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Influence of polled direction on the stress distribution in piezoelectric materials

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Abstract. In this paper, the influence of the polled direction of piezoelectric materials on the stress distribution is studied under time-harmonic dynamical load (time-harmonic Lamb's problem). The system considered in this study consists of piezoelectric covering layer and piezoelectric half-plane, and the harmonic dynamical load acts on the free face of the covering layer. The investigations are carried out by utilizing the exact equations of motion and relations of the linear theory of electro-elasticity. The plane-strain state is considered. It is assumed that the perfect contact conditions between the covering layer and half-plane are satisfied. The boundary value problems under consideration are solved by employing Fourier exponential transformation techniques with respect to coordinates directed along the interface line. Numerical results on the influence of the polled direction of the piezoelectric materials such as PZT-5A, PZT-5H, PZT-4 and PZT-7A on the normal stresses, shear stresses and electric potential acting on the interface plane are presented and discussed. As a result of the analyses, it is established that the polled directions of the piezoelectric materials play an important role on the values of the studied stresses and electric potential.

Keywords: piezoelectric layered material; polled direction; time-harmonic dynamical loading; stress distribution; resonance behavior

1. Introduction

Piezoelectric materials are widely used in electromechanical and electronic devices systems such as transducers, sensors and actuators. Because of their suitable properties, piezoelectric structural materials can have a function as deployed sensors and actuators for observing and controlling the response of a structure. In these devices, both electrical and mechanical loads applied on the piezoelectric system can cause to quite high stresses. To understand their dynamical behaviors, many research have been performed so far.

The problems with the half-plane boundary, Sosa and Castro (1994) investigated a piezoelectric half-plane indented by a concentrated line force and a concentrated line electric charge by using the state space approach. Fan *et al*. (1996) investigated the two-dimensional contact on a piezoelectric half-plane. In this study, the stress and electric field distributions in the system under

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contact loads at the surface was investigated by using Stroh's formalism. Gao and Fan (1998) studied the two-dimensional Green's functions for a transversely isotropic piezoelectric half-plane. The principle of analytical continuation of complex potentials was used. Chen (2000) studied the transversely isotropic piezoelectric half-space subjected to a rigid smooth punch with an arbitrarily shaped profile by using the complex potential function method.

A crack embedded in a half-plane piezoelectric solid with traction-induction free and electric-open boundary conditions was analyzed in Yang *et al*. (2007). The plane problem considered multilayered piezoelectric laminates were investigated by Ruan *et al*. (2000) and by Tarn and Huang (2002), Borrelli *et al*. (2006). Ma *et al*. (2014) have investigated the two-dimensional sliding frictional contact of a piezoelectric half-plane. Plane strain state cases are discussed under the action of a rigid flat or a triangular punch. Fourier integral transform and the superposition theorem were used. The unknown contact pressure and surface electric charge distribution were determined.

The study of Lamb's problem for the system consisting of the elastic layer and elastic half-space were made in recent papers by Akbarov (2006a, 2006b, 2006c, 2006d, 2013), Akbarov and Guler (2007), Akbarov and Ilhan (2008, 2009, 2010), Ilhan (2012), Akbarov and Salmanova (2009), Akbarov *et al*. (2013), Akbarov *et al*. (2005), Emiroglu *et al*. (2009). In a paper by Akbarov and Ilhan (2013) the first attempt was made to study the time-harmonic Lamb's problem for a system consisting of a covering piezoelectric layer and piezoelectric half-plane. However, in the paper by Akbarov and Ilhan (2013) it was assumed that the polled direction of the materials of the constituents of the system directed along the direction which is perpendicular to the interphase plane. But in many real cases the polled direction of the materials cannot coincide and how this statement can effect on the dynamic stress field caused by linearly-located time harmonic forces acting on the covering layer is the novelty of the present paper.

2. Formulation of the problem

As in the paper by Akbarov and Ilhan (2013), consider a system consisting of a half-plane and covering layer and assume that a linearly-located time harmonic force acts on the upper free face plane of the covering layer (see Fig. 1(a)). It is required to determine the dynamical response of the considered system under the plane-strain state in the *Oxz* plane. Note that the covering layer and half-plane occupy the regions $\{-\infty<x<+\infty,\ -h<z<0\}$ and $\{-\infty<x<+\infty,\ -\infty<z<-h\}$, respectively. The materials of the constituents are taken piezoelectric ones.

To distinguish between the values related to the covering layer and half-plane, we will use the upper indices (1) and (2), respectively.

It is presume that the materials of the constituents are transversally isotropic. We write the equations of motion based on the linear theory of electro-elasticity (Eringen and Maugin 1990, Yang 2005).

$$\frac{\partial \sigma_{xx}^{(k)}}{\partial x}+\frac{\partial \sigma_{xz}^{(k)}}{\partial z}=\rho^{(k)}\frac{\partial^2 u^{(k)}}{\partial t^2},\quad \frac{\partial \sigma_{xz}^{(k)}}{\partial x}+\frac{\partial \sigma_{zz}^{(k)}}{\partial z}=\rho^{(k)}\frac{\partial^2 w^{(k)}}{\partial t^2},$$
$$\frac{\partial D_x^{(k)}}{\partial x}+\frac{\partial D_z^{(k)}}{\partial z}=0,\qquad k=1,2 \tag{1}$$

Consider the formulation of the electro-mechanical relations for the piezoelectric materials of the constituents. If the polled direction of the *k*-th piezoelectric material is directed along the *Oz*

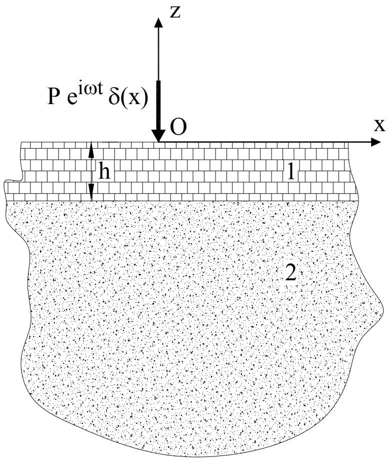


Fig. 1(a) Geometry of the considered system

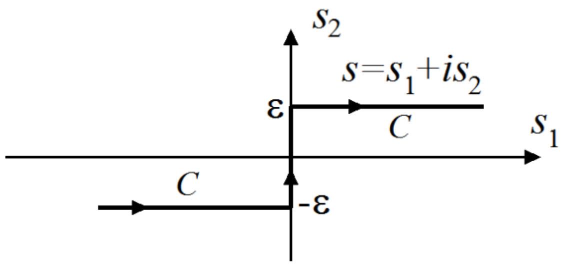


Fig. 1(b) Sommerfeld contour

axis (Fig. 1(a)), the electro-mechanical relations are written in the following form

$$\sigma_{xx}^{(k)} = c_{11}^{(k)}\gamma_{xx} + c_{13}^{(k)}\gamma_{zz} - e_{31}^{(k)}E_z^{(k)}, \ \ \sigma_{zz}^{(k)} = c_{13}^{(k)}\gamma_{xx} + c_{33}^{(k)}\gamma_{zz} - e_{33}^{(k)}E_z^{(k)},$$

$$\sigma_{xz}^{(k)} = c_{44}^{(k)}\left(\gamma_{xz}+\gamma_{zx}\right) - e_{15}^{(k)}E_x^{(k)}, \ D_x^{(k)} = e_{15}^{(k)}\left(\gamma_{xz}+\gamma_{zx}\right) + \varepsilon_{11}^{(k)}E_x^{(k)},$$

$$D_z^{(k)} = e_{31}^{(k)}\left(\gamma_{xx}\right) + e_{33}^{(k)}\left(\gamma_{zz}\right) + \varepsilon_{33}^{(k)}E_z^{(k)}. \tag{2}$$

But in the case where the polled direction of the *k*-th piezoelectric material is directed along the *Ox* axis (Fig. 1(a)) the electro-mechanical relations for this material are written as follows

$$\sigma_{xx}^{(k)} = c_{33}^{(k)}\gamma_{xx} + c_{13}^{(k)}\gamma_{zz} + e_{33}^{(k)}E_x^{(k)}, \ \sigma_{zz}^{(k)} = c_{13}^{(k)}\gamma_{xx} + c_{11}^{(k)}\gamma_{zz} + e_{31}^{(k)}E_x^{(k)},$$

$$\sigma_{xz}^{(k)} = c_{44}^{(k)}\left(\gamma_{xz}+\gamma_{zx}\right) + e_{15}^{(k)}E_z^{(k)}, \ D_x^{(k)} = e_{33}^{(k)}\left(\gamma_{xx}\right) + e_{31}^{(k)}\left(\gamma_{zz}\right) - \varepsilon_{33}^{(k)}E_x^{(k)},$$

$$D_z^{(k)} = e_{15}^{(k)}\left(\gamma_{xz}+\gamma_{zx}\right) - \varepsilon_{11}^{(k)}E_z^{(k)}. \tag{3}$$

We also write the strain-displacement and electric field vector-electric potential relations as follows

$$\gamma_{xx}^{(k)} = \frac{\partial u^{(k)}}{\partial x}, \gamma_{zz}^{(k)} = \frac{\partial w^{(k)}}{\partial z}, \gamma_{xz}^{(k)} = \gamma_{zx}^{(k)} = \frac{\partial u^{(k)}}{\partial z} + \frac{\partial w^{(k)}}{\partial x}, E_x^{(k)} = -\frac{\partial \varphi^{(k)}}{\partial x}, E_z^{(k)} = -\frac{\partial \varphi^{(k)}}{\partial z}. \tag{4}$$

In Eqs. (1), (2), (3) and (4) $c_{11}^{(k)}$, $c_{13}^{(k)}$, $c_{33}^{(k)}$ and $c_{44}^{(k)}$ are elastic constants well known from classical linear theory of elasticity; $D_x^{(k)}$ and $D_z^{(k)}$ are components of the electric displacement vector; $E_x^{(k)}$ and $E_z^{(k)}$ are components of the electric field vector; $\varphi^{(k)}$ is an electric potential; $e_{31}^{(k)}$, $e_{33}^{(k)}$ and $e_{15}^{(k)}$ are piezoelectric constants; $\varepsilon_{11}^{(k)}$ and $\varepsilon_{33}^{(k)}$ are dielectric constants; $u^{(k)}$ and $w^{(k)}$ are components of the displacement vector in the direction of the *Ox* and *Oz* axes respectively, $\gamma_{xx}^{(k)}$, $\gamma_{zz}^{(k)}$ and $\gamma_{xz}^{(k)}$ are components of the strain tensor, and $\sigma_{xx}^{(k)}$, $\sigma_{zz}^{(k)}$ and $\sigma_{xz}^{(k)}$ are components of the stress tensor.

Assume that the following perfect contact conditions are satisfied on the interface plane between the covering layer and half plane

$$\sigma_{xz}^{(1)}\Big|_{z=-h} = \sigma_{xz}^{(2)}\Big|_{z=-h},\ \sigma_{zz}^{(1)}\Big|_{z=-h} = \sigma_{zz}^{(2)}\Big|_{z=-h},\ u^{(1)}\Big|_{z=-h} = u^{(2)}\Big|_{z=-h},\ w^{(1)}\Big|_{z=-h} = w^{(2)}\Big|_{z=-h}, \tag{5}$$

$$\varphi^{(1)}\Big|_{z=-h} = \varphi^{(2)}\Big|_{z=-h},\ D_z^{(1)}\Big|_{z=-h} = D_z^{(2)}\Big|_{z=-h}. \tag{6}$$

On the free upper face plane of the covering layer the mechanical load conditions are given below

$$\sigma_{xz}^{(1)}\Big|_{z=0} = 0,\ \sigma_{zz}^{(1)}\Big|_{z=0} = -P\mathrm{e}^{\mathrm{i}\omega t}\delta(x) \tag{7}$$

where $\delta(x)$ is a Dirac delta function.

In a further discussion the following cases will be considered:

Case 1. The face plane of the covering layer is electroded and grounded, i.e.

$$\varphi^{(1)}\Big|_{z=0} = 0 \tag{8}$$

Case 2. The face plane of the covering layer is unelectroded, i.e.

$$D_z^{(1)}\Big|_{z=0} = 0. \tag{9}$$

In addition, there are the following boundedness conditions

$$\left|u^{(2)}\right|;\ \left|w^{(2)}\right|;\ \left|\varphi^{(2)}\right| < M = \mathrm{const}\ \text{ as }\ z \to -\infty. \tag{10}$$

This completes the formulation of problem. It should be noted that the formulated problem in the case where the electro-mechanical relation (Eq. (2)) occurs for both covering layer and half-plane materials simultaneously, has been considered in the paper by Akbarov and Ilhan (2013). The case where the electro-mechanical relations of the both materials of the covering layer and half-plane are given through the Eq. (3), as well as the case where the electro-mechanical relations of the covering layer material (half-plane material) are given through the Eq. (2) (through the Eq. (3)). However, the electro-mechanical relations for the half-plane material (covering layer material) are given through the Eq. (3) (through the Eq. (2)) will be investigated in the present paper.

3. Method of solution

According to the boundary condition in Eq. (7) we will describe the sought values as:

$$u^{(k)} = \overline{u}^{(k)}(x,z)\mathrm{e}^{\mathrm{i}\omega t},\ w^{(k)} = \overline{w}^{(k)}(x,z)\mathrm{e}^{\mathrm{i}\omega t},\ \varphi^{(k)} = \overline{\varphi}^{(k)}(x,z)\mathrm{e}^{\mathrm{i}\omega t}, \tag{11}$$

and consider the finding the amplitudes $\overline{u}^{(k)}(x,z)$, $\overline{w}^{(k)}(x,z)$ and $\overline{\varphi}^{(k)}(x,z)$ in the case where the electro-mechanical relation (Eq. (4)) occur. Note the solution procedure related to the case where the electro-mechanical relation (Eq. (2)) occurs was considered in the paper by Akbarov and Ilhan (2013).

Thus, according to Eq. (11), (4) and (3), we obtain the following equations for the amplitudes $\overline{u}^{(k)}(x,z)$, $\overline{w}^{(k)}(x,z)$ and $\overline{\varphi}^{(k)}(x,z)$ from Eq. (1) as

$$\frac{c_{33}^{(k)}}{c_{44}^{(k)}}\frac{\partial^2 u^{(k)}}{\partial x^2}+\frac{c_{13}^{(k)}}{c_{44}^{(k)}}\frac{\partial^2 w^{(k)}}{\partial x\partial z}+\frac{e_{33}^{(k)}}{c_{44}^{(k)}}\frac{\partial^2 \varphi^{(k)}}{\partial x^2}+\left(\frac{\partial^2 u^{(k)}}{\partial z^2}+\frac{\partial^2 w^{(k)}}{\partial x\partial z}\right)+\frac{e_{15}^{(k)}}{c_{44}^{(k)}}\frac{\partial^2 \varphi^{(k)}}{\partial z^2}=-\left(\frac{\omega}{c_2^{(k)}}\right)^2 u^{(k)},$$

$$\left(\frac{\partial^2 u^{(k)}}{\partial x\partial z}+\frac{\partial^2 w^{(k)}}{\partial x^2}\right)+\frac{e_{15}^{(k)}}{c_{44}^{(k)}}\frac{\partial^2 \varphi^{(k)}}{\partial x\partial z}+\frac{c_{13}^{(k)}}{c_{44}^{(k)}}\frac{\partial^2 u^{(k)}}{\partial x\partial z}+\frac{c_{11}^{(k)}}{c_{44}^{(k)}}\frac{\partial^2 w^{(k)}}{\partial z^2}+\frac{e_{31}^{(k)}}{c_{44}^{(k)}}\frac{\partial^2 \varphi^{(k)}}{\partial x\partial z}=-\left(\frac{\omega}{c_2^{(k)}}\right)^2 w^{(k)} \tag{12}$$

$$\frac{e_{15}^{(k)}}{c_{44}^{(k)}}\left(\frac{\partial^2 u^{(k)}}{\partial z^2}+\frac{\partial^2 w^{(k)}}{\partial x\partial z}\right)-\frac{\varepsilon_{11}^{(k)}}{c_{44}^{(k)}}\frac{\partial^2 \varphi^{(k)}}{\partial z^2}+\frac{e_{33}^{(k)}}{c_{44}^{(k)}}\left(\frac{\partial^2 u^{(k)}}{\partial x^2}\right)+\frac{e_{31}^{(k)}}{c_{44}^{(k)}}\left(\frac{\partial^2 w^{(k)}}{\partial x\partial z}\right)-\frac{\varepsilon_{33}^{(k)}}{c_{44}^{(k)}}\frac{\partial^2 \varphi^{(k)}}{\partial x^2}=0$$

where $c_2^{(k)}=\sqrt{c_{44}^{(k)}/\rho^{(k)}}$. In Eq. (12) and the subsequent equations, the over-bars on the sought values are omitted for the sake of brevity. The contact conditions (5) and (6), the boundary conditions (8) and (9), and the first boundary condition in (7) also hold for the amplitudes of the corresponding values. In this case, the second boundary condition in (7) is transformed into the following one

$$\sigma_{zz}^{(1)}\Big|_{z=0}=-P\delta(x)\,. \tag{13}$$

To proceed further, we define the dimensionless coordinates and dimensionless frequency as follows

$$x'=\frac{x}{h},\ z'=\frac{z}{h},\ \Omega=\frac{\omega h}{c_2^{(1)}}\,. \tag{14}$$

The prime on the x' and z' will be omitted below. Now we consider the solutions to Eq. (12). For this purpose, as in the paper Akbarov and Ilhan (2013) we employ the exponential Fourier transformation with respect to the x coordinate defined as

$$f_F(s,z)=\int_{-\infty}^{+\infty} f(x,z)\mathrm{e}^{-\mathrm{i}sx}\mathrm{d}x\,. \tag{15}$$

in Eq. (12) and given the corresponding boundary and contact conditions. From (12), (13) and (15) we obtain the following relations

$$\frac{d^2u_F^{(k)}}{dz^2}+\frac{e_{15}^{(k)}}{c_{44}^{(k)}}\frac{d^2\varphi_F^{(k)}}{dz^2}+\left(\frac{c_{13}^{(k)}}{c_{44}^{(k)}}+1\right)(is)\frac{d\,w_F^{(k)}}{dz}+\left(\left(\frac{c_{33}^{(k)}}{c_{44}^{(k)}}\right)(-s^2)+\left(\Omega\frac{c_2^{(1)}}{c_2^{(k)}}\right)^2\right)u_F^{(k)}+\frac{e_{33}^{(k)}}{c_{44}^{(k)}}(-s^2)\varphi_F^{(k)}=0,$$

$$\frac{c_{11}^{(k)}}{c_{44}^{(k)}}\frac{d^2w_F^{(k)}}{dz^2}+\left(\frac{c_{13}^{(k)}}{c_{44}^{(k)}}+1\right)(is)\frac{du_F^{(k)}}{dz}+\left(\frac{e_{15}^{(k)}+e_{31}^{(k)}}{c_{44}^{(k)}}\right)(is)\frac{d\varphi_F^{(k)}}{dz}+\left((-s^2)+\left(\Omega\frac{c_2^{(1)}}{c_2^{(k)}}\right)^2\right)w_F^{(k)}=0$$

$$\frac{e_{15}^{(k)}}{c_{44}^{(k)}}\frac{d^2u_F^{(k)}}{dz^2}-\frac{\varepsilon_{11}^{(k)}}{c_{44}^{(k)}}\frac{d^2\varphi_F^{(k)}}{dz^2}+\left(\frac{e_{15}^{(k)}+e_{31}^{(k)}}{c_{44}^{(k)}}\right)(is)\frac{dw_F^{(k)}}{dz}+\frac{e_{33}^{(k)}}{c_{44}^{(k)}}(-s^2)u_F^{(k)}-\frac{\varepsilon_{33}^{(k)}}{c_{44}^{(k)}}(-s^2)\varphi_F^{(k)}=0 \tag{16}$$

$$\sigma_{Fzz}^{(1)}\Big|_{z=0}=-P\,. \tag{17}$$

The contact conditions, the boundary conditions, and the first boundary condition in Eq. (7) also hold for the Fourier transformation Eq. (15) of the amplitudes of the corresponding values.

For the particular solution of Eq. (16), we select the components of the displacement vector and electric potential as follows

$$u_F^{(k)} = A^{(k)}\,\mathrm{e}^{\lambda^{(k)}z}\,,\; w_F^{(k)} = B^{(k)}\,\mathrm{e}^{\lambda^{(k)}z}\,,\; \varphi_F^{(k)} = C^{(x)}\,\mathrm{e}^{\lambda^{(k)}z}\,. \tag{18}$$

Substituting Eq. (18) into Eq. (16) and doing the corresponding mathematical manipulations yield the following equations

$$\left((\lambda^{(k)})^2+\left(\frac{c_{33}^{(k)}}{c_{44}^{(k)}}(-s^2)+\left(\Omega\frac{c_2^{(1)}}{c_2^{(k)}}\right)^2\right)\right)A^{(k)}+\left(\frac{c_{13}^{(k)}}{c_{44}^{(k)}}+1\right)(is)\lambda^{(k)}B^{(k)}+\left(\frac{e_{15}^{(k)}}{c_{44}^{(k)}}(\lambda^{(k)})^2+\frac{e_{33}^{(k)}}{c_{44}^{(k)}}(-s^2)\right)C^{(k)}=0$$

$$\left(\frac{c_{13}^{(k)}}{c_{44}^{(k)}}+1\right)(is)\lambda^{(k)}A^{(k)}+\left(\frac{c_{11}^{(k)}}{c_{44}^{(k)}}(\lambda^{(k)})^2+\left((-s^2)+\left(\Omega\frac{c_2^{(1)}}{c_2^{(k)}}\right)^2\right)\right)B^{(k)}+\left(\frac{e_{15}^{(k)}}{c_{44}^{(k)}}+\frac{e_{31}^{(k)}}{c_{44}^{(k)}}\right)(is)\lambda^{(k)}C^{(k)}=0$$

$$\left(\frac{e_{15}^{(k)}}{c_{44}^{(k)}}(\lambda^{(k)})^2+\frac{e_{33}^{(k)}}{c_{44}^{(k)}}(-s^2)\right)A^{(k)}+\left(\frac{e_{15}^{(k)}}{c_{44}^{(k)}}+\frac{e_{31}^{(k)}}{c_{44}^{(k)}}\right)(is)\lambda^{(k)}B^{(k)}+\left(-\frac{\varepsilon_{11}^{(k)}}{c_{44}^{(k)}}(\lambda^{(k)})^2-\frac{\varepsilon_{33}^{(k)}}{c_{44}^{(k)}}(-s^2)\right)C^{(k)}=0 \tag{19}$$

We obtain the characteristic equation from Eq. (19) so as to determine the values of $\lambda^{(k)}$

$$\left(\Lambda^{(k)}\right)^3+\left(\Lambda^{(k)}\right)^2a_4^{(k)}+\Lambda^{(k)}a_2^{(k)}+a_0^{(k)}=0\,, \tag{20}$$

where

$$\Lambda^{(k)}=\left(\lambda^{(k)}\right)^2,\;\; a_4^{(k)}=\frac{d_2^{(k)}}{d_3^{(k)}},\; a_2^{(k)}=\frac{d_1^{(k)}}{d_3^{(k)}}\;,\; a_0^{(k)}=\frac{d_0^{(k)}}{d_3^{(k)}}\,,$$

$$d_3^{(k)}=-\left(\frac{e_{15}^{(k)}}{c_{44}^{(k)}}\right)^2\frac{c_{11}^{(k)}}{c_{44}^{(k)}}-\frac{c_{11}^{(k)}}{c_{44}^{(k)}}\frac{\varepsilon_{11}^{(k)}}{c_{44}^{(k)}}$$

$$\begin{aligned} d_2^{(k)}=&-\Omega^2\left(\frac{c_2^{(1)}}{c_2^{(k)}}\right)^2\left(\left(\frac{e_{15}^{(k)}}{c_{44}^{(k)}}\right)^2-\frac{c_{11}^{(k)}}{c_{44}^{(k)}}\frac{\varepsilon_{11}^{(k)}}{c_{44}^{(k)}}-\frac{\varepsilon_{11}^{(k)}}{c_{44}^{(k)}}\right)+s^2\left(-\left(\frac{e_{31}^{(k)}}{c_{44}^{(k)}}\right)^2-\left(\frac{c_{13}^{(k)}}{c_{44}^{(k)}}\right)^2\frac{\varepsilon_{11}^{(k)}}{c_{44}^{(k)}}\right)\\ &+s^2\left(\left(\frac{e_{15}^{(k)}}{c_{44}^{(k)}}\right)^2+\frac{c_{11}^{(k)}}{c_{44}^{(k)}}\frac{\varepsilon_{11}^{(k)}}{c_{44}^{(k)}}+\frac{\varepsilon_{11}^{(k)}}{c_{44}^{(k)}}+\frac{c_{11}^{(k)}}{c_{44}^{(k)}}\frac{\varepsilon_{33}^{(k)}}{c_{44}^{(k)}}+\frac{c_{11}^{(k)}}{c_{44}^{(k)}}\frac{c_{33}^{(k)}}{c_{44}^{(k)}}\frac{\varepsilon_{11}^{(k)}}{c_{44}^{(k)}}\right)\\ &-2s^2\left(\frac{c_{13}^{(k)}}{c_{44}^{(k)}}\frac{\varepsilon_{11}^{(k)}}{c_{44}^{(k)}}+\frac{e_{15}^{(k)}}{c_{44}^{(k)}}\frac{e_{31}^{(k)}}{c_{44}^{(k)}}-\frac{e_{33}^{(k)}}{c_{44}^{(k)}}\frac{e_{15}^{(k)}}{c_{44}^{(k)}}\frac{c_{11}^{(k)}}{c_{44}^{(k)}}\right)\end{aligned}$$

$$\begin{aligned} d_1^{(k)}=&-\Omega^4\left(\frac{c_2^{(1)}}{c_2^{(k)}}\right)^4\frac{\varepsilon_{11}^{(k)}}{c_{44}^{(k)}}+s^4\left(-\left(\frac{e_{33}^{(k)}}{c_{44}^{(k)}}\right)^2\frac{c_{11}^{(k)}}{c_{44}^{(k)}}+\left(\frac{e_{15}}{c_{44}^{(k)}}\right)^2\frac{c_{33}^{(k)}}{c_{44}^{(k)}}+\left(\frac{e_{31}^{(k)}}{c_{44}^{(k)}}\right)^2\frac{c_{33}^{(k)}}{c_{44}^{(k)}}\right)\\ &+s^4\left(\left(\frac{c_{13}^{(k)}}{c_{44}^{(k)}}\right)^2\frac{\varepsilon_{33}^{(k)}}{c_{44}^{(k)}}+\left(\frac{e_{15}^{(k)}}{c_{44}^{(k)}}\right)^2+\left(\frac{e_{31}^{(k)}}{c_{44}^{(k)}}\right)^2-2\frac{\varepsilon_{11}^{(k)}}{c_{44}^{(k)}}+2\frac{e_{31}^{(k)}}{c_{44}^{(k)}}\frac{e_{15}^{(k)}}{c_{44}^{(k)}}\frac{c_{33}^{(k)}}{c_{44}^{(k)}}-4\frac{e_{33}^{(k)}}{c_{44}^{(k)}}\frac{e_{15}^{(k)}}{c_{44}^{(k)}}\right)\end{aligned}$$

$$+s^4\left(-\frac{c_{11}^{(k)}}{c_{44}^{(k)}}\frac{\varepsilon_{33}^{(k)}}{c_{44}^{(k)}}-2\frac{c_{33}^{(k)}}{c_{44}^{(k)}}\frac{\varepsilon_{11}^{(k)}}{c_{44}^{(k)}}-\frac{\varepsilon_{33}^{(k)}}{c_{44}^{(k)}}-\frac{c_{33}^{(k)}}{c_{44}^{(k)}}\frac{c_{11}^{(k)}}{c_{44}^{(k)}}\frac{\varepsilon_{33}^{(k)}}{c_{44}^{(k)}}+2\frac{c_{13}^{(k)}}{c_{44}^{(k)}}\frac{\varepsilon_{33}^{(k)}}{c_{44}^{(k)}}+2s^4\frac{e_{31}^{(k)}}{c_{44}^{(k)}}\frac{e_{15}^{(k)}}{c_{44}^{(k)}}\right)$$

$$+s^2\Omega^2\left(\frac{c_2^{(1)}}{c_2^{(k)}}\right)^2\left(-\left(\frac{e_{15}^{(k)}}{c_{44}^{(k)}}\right)^2-\left(\frac{e_{31}^{(k)}}{c_{44}^{(k)}}\right)^2-2\frac{e_{31}^{(k)}}{c_{44}^{(k)}}\frac{e_{15}^{(k)}}{c_{44}^{(k)}}+2\frac{e_{33}^{(k)}}{c_{44}^{(k)}}\frac{e_{15}^{(k)}}{c_{44}^{(k)}}+3\frac{\varepsilon_{11}^{(k)}}{c_{44}^{(k)}}+\frac{\varepsilon_{33}^{(k)}}{c_{44}^{(k)}}+\frac{\varepsilon_{33}^{(k)}}{c_{44}^{(k)}}\frac{c_{11}^{(k)}}{c_{44}^{(k)}}+\frac{c_{33}^{(k)}}{c_{44}^{(k)}}\frac{\varepsilon_{11}^{(k)}}{c_{44}^{(k)}}\right)$$

$$d_0^{(k)}=+s^6\left(2\left(\frac{e_{33}^{(k)}}{c_{44}^{(k)}}\right)^2+2\frac{\varepsilon_{33}^{(k)}}{c_{44}^{(k)}}+2\frac{c_{33}^{(k)}}{c_{44}^{(k)}}\frac{\varepsilon_{33}^{(k)}}{c_{44}^{(k)}}\right)+s^2\Omega^4\left(\frac{c_2^{(1)}}{c_2^{(k)}}\right)^4\frac{\varepsilon_{33}^{(k)}}{c_{44}^{(k)}}$$

$$-s^4\Omega^2\left(\frac{c_2^{(1)}}{c_2^{(k)}}\right)^2\left(\left(\frac{e_{33}^{(k)}}{c_{44}^{(k)}}\right)^2+\frac{c_{33}^{(k)}}{c_{44}^{(k)}}\frac{\varepsilon_{33}^{(k)}}{c_{44}^{(k)}}+3\frac{\varepsilon_{33}^{(k)}}{c_{44}^{(k)}}\right) \tag{21}$$

From Eq. (19) it can be stated that

$$\Lambda_1^{(k)}=\frac{\psi^{(k)}}{\left|\psi^{(k)}\right|}2\sqrt{\frac{-P^{(k)}}{3}}\cos\left(\frac{\psi^{(k)}}{3}\right)-\frac{a_4^{(k)}}{3},\ \psi^{(k)}=\arctan\left(2\frac{\sqrt{-D^{(k)}}}{-Q^{(k)}}\right),$$

$$\Lambda_2^{(k)}=\frac{\psi^{(k)}}{\left|\psi^{(k)}\right|}2\sqrt{\frac{-P^{(k)}}{3}}\left(-\cos\left(\frac{\psi^{(k)}}{3}\right)-\sqrt{3}\sin\left(\frac{\psi^{(k)}}{3}\right)\right)-\frac{a_4^{(k)}}{3},$$

$$\Lambda_3^{(k)}=\frac{\psi^{(k)}}{\left|\psi^{(k)}\right|}2\sqrt{\frac{-P^{(k)}}{3}}\left(-\cos\left(\frac{\psi^{(k)}}{3}\right)+\sqrt{3}\sin\left(\frac{\psi^{(k)}}{3}\right)\right)-\frac{a_4^{(k)}}{3},$$

$$P^{(k)}=-\frac{\left(a_4^{(k)}\right)^2}{3}+a_2^{(k)},\ Q^{(k)}=\frac{2\left(a_4^{(k)}\right)^3}{27}-\frac{a_2^{(k)}a_4^{(k)}}{3}+a_0^{(k)},\ D^{(k)}=\frac{\left(Q^{(k)}\right)^2}{4}+\frac{\left(P^{(k)}\right)^3}{27}. \tag{22}$$

We will consider the cases where

$$\Lambda_1^{(k)}\neq\Lambda_2^{(k)}\neq\Lambda_3^{(k)}, \tag{23}$$

According to the condition in Eq. (23), it can be written that

$$\lambda_1^{(k)}=\sqrt{\Lambda_1^{(k)}},\lambda_2^{(k)}=-\sqrt{\Lambda_1^{(k)}},\lambda_3^{(k)}=\sqrt{\Lambda_2^{(k)}},\lambda_4^{(k)}=-\sqrt{\Lambda_2^{(k)}},\lambda_5^{(k)}=\sqrt{\Lambda_3^{(k)}},\lambda_6^{(k)}=-\sqrt{\Lambda_3^{(k)}}. \tag{24}$$

From the foregoing results we can present the general solution of the Eq. (16) for the covering layer as follows

$$u_F^{(1)}=\sum_{n=1}^{6}A_n^{(1)}\mathrm{e}^{\lambda_n^{(1)}z},\ w_F^{(1)}=\sum_{n=1}^{6}A_n^{(1)}\beta_n^{(1)}\mathrm{e}^{\lambda_n^{(1)}z},\ \varphi_F^{(1)}=\sum_{n=1}^{6}A_n^{(1)}\delta_n^{(1)}\mathrm{e}^{\lambda_n^{(1)}z}.$$

$$\sigma_{xxF}^{(1)}=c_{13}^{(1)}\sum_{n=1}^{6}A_n^{(1)}\left(\frac{c_{33}^{(1)}}{c_{13}^{(1)}}(\mathrm{i}s)+\beta_n^{(1)}\lambda_n^{(1)}+\frac{e_{33}^{(1)}}{c_{13}^{(1)}}\mathrm{i}s\delta_n^{(1)}\right)\mathrm{e}^{\lambda_n^{(1)}z},$$

$$\sigma_{zzF}^{(1)}=c_{13}^{(1)}\sum_{n=1}^{6}A_n^{(1)}\left((\mathrm{i}s)+\frac{c_{11}^{(1)}}{c_{13}^{(1)}}\beta_n^{(1)}\lambda_n^{(1)}+\frac{e_{31}^{(1)}}{c_{13}^{(1)}}\mathrm{i}s\delta_n^{(1)}\right)\mathrm{e}^{\lambda_n^{(1)}z},$$

$$\sigma_{xzF}^{(1)} = c_{44}^{(1)} \sum_{n=1}^{6} A_n^{(1)} \left(\lambda_n^{(1)} + (\mathrm{i}s)\beta_n^{(1)} + \frac{e_{15}^{(1)}}{c_{44}^{(1)}} \delta_n^{(1)} \lambda_n^{(1)} \right) \mathrm{e}^{\lambda_n^{(1)} z} \ ,$$

$$D_{xF}^{(1)} = \sum_{n=1}^{6} A_n^{(1)} \left(e_{33}^{(1)} \mathrm{i}s + e_{31}^{(1)} \beta_n^{(1)} \lambda_n^{(1)} - \varepsilon_{33}^{(1)} \mathrm{i}s \delta_n^{(1)} \right) \mathrm{e}^{\lambda_n^{(1)} z} \ ,$$

$$D_{zF}^{(1)} = \sum_{n=1}^{6} A_n^{(1)} \left(e_{15}^{(1)} (\lambda_n^{(1)} + \mathrm{i}s\beta_n^{(1)}) - \varepsilon_{11}^{(1)} \delta_n^{(1)} \lambda_n^{(1)} \right) \mathrm{e}^{\lambda_n^{(1)} z} \ . \tag{25}$$

Taking into account the condition Eq. (10), the general solution of the Eq. (16) for the half-plane is

$$u_F^{(2)} = \sum_{n=1;3;5} A_n^{(2)} \, \mathrm{e}^{\lambda_n^{(2)} z} \ , \ w_F^{(2)} = \sum_{n=1;3;5} A_n^{(2)} \beta_n^{(2)} \, \mathrm{e}^{\lambda_n^{(2)} z} \ , \ \varphi_F^{(2)} = \sum_{n=1;3;5} A_n^{(2)} \delta_n^{(2)} \, \mathrm{e}^{\lambda_n^{(2)} z} \ ,$$

$$\sigma_{xxF}^{(2)} = c_{13}^{(2)} \sum_{n=1}^{6} A_n^{(2)} \left(\frac{c_{33}^{(2)}}{c_{13}^{(2)}} (\mathrm{i}s) + \beta_n^{(2)} \lambda_n^{(2)} + \frac{e_{33}^{(2)}}{c_{13}^{(2)}} \mathrm{i}s \delta_n^{(2)} \right) \mathrm{e}^{\lambda_n^{(2)} z} \ ,$$

$$\sigma_{zzF}^{(2)} = c_{13}^{(2)} \sum_{n=1}^{6} A_n^{(2)} \left((\mathrm{i}s) + \frac{c_{11}^{(2)}}{c_{13}^{(2)}} \beta_n^{(2)} \lambda_n^{(2)} + \frac{e_{31}^{(2)}}{c_{13}^{(2)}} \mathrm{i}s \delta_n^{(2)} \right) \mathrm{e}^{\lambda_n^{(2)} z} \ ,$$

$$\sigma_{xzF}^{(2)} = c_{44}^{(2)} \sum_{n=1}^{6} A_n^{(2)} \left(\lambda_n^{(2)} + (\mathrm{i}s)\beta_n^{(2)} + \frac{e_{15}^{(2)}}{c_{44}^{(2)}} \delta_n^{(2)} \lambda_n^{(2)} \right) \mathrm{e}^{\lambda_n^{(2)} z} \ ,$$

$$D_{xF}^{(2)} = \sum_{n=1}^{6} A_n^{(2)} \left(e_{33}^{(2)} \mathrm{i}s + e_{31}^{(2)} \beta_n^{(2)} \lambda_n^{(2)} - \varepsilon_{33}^{(2)} \mathrm{i}s \delta_n^{(2)} \right) \mathrm{e}^{\lambda_n^{(2)} z} \ ,$$

$$D_{zF}^{(2)} = \sum_{n=1}^{6} A_n^{(2)} \left(e_{15}^{(2)} (\lambda_n^{(2)} + \mathrm{i}s\beta_n^{(2)}) - \varepsilon_{11}^{(2)} \delta_n^{(2)} \lambda_n^{(2)} \right) \mathrm{e}^{\lambda_n^{(2)} z} \ . \tag{26}$$

From the Eq. (19) we determine the constants $\beta_n^{(k)}$ and $\delta_n^{(k)}$ which enter the expressions Eq. (25) and Eq. (26) as follows

$$\beta_n^{(k)} = \left(\frac{ -\left(\frac{c_{13}^{(k)}}{c_{44}^{(k)}} + 1 \right) (-s) \lambda_n^{(k)} }{ \left(\left(\frac{e_{15}^{(k)} + e_{31}^{(k)}}{c_{44}^{(k)}} \right) (-s) \lambda_n^{(k)} \right) } + \frac{ \left(\frac{e_{15}^{(k)}}{c_{44}^{(k)}} \left(\lambda_n^{(k)} \right)^2 + \frac{e_{33}^{(k)}}{c_{44}^{(K)}} (-s^2) \right) }{ \left(-\frac{\varepsilon_{11}^{(k)}}{c_{44}^{(k)}} \left(\lambda_n^{(k)} \right)^2 - \frac{\varepsilon_{33}^{(k)}}{c_{44}^{(k)}} (-s^2) \right) } \right) \times$$

$$\left(\frac{ \frac{c_{11}^{(k)}}{c_{44}^{(k)}} \left(\lambda_n^{(k)} \right)^2 + \left((-s^2) + \Omega^2 \left(\frac{c_2^{(1)}}{c_2^{(k)}} \right)^2 \right) }{ \left(\left(\frac{e_{15}^{(k)} + e_{31}^{(k)}}{c_{44}^{(k)}} \right) (-s) \lambda_n^{(k)} \right) } - \frac{ \left(\left(\frac{e_{15}^{(k)} + e_{31}^{(k)}}{c_{44}^{(k)}} \right) (-s) \lambda_n^{(k)} \right) }{ \left(-\frac{\varepsilon_{11}^{(k)}}{c_{44}^{(k)}} \left(\lambda_n^{(k)} \right)^2 - \frac{\varepsilon_{33}^{(k)}}{c_{44}^{(k)}} (-s^2) \right) } \right)^{-1} ,$$

$$\delta_n^{(k)}=\left(\frac{-\left(\frac{c_{13}^{(k)}}{c_{44}^{(k)}}+1\right)(-s)\lambda_n^{(k)}}{\left(\frac{c_{11}^{(k)}}{c_{44}^{(k)}}\left(\lambda_n^{(k)}\right)^2+\left((-s^2)+\Omega^2\left(\frac{c_2^{(1)}}{c_2^{(k)}}\right)^2\right)\right)}+\frac{\left(\frac{e_{15}^{(k)}}{c_{44}^{(k)}}\left(\lambda_n^{(k)}\right)^2+\frac{e_{33}^{(k)}}{c_{44}^{(k)}}(-s^2)\right)}{\left(\frac{e_{15}^{(k)}}{c_{44}^{(k)}}+\frac{e_{31}^{(k)}}{c_{44}^{(k)}}\right)(-s)\lambda_n^{(k)}}\right)\times$$

$$\left(\frac{\left(\frac{e_{15}^{(k)}}{c_{44}^{(k)}}+\frac{e_{31}^{(k)}}{c_{44}^{(k)}}\right)(-s)\lambda_n^{(k)}}{\left(\frac{c_{11}^{(k)}}{c_{44}^{(k)}}\left(\lambda_n^{(k)}\right)^2+\left((-s^2)+\Omega^2\left(\frac{c_2^{(1)}}{c_2^{(k)}}\right)^2\right)\right)}-\frac{\left(-\frac{\varepsilon_{11}^{(k)}}{c_{44}^{(k)}}\left(\lambda_n^{(k)}\right)^2-\frac{\varepsilon_{33}^{(k)}}{c_{44}^{(k)}}(-s^2)\right)}{\left(\frac{e_{15}^{(k)}}{c_{44}^{(k)}}+\frac{e_{31}^{(k)}}{c_{44}^{(k)}}\right)(-s)\lambda_n^{(k)}}\right)^{-1} \quad (27)$$

If the Eq. (20) has repeated roots, the foregoing procedures are done with the use of the well-known solution rules of ordinary differential equations.

Consider satisfaction of boundary conditions Eqs. (7)-(8) (or (9)) and contact conditions Eq. (5) and Eq. (6) from which we obtain the following algebraic equations for determination of the unknown constants $A_n^{(1)}$ $(n=1,2,3,4,5,6)$ and $A_k^{(2)}$ $(k=1,3,5)$.

$$\sigma_{xzF}^{(1)}\Big|_{z=0}=0\Rightarrow\ \sum_{n=1}^{6}A_n^{(1)}\alpha_{1n}^{(1)}=0,\ \sigma_{zzF}^{(1)}\Big|_{z=0}=-P\Rightarrow\ \sum_{n=1}^{6}A_n^{(1)}\alpha_{2n}^{(1)}=-P, \quad (28)$$

$$\varphi_F^{(1)}\Big|_{z=0}=0\Rightarrow\ \sum_{n=1}^{6}A_n^{(1)}\alpha_{3n}^{(1)}=0,\ \text{or} \quad (29)$$

$$D_{zF}^{(1)}\Big|_{z=0}=0\Rightarrow\ \sum_{n=1}^{6}A_n^{(1)}\alpha_{3n}^{(1)}=0, \quad (30)$$

$$\sigma_{xzF}^{(1)}\Big|_{z=-h}-\sigma_{xzF}^{(2)}\Big|_{z=-h}=0\Rightarrow\ \sum_{n=1}^{6}A_n^{(1)}\alpha_{4n}^{(1)}-\sum_{k=1;3;5}A_k^{(2)}\alpha_{4k}^{(2)}=0,$$

$$\sigma_{zzF}^{(1)}\Big|_{z=-h}-\sigma_{zzF}^{(2)}\Big|_{z=-h}=0\Rightarrow\ \sum_{n=1}^{6}A_n^{(1)}\alpha_{5n}^{(1)}-\sum_{k=1;3;5}A_k^{(2)}\alpha_{5k}^{(2)}=0,$$

$$u_F^{(1)}\Big|_{z=-h}-u_F^{(2)}\Big|_{z=-h}=0\Rightarrow\ \sum_{n=1}^{6}A_n^{(1)}\alpha_{6n}^{(1)}-\sum_{k=1;3;5}A_k^{(2)}\alpha_{6k}^{(2)}=0,$$

$$w_F^{(1)}\Big|_{z=-h}-w_F^{(2)}\Big|_{z=-h}=0\Rightarrow\ \sum_{n=1}^{6}A_n^{(1)}\alpha_{7n}^{(1)}-\sum_{k=1;3;5}A_k^{(2)}\alpha_{7k}^{(2)}=0,$$

$$\varphi_F^{(1)}\Big|_{z=-h}-\varphi_F^{(2)}\Big|_{z=-h}=0\Rightarrow\ \sum_{n=1}^{6}A_n^{(1)}\alpha_{8n}^{(1)}-\sum_{k=1;3;5}A_k^{(2)}\alpha_{8k}^{(2)}=0,$$

$$D_{zF}^{(1)}\Big|_{z=-h}-D_{zF}^{(2)}\Big|_{z=-h}=0\Rightarrow\ \sum_{n=1}^{6}A_n^{(1)}\alpha_{9n}^{(1)}-\sum_{k=1;3;5}A_k^{(2)}\alpha_{9k}^{(2)}=0. \quad (31)$$

Expressions for $\alpha_{mn}^{(1)}$ (m=1,2,...,9, n=1,2,...,6) and $\alpha_{rk}^{(2)}$ (r=4,5,...,9, k=1,3,5) can be determined from the Eqs. (25)-(26). Thus, after determination of the unknowns $A_n^{(1)}$ (n=1,2,3,4,5,6) and $A_k^{(2)}$ (k=1,3,5) from the Eqs. (28)-(29)-(31) for Case 1 or from the Eqs. (28)-(30)-(31) for Case 2 we determine completely the Fourier transformations of the sought values. According to the inverse Fourier transformation, the originals of the stresses and displacements can be presented formally as follows

$$\left\{\sigma_{xx}^{(k)},\sigma_{xz}^{(k)},\sigma_{zz}^{(k)},u^{(k)},w^{(k)},\varphi^{(k)}\right\}=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\left\{\sigma_{xxF}^{(k)},\sigma_{xzF}^{(k)},\sigma_{zzF}^{(k)},u_F^{(k)},w_F^{(k)},\varphi^{(k)}\right\}\mathrm{e}^{\mathrm{i}sx}\,\mathrm{d}s\ . \tag{32}$$

Using the symmetry properties of the stresses and displacements with respect to x=0 we can simplify the equation Eq. (32) in the following manner

$$\left\{\sigma_{xx}^{(k)},\sigma_{zz}^{(k)},w^{(k)},\varphi^{(k)}\right\}=\frac{1}{\pi}\int_{0}^{+\infty}\left\{\sigma_{xxF}^{(k)},\sigma_{zzF}^{(k)},w_F^{(k)},\varphi^{(k)}\right\}\cos(sx)\mathrm{d}s\,,$$

$$\left\{\sigma_{xz}^{(k)},u^{(k)}\right\}=\frac{i}{\pi}\int_{0}^{+\infty}\left\{\sigma_{xzF}^{(k)},u_F^{(k)}\right\}\sin(sx)\mathrm{d}s\,. \tag{33}$$

According to the paper by Akbarov and Ilhan (2013), we will evaluate the wave-number integrals (Eq. (33)) along the Sommerfeld contour (Fig. 1(b)), by using Cauchy’s theorem, the contour [−∞,+ ∞] of integration is deformed into the contour C (Fig. 1(b)) in the complex plane $s=s_1+\mathrm{i}s_2$. The advantage of the Sommerfeld contour method for calculation of the mentioned integrals is discussed in the paper by Akbarov and Ilhan (2013).

Thus, the sought values are determined from the relation

$$\left\{\sigma_{xx}^{(k)},\sigma_{xz}^{(k)},\sigma_{zz}^{(k)},u^{(k)},w^{(k)},\varphi^{(k)}\right\}=\frac{1}{2\pi}\mathrm{Re}\left\{\int_{C}\left\{\sigma_{xxF}^{(k)},\sigma_{xzF}^{(k)},\sigma_{zzF}^{(k)},u_F^{(k)},w_F^{(k)},\varphi^{(k)}\right\}\mathrm{e}^{\mathrm{i}sx}\,\mathrm{d}s\right\}. \tag{34}$$

According to Fig. 1(b), we can write the following relation

$$\int_C f(s)\mathrm{e}^{\mathrm{i}sx}\,\mathrm{d}s=\int_{-\infty}^{0}f(s_1-\mathrm{i}\varepsilon)\mathrm{e}^{\mathrm{i}(s_1-\mathrm{i}\varepsilon)x}\,\mathrm{d}s_1+\mathrm{i}\int_{-\varepsilon}^{\varepsilon}f(\mathrm{i}s_2)\mathrm{e}^{-s_2x}\,\mathrm{d}s_2+\int_{0}^{\infty}f(s_1+\mathrm{i}\varepsilon)\mathrm{e}^{\mathrm{i}(s_1+\mathrm{i}\varepsilon)x}\,\mathrm{d}s_1\ , \tag{35}$$

Using the transformations

$$\int_{-\infty}^{0}f(s_1-\mathrm{i}\varepsilon)\mathrm{e}^{\mathrm{i}(s_1-\mathrm{i}\varepsilon)x}\,\mathrm{d}s_1=-\int_{0}^{-\infty}f(s_1-\mathrm{i}\varepsilon)\mathrm{e}^{\mathrm{i}(s_1-\mathrm{i}\varepsilon)x}\,\mathrm{d}s_1=\int_{0}^{\infty}f(-s_1-\mathrm{i}\varepsilon)\mathrm{e}^{\mathrm{i}(-s_1-\mathrm{i}\varepsilon)x}\,\mathrm{d}s_1=$$

$$\int_{0}^{\infty}f(-s_1-\mathrm{i}\varepsilon)(\cos((-s_1-\mathrm{i}\varepsilon)x)+\mathrm{i}\sin((-s_1-\mathrm{i}\varepsilon)x))\mathrm{d}s_1\ ,$$

$$\int_{0}^{\infty}f(s_1+\mathrm{i}\varepsilon)\mathrm{e}^{\mathrm{i}(s_1+\mathrm{i}\varepsilon)x}\,\mathrm{d}s_1=\int_{0}^{\infty}f(s_1+\mathrm{i}\varepsilon)(\cos((s_1+\mathrm{i}\varepsilon)x)+\mathrm{i}\sin((s_1+\mathrm{i}\varepsilon)x))\mathrm{d}s_1\ ,$$

the integral (Eq. (35)) can be rewritten as follows

$$\int_C f(s)\mathrm{e}^{\mathrm{i}sx}\,\mathrm{d}s=\int_{0}^{\infty}\left[f(-s_1-\mathrm{i}\varepsilon)+f(s_1+\mathrm{i}\varepsilon)\right]\cos((s_1+\mathrm{i}\varepsilon)x)\mathrm{d}s_1$$

$$+\mathrm{i}\int_{0}^{\infty}\left[f(s_1+\mathrm{i}\varepsilon)-f(-s_1-\mathrm{i}\varepsilon)\right]\sin((s_1+\mathrm{i}\varepsilon)x)\mathrm{d}s_1 + \int_{-\varepsilon}^{+\varepsilon} f(\mathrm{i}s_2)e^{-s_2 x}\mathrm{d}s_2 \ . \quad (36)$$

Taking the fact that the values of the integral $\int_C f(s)e^{\mathrm{i}sx}\mathrm{d}s$ are independent on the values of the parameter $\varepsilon>0$ into account, as usual (see, for example Jensen *et al.* 2011, Tsang 1978), to simplify the calculation procedure of the integral $\int_C f(s)e^{\mathrm{i}sx}\mathrm{d}s$, where the parameter ε is assumed as a small parameter. According to this assumption and according to the theorem given in Akbarov and Ilhan (2013) and to the relation $\left|\int_{-\varepsilon}^{+\varepsilon} f(is_2)e^{-s_2x}\mathrm{d}s_2\right| = O(\varepsilon)$, we use the following approximate expressions for calculation of the integral $\int_C f(s)e^{\mathrm{i}sx}\mathrm{d}s$:

For the even functions

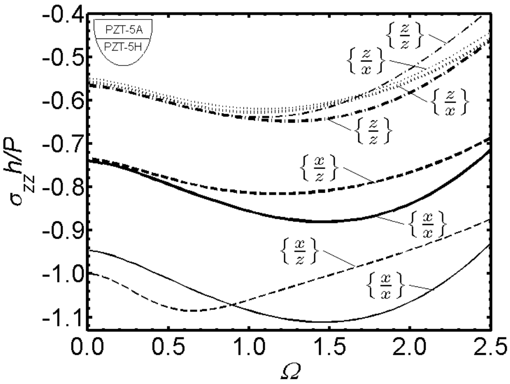
$$\int_C f(s)e^{\mathrm{i}sx}\mathrm{d}s \approx 2\int_0^{\infty} f(s_1+\mathrm{i}\varepsilon)\cos((s_1+\mathrm{i}\varepsilon)x)\mathrm{d}s_1 \ , \quad (37)$$

For the odd functions

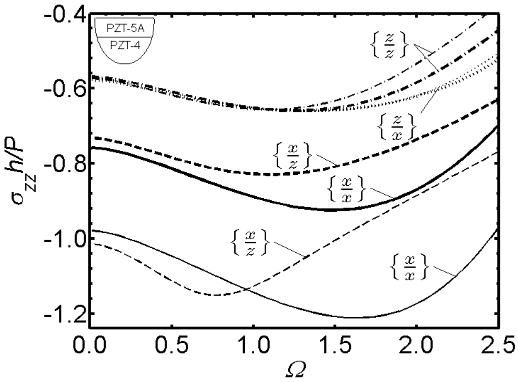
$$\int_C f(s)e^{\mathrm{i}sx}\mathrm{d}s \approx 2\mathrm{i}\int_0^{\infty} f(s_1+\mathrm{i}\varepsilon)\sin((s_1+\mathrm{i}\varepsilon)x)\mathrm{d}s_1 \ . \quad (38)$$

The accuracy of the expressions Eq. (37) and Eq. (38) with respect to values of the parameter ε has been discussed in the paper by Akbarov and Ilhan (2013) and therefore we here do not consider this question. Under the calculation procedure the improved integral $\int_0^{+\infty}(\bullet)\mathrm{d}s_1$ in (37) and in Eq. (38) is replaced by the corresponding definite integral $\int_0^{+S_1^*}(\bullet)\mathrm{d}s_1$. The values of S_1^* are determined from the convergence requirement of the corresponding improved integrals. Note that under calculation of the latter integral, the interval $\left[0,+S_1^*\right]$ is further divided into a certain number of shorter intervals, which are used in the Gauss integration algorithm. In this integration procedure the values of the integrated expressions, i.e., the values of the unknowns $A_1^{(1)}(s), A_2^{(1)}(s),...,A_6^{(1)}(s)$, $A_1^{(2)}(s), A_3^{(2)}(s), A_5^{(2)}(s)$ in Gauss's integration points are determined through Eqs. (28) – (31). In the aforementioned integration procedure it is assumed that in each of the shorter intervals the sampling interval of the numerical integration Δs_1 must satisfy the relation $|\Delta s_1| << \min\{\varepsilon, 1|x|\}$.

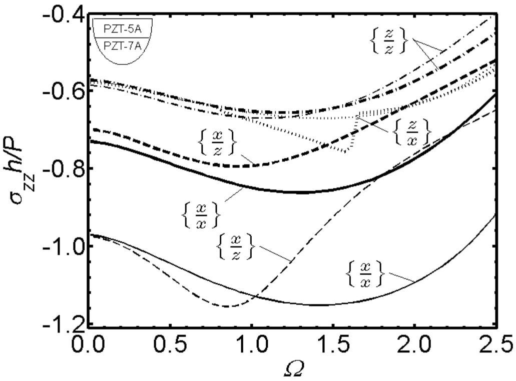
All these procedures are carried out automatically in the PC by using the programs constructed by the author. At the same time, we note that all numerical results which will be discussed below are obtained in the case where $S_1^* = 30$ and $\varepsilon=0.01$ for some pairs of piezoelectric materials which are given in Yang (2005).



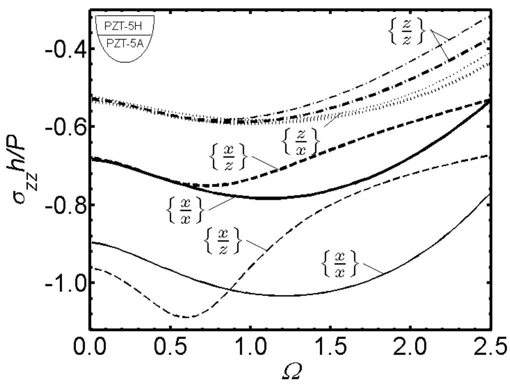
(a) Covering layer material is PZT-5A and half-plane PZT-5H



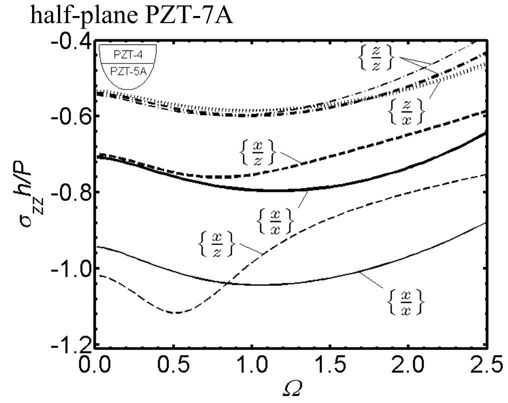
(b) Covering layer material is PZT-5A and half-plane PZT-4



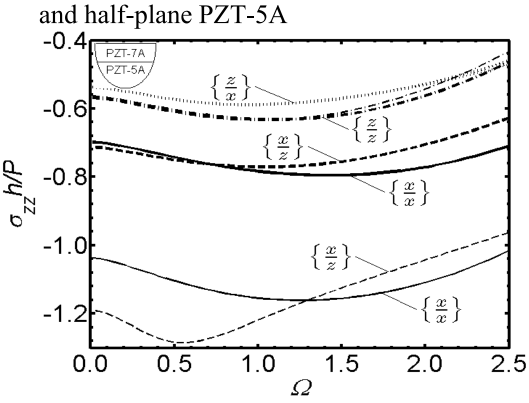
(c) Covering layer material is PZT-5A and half-plane PZT-7A



(d) Covering layer material is PZT-5H and half-plane PZT-5A



(e) Covering layer material is PZT-4 and half-plane PZT-5A



(f) Covering layer material is PZT-7A and half-plane PZT-5A

Fig. 2 Graphs of the dependence between normal stress σ_{zz} (x=0, z=−h) and dimensionless frequency Ω

4. Numerical results

In this section, some numerical results are presented to examine the influence of the polled direction of the piezoelectric materials such as PZT-5A, PZT-5H, PZT-4 and PZT-7A on the normal stresses, shear stresses and electric potential acting on the interface plane. Although it is possible to calculate the problem unknowns in anywhere on the O_{xz} plane, we will focus interface plane i.e., $z=-h$ to consider the normal and shear stresses and electric potential.

We obtain the numerical results with respect to the dependence between the stresses, electric potential and dimensionless frequency Ω determined by expression (14), where

$$\sigma_{zz} = \sigma_{zz}^{(1)}\Big|_{(x=0;z=-h)} = \sigma_{zz}^{(2)}\Big|_{(x=0;z=-h)} \quad \sigma_{xz} = \sigma_{xz}^{(1)}\Big|_{(x=0.7;z=-h)} = \sigma_{xz}^{(2)}\Big|_{(x=0.7;z=-h)} ,$$

$$\varphi = \varphi^{(1)}\Big|_{(x=0;z=-h)} = \varphi^{(2)}\Big|_{(x=0;z=-h)} . \tag{39}$$

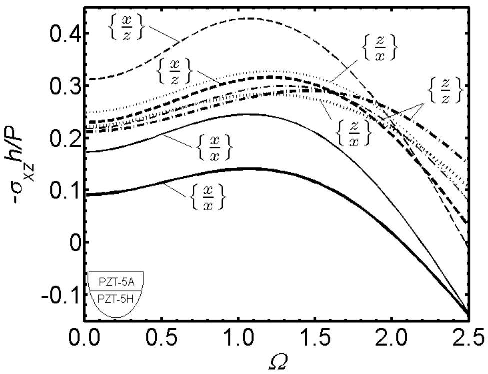
The graphs of these response are given in Figs. 2-4 and in these figures the symbol *a/b* indicates that the polled direction of the covering layer (half-plane) material with the *a* (with the *b*) direction. For instance, the symbol *x/z* means that the polled direction of the covering layer material (half-plane) material coincides with the direction of the *Ox* (of the *Oz*) axis. Moreover, in these figures the graphs drawn with thin lines relate to Case 1, i.e., the case where the condition Eq. (8) takes place, but the graphs drawn with thick lines relate to Case 2, i.e., the case where the condition Eq. (9) takes place.

Figs. 2(a), 2(b) and 2(c) (Figs. 3(a), 3(b) and 3(c)) show the frequency response of the normal stress σ_{zz} (of the shear stress σ_{xz}) in the cases where the half-plane material is PZT-5H, PZT-4 and PZT-7A respectively, but the covering layer material is PZT-5A. The graphs of the frequency response of the normal stress σ_{zz} (of the shear stress σ_{xz}) obtained in the cases where the covering layer material is PZT-5H, PZT-4 and PZT-7A, but the half-plane material is PZT-5A, are given in Figs. 2(d), 2(e) and 2(f) (in Figs. 3(d), 3(e) and 3(f)), respectively. The numerical results related to the frequency response of the dimensionless electric potential $\Phi(=\varphi^{(1)}(x=0,z)e_{15}^{(1)}/(C_{44}^{(1)}h))$ are given in Figs. 4(a), 4(b) and 4(c) in the cases where the half-plane material is PZT-5H, PZT-4 and PZT-7A respectively, but the covering layer material is PZT-5A, and in the Figs. 4(d), 4(e) and 4(f) in the cases where the covering layer material is PZT-5H, PZT-4 and PZT-7A respectively, but the half-plane material is PZT-5A. Note that in these figures the graphs indicated by the symbol *z/z* are the results obtained in the paper by Akbarov and Ilhan (2013).

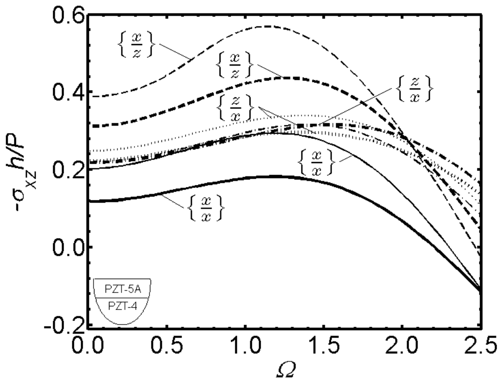
To simplify the formulation of the conclusions which follow from the foregoing results, we introduce the notations $\sigma_{zz}\big|_{a/b}$, $\sigma_{xz}\big|_{a/b}$ and $\Phi\big|_{a/b}$ which indicate the stresses σ_{zz}, σ_{xz} and dimensionless electric potential Φ in the case where the polled direction of the covering layer (half-plane) material is the *a* (the *b*) direction. For instance, the notation $\sigma_{zz}\big|_{x/z}$ indicates the values of the normal stress σ_{zz} in the case where the polled direction of the covering layer (half-plane) material coincides with the *Ox* (the *Oz*) axis direction. Thus, taking the foregoing assumption and notation we attempt to formulate the related results.

The polled direction of the covering layer and half-plane materials can influence significantly on the values of the studied interphase stresses and dimensionless electric potential. The character of this influence can be determined according to the following conclusions which are made according to the foregoing numerical results:

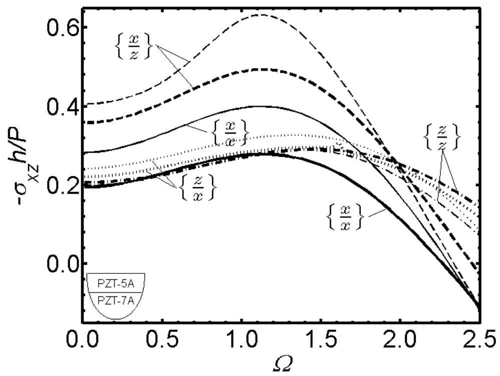
Figs. 2 and 3 show that in the all considered cases dependencies among the stresses σ_{zz}, σ_{xz} and Ω have non-monotonic character, i.e., there exists such value of the dimensionless frequency Ω (denote it by Ω_{res}) under which the absolute values of the stresses σ_{zz} and σ_{xz} have its absolute maximum. The values of the Ω_{res} depend significantly not only on the selected pairs of materials but also on the polled directions of these materials.



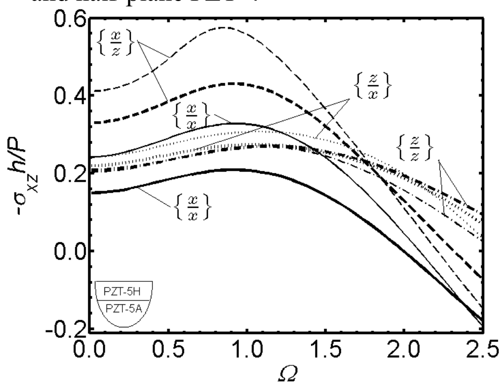
(a) Covering layer material is PZT-5A and half-plane PZT-5H



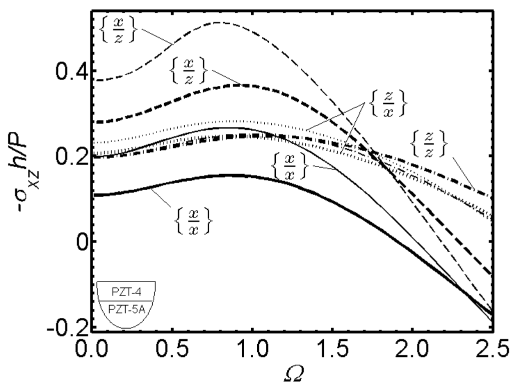
(b) Covering layer material is PZT-5A and half-plane PZT-4



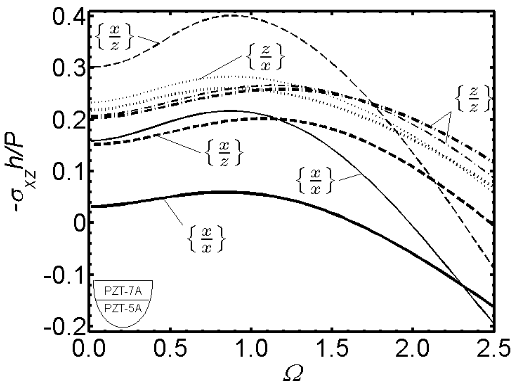
(c) Covering layer material is PZT-5A and half-plane PZT-7A



(d) Covering layer material is PZT-5H and half-plane PZT-5A



(e) Covering layer material is PZT-4 and half-plane PZT-5A

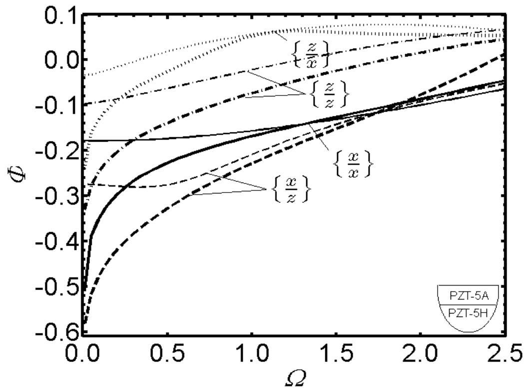


(f) Covering layer material is PZT-7A and half-plane PZT-5A

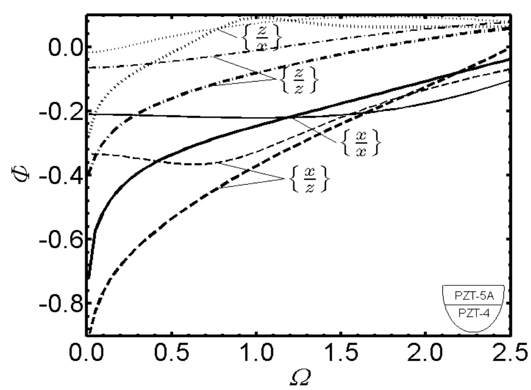
Fig. 3 Graphs of the dependence between shear stress σ_{xz} (x=0, z=−0.7h) and dimensionless frequency Ω

It can be seen from Figs. 2-4 that for the all selected pair of materials both Case 1 and Case 2 the following relations are satisfied

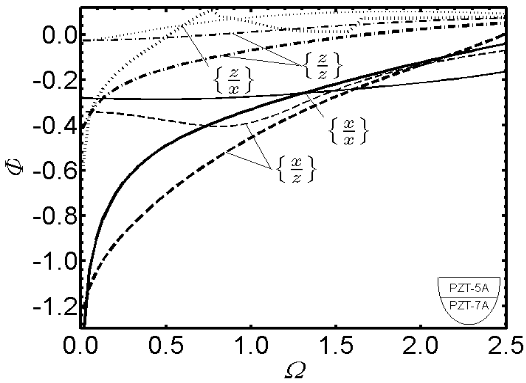
$$\left|\sigma_{zz}\big|_{x/x}\right| > \left|\sigma_{zz}\big|_{z/z}\right|,\ \left|\sigma_{xz}\big|_{x/x}\right| < \left|\sigma_{xz}\big|_{z/z}\right|,\ \left|\Phi\big|_{x/x}\right| > \left|\Phi\big|_{z/z}\right|;$$



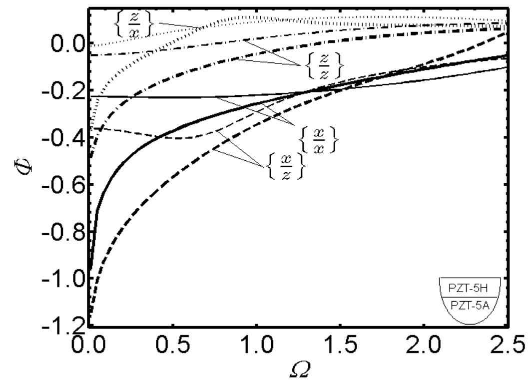
(a) Covering layer material is PZT-5A and half-plane PZT-5H



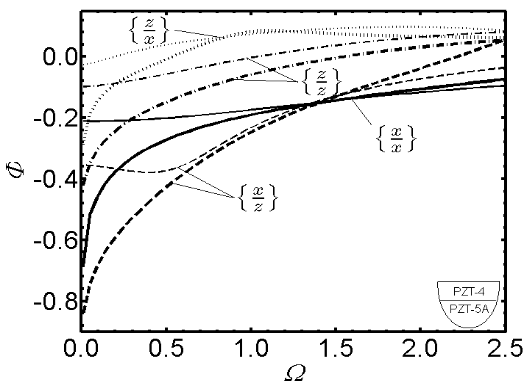
(b) Covering layer material is PZT-5A and half-plane PZT-4



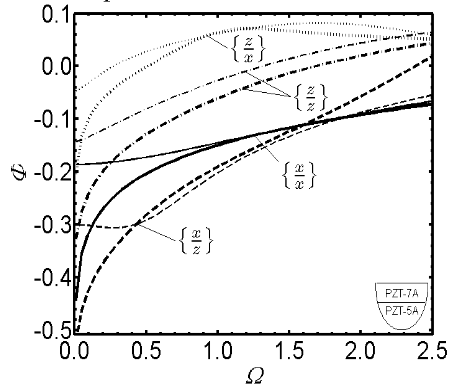
(c) Covering layer material is PZT-5A and half-plane PZT-7A



(d) Covering layer material is PZT-5H and half-plane PZT-5A



(e) Covering layer material is PZT-4 and half-plane PZT-5A



(f) Covering layer material is PZT-7A and half-plane PZT-5A

Fig. 4 Graphs of the dependence between dimensionless electric potential $\Phi(=\varphi^{(1)}(x=0,z=-h)e_{15}^{(1)}/C_{44}^{(1)}h)$ and dimensionless frequency Ω

And for the all selected pairs of materials in Case 2 there exists such value of dimension frequency Ω (denote it by Ω*), according to which the following relations take place:

$\left|\sigma_{zz}\right|_{x/z}\Big| > \Big|\sigma_{zz}\big|_{x/x}\Big|$ if $\Omega < \Omega^{*}_{\sigma_{zz}}$, $\Big|\sigma_{zz}\big|_{x/z}\Big| < \Big|\sigma_{zz}\big|_{x/x}\Big|$ if $\Omega > \Omega^{*}_{\sigma_{zz}}$,

$\Big|\Phi\big|_{x/z}\Big| > \Big|\Phi\big|_{x/x}\Big|$ if $\Omega < \Omega^{*}_{\Phi}$, $\Big|\Phi\big|_{x/z}\Big| < \Big|\Phi\big|_{x/x}\Big|$ if $\Omega > \Omega^{*}_{\Omega}$ and

$\Big|\sigma_{xz}\big|_{x/z}\Big| > \Big|\sigma_{xz}\big|_{x/x}\Big|$ for the all considered values of the Ω;

In addition to Case1 for the all selected pairs of materials the following relations are satisfied: $\Big|\sigma_{zz}\big|_{x/x}\Big| > \Big|\sigma_{zz}\big|_{x/z}\Big|$, $\Big|\sigma_{xz}\big|_{x/x}\Big| < \Big|\sigma_{xz}\big|_{x/z}\Big|$ and $\Big|\Phi\big|_{x/x}\Big| < \Big|\Phi\big|_{x/z}\Big|$ for $\Omega < \Omega'_{\Phi}$, $\Big|\Phi\big|_{x/x}\Big| > \Big|\Phi\big|_{x/z}\Big|$ for $\Omega > \Omega'_{\Phi}$, the values of the Ω'_{Φ} depend on the selected pairs of the materials and its polled directions;

Also, in Case1 and Case 2 for the all selected pairs of materials the following relations are satisfied:

$\Big|\sigma_{zz}\big|_{x/z}\Big| > \Big|\sigma_{zz}\big|_{z/x}\Big|$ for the all considered values of Ω , $\Big|\sigma_{xz}\big|_{x/z}\Big| > \Big|\sigma_{xz}\big|_{z/x}\Big|$ for $\Omega < \Omega''_{\sigma_{xz}}$, $\Big|\sigma_{xz}\big|_{x/z}\Big| < \Big|\sigma_{xz}\big|_{z/x}\Big|$ for $\Omega > \Omega''_{\sigma_{xz}}$ and $\Big|\Phi\big|_{x/z}\Big| > \Big|\Phi\big|_{z/x}\Big|$ for the all considered values of Ω.

5. Conclusions

The influence of the polled direction of piezoelectric materials on the stress distribution is studied. Numerical results on the influence of the polled direction of the piezoelectric materials such as PZT-5A, PZT-5H, PZT-4 and PZT-7A on the normal stresses, shear stresses and electric potential acting on the interface plane are presented and discussed in detail. The final analysis show that the polled direction of the covering layer and half-plane materials can influence significantly on the values of the studied interphase stresses and dimensionless electric potential.

Absolute maximum values of normal stresses, shear stresses and electric potential depend not only on the selected pair of materials but also polled directions of these materials. When the covering layer and half-plane material is polled with the direction of the *Ox*, absolute maximum values of the normal stresses, shear stresses and electric potential are greater than those of other cases.

References

Akbarov, S.D., Emiroglu, I. and Tasci, F. (2005), "The Lamb's problem for a half-space covered with the pre-stretched layer", *Int. J. Mech. Sci.*, **47**(9), 1326-1349.

Akbarov, S.D. (2006a), "On the dynamical axisymmetric stress field in a finite pre-stretched bilayered slab resting on a rigid foundation", *J. Sound Vib.*, **294**(1-2), 221-237.

Akbarov, S.D. (2006b), "The influence of the third order elastic constants on the dynamical interface stress field in a half-space covered with a pre-stretched layer", *Int. J. Nonlin. Mech.*, **41**(3), 417-425.

Akbarov, S.D. (2006c), "Dynamical (time-harmonic) axisymmetric interface stress field in the finite pre-strained half-space covered with the finite pre-stretched layer", *Int. J. Eng. Sci.*, **44**(1-2), 93-112.

Akbarov, S.D. (2006d), "Frequency response of the axisymmetrically finite pre-stretched slab from incompressible functionally graded material on a rigid foundation", *Int. J. Eng. Sci.*, **44**(8-9), 484-500.

Akbarov, S.D. and Guler, C. (2007), "On the stress field in a half-plane covered by the pre-stretched layer under the action of arbitrary linearly located time-harmonic forces", *Appl. Math. Model.*, **31**(11), 2375-2390.

Akbarov, S.D. and Ilhan, N. (2008), "Dynamics of a system comprising an orthotropic layer and pre-stressed orthotropic half-plane under the action of a moving load", *Int. J. Solid. Struct*., **45**(14-15), 4222-4235.

Akbarov, S.D. and Ilhan, N. (2009), "Dynamics of a system comprising an orthotropic layer and pre-stressed orthotropic half-plane under the action of an oscillating moving load", *Int. J. Solid. Struct*., **46**(21), 3873-3881.

Akbarov, S.D. and Salmanova, K.A. (2009), "On the dynamics of a finite pre-strained bi-layered slab resting on a rigid foundation under the action of an oscillating moving load", *J. Sound Vib.*, **327**, 454-472.

Akbarov, S.D. and Ilhan, N. (2010), "Time-harmonic dynamical stress field in a system comprising a pre-stressed orthotropic layer and pre-stressed orthotropic half-plane", *Arch. Appl. Mech.*, **80**(11), 1271-1286.

Akbarov, S.D. (2013), "On the axisymmetric time-harmonic lamb's problem for a system comprising a half-space and a covering layer with finite initial strains", *CMES-Comput. Model. Eng. Sci.*, **95**(3), 173-205.

Akbarov, S.D. and Ilhan, N. (2013), "Time-harmonic lamb's problem for a system comprising a piezoelectric layer and piezoelectrıc half-plane", *J. Sound Vib.*, **332,** 5375-5392.

Akbarov, S.D., Hazar, E. and Eroz, M. (2013), "Forced vibration of the pre-stressed and imperfectly bonded bi-layered plate strip resting on a rigid foundation", *CMC: Comput. Mater. Contin.*, **36**(1), 23-48.

Borrelli, A., Horgan, C.O. and Patria, M.C. (2006), "Saint-Venant end effects for plane deformations of linear piezoelectric solids", *Int. J. Solid. Struct*., **43**, 943-956.

Chen, W.Q. (2000), "On piezoelastic contact problem for a smooth punch", *Int. J. Solid. Struct*., **37**, 2331-2340.

Emiroglu, I., Tasci, F. and Akbarov, S.D. (2009), "Dynamical response of two-axially pre-strained system comprising of a covering layer and half-space to rectangular time-harmonic forces.", *CMES-Comput. Model. Eng. Sci.*, **49**, 47-68.

Eringen, A.C. and Maugin, G.A. (1990), *Electrodynamics of Continua*, Vol 1, Springer.

Fan, H., Sze, K.Y. and Yang, W. (1996), "Two-dimensional contact on a piezoelectric half-space", *Int. J. Solid. Struct*., **33**, 1305-1315.

Gao, C.F. and Fan, W.X. (1998), "Green's functions for the plane problem in a half-infinite piezoelectric medium", *Mech. Res. Commun*., **25**, 69-74.

Ilhan, N. (2012), "The critical speed of a moving time-harmonic load acting on a system consisting a pre-stressed orthotropic covering layer and a pre-stressed half-plane", *Appl. Math. Model.*, **36**(8), 3663-3672.

Jensen, F.B., Kuperman, W.A., Porter, M.B. and Schmidt, H. (2011), *Computational Ocean Acoustic*, Sd. Ed, Springer,.

Ma, J., Ke. L.L. and Wang, Y.S. (2014), "Electro-mechanical sliding frictional contact of a piezoelectric half-plane under a rigid conducting punch", *Appl. Math. Model.*, **38**, 5471-5489.

Ruan, X., Danforth, S.C., Safari, A. and Chou, T. (2000), "Saint-Venant end effects in piezoelectric materials", *Int. J. Solid. Struct.*, **37**, 2625-2637.

Sosa, H.A. and Castro, M.A. (1994), "On concentrated loads at the boundary of a piezoelectric half-plane", *J. Mech. Phys. Solid.*, **42**, 1105-1122.

Tarn, J.Q. and Huang, L.J. (2002), "Saint-Venant end effects in multilayered piezoelectric laminates", *Int. J. Solid. Struct.*, **39**, 4979-4998.

Tsang, L. (1978), "Time-harmonic solution of the elastic head wave problem incorporating the influence of Rayleigh poles", *J. Acoust. Soc. Am*., **65**(5), 1302-1309.

Yang, J. (2005), *An Introduction to the theory of Piezoelectricity*, Springer.

Yang P.S., Liou J.Y. and Sung J.C. (2007), "Analysis of a crack in a half-plane piezoelectric solid with traction-induction free boundary", *Int. J. Solid. Struct.*, **44**, 8556-8578.

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