizontal vibration	
	Day time	Night	Day time	Night
-20 m	53.95	41.60	39.79	31.48
0 m	66.68	55.30	49.11	37.71
30 m	64.13	56.25	45.21	41.98



(a) Vertical vibration (b) Horizontal vibration

Fig.4 The changes of vertical and horizontal vibration with the floors

4. Numerical models for vibration performance of the museum

Although the actual vibrations can be obtained in site measurement, it is impossible to get all the responses of the building by tests. Besides, it is also difficult to obtain an relatively accurate effect of the metro on the building, due to inevitable impact of other factors, such as traffic flow, people, building construction or other outside interference. Therefore, it is necessary to establish a numerical method for investigating the vibration performances of Chengdu museum caused by metro. Firstly, the unified finite element model of the tunnel-surrounding soil-building floor-building structure should be proposed with software ANSYS, including elements types, material properties of soils and structures, parameters of vibration isolators, the train moving load and the treatment of boundary conditions. Secondly, the model should be compared with measured results.

4.1 The establishment of the coupled tunnel-soil-structure finite element model

According to the topographic map of field and the related parameters of structures, a coupled tunnel-soil-structure 3D finite element model is established with the finite element software ANSYS, as shown in Fig. 5. Each part is described detailedly as follow.

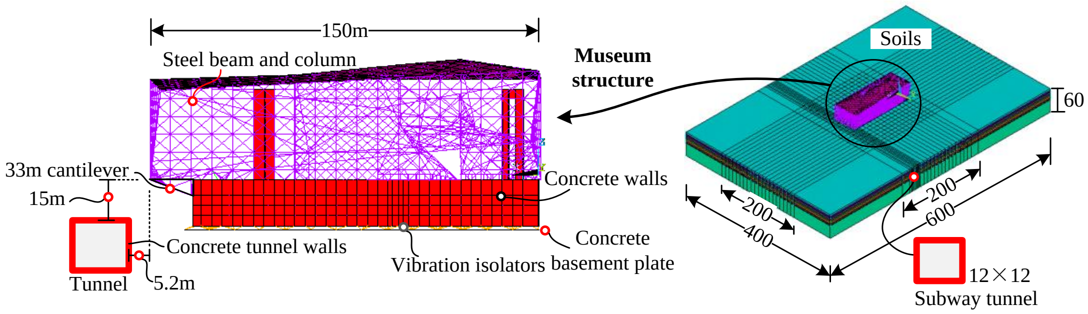
Three dimensional solid elements SOLID45 (8 nodes) is used for simulating soil. The material parameters are referred to section 4.2. The square tunnel with 12 m side length is established with shell element SHELL63 (4 nodes), and the buried depth of the tunnel roof is 15 m from the

Table 5 Parameters of each element in model

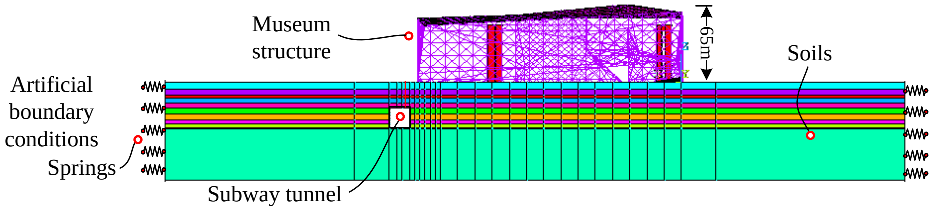
Type	Material	Thickness (m)	Mass (kN/m^3)	Elasticity modulus (GPa)	Poisson's ratio
Basement plate	Concrete	1.5	25	35	0.2
Tunnel wall	Concrete	0.3	25	30	0.2
Plate and wall of structure	Concrete	--	25	30	0.2
Column and beam of structure	Steel	--	78.5	206	0.3

Table 6 Stiffness parameters of vibration isolators (kN/m)

Type	*x*-direction	*y*-direction	*z*-direction
I	3×10^6	3×10^6	7×10^9
II	5×10^6	5×10^6	7×10^9
III	7×10^6	7×10^6	7×10^9
IV	1×10^9	1×10^9	1×10^{12}



(a) The coupled tunnel-soli-structure finite element model



(b) Boundary conditions

Fig. 5 Description of finite element model

ground. Concrete building basement plate is also simulated with shell element SHELL63. This shell element is easy to couple with solid element, achieving the interaction of soils and structure. The concrete parameters of tunnel and basement plate are shown in Table 5.

The local amplification of the building is shown in Fig. 5(a). 3D linear beam element beam188 is used for the steel columns and beams of structure model. The shell element SHELL181 (4 nodes) is adopted for concrete floors and walls. The material parameters of structure members are shown in Table 5. 3D linear element LINK180 (2 nodes) is adopted for simulating vibration isolators (Fig. 5(b)), which is usually used to simulate spring or cable. The values of stiffness are

Table 7 The dynamics parameters of soils

Soil type	H (m)	ρ (kN/m^3)	E_d (MPa)	G (MPa)	v_d	c_p (m/s)	c_s (m/s)
miscellaneous fill	4	17.5	98	34	0.444	440	140
Loose gravel	3	20	584	209	0.399	780	320
Little dense gravel	2	21	691	248	0.396	820	340
Loose gravel	3	20	584	209	0.399	780	320
Moderately dense gravel	6	21.5	989	359	0.377	900	400
Dense gravel	4	22	1500	640	0.366	1000	460
Moderately dense gravel	5	21.5	1100	380	0.377	900	400
Moderately weathered mudstone	23	23	3907	1501	0.301	1500	800

shown in Table 6 according to the data provided by manufacturer (Yang *et al.* 2011). Rigid beams are used to connect basement plate and vibration isolators. The quality of the building is applied to each node with uniformly distributed loading.

4.2 The dynamic parameters of soil

Kouroussis *et al.* (2013) found that in the practical range, density and Poisson's ratio had little effect on the results, while shear modulus and structural damping were the main parameters that influenced the vibration level. Therefore, the dynamic parameters of soil are obtained from the site measurement, according to the geotechnical engineering investigation report of Chengdu museum (AVIC Geotechnical Engineering institute 2009). The values of the equivalent shear wave velocity are 255-267 m/s. In order to facilitate the modeling and calculation, the soil is simplified to eight layers from top to bottom in this area according to the assumptions proposed by Shen *et al.* 2007. The dynamic parameters of each layer soil are listed in Table 7. E_d and v_d are dynamic elastic modulus and dynamic Poisson's ratio, G is the shear modulus, H and ρ are the thickness and density of each soil layer, and c_p and c_s are longitudinal wave and transverse wave respectively. According to the related research on the soil damping carried out by Wang *et al.* (2007), Rayleigh damping is used, and the damping coefficient of soil is suggested as 0.05.

4.3 The finite element mesh size and the basis for mesh division

The finite element meshes for dynamic analysis need to follow the rules below: in order to obtain more accurate results, the finite element meshes should be able to simulate the wave shape. Yang *et al.* (1996) carried out the impact analysis of the finite element mesh sizes, based on the vibration behaviors induced by a harmonic load in a semi-infinite free surface. Two conclusions were summarized as follow: 1) The range of mesh should be $2\lambda_s$ - $3\lambda_s$; 2) The mesh length should be λ_s /6 -λ_s /12, and no more than λ_s /12 near the vibration source. λ_s is the wave length $\lambda_s=2\pi C_s/\omega$, depending on the shear wave velocity C_s and angular velocity ω. For the area of Chengdu museum, the shear wave velocity of C_s is 260 m/s and ω is 10πrad/s, so λ_s is about 52 m. In order to reduce the influence of reflection wave at the virtual boundary, the size of soil model is chosen as $X\times Y\times Z$=600 m×400 m×60 m (as shown in Fig. 5(a)), which is much larger than structures and tunnels. The meshes near the vibration source are denser. With the increasing of the distance from vibration source, the meshes get sparse gradually, and the size of elements ranges from 0.5 to 5 m. The maximum sizes are controlled within 5 m×5 m.

4.4 The treatment of boundary conditions

When the process of wave propagation in semi-infinite medium is simulated by finite element method, the reflection of stress wave will be caused on the sectional boundary, leading to the false calculated results. In order to minimize this effect, spring damping model is arranged around the boundary, and the bottom of soils is the fixed (Wang 2007). The element COMBIN14 is used as spring damping element for boundary simulation of the model, which combines the damping and spring. According to Wei (2012), the artificial boundary conditions of viscoelasticity are used for calculation model, as shown in Eq. (4)

$$C_b=\rho c,\quad K_b=\alpha G/R \tag{4}$$

Where, C_b and K_b are the viscous damper and elastic spring applied to artificial boundary. ρ is the density of soil. c is the wave velocity (longitudinal wave and transverse wave). The values of parameter α are respectively 2 and 1.5 for longitudinal and transverse springs. R is the distance form vibration source to the artificial boundary. G is the shear modulus of soil.

4.5 The verification of train load

The software DRVB developed by Beijing Jiaotong University (Zhang *et al.* 2005, Xia *et al.* 2007) is used to obtain the vibration load of train, considering the coupling effect of wheels and tracks. These dynamic loads are applied on the nodes of tunnel bottom floor in finite element model to simulate the running train load. The normal traveling speed of 90 km/h is selected for trains. The vertical vibration level VL_z of tunnel wall in various frequencies can be obtained based on the results of time history analysis of tunnel wall with Fourier transform. The calculated method of VL_z is described in section 3.3. According to the established finite element model, the VL_z of tunnel wall considering comprehensive weighting calculated by Eq. (2) is 72.8 dB.

In order to verify the train load, the value of tunnel wall vibration calculated by empirical formula is used for comparing with the result calculated by finite element model. The ground vibration caused by metro vibration is consisted of transverse wave, longitudinal wave, and surface wave, which is the synthesis of complex wave phenomena. Because of the complex influencing factors, the varied vibration mechanism and propagation patterns, it is difficult to accurately determine the transfer function during the propagation process. Therefore, the VL_z of tunnel wall when train passing by is predicted by analogy with calculating analysis method, based on the actual situation and environmental characteristics of Chengdu metro Line 2 project. According to the domestic and foreign research data, the impact assessment method of environmental vibration and the verified results of Beijing metro Line 5 projects are referred. The value of VL_z is obtained by empirical formula Eq. (5), according to the modification with vehicle speed, axle load, floating slab, and wheel and rail conditions.

$$VL_z=VL_0+\Delta L_t+\Delta L_s+\Delta L_r+\Delta L_p \tag{5}$$

Where, VL_z is the *z*-direction vibration level considering comprehensive weighting at the receiving point (dB). VL_0 is the original vibration level of standard line (dB). ΔL_t is the corrected value of axle weight (dB). ΔL_s is the corrected value of train running speed (dB); ΔL_r is the corrected value of wheel and rail conditions (dB). ΔL_p is the corrected value of floating slab.

For Beijing metro line 5, the running speed of the train at the testing cross-section is about 70 km/h, and the axle load of the train is about 14 t with a total of six carriages. The vibration level

Table 8 Prediction of tunnel wall vibration caused by metro

Methods	The results of finite element method considering comprehensive weighting	The results of the empirical formula method
VL_z	72.8dB	73.2dB

Table 9 Comparison of vertical vibration accelerations between test records a_{mea} and the finite element results a_{fem}

Measured points floors	Acceleration（mm/s^2）	
	a_{mea}	a_{fem}
−4	1.83	1.88
−3	1.36	1.99
−1	2.30	1.55
0	1.91	2.15
1	3.61	2.73
2	4.42	2.82
3	2.84	3.02
4	4.64	3.36
5	5.10	3.43

VL_z of tunnel wall at Tiantan east station is evaluated as 84.0 dB, which is used for the original vibration level VL_0. The axle load and wheel and rail conditions of Chengdu metro Line 2 are similar to the Beijing metro line 5 without correction. It is only need to modify the train running speed and floating slab condition (Beijing metro line 5 does not have floating slab, while Chengdu metro Line 2 has floating slab). The vertical vibration of tunnel walls can be reduced by 13 dB with floating slab (Yang *et al.* 2011), so ΔL_p=-13 dB. The corrected value of train running speed ΔL_s is calculated by Eq. (6).

$$\Delta L_s=20\lg(v/v_0) \tag{6}$$

Where, v is the design running speed of Chengdu metro Line 2. v_0 is the train standard running speed of 70 km/h. When v=90 km/h, ΔL_s=2.2 dB. Therefore, by the correction of running speed, floating slab, axle load and wheel and rail conditions, the z-direction vibration level VL_z of tunnel wall induced by Chengdu metro Line 2 is 73.2 dB calculated by Eq. (5).

The values calculated by empirical method and finite element method are listed in Table 8. It can be seen that the calculation results of finite element method and empirical formula are relatively consistent, indicating the method is feasible.

4.6 Model checking

In order to verify the finite element model, the calculated responses induced by metro at the same position of site measurement are compared, as shown in Table 9. The results of finite element analysis are in good agreement with measured values in order of magnitude, indicating the modeling in Sections 4.1~4.5 is relatively reasonable and the numerical model can be used to predict the vibration responses. The reasons of larger differences at individual positions are that part of museum project is still under construction, and the nearby traffic flow, vehicle, people or

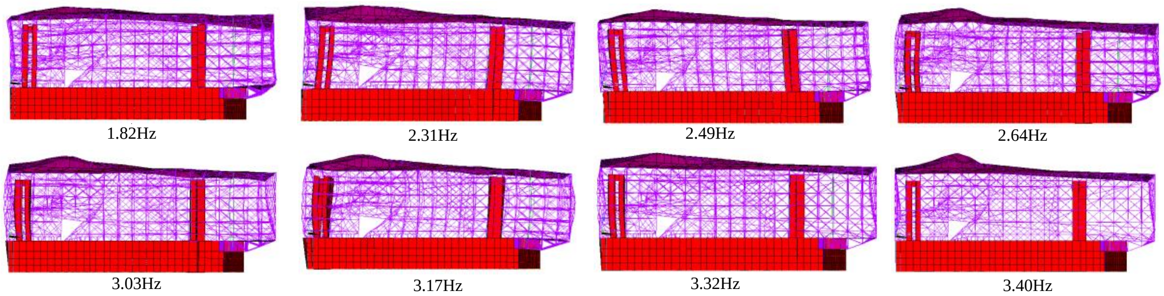


Fig. 6 The vibration modes of structures

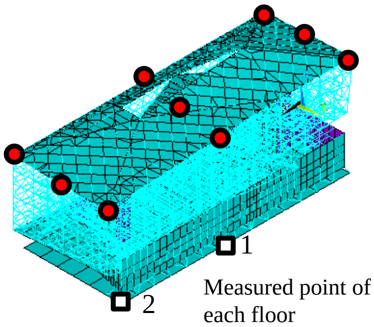


Fig. 7 The measured points in finite element model

other outside interference (for example, staffs walk around the measurement site) will have great impacts on the measured results. The finite element model is relatively ideal and simplified model, however, the interferences are inevitable for site measurement of vibrations. The numerical results are difficult to consistent with tests quite well.

5. Vibration prediction and analyses of the museum

Based on the finite element model, the vibration analyses are carried out to investigate the effect of metro on museum structure.

5.1 Mode analysis and response of building

The typical natural frequencies and the related mode shapes obtained from a modal analysis are shown in Fig. 6. The first order vibration mode is overall horizontal vibration in y-direction. The second order vibration mode is overall horizontal vibration in x-direction, and torsional vibration for third and fourth orders. The fifth, sixth, seventh modes of structure are vertical vibration, and the vibrations mainly occur at the cantilever on the right and larger span in the middle. The rest vibration modes are local vibrations.

5.2 Time-domain analysis of vibration responses of the building caused by running metro

In order to simulate the real vibration of museum building, the calculated case is consistent with the actual situation that: the normal traveling speed of 90 km/h is selected for trains; the isolation trenches are arranged around the building structure with four layers depth underground; a cantilever of 33 m span with five floors height is set near the tunnel side with transfer truss supporting the upper structures; the vibration isolators are set at the basement (they are used for both seismic isolation and traffic vibration isolation), as shown in Fig. 5. Nine key points are taken at each layer of the building as the extraction points of vibration responses, whose positions are marked in Fig. 7 (circles).

The simulated dynamic loads of metro train (described in Section 4.5) are applied to the coupling tunnel-soil-structure finite element model for time-domain analysis. The size of soil model is chosen as $X \times Y \times Z$=600 m×400 m×60 m, in which, the effective area is 200 m×200 m×60 m. The effective length of tunnel is 200 m, and the normal traveling speed of the train is 90 km/h, so time history of 20 s is determined. From the analysis in section 3.3, the vertical vibration of the building is larger than the horizontal vibration, indicating the vibrations of building caused by metro are characterized by vertical vibrations. Therefore, vertical vibrations are used for the following analyses. The vibration accelerations of each floor are calculated by dynamics analysis, and the typical time history curves are shown in Fig.8 (the measured point 1 and 2 are shown in Fig. 7 (square)). The vertical accelerations of key points of each floor are collected. The concerned peak acceleration values of the underground and upper structures are listed in Table 10.

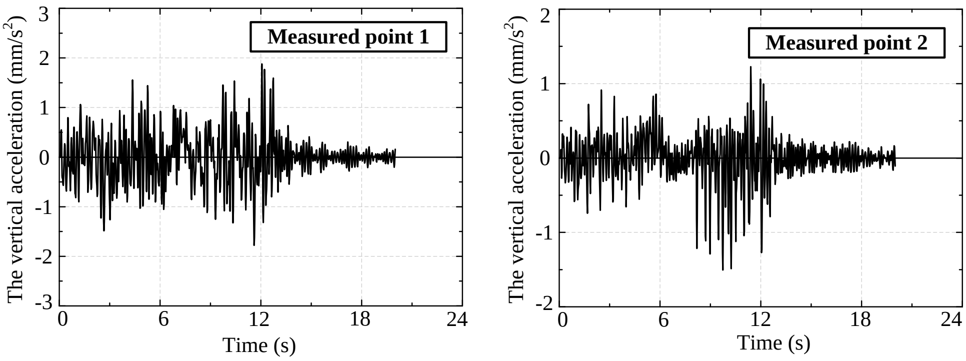


Fig. 8 The typical time history curves

Table 10 The maximum acceleration values of each floor

Floors	Vertical acceleration (mm/s^2)
Floor 4 underground	1.88
Floor 3 underground	1.99
Floor 2 underground	1.06
Floor 1 underground	1.55
Floor 1 on the ground	2.73
Floor 2 on the ground	2.82
Floor 3 on the ground	3.02
Floor 4 on the ground	3.36
Floor 5 on the ground	3.43

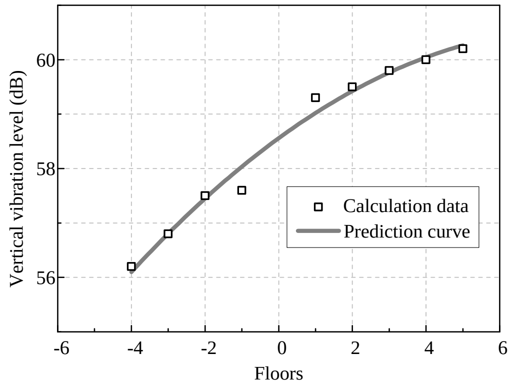
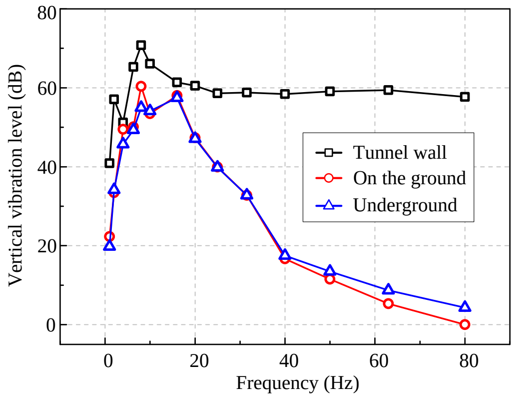
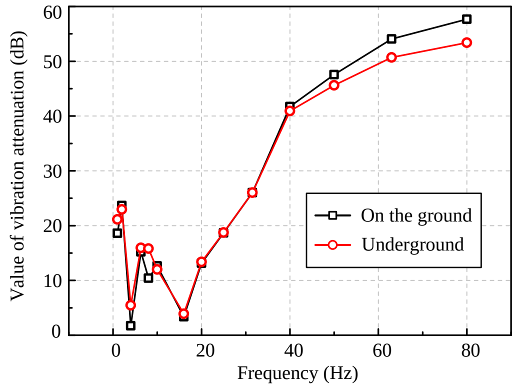


Fig. 9 The prediction curve of vertical vibration level with the floors



(a) Distribution of vertical vibration level VAL_i



(b) Distribution of vertical vibration attenuation

Fig. 10 The *z*-direction vibration distributions and transfer functions of tunnel walls to the building calculated by time-domain analysis

The results of time history analysis of each measured point are dealt with Fourier transform. The maximum values of vertical acceleration of each floor in various frequencies are obtained. Based on the calculated method of vibration level by Eq. (3), the vibration level of each frequency VAL_i is calculated. The vertical vibration level VL_z considering comprehensive weighting of each measured point are calculated by Eq. (2). According to the maximum VL_z of each floor, the changing laws of vibration responses with the floors induced by metro are analyzed based on statistical regression method. The maximum vertical vibration level VL_z of each floor is the function, and the corresponding floor is the variable. The fitting formula of environmental vibration is proposed for predicting the vibration behaviors of structure, as shown in Eq. (7).

$$VL_z = 58.557 + 0.493h - 0.03h^2 \tag{7}$$

Where, h is floor number, and the value range is from -4 underground to 5 on the ground. This formula shows the nonlinear relationship of vibration level with the floors. The peak vibration responses increase monotonically with the increasing of building floors. The prediction curve and calculated data are compared in Fig. 9.

Table 11 The vertical vibration results of the building

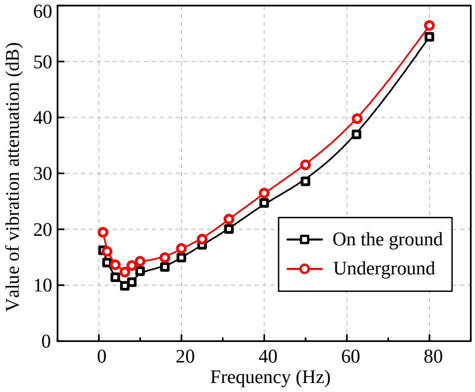
Position	The maximum acceleration (mm/s^2)	VL_z (dB)
On the ground	3.43	62.0
Underground	1.99	58.8

In order to compare the vibration behaviors of the structure on the ground and underground, the maximum responses of the structure on the ground (1~5 floors) and underground (-1~-4 floors) in various frequencies are obtained respectively by Fourier transform. According to Eq. (3), the vibration level of each frequency VAL_i is calculated. Therefore, the distribution laws of vertical vibration level on the ground and underground are shown in Fig. 10(a). The frequencies corresponding to the peak vibration levels are around 6~17 Hz, which are consistent with experimental results. By comparing with the distribution curve of tunnel wall vertical vibration level, the values of soil vibration attenuation (the transfer function of vibration induced by metro from tunnel walls to the building calculated by time-domain analysis) under different frequencies can be obtained, as shown in Fig. 10(b). The damping effect of soil generally increases with the increasing of frequency after 16 Hz.

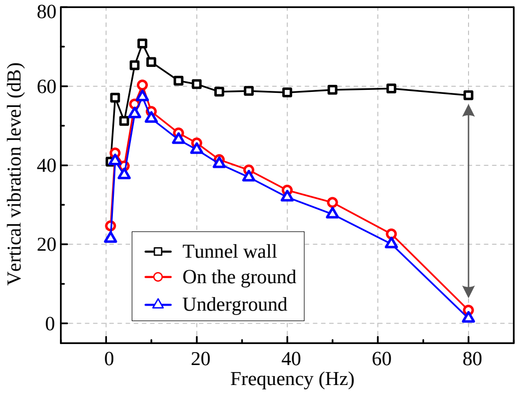
The vertical vibration VL_z analysis results of underground and upper structures considering comprehensive weighting are listed in Table 11. According to the results of finite element method, the following conclusions of the vertical vibration can be drawn: (1) Based on the calculated results of time-domain, the maximum value of vertical acceleration is 3.43 mm/s^2 for upper structures, and 1.99 mm/s^2 for the underground part. These values satisfy the vibration control indices of the museum proposed in section 2. (2) By the transmission effects of the vibration from tunnel, soil, basement plate to building structures, the vertical vibrations VL_z are reduced obviously by 15-19 dB comparing with tunnel wall (VL_z of tunnel wall is 72.8 dB). The z-direction vibration level considering weighting is 62.0 dB for upper structures, and 58.8 dB for the underground part, which meet the vibration reference values in section 2 (67.0 dB for upper structures, 65.0 dB for the underground part).

5.3 Frequency -domain analysis of vibration responses and the vertical vibration transmission from tunnel wall to building

In order to investigate the vertical vibration transmission behaviors, frequency-domain analyses are also carried out. The appropriate vertical harmonic forces are applied to the intermediate points in the metro center line of the finite element model, whose amplitude is 100 kN, and frequency range is 1-80 Hz. As the vertical acceleration responses of control points on each floor subjected to the vertical harmonic force are larger than the horizontal responses, namely the vibration damping effects caused by soils on the horizontal vibration of buildings are more obvious than vertical responses during the concerned frequency range of metro vibration. Therefore, the analyses below only focus on the vertical vibrations of key points inside the building. The vibration transfer functions of the tunnel wall to structure are obtained, as shown in Fig. 11(a), which indicate the attenuation of vibrations. For upper structures, the maximum damping effect is 54.2 dB when the vibration frequency is 80 Hz, and the minimum damping effect is 9.86 dB when the vibration frequency is 6.3 Hz; for underground structures, the maximum damping effect is 56.4 dB when the vibration frequency is 80 Hz, and the minimum damping effect is 12.3 dB when the vibration frequency is 6.3 Hz. Comparing the vertical vibration levels of the tunnel wall and vertical



(a) Distribution of vibration transfer function



(b) Distribution of vertical vibration level

Fig. 11 The *z*-direction vibration transfer function of tunnel walls to the building calculated by frequency-domain

Table 12 Vertical vibration level of the building calculated by frequency -domain analysis

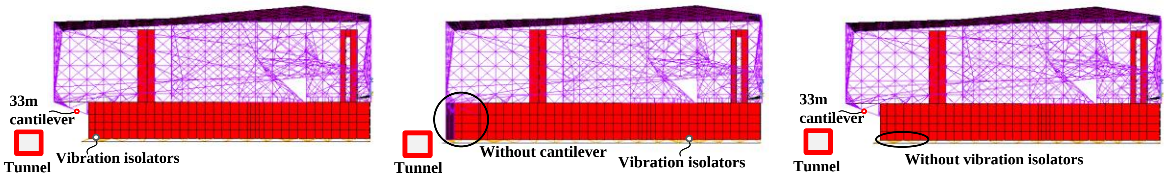
frequency (Hz)	1	2	4	6.3	8	10	16	20	25	31.5	40	50	63	80	VL_z
Tunnel wall (dB)	40.9	57.1	51.2	65.3	70.8	66.1	61.4	60.5	58.6	58.8	58.4	59.1	59.4	57.7	72.8
On the ground (dB)	24.7	43.1	39.8	55.4	60.3	53.6	48.2	45.6	41.4	38.8	33.7	30.6	22.6	3.3	62.1
Underground (dB)	21.4	41.1	37.6	53.0	57.3	51.8	46.5	44.0	40.4	37.0	31.9	27.6	20.1	1.3	59.3

vibration attenuation, the vibration levels of buildings on the ground and underground parts under different frequencies are obtained (Fig. 11(b)). The distributions are basically the same as the results calculated by time-domain analysis.

According to vertical vibration levels of structure under different frequencies, the vertical vibration levels VL_z considering comprehensive weighting correction of the buildings on the ground and underground parts are obtained by Eq. (2), which are listed in Table 12. The calculated vertical vibration level VL_z of upper structure is 62.1 dB, and 59.3 dB for the underground part, which satisfy the reference values of vibration proposed in section 2 (67.0 dB for upper structures, 65.0 dB for the underground part). Compared with the VL_z of tunnel wall (72.8 dB), the vibrations are reduced substantially, indicating the obvious effect of propagate path on vibration attenuation. Compared with the calculated results of time-domain analysis in Table 11, the results of frequency-domain analyses are slightly larger. The probable reason is that the attenuation of high frequency components of train load is faster, and when Fourier transform is used for time history data, some frequency components are filtered out.

5.4 The measures of structural vibration isolation for the museum

The isolation and reduction methods of metro vibration can be divided into three categories according to the locations, including the vibration reduction at source, isolation at the soil medium

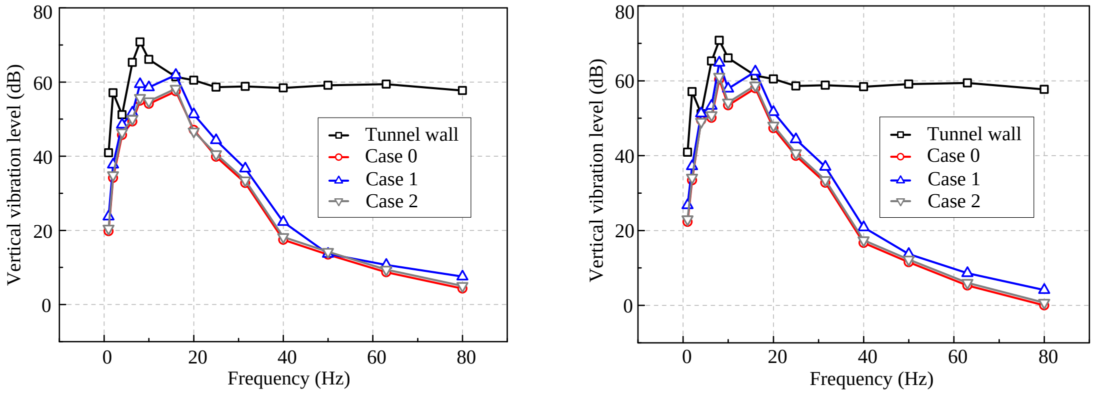


(a) Case 0: with cantilever and vibration isolators

(b) Case 1: without cantilever and with vibration isolators

(c) Case 2: with cantilever and vibration without isolators

Fig. 12 The measures of structural vibration isolation for museum



(a) Underground

(b) On the ground

Fig. 13 Comparison of vertical vibration level distribution

of propagation (Hung *et al.* 2001) and the isolation of foundation and structures (structural vibration isolation). The method of the vibration reduction at source is called active vibration isolation. The other two methods are called inactive vibration isolation. For Chengdu museum, in order to study the damping effect of cantilever structure and vibration isolators on the museum building, two other cases are supplemented in following section, compared with the actual case (the case with cantilever and vibration isolators described in section 5.2, called Case 0, shown in Fig. 12(a)). The traveling speeds of train and isolation trenches arrangements are the same for three cases. For Case 1, the vibration isolators are set at the basement, however, there is no cantilever set near the tunnel side, as shown in Fig. 12(b). For Case 2, a cantilever with 33m span and five floors height is set near the tunnel side with transfer truss supporting the upper structure, however, there is on vibration isolator set at the basement, as shown in Fig. 12(c).

The simulated dynamic loads of metro train (described in Section 4.5) are applied to the coupled tunnel-soil-structure finite element model for time-domain analysis. The results of time history analysis are dealt with Fourier transform to obtain the maximum z-direction vibration level VAL_i under each frequency. Then, the distributions of vertical vibration level VAL_i on the ground and underground are shown in Fig. 13. From the comparative analysis, the distributions of three cases are nearly the same. Around 10Hz~16Hz, vertical vibration levels of case 1 are much larger than the other two cases.

Based on the distribution curves, the vertical vibration levels VL_z considering comprehensive weighting of underground and upper structures are calculated, as shown in Table 13. The damping

Table 13 The results of finite element calculation of different cases

Cases	The maximum acceleration (mm/s^2)		VL_z (dB)	
	On the ground	Underground	On the ground	Underground
Standard	5.00	2.50	67.0	65.0
Case 0	3.43	1.99	62.0	58.8
Case 1	4.80	2.77	66.3	62.9
Case 2	3.85	2.26	62.6	59.5

effect of cantilevered structure and vibration isolators on the museum building are compared. It can be drawn that all the cases meet the requirements of vibration control. However, the *z*-direction vibration level VL_z of case 1 nearly reaches the maximum limit. For the comparison of the cantilever effect, the vertical acceleration of structure with cantilever (case 0) is attenuated by more than 40% than the results without cantilever (case 1). The attenuation of vertical vibration level on the ground is 4.3 dB, and 4.1 dB for underground part. The damping effect of a 33 m-span cantilever is obvious to control vertical vibration. For the comparison of vibration isolator effect, the changes of the vertical vibration responses of floors are small with no obvious law. The vertical acceleration of structure with vibration isolators (the original case) is attenuated by less than 13% than the results without vibration isolators (case 1). The attenuation of vertical vibration level is about 0.6 dB. The data indicate that the damping effect of vibration isolators on vertical vibration is not obvious.

6. Conclusions

In this paper, in order to investigate and evaluate the vibration performances of museum induced by metro, vibration experiments and numerical simulations were carried out, based on the practical engineering Chengdu museum. The main findings in this paper were summarized as follows:

- Form the results of vibration experiment, by the transmission effects of the vibration from tunnel, soil, basement plate to building structures, the dynamic responses of museum building measured in site can meet the vibration reference values. In general, the effects of metro on structure are relatively obvious. The vibrations of structure on the ground are larger than the ones underground. The vertical vibration responses almost increase with the increasing of building floors, while weak floors exist for the curve of horizontal vibration. The vertical vibration is larger than the horizontal vibration, indicating the vertical vibration is in control for vibration evaluation caused by metro.
- The coupled tunnel-soil-structure finite element model is established. The simulated results of numerical models are compared with the actual measured data. It provided a useful tool for investigating the vibration performances of structures. The reasons of larger differences at individual positions are that part of museum project is still under construction, and the nearby traffic flow, vehicle, people or other outside interference (for example, staffs walk around the measurement site) will have great impacts on the measured results. The finite element model is relatively ideal and simplified model, however, the interferences are inevitable for site measurement of vibrations.
- From the whole process of time-domain analysis, the frequencies corresponding to the peak

vibration level are around 6~17 Hz, and then the vertical vibration responses increase monotonically with the increasing of building floors. Based on the regression analysis of calculated results, the possible tendency of vibration level is proposed.

• From the results of the frequency-domain analyses, the vertical vibration transmission behaviors of tunnel to structure show that the damping effect generally increases with the increasing of the frequency after 6.3 Hz. For the vertical vibration of structure caused by tunnel wall vibration, the maximum damping effect is around 53-56 dB when the vibration frequency is 80 Hz, and the minimum damping effect is around 10-13 dB when the vibration frequency is 6.3 Hz.

• The damping effect of a 33 m-span cantilever on vertical vibration of structure is obvious. The vertical acceleration of structure with cantilever is attenuated by more than 40%, and the vertical vibration level is attenuated by 4 dB. However, the damping effect of vibration isolators is not obvious. The attenuation of vertical vibration level is about 0.6 dB.

Acknowledgements

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