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Performance of rotational mode based indices in identification of added mass in beams

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Abstract. This study investigates the identification of added mass and its location in the glass fiber reinforced polymer (GFRP) beam structures. The main emphasis of this paper is to ascertain the importance of inclusion of rotational degrees of freedom (dofs) in the introduction of added mass or damage identification. Two identification indices that include the rotational dofs have been introduced in this paper: the modal force index (MFI) and the modal rotational curvature index (MRCI). The MFI amplifies damage signature using undamaged numerical stiffness matrix which is related to changes in the altered mode shapes from the original mode shapes. The MRCI is obtained by using a higher derivative of rotational mode shapes. Experimental and numerical results are compared with the existing methods leading to a conclusion that the contributions of the rotational modes play a key role in the identification of added mass. The authors believe that the similar results are likely in the case of damage identification also.

Keywords: modal force index (MFI); modal rotational curvature index (MRCI); rotational mode shape; added mass; glass fiber reinforced polymer (GFRP)

1. Introduction

In the last few decades, demand for health monitoring of composite structures has been steady growing among the aerospace, mechanical and windmill engineering communities. During the service conditions, the structure may experience high fatigue loads, which could lead to a catastrophic failure. To prevent these sudden failures, it is important to identify the damage at early stages. Therefore, structural health monitoring (SHM) plays a vital role in the condition assessment of structures and thus reduces the inspection and repair costs.

Damage identification is an integral part of the SHM process. It can be classified as local and global methods. Global methods are preferred because the vicinity of damage is not required. Also, it is used in real time applications. Several damage (added mass) identification methods and indices based on vibration data have been proposed in literature.

One of the identification methods used for the damage detection is the frequency response function (FRF) method. Kessler *et al*. (2002) investigated the damage detection in graphite/epoxy

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structures with different types of damage using the change in FRF. Fanning and Carden (2004) presented a methodology for detecting added mass by measuring the FRF of the structure. Liu *et al*. (2009) proposed a structural damage localization method for a cantilever beam using the imaginary parts of FRF shapes. Esfandiari *et al*. (2010) a finite element (FE) model based updating FRF to identify the change of stiffness and mass location and the severity of damage in a truss structure. Liu *et al*. (2012) measured strain FRF (SFRF) using fiber bragg grating (FBG) sensors to detect damage location and its extent. These methods have had limited success because of the difficulty in extracting the damage information contaminated by noise in the signal.

Modal based methods, on the other hand, use parameters unique to the structure independent of the excitation. Therefore, it is possible to use these methods more effectively in extracting the feeble signatures that arise from the damage. Essentially, these methods could be classified into two categories: one that uses the shape changes in the mode and the other that uses changes in the force extracted from the mode.

Pandey *et al.* (1991) introduced an index based on absolute changes in the mode shape curvature to detect the damage in cantilever and simply supported beam models. Doebling *et al.* (1998) summarized damage indices such as change in natural frequencies, mode shapes, modal curvature and modal flexibility. Lestari *et al*. (2007) examined the effectiveness of the modal curvature method for different damage types in carbon/epoxy composite beams. Qiao *et al*. (2007) introduced damage indices utilizing three consecutive modal curvatures (modes 3-5) to identify the delamination in a composite plate. But, changes observed in the modal curvature are generally weak making it difficult to identify the damage signatures. Higher derivatives would be necessary to sharpen the damage signatures.

Whalen *et al*. (2008) observed their study that higher derivatives of mode shapes (e.g., modal curvature, third derivative, and fourth derivative) were better than the mode shapes in indicating the presence and location of damage. Abdo (2012) introduced an index based on the fourth derivative of lower and higher order mode shapes for damage detection in simply supported and cantilever steel plates. It was found in his study that the method is quite sensitive to measurement noise. Higher the derivative, higher is the sensitivity to measurement noise. Therefore, a higher order derivative would need enough sampling points to enhance the scaling of damage features. Moreno-Garcia *et al*. (2014) proposed an optimal spatial sampling while computing the higher order derivatives which would minimize the total error. This could solve the problem to a certain extent. Since the measurement noise is also enhanced while taking the derivatives, one has to ensure that the signature enhancement due to computation of derivatives is higher than the measurement noise.

Kosmatka and Ricles (1999) demonstrated a damage identification method using residual force vector (RFV) extracted from experimentally measured modal test data. A reduced order model (ROM) of a three dimensional 10-bay space truss was used to demonstrate the effectiveness of the method. Gupta *et al*. (2008) proposed a damage force index (DFI) which is a modified form of the residual force index to detect the damage on beam and plate structures using a FE based ROM. However, ROM based force index which uses mainly the transverse dofs may camouflage the damage effect and therefore, may suffer from the required sensitivity necessary to identify the damage and its location. Since bending is the primary action in beams, the rotational dofs play a vital role compared to the direct transverse dofs. Abdo and Hori (2002) investigated a numerical study of structural damage detection of bar, beam and plate elements using the change in slope of mode shapes. Homaei *et al*. (2014) utilized rotational dofs to compute multiple damage localization index based on mode shapes (MDLIBMS) to locate the multiple damages in beam like structures. Therefore, force based index, computed by including the rotational dofs also along with the transverse dofs, would enhance the damage signature.

Conventional method of conducting an experimental study for identifying the damage is to use different specimens from the same batch of material. There are two problems that are generally associated with this technique (i) variations in material/geometric properties can occur from specimen to specimen (ii) location of the crack type damage in the specimen cannot be shifted within the same specimen for different crack location studies. To avoid these difficulties, Dinh *et al*. (2011) proposed an algorithm to extract the stiffness of beam models by the use of added mass. Toyosaki *et al*. (2012) examined damage identification using added masses and the effect due to change in masses before and after the damage. The use of added mass offers flexibility in moving the damage (added mass) to different locations without changing the original characteristics of the structure.

Summarizing the observations from the above literature survey, (1) use of only transverse degrees of freedom dofs may camouflage the damage effect (2) rotational dofs are sensitive to damage localization (3) one has to ensure that the signature enhancement due to computation of derivatives is higher than the measurement noise (4) force based index would require addition of rotational dofs in its computation in addition to the translational dofs to enhance the damage signature (5) adding a movable mass offers versatility of moving the damage to different locations without changing the original characteristics of the structure.

The objective of this study is to identify the added mass and its location in a GFRP beam using two proposed damage indices, the MFI and the MRCI. The importance of this paper is to ascertain the significance of inclusion of rotational degrees of freedom (dofs) and the introduction of added mass in the damage identification methodology.

This paper is organized as follows: Section 1 introduces the various damage identification methods. Section 2 defines the two proposed damage indices, the modal force index (MFI) and the modal rotational curvature index (MRCI) and explains the procedure for computing them. Section 3 describes the experiments that have been conducted, the setup and the specimen details. The modal analysis procedure used in this work is also elaborated. The experimental results and the numerical results are presented and discussed in Section 4 ending with concluding remarks in section 5.

2. Theoretical background

The eigen value equation for a multi-degree of freedom (MDOF) undamped dynamic system can be written as

$$\left(\mathrm{K}-\mathrm{M}\lambda\right)\Phi=0 \tag{1}$$

where K and M are global stiffness and mass matrices of the beam structure respectively. λ and Φ are the natural frequencies and the mode shapes respectively. The added mass is fixed to the beam at a particular location. At that location, the mass as well as the stiffness of the beam increases locally. This change leads to changes in the natural frequencies and the mode shapes of the system. The corresponding eigen value equation of a dynamic system with damage (or added mass) can be represented by Cawley and Adams (1979)

$$\left(\mathrm{K}_d-\mathrm{M}_d\lambda_d\right)\Phi_d=0 \tag{2}$$

where K_d and M_d are global stiffness and mass matrices of the beam structure with added mass respectively. λ_d and Φ_d are altered natural frequencies and mode shapes respectively. d subscript is used to indicate damage which is akin to the added mass in this study. Changes in natural frequencies could help get a preliminary idea about the existence of damage and its location in the beam. However, it is far from a reliable identification. In order to understand damage localization better, it requires additional spatial information like the mode shapes. Two indices are introduced here for the identification of damage (added mass) location in a beam structure that relies on the addition of rotational modes in their calculation.

2.1 Modal force index (MFI)

The modal force index (MFI) for localization of damage (added mass) to the beam element can be expressed by

$$\mathrm{F}^e_{u,j} = \mathrm{K}^e \Phi^e_{u,j} \quad \text{and} \quad \mathrm{F}^e_{d,j} = \mathrm{K}^e \Phi^e_{d,j} \tag{3, 4}$$

where F, is the elemental force vector and $\Phi^e_j = \begin{bmatrix} w_{i,j} & \theta_{i,j} & w_{i+1,j} & \theta_{i+1,j} \end{bmatrix}^T$ are the mode shapes corresponding to a complete set of dofs that include both transverse and rotational dofs for each element. i, j and e denote the node, mode and element respectively. u and d represent the undamaged and damaged state respectively. K^e is the stiffness matrix of the beam element as shown in Eq. (5).

$$\mathrm{K}^e = \frac{\mathrm{EI}_e}{l_e} \begin{bmatrix} 12 & 6l_e & -12 & 6l_e \\ 6l_e & 4l_e^2 & -6l_e & 2l_e^2 \\ -12 & -6l_e & 12 & -6l_e \\ 6l_e & 2l_e^2 & -6l_e & 4l_e^2 \end{bmatrix} \tag{5}$$

where E, I_e and l_e represents the modulus of elasticity, area moment of inertia of the beam section and the elemental length respectively. The rotational mode shape for i^{th} node is a derivative of transverse mode shapes of nodes between i-1$^{\text{th}}$ and i+1$^{\text{th}}$ given by first derivative central difference scheme as in Eq. (6). Since the error in this method is of second order $O(h^2)$, the accuracy increases when computing rotational mode shapes. The boundary conditions are: the displacement and slope are zero at the fixed end and the curvature is zero at the free end for the cantilever configurations.

$$\theta_{i,j} = \frac{w_{i+1,j} - w_{i-1,j}}{2h} \tag{6}$$

where w and θ are transverse and rotational dofs of each nodal point. h is the distance between i^{th} and i+1$^{\text{th}}$ node. Since the modal force index scales the damage signature using the undamaged numerical stiffness matrix, which is related to the change in altered mode shape from the original mode shape.

In the force vector, moments are normalized with respect to the elemental length in order to maintain unit constancy. Then, the L_2-norm is applied to calculate the equivalent force for each element. The difference between the damaged and undamaged forces is then computed that defines the normalized MFI for each beam element. Taking the mean of normalized MFI of all the modes excluding first mode enhances the damage signature. The damage identification algorithm based

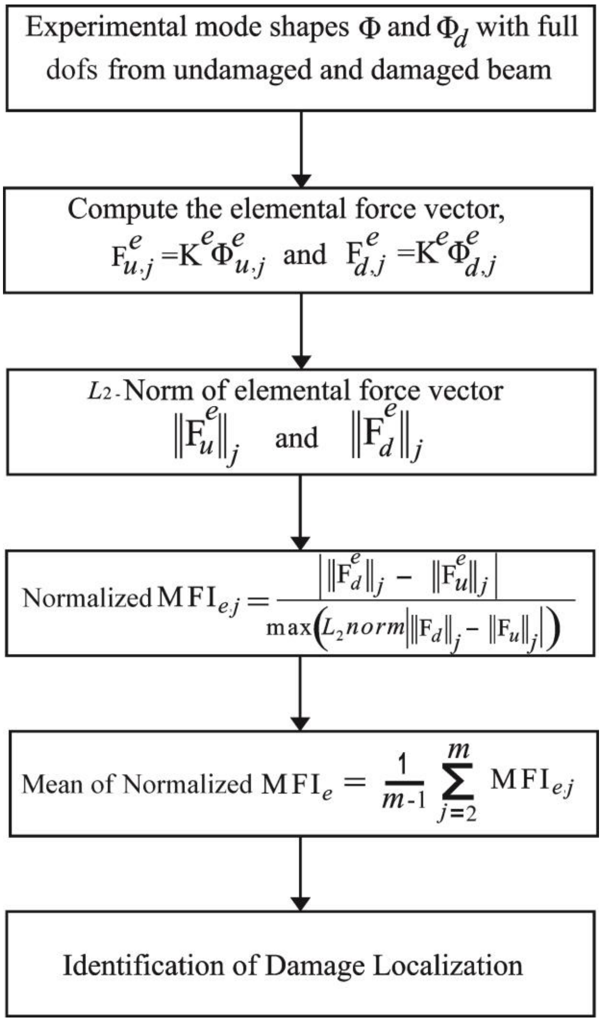


Fig. 1 Damage identification algorithm based on modal force index

on MFI can be seen in Fig. 1. Results of the proposed MFI have been compared with the residual force vector (RFV) proposed by Kosmatka and Ricles (1999) model and are presented in Section 4.1.

2.2 Modal rotational curvature index (MRCI)

The major assumption that is generally accepted (unless shear deformations dominate) in this computation is that the beam follows Euler-Bernoulli beam theory. The modal rotational curvature from 2nd node to n-1th node is obtained by using the second derivative central difference scheme for rotational mode shape is given by

$$\varphi_{i,j} = \frac{\theta_{i+1,j} - 2\theta_{i,j} + \theta_{i-1,j}}{h^2} \tag{7}$$

The rotational curvature at the first node is calculated by using second derivative forward difference scheme

$$\varphi_{1,j} = \frac{\theta_{3,j} - 2\theta_{2,j} + \theta_{1,j}}{h^2} \tag{8}$$

Thus, MRCI is obtained at all the nodes. The difference between the undamaged and damaged modal rotational curvature is calculated at each node and normalized with respect to maximum value of L_2-norm of modal rotational curvature difference. Although computational error accumulates in the subsequent numerical operations, the signature enhancement is higher than the computation error. Thus, the normalized MRCI is computed as

$$\text{Normalized MRCI, } \varphi_{i,j} = \frac{\varphi_{d,ij} - \varphi_{u,ij}}{max\left\|\varphi_{d,j} - \varphi_{u,j}\right\|_2} \tag{9}$$

The overall MRCI is now calculated as the mean of normalized MRCI of all the modes excluding the first mode as given below.

$$\text{Mean of normalized MRCI, } \varphi_i = \frac{1}{m-1}\sum_{j=2}^{m} MRCI_{i,j} \quad \text{where } j>1. \tag{10}$$

MRCI is used for identifying the location of added mass at different positions on the cantilever beam that is chosen as an example for this study. Numerical and experimental results are compared with an existing model proposed by Pandey *et al*. (1991). The results are presented in Sections 4.2 and 4.3.

3. Experimental modal analysis

3.1 Material properties of the GFRP beam

A GFRP specimen is used to conduct the test with undamaged and damaged (added mass included case) as shown in Fig. 2. The beam is made up of bidirectional glass fiber woven mat $[0^{\circ}/90^{\circ}]_8$ stacked up in a layer by layer process and bonded together using an epoxy matrix. Specimens of desired shape are cut from the same batch of laminate. The GFRP beam is set up in a cantilever configuration. The free span, width and thickness are 450 mm, 26 mm and 2.93 mm respectively. In the analysis, the beam is considered to be transversely isotropic and to follow the Euler-Bernouli beam assumption. The material properties of the GFRP beam are estimated by using a finite element (FE) model updating method. The elastic moduli thus computed are E_1=E_2=14.53 GPa and density, ρ=1835 kg/m^3 respectively.

3.2 Added mass

The added mass is prepared in the form of two aluminium strips that could be tightened to the beam using screws at the desired locations of the beam as shown in Fig. 3(b). This provides the versatility of moving the damage (added mass) for different damage location studies in the beam structure without altering the original structural characteristics. The weight of the added mass is

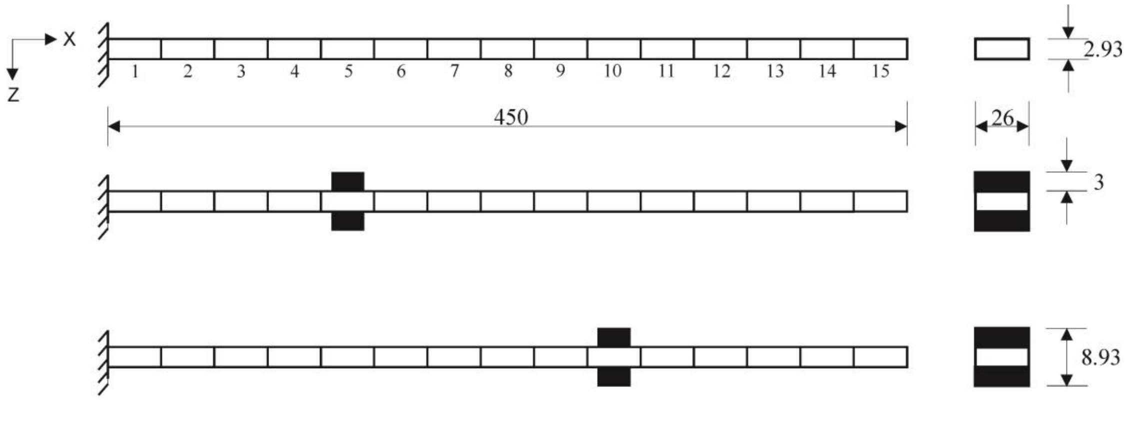


Fig. 2 GFRP beam with (a) undamaged specimen (b) added mass at 5th element (c) added mass at 10thelement and their cross sections

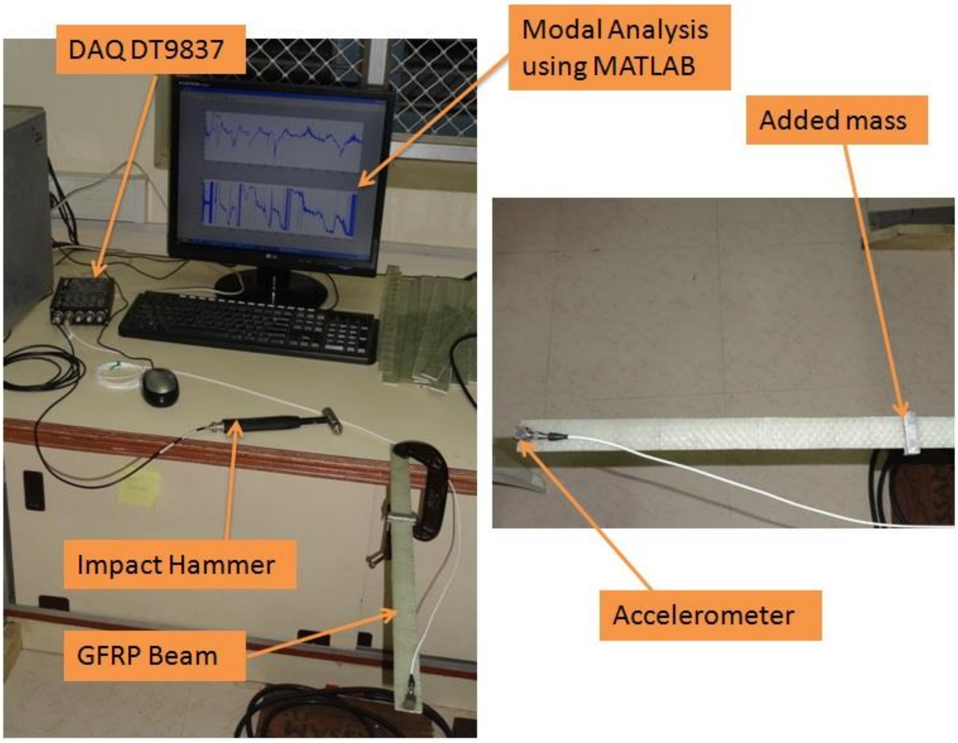


Fig. 3 (a) Experimental modal analysis test setup (b) Added mass is attached with GFRP beam

about 12% of the beam weight. The undamaged condition of the beam is preserved while altering the beam structure using the added mass to simulate damaged condition. The beam is divided into 15 zones (or elements) of equal length for assessing the performance of the proposed damage identification indices with the added mass (akin to damage) located at different positions in the beam. The following cases are considered in this study as shown in Fig. 2.

1. undamaged case with no added mass in it
2. added mass at the 5th element (closer to the fixed end)
3. added mass at the 10th element (closer to the free end)

3.3 Experimental setup

An integrated experimental modal analysis setup is developed to perform the modal analysis using MATLAB® as shown in Fig. 3(a). A data acquisition (DAQ) board (Data Translation DT9837) is used to acquire the signals from accelerometer and impact hammer and it could interface with MATLAB® to perform the modal analysis. A uniaxial accelerometer (Kistler model 8640A50) weighing 3 gms with a sensitivity of 98.2 mV/g is used to measure the acceleration signals. It is attached at the free end of beam using wax glue as shown in Fig. 3(b). An impact hammer (Kistler model 9722A500), with a sensitivity of 10 mV/N force, is used to excite the beam structure.

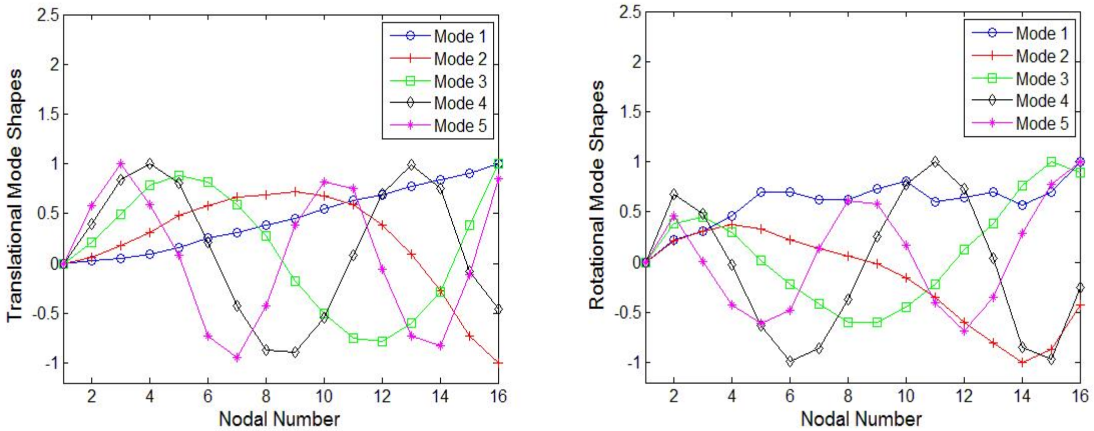
3.3.1 Experimental procedure

The GFRP beam is setup in a cantilever configuration and a rowing-hammer method is adopted. In this method, the hammer is moved to each node along the length of the beam while the accelerometer is fixed at one location. In this case, the accelerometer is fixed at the free end. The impulse hammer excites the beam at different locations and the accelerometer measures the corresponding acceleration signals. The excitation and the accelerometer signals are obtained from both the impulse hammer and accelerometer respectively using the DAQ board interface. The total sampled time is 0.964 sec with the sampling frequency of 10375 Hz. An exponential window and a rectangular window are applied to the acceleration and the force signals in time domain to remove the leakage errors when truncating the time signals. The processed time signals are transformed into frequency domain using fast Fourier transform (FFT). The frequency response function (FRF) is calculated as the ratio of the acceleration response to the impulse force in the frequency domain. It is also commonly referred to as accelerance or inertance. Rational fraction polynomial (RFP) curve fitting technique is employed to extract the modal parameters such as natural frequencies (see Table 1) and the corresponding transverse mode shapes are obtained from the measured FRF data using the method proposed by Ewins (2000) as shown in Fig. 4(a). Since it is difficult to measure rotational mode shapes directly, they are computed by taking the first derivative of the transverse mode shapes. The computed rotational mode shapes are shown in Fig. 4(b).

Table 1 First five measured and numerical natural frequencies

	Measured frequency (Hz)			Numerical frequency (Hz)		
Mode	Undamaged	D1*	D2**	Undamaged	D1*	D2**
1	6.47	6.48	6.1	6.57	6.85	6.12
2	42.47	39.70	39.46	41.21	38.86	41.24
3	121.64	109.13	115.18	115.41	109.92	115.4
4	239.32	231.81	238.63	226.2	224.23	225.38
5	394.17	392.42	367.1	374.05	373.12	378.09

D1*: added mass at 5th element; D2**: added mass at 10th element



(a) Measured transverse mode shapes (b) Computed rotational mode shapes

Fig. 4 Measured transverse mode shapes and computed rotational mode shapes for the undamaged case

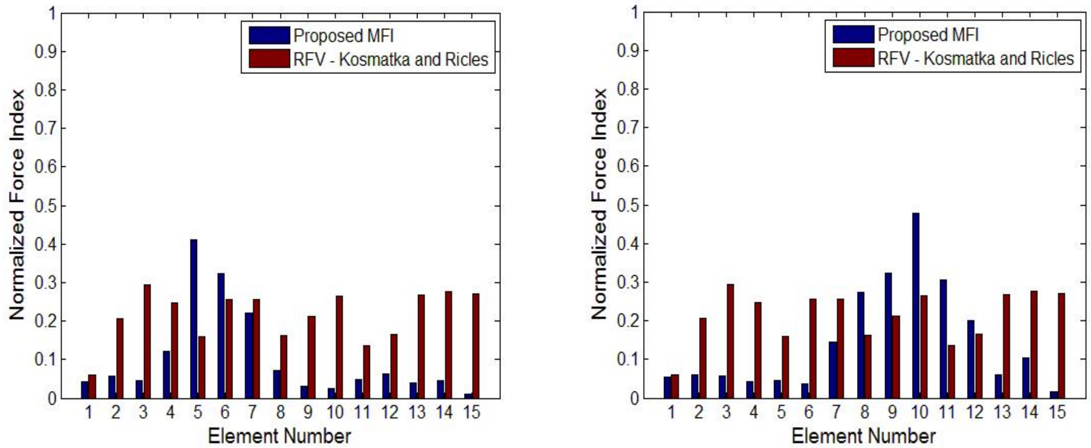
4. Results and discussions

Experimental modal analysis is performed and the first five transverse mode shapes are extracted. They are normalized with respect to maximum absolute value of each mode. This is shown in Fig. 4(a). The modal parameters of the undamaged beam are compared with different damage cases (added mass attached at different locations). A slight increase in measured first natural frequency from 6.47 Hz to 6.48 Hz is observed when the added mass is moving towards the fixed end. This indicates that the increase in stiffness of the structure due to the addition of added mass is relatively higher than the increase in the equivalent mass of the structure. On the other hand, when the added mass is moving towards the free end, the natural frequency is reduced to 6.1 Hz. This shows that the increase in equivalent mass is higher than the increase in stiffness of the structure. The feeble changes in natural frequencies could help to get an initial idea about damage presence and approximate damage location in the beam. But it is not sufficient to identify the exact added mass or damage extent and its location. Therefore, the proposed indices employ rotational mode shapes and their higher derivatives to localize the added mass or damage in the beam.

4.1 Modal force index (MFI)

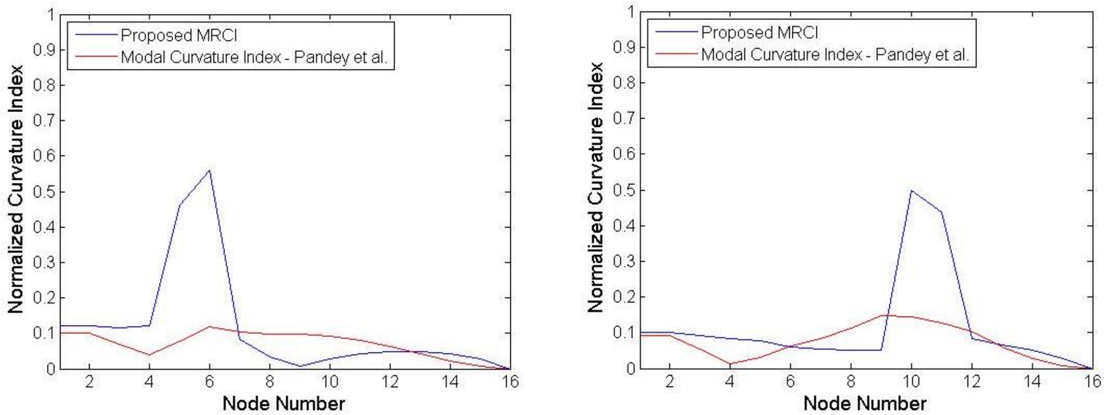
The proposed MFI is calculated using the algorithm shown in Fig. 1 in the cases of added mass located at the 5th element and at the 10th element respectively. In order to emphasize the contribution of the rotational modes, the proposed MFI thus calculated is compared with an existing index introduced by Kosmatka and Ricles (1999) using ROM. This index, called the residue force vector (RFV), does not consider the rotational dofs.

Though all mode shapes could carry some signature of the damage, the intensity varies with modes. First mode is least influenced by the presence of the damage since the bending action in the first mode is less sharp than in the other modes. Hence, in the present study, the average of all the modes excluding the first mode is employed to identify the damage locations. The resulting average which does not consider the first mode shows a sharper signature. The proposed MFI



(a) added mass at 5th element (b) added mass at 10th element

Fig. 5 Results of mean of normalized MFI and the RFV proposed by Kosmatka and Rickles for the cases of the added mass at 5th and at 10th element individually



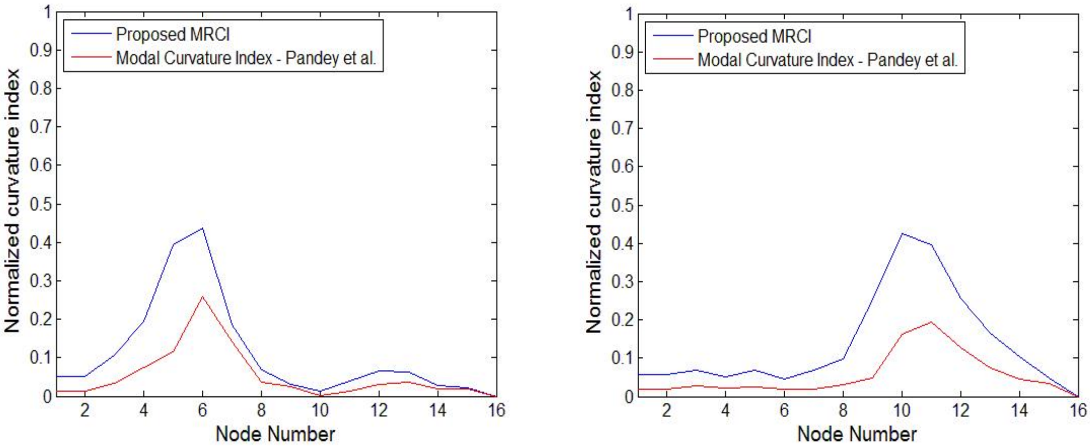
(a) added mass at 5th element (b) added mass at 10th element

Fig. 6 The numerical results of mean of normalized MRCI and MCI proposed by Pandey *et al*. for the cases of the added mass at 5th and at 10th element individually

scores over the RFV results in indicating the distinct features of the damage location unambiguously for both the cases in which the added mass is attached at 5th and 10th element individually. A comparison of the proposed MFI and the RFV results are shown in Figs. 5(a)-(b) to substantiate this claim. This vindicates the importance of inclusion of rotational modes in the calculation of MFI.

4.2 Modal rotational curvature index (MRCI) - numerical analysis

A numerical finite element (FE) simulation is carried out for the cantilever beam discretized into 15 beam elements as shown in Fig. 2. Both the transverse and the rotational mode shapes are extracted. The modal rotational curvature index (MRCI) is calculated using the rotational mode



(a) added mass at 5th element (b) added mass at 10th element

Fig. 7 The experimental results of mean of normalized MRCI and MCI proposed by Pandey *et al*. for the cases of the added mass at 5th and at 10th element individually.

shapes and normalized as in Eq. (9). The normalized MRCI is now suitable for comparison with an existing method such as the modal curvature index (MCI) proposed by Pandey *et al*. (1991). In the first case of added mass located at 5th element, the mean of normalized MRCI of all the modes excluding the first is computed and presented in Fig. 6(a). Sharp changes in the MRCI magnitude can be distinctly seen. This is expected since MRCI utilizes a higher derivative, which is sensitive to the damage location.

Similarly in the other case of added mass at 10th element, substantial changes can be seen in Fig. 6(b). In both the cases, the MRCI wins over the MCI method indicating the power of using only rotational modes in the calculation of the indices. It should also be emphasized that it required only coarse grid measurements to identify the damage location effectively. It is important now to test the effectiveness of this index under experimental conditions that is usually contaminated with measurement noise in the measured signals.

4.3 Modal rotational curvature index (MRCI) – experimental analysis

Experiments are conducted to validate the MRCI for added mass type damage in the beam. In this experimental study, the first five modes extracted for each case of added mass (1) attached to the 5th element and (2) attached to the 10th element. The modes thus extracted are considered for the analysis. Since rotational mode shapes are closely associated with bending action in beams, it is expected that rotational mode shapes should carry the bulk of the damage signature. This results in signature enhancement. This enhancement also helps in partially removing the measurement noise related masking. Moreover, in order to reduce the error in derivative computation, the central difference scheme is employed so that the order of error is h^2. The experimental results of MRCI shown in Figs. 7(a)-(b) show this enhancement compared to MCI proposed by Pandey *et al*. (1991). Thus, we see an increased effectiveness and reliability in damage identification.

5. Conclusions

This work primarily focuses on the identification of added mass and its location in a GFRP beam structure. The effectiveness of using rotational modes separately is tested and substantiated.

To understand the role of rotational dofs, two indices are introduced viz. modal force index (MFI) and the modal rotational curvature index (MRCI). The underlying principles behind these two proposed indices are simple yet robust. The results of this study have clearly shown through an example of a cantilever beam both by means of experimental investigations as well as by numerical simulations that rotational modes play a vital role in enhancing the signature of damage (added mass). The results of MFI that includes both the transverse and the rotational dofs indicates substantial changes in the added mass locations compared to the RFV index which utilizes only transverse dofs. The results also reveal that the use of higher derivatives of rotational mode shapes, as in the computation of MRCI, further enhances performance when compared to the MCI which considers the lower derivatives.

The study also shows that it is important to extract sufficient number of modes from the experimental data to confirm and identify the location of structural change. Ongoing study is to introduce a measure of local change in the characteristics of a structure and to identify the extent and location of multiple local changes in the structure. The authors feel that the proposed indices perform better since the gradients and curvatures are used in the damage identification that directly correlate with the force and stress resultants in the structure. Both the damage indices rely on the comparisons with experimental data of the healthy structures, which limit the application of the proposed methods.

References

Abdo, M.A.B. and Hori, M. (2002), “A numerical study of structural damage detection using changes in the rotation of mode shapes”, *J. Sound Vib.*, **251**(2), 227-239.

Abdo, M.A.B. (2012), “Damage detection in plate-like structures using high-order mode shape derivatives”, *Int. J. Civil Struct. Eng.*, **2**(3), 801-816.

Cawley, P. and Adams, R.D. (1979), “The location of defects in structures from measurements of natural frequencies”, *J. Strain Anal. Eng.*, **14**, 49-57.

Dinh, H.M., Nagayama, T. and Fujino, Y. (2011), “Structural parameter identification by use of additional known masses and its experimental application”, *Struct. Control Hlth. Monit.*, **19**(3), 1-15.

Doebling, S.W., Farrar, C.R. and Prime, M.B. (1998), “A summary of vibration-based damage identification methods”, *Shock Vib. Dig.*, **30**, 91-105.

Esfandiari, A., Bakhtiari-Nejad, F., Sanayei, M. and Rahai, A. (2010), “Structural finite element model updating using transfer function data”, *Comput. Struct*., **88**, 54-64.

Ewins, D.J. (2000), *Modal Testing: Theory, Practice and Application*, Research Studies Press, England.

Fanning, P.J. and Carden, P.E. (2004), “Experimentally validated added mass identification algorithm based on frequency response functions”, *J. Eng. Mech.*, ASCE, **130**(9), 1045-1051.

Gupta, P., Giridhara, G. and Gopalakrishnan, S. (2008), “Damage detection based on damage force indicator using reduced-order FE models”, *Int. J. Comput. Meth. Eng. Sci. Mech.*, **9**(3), 154-170.

Homaei, F., Shojaee, S. and Amiri, G.G. (2014), “A direct damage detection method using Multiple Damage Localization Index Based on Mode Shapes criterion”, *Struct. Eng. Mech.*, **49**(2), 183-202.

Kessler, S.S., Spearing, S.M., Atalla, M.J., Cesnik, C.E.S. and Soutis, C. (2002), “Damage detection in composite materials using frequency response methods”, *Compos. Part B-Eng.*, **33**, 87-95.

Kosmatka, J.B. and Ricles, J.M. (1999), “Damage detection in structures by modal vibration

characterization", *J. Struct. Eng.*, ASCE, **125**(12), 1384-1392.
Lestari, W., Qiao, P. and Hanagud, S. (2007), "Curvature mode shape-based damage assessment of carbon/ epoxy composite beams", *J. Intel. Mater. Syst. Struct.*, **18**, 189-208.
Liu, X., Lieven, N.A.J. and Escamilla-Ambrosio, P.J. (2009), "Frequency response function shape-based methods for structural damage localization", *Mech. Syst. Signal Pr.*, **23**, 1243-1259.
Liu, M., Zhou, Z., Tan, Y., Ling, A. and Ke, M. (2012), "Fiber bragg grating sensors-based mechanical structure damage identification", *Mechanika*, **18**(1), 108-112.
Moreno-Garcia, P., Araujo dos Santos, J.V. and Lopes, H. (2014), "A new technique to optimize the use of mode shape derivatives to localize damage in laminated composite plates", *Compos. Struct.*, **108,** 548-554.
Pandey, A.K., Biswas, M. and Samman, M.M. (1991), "Damage detection from changes in curvature mode shapes", *J. Sound Vib.*, **145**(2), 321-332.
Qiao, P., Lu, K., Lestari, W. and Wang, J. (2007), "Curvature mode shape-based damage detection in composite laminated plates", *Compos. Struct.*, **80**, 409-428.
Toyosaki, Y., Nariyuki, Y., Mitsuya, S. and Konishi, T. (2012), "Identification of damage to beam structures using modal data", *Proceedings of the 15th World Conference on Earthquake Engineering,* Lisbon, Portugal, September.
Whalen, T.M. (2008), "The behavior of higher order mode shape derivatives in damaged, beam-like structures", *J. Sound Vib.*, **309**, 426-464.

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