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A mechanical response of functionally graded nanoscale beam: an assessment of a refined nonlocal shear deformation theory beam theory

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Abstract. This paper presents a nonlocal shear deformation beam theory for bending, buckling, and vibration of functionally graded (FG) nanobeams using the nonlocal differential constitutive relations of Eringen. The developed theory account for higher-order variation of transverse shear strain through the depth of the nanobeam, and satisfy the stress-free boundary conditions on the top and bottom surfaces of the nanobeam. A shear correction factor, therefore, is not required. In addition, this nonlocal nanobeam model incorporates the length scale parameter which can capture the small scale effect and it has strong similarities with Euler–Bernoulli beam model in some aspects such as equations of motion, boundary conditions, and stress resultant expressions. The material properties of the FG nanobeam are assumed to vary in the thickness direction. The equations of motion are derived from Hamilton's principle. Analytical solutions are presented for a simply supported FG nanobeam, and the obtained results compare well with those predicted by the nonlocal Timoshenko beam theory.

Keywords: nanobeam; nonlocal elasticity theory; bending; buckling; vibration; functionally graded materials

1. Introduction

Nanotechnology is primarily concerned with fabrication of functionally graded materials and engineering structures at a nanoscale, which enables a new generation of materials with revolutionary properties and devices with enhanced functionality. One of these structures is the nanobeam, which has been used widely in systems and devices such as nanowires, nano-probes, atomic force microscope (AFM), nanoactuators and nanosensors. The understanding of

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mechanical behavior of nanobeam is essential in developing of such structures due to their great potential engineering applications. Hence, size effects are significant in the mechanical behavior of these structures in which dimensions are small and comparable to molecular distances. These effects can be captured using size-dependent continuum mechanics such as strain gradient theory (Nix and Gao 1980), modified couple stress theory (Ma *et al.* 2008), and nonlocal elasticity theory (Eringen 1972, 1983). Unlike classical theories, the nonlocal theories contain internal material length scale parameters that can capture size effects at the nano scale. A review of various nonlocal models can be found in Bazant and Jirasek (2002).

The nonlocal elasticity theory of Eringen (1972, 1983) was developed by several authors as a response to the inability of local elasticity to handle elastic problems with sharp geometrical singularities (for example, a sharp crack-tip). The Eringen model was applied to Euler–Bernoulli micro and nanobeams by Peddieson *et al*. (2003), Sudak (2003) and Amara *et al*. (2010) to the study of column buckling of carbon nanotubes and by Pisano *et al*. (2003) for the study of an elastic bar in tension. Reddy (2007) reformulated different nonlocal beam theories including Euler–Bernoulli, Timoshenko, Reddy (1984), Levinson (1981) to evaluate the static bending, vibration, and buckling responses of nanobeams. Adda Bedia *et al*. (2015) studied the thermal buckling characteristics of armchair single-walled carbon nanotube embedded in a one-parameter elastic medium by proposing a new nonlocal first-order shear deformation theory.

A new class of composites that called functionally graded materials (FGMs) has a great practical importance because of their vast applications in many industrial and engineering fields (Ait Yahia *et al.* 2015, Attia *et al.* 2015, Khalfi *et al.* 2014, Bachir Bouiadjra *et al.* 2013, Bessaim *et al.* 2013, Fekrar *et al.* 2014). Recently, the application of FG materials has broadly been spread in nano-structures such as nano-electromechanical systems (NEMS), thin films in the form of shape memory alloys, and atomic force microscopes (AFMs) to achieve high sensitivity and desired performance. With the rapid development of technology, functionally graded (FG) beams and plates have been started to use in micro/nanoelectromechanical systems (MEMS/NEMS), such as the components in the form of shape memory alloy thin films with a global thickness in micro- or nano-scale (Fu *et al.* 2003, Witvrouw and Mehta 2005, Lü *et al.* 2009), electrically actuated MEMS devices (Hasanyan *et al.* 2008, Mohammadi-Alasti *et al.* 2011, Zhang and Fu 2012), and atomic force microscopes (AFMs) (Rahaeifard *et al.* 2009). Since the dimension of these structural devices typically falls below micron- or nano-scale in at least one direction, an essential feature triggered in these devices is that their mechanical properties such as Young’s modulus, flexural rigidity, and so on are size-dependent. So far, only a few works have been reported for FG nanobeams based on the nonlocal elasticity theory. Pisano *et al*. (2009ab) exploited the nonlocal finite element method for analyzing homogeneous and nonhomogeneous nonlocal elastic 2D problems. Janghorban and Zare (2011) investigated nonlocal free vibration axially FG nanobeams by using differential quadrature method. Eltaher *et al*. (2012) studied free vibration of FG nanobeam based on the nonlocal Euler-Bernoulli beam theory. Belkorissat *et al*. (2015) analysed the vibration properties of FG nano-plate using a new nonlocal refined four variable model. Recently, Larbi Chaht *et al*. (2015) studied the static bending and buckling of a FG nanobeam using the nonlocal sinusoidal beam theory.

Therefore, based on the above discussion it can be seen that a very limited literature is available for micro/nano-scale FG structures. That gives us a strong encouragement to understand the mechanical behavior of FG nanobeams in the design of nanodevices. The aim of this paper is to propose a refined nonlocal beam theory for bending, buckling, and vibration of FG nanobeams. This theory is based on assumption that the in-plane and transverse displacements consist of

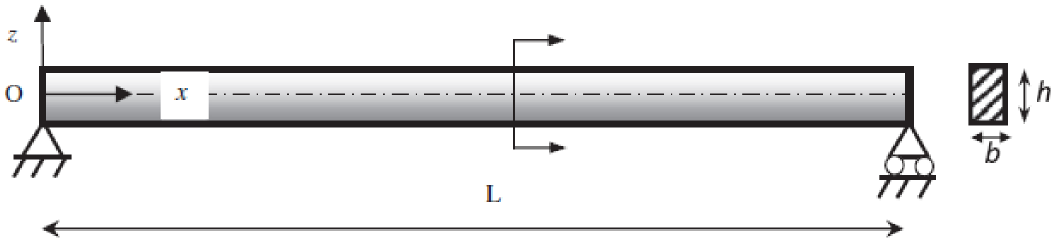

Fig. 1 Gradation of material properties through the thickness of the FG beam

bending and shear components, in which the bending components do not contribute toward shear forces and, likewise, the shear components do not contribute toward bending moments. In addition, the small scale effect is taken into account by using the nonlocal constitutive relations of Eringen. The most interesting feature of this theory is that it accounts for a quadratic variation of the transverse shear strains across the thickness, and satisfies the zero traction boundary conditions on the top and bottom surfaces of the beam without using shear correction factors. In addition, it has strong similarities with Euler–Bernoulli beam theory in some aspects such as equations of motion, boundary conditions, and stress resultant expressions. The material properties of the FG nanobeam are assumed to vary in the thickness direction. Based on the nonlocal constitutive relations of Eringen, equations of motion of FG nanobeams are derived using Hamilton's principle. To illustrate the accuracy of the present theory, the obtained results are compared with those predicted by the Euler–Bernoulli beam theory and Timoshenko beam theory. Finally, the influences of nonlocal parameter, power law index, and aspect ratio on the bending, buckling and vibration responses of FG nanobeam are discussed.

2. Theoretical formulations

The theoretical formulation of a uniform FG nanobeam based on certain kinematical and physical assumptions is presented. The variationally correct forms of differential equations and boundary conditions, based on the assumed displacement field are obtained using the principle of virtual work. As is seen in Fig. 1, the beam under consideration occupies the region

$$0 \le x \le L; \quad -b/2 \le y \le b/2; \quad -h/2 \le z \le h/2 \tag{1}$$

where x, y, z are Cartesian coordinates, L is the length, b is the width, and h is the total depth of nanobeam. The nanobeam is subjected to the distributed transverse load $q(x)$ and an axial compressive force N_0.

2.1 Functionally graded materials

It is assumed that material properties of the FG nanobeam, such as Young's modulus (E), Poisson's ratio (v), the shear modulus (G), and the mass density (ρ), vary continuously through the nanobeam thickness according to power-law form (El Meiche *et al.* 2011, Eltaher *et al.* 2012, Larbi Chaht *et al.* 2015, Tounsi *et al.* 2013a, Bouderba *et al.* 2013, Houari *et al.* 2013, Saidi *et al.* 2013, Ould Larbi *et al.* 2013, Belabed *et al.* 2014, Bousahla *et al.* 2014, Hebali *et al.* 2014, Bourada *et al.* 2015, Hamidi *et al.* 2015), which can be described by

$$P(z) = \left(P_t - P_b\right)\left(\frac{z}{h} + \frac{1}{2}\right)^k + P_b \tag{2}$$

where P_t and P_b are the corresponding material property at the top and bottom surfaces of the nanobeam, k is a non-negative number that dictates the material variation profile through the thickness of the nanobeam.

2.2 Basic assumptions

The displacement field of the proposed theory is chosen based on the following assumptions

(i) The displacements are small in comparison with the FG nanobeam thickness and, therefore, strains involved are infinitesimal.

(ii) The transverse displacement w includes two components of bending w_b, and shear w_s. These components are functions of coordinate x only.

$$w(x,z) = w_b(x) + w_s(x) \tag{3}$$

(iii) The transverse normal stress σ_z is negligible in comparison with in-plane stresses σ_x.

(iv) The displacement u in x-direction consists of extension, bending, and shears components.

$$u = u_0 + u_b + u_s \tag{4}$$

The bending component u_b is assumed to be similar to the displacement given by the classical beam theory. Therefore, the expression for u_b can be given as

$$u_b = -z\frac{\partial w_b}{\partial x} \tag{5}$$

The shear component u_s gives rise, in conjunction with w_s, to a sinusoidal variations of shear strain γ_{xz} and hence to shear stress τ_{xz} through the thickness of the nanobeam in such a way that shear stress τ_{xz} is zero at the top and bottom faces of the nanobeam. Consequently, the expression for u_s can be given as (Benachour *et al.* 2011, Zidi *et al.* 2014)

$$u_s = z\left[\frac{1}{4} - \frac{5}{3}\left(\frac{z}{h}\right)^2\right]\frac{\partial w_s}{\partial x} \tag{6}$$

2.3 Kinematics

Based on the assumptions made in the preceding section, the displacement field can be obtained using Eqs. (3)-(6) as

$$u(x,z,t) = u_0(x,t) - z\frac{\partial w_b}{\partial x} + z\left[\frac{1}{4} - \frac{5}{3}\left(\frac{z}{h}\right)^2\right]\frac{\partial w_s}{\partial x} \tag{7a}$$

$$w(x,z,t) = w_b(x,t) + w_s(x,t) \tag{7b}$$

The strains associated with the displacements in Eq. (7) are

$$\varepsilon_x = \varepsilon_x^0 + z\,k_x^b + f(z)\,k_x^s \text{ and } \gamma_{xz} = g(z)\,\gamma_{xz}^s \tag{8}$$

where

$$\varepsilon_x^0 = \frac{\partial u_0}{\partial x}\,,\ \ k_x^b = -\frac{\partial^2 w_b}{\partial x^2}\,,\ \ k_x^s = -\frac{\partial^2 w_s}{\partial x^2}\,,\ \ \gamma_{xz}^s = \frac{\partial w_s}{\partial x}\,,\ \ f = \frac{-1}{4}z + \frac{5}{3}z\left(\frac{z}{h}\right)^2,\ \ g = \frac{5}{4} - 5\left(\frac{z}{h}\right)^2 \tag{9}$$

2.4 Constitutive relations

Response of materials at the nanoscale is different from those of their bulk counterparts. Nonlocal elasticity is first considered by Eringen (1972, 1983). He assumed that the stress at a reference point is a functional of the strain field at every point of the continuum. Eringen (1972, 1983) proposed a differential form of the nonlocal constitutive relation as

$$\sigma_x - \mu \frac{d^2\sigma_x}{dx^2} = E\varepsilon_x \tag{10a}$$

$$\tau_{xz} - \mu \frac{d^2\tau_{xz}}{dx^2} = G\gamma_{xz} \tag{10b}$$

where $\mu=(e_0a)^2$ is the nonlocal parameter, e_0 is a constant appropriate to each material and a is an internal characteristic length. In general, a conservative estimate of the nonlocal parameter is $e_0a<2.0$ nm for a single wall carbon nanotube (Wang 2005, Benzair *et al.* 2008, Heireche *et al.* 2008a,b,c, Tounsi *et al.* 2008, Benzair *et al.* 2008, Zidour *et al.* 2012, Tounsi *et al.* 2013b,c,d, Berrabah *et al.* 2013, Boumia *et al.* 2014, Zidour *et al.* 2014, Semmah *et al.* 2014, Baghdadi *et al.* 2014, Benguediab *et al.* 2014).

2.4 Equations of motion

Using the dynamic version of principle of virtual work (Ait Amar Meziane *et al.* 2014, Mahi *et al.* 2014, Tounsi *et al.* 2015), variationally consistent governing differential equations for the FG nanobeam under consideration are obtained. The principle of virtual work when applied to the FG nanobeam leads to

$$\begin{aligned}&\int_0^L\!\!\int_A \left(\sigma_x\delta\,\varepsilon_x + \tau_{zx}\delta\,\gamma_{zx}\right)dA\,dx - \int_0^L\!\!\int_A \rho\left[\ddot{u}_0\delta\,u_0 + (\ddot{w}_b + \ddot{w}_s)\delta\,(w_b + w_s)\right]dA\,dx\\ &-\int_0^L q\delta\left(w_b + w_s\right)dx - \int_0^L N_0\,\frac{d\left(w_b + w_s\right)}{dx}\,\frac{d\delta\left(w_b + w_s\right)}{dx}\,dx = 0\end{aligned} \tag{11}$$

Collecting the coefficients of $\delta\,u_0$, $\delta\,w_b$ and $\delta\,w_s$ in Eq. (11), equations of motion are obtained as

$$\delta\,u_0:\ \ \frac{dN}{dx} = I_0\ddot{u}_0 \tag{12a}$$

$$\delta w_b : \frac{d^2 M_b}{dx^2} + q - N_0 \frac{d^2(w_b + w_s)}{dx^2} = I_0(\ddot{w}_b + \ddot{w}_s) - I_2 \frac{d^2 \ddot{w}_b}{dx^2} \tag{12b}$$

$$\delta w_s : \frac{d^2 M_s}{dx^2} + \frac{dQ}{dx} + q - N_0 \frac{d^2(w_b + w_s)}{dx^2} = I_0(\ddot{w}_b + \ddot{w}_s) - \frac{I_2}{84} \frac{d^2 \ddot{w}_s}{dx^2} \tag{12c}$$

where N, M_b, M_s and Q are the stress resultants defined as

$$(N, M_b, M_s) = \int_A (1, z, f)\, \sigma_x dA \text{ and } Q = \int_A g \tau_{xz} dA \tag{13}$$

and (I_0, I_2) are mass inertias defined as

$$(I_0, I_2) = \int_A (1, z^2) \rho(z) dA \tag{14}$$

when the shear deformation effect is neglected (w_s=0), the equilibrium equations in Eq. (12) recover those derived from the Euler-Bernoulli beam theory.

By substituting Eq. (8) into Eq. (10) and the subsequent results into Eq. (13), the stress resultants are obtained as

$$N - \mu \frac{d^2 N}{dx^2} = A \frac{du_0}{dx} - B \frac{d^2 w_b}{dx^2} - B_s \frac{d^2 w_s}{dx^2} \tag{15a}$$

$$M_b - \mu \frac{d^2 M_b}{dx^2} = B \frac{du_0}{dx} - D \frac{d^2 w_b}{dx^2} - D_s \frac{d^2 w_s}{dx^2} \tag{15b}$$

$$M_s - \mu \frac{d^2 M_s}{dx^2} = B \frac{du_0}{dx} - D \frac{d^2 w_b}{dx^2} - H_s \frac{d^2 w_s}{dx^2} \tag{15c}$$

$$Q - \mu \frac{d^2 Q}{dx^2} = A_s \frac{dw_s}{dx} \tag{15d}$$

where the stiffness components are given as

$$\begin{aligned}
&\{A, B, D, \overline{E}, F, H\} = \int_A \{1, z, z^2, z^3, z^4, z^6\} E(z) dA, \\
&B_s = -\frac{1}{4} B + \frac{5}{3h^2} \overline{E}, \\
&D_s = -\frac{1}{4} D + \frac{5}{3h^2} F, \\
&H_s = \frac{1}{16} D - \frac{5}{6h^2} F + \frac{25}{9h^4} H, \\
&\{A_{55}, D_{55}, F_{55}\} = \int_A \{1, z^2, z^4\} G(z) dA, \\
&A_s = \frac{25}{16} A_{55} - \frac{25}{2h^2} D_{55} + \frac{25}{h^4} F_{55},
\end{aligned} \tag{16}$$

By substituting Eq. (15) into Eq. (12), the nonlocal equations of motion can be expressed in terms of displacements (u_0, w_b, w_s) as

$$A\frac{d^2u_0}{dx^2}-B\frac{d^3w_b}{dx^3}-B_s\frac{d^3w_s}{dx^3}=I_0\left(\ddot{u}_0-\mu\frac{d^2\ddot{u}_0}{dx^2}\right) \tag{17a}$$

$$\begin{aligned}&B\frac{d^3u_0}{dx^3}-D\frac{d^4w_b}{dx^4}-D_s\frac{d^4w_s}{dx^4}+q-\mu\frac{d^2q}{dx^2}-N_0\left(\frac{d^2(w_b+w_s)}{dx^2}-\mu\frac{d^4(w_b+w_s)}{dx^4}\right)\\&=I_0\left((\ddot{w}_b+\ddot{w}_s)-\mu\frac{d^2(\ddot{w}_b+\ddot{w}_s)}{dx^2}\right)-I_2\left(\frac{d^2\ddot{w}_b}{dx^2}-\mu\frac{d^4\ddot{w}_b}{dx^4}\right)\end{aligned} \tag{17b}$$

$$\begin{aligned}&B_s\frac{d^3u_0}{dx^3}-D_s\frac{d^4w_b}{dx^4}-H_s\frac{d^4w_s}{dx^4}+A_s\frac{d^2w_s}{dx^2}+q-\mu\frac{d^2q}{dx^2}-N_0\left(\frac{d^2(w_b+w_s)}{dx^2}-\mu\frac{d^4(w_b+w_s)}{dx^4}\right)\\&=I_0\left((\ddot{w}_b+\ddot{w}_s)-\mu\frac{d^2(\ddot{w}_b+\ddot{w}_s)}{dx^2}\right)-\frac{I_2}{84}\left(\frac{d^2\ddot{w}_s}{dx^2}-\mu\frac{d^4\ddot{w}_s}{dx^4}\right)\end{aligned} \tag{17c}$$

The equations of motion of local beam theory can be obtained from Eq. (17) by setting the nonlocal parameter μ equal to zero.

3. Analytical solution of simply supported FG nanobeam

The above equations of motion are analytically solved for bending, buckling and free vibration problems. The Navier solution procedure is used to determine the analytical solutions for a simply supported FG nanobeam. The solution is assumed to be of the form

$$\begin{Bmatrix}u_0\\w_b\\w_s\end{Bmatrix}=\sum_{m=1}^{\infty}\begin{Bmatrix}U_n\cos(\alpha x)e^{i\omega t}\\W_{bn}\sin(\alpha x)e^{i\omega t}\\W_{sn}\sin(\alpha x)e^{i\omega t}\end{Bmatrix} \tag{18}$$

where U_n, W_{bn}, and W_{sn} are arbitrary parameters to be determined, ω is the eigenfrequency associated with n th eigenmode, and $\alpha=n\pi/L$. The transverse load q is also expanded in the Fourier sine series as

$$q(x)=\sum_{n=1}^{\infty}Q_n\sin\alpha x,\quad Q_n=\frac{2}{L}\int_0^L q(x)\sin(\alpha x)\,dx \tag{19}$$

The Fourier coefficients Q_n associated with some typical loads are given

$$Q_n=q_0,\quad n=1\quad \text{for sinusoidal load,} \tag{20a}$$

$$Q_n=\frac{4q_0}{n\pi},\quad n=1,3,5.....\ \text{for uniform load,} \tag{20b}$$

$$Q_n = \frac{2q_0}{L}\sin\frac{n\pi}{2},\ n = 1,2,3....\ \text{for point load } Q_0 \text{ at the midspan,} \tag{20c}$$

Substituting the expansions of u_0, w_b, w_s and q from Eqs. (18) and (19) into Eq. (17), the analytical solutions can be obtained from the following equations

$$\left(\begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22}-\bar{P} & S_{23}-\bar{P} \\ S_{13} & S_{23}-\bar{P} & S_{33}-\bar{P} \end{bmatrix} - \lambda\omega^2 \begin{bmatrix} m_{11} & 0 & 0 \\ 0 & m_{22} & m_{23} \\ 0 & m_{23} & m_{33} \end{bmatrix}\right)\begin{Bmatrix} U_n \\ W_{bn} \\ W_{sn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ \lambda Q_n \\ \lambda Q_n \end{Bmatrix} \tag{21}$$

where

$$S_{11} = A\alpha^2,\ S_{12} = -B\alpha^3,\ S_{13} = -B_s\alpha^3,\ S_{22} = D\alpha^4,\ S_{23} = D_s\alpha^4,\ S_{33} = H_s\alpha^4 + A_s\alpha^2,$$

$$m_{11} = m_{23} = I_0,\ m_{22} = I_0 + I_2\alpha^2,\ m_{33} = I_0 + \frac{I_2}{84}\alpha^2,$$

$$\bar{P} = \lambda N_0\alpha^2,\ \lambda = 1 + \mu\alpha^2 \tag{22}$$

4. Results and discussion

This section is divided into two parts. The first one presents a verification of the proposed nonlocal model with those previously published. The second section shows the effects of nonlocal parameter, power law index, and aspect ratio on the bending, buckling and vibration responses of FG nanobeam.

In the following analysis, two FG nanobeams are investigated. The first FG nanobeam has the following material properties: E_t=0.25 **TPa**, E_b=1 **TPa**, v_t=v_b=0.3 (Larbi Chaht *et al.* 2015). The second FG nanobeam is composed of steel and alumina (Al_2O_3). The bottom surface of the beam is pure steel, whereas the top surface of the beam is pure alumina. The material properties are as follows: E_t=390 **GPa**, E_b=210 **GPa**, ρ_t=3960 **kg/m^3**, ρ_b=7800 **kg/m^3**, v_t=v_b=0.3 (Eltaher *et al.* 2012). The shear correction factor is taken as 5/6 for Timoshenko beam theory. For convenience, the following nondimensionalizations are used:

- $\bar{w} = 100w\dfrac{E_t I}{q_0 L^4}$ for uniform load;
- $\bar{\omega} = \omega L^2\sqrt{\dfrac{\rho_t A}{E_t I}}$ frequency parameter;
- $\bar{N} = N_{cr}\dfrac{L^2}{E_t I}$ critical buckling load parameter:

4.1 Comparative studies

In order to demonstrate the accuracy of the present closed-form exact solution, some comparisons of the present results with those available in the literature has been carried out.

Table 1 Dimensionless transverse deflections ($\overline{w}$) of the FG nanobeam for uniform load

L/h	k	Nonlocal parameter, e_0a (nm)														
		0			0.5			1			1.5			2		
		TBT[(a)]	SBT[(a)]	Present	TBT[(a)]	SBT[(a)]	Present	TBT[(a)]	SBT[(a)]	Present	TBT[(a)]	SBT[(a)]	Present	TBT[(a)]	SBT[(a)]	Present
10	0	5.3383	5.3381	5.3383	5.4659	5.4659	5.4659	5.8487	5.8485	5.8487	6.4867	6.4865	6.4867	7.3798	7.3797	7.3799
	0.3	3.2169	3.2178	3.2181	3.2938	3.2946	3.2951	3.5245	3.5254	3.5258	3.9090	3.9102	3.9104	4.4472	4.4482	4.4488
	1	2.4194	2.4193	2.4194	2.4772	2.4772	2.4773	2.6508	2.6508	2.6509	2.9401	2.9401	2.9401	3.3451	3.3449	3.3452
	3	1.9249	1.9234	1.9234	1.9710	1.9693	1.9694	2.1091	2.1074	2.1074	2.3393	2.3373	2.3375	2.6615	2.6596	2.6595
	10	1.5799	1.5790	1.5790	1.6176	1.6169	1.6168	1.7310	1.7301	1.7301	1.9190	1.9190	1.9189	2.1843	2.1831	2.1831
30	0	5.2227	5.2228	5.2228	5.2366	5.2366	5.2367	5.2784	5.2786	5.2785	5.3480	5.3480	5.3481	5.4455	5.4456	5.4455
	0.3	3.1486	3.1473	3.1475	3.1570	3.1557	3.1559	3.1822	3.1809	3.1811	3.2241	3.2230	3.2230	3.2829	3.2815	3.2818
	1	2.3732	2.3731	2.3732	2.3795	2.3795	2.3795	2.3985	2.3984	2.3985	2.4301	2.4301	2.4302	2.4744	2.4744	2.4744
	3	1.8894	1.8892	1.8892	1.8944	1.8943	1.8943	1.9095	1.9094	1.9094	1.9347	1.9344	1.9346	1.9700	1.9698	1.9698
	10	1.5489	1.5488	1.5488	1.5530	1.5530	1.5529	1.5654	1.5653	1.5653	1.5860	1.5861	1.5860	1.6149	1.6149	1.6149
100	0	5.2096	5.2097	5.2096	5.2108	5.2110	5.2109	5.2146	5.2146	5.2146	5.2208	5.2210	5.2209	5.2296	5.2296	5.2296
	0.3	3.1408	3.1394	3.1395	3.1416	3.1404	3.1403	3.1438	3.1426	3.1425	3.1476	3.1465	3.1463	3.1529	3.1517	3.1515
	1	2.3679	2.3680	2.3679	2.3685	2.3686	2.3685	2.3702	2.3702	2.3702	2.3730	2.3731	2.3731	2.3770	2.3771	2.3770
	3	1.8853	1.8853	1.8854	1.8858	1.8858	1.8858	1.8871	1.8871	1.8872	1.8894	1.8893	1.8894	1.8926	1.8926	1.8926
	10	1.5453	1.5453	1.5454	1.5457	1.5457	1.5458	1.5468	1.5468	1.5469	1.5487	1.5487	1.5487	1.5513	1.5513	1.5513

[(a)]Taken from Larbi Chaht *et al*. (2015)

Table 2 Dimensionless critical buckling load ($\overline{N}$) of the FG nanobeam

L/h	k	Nonlocal parameter, e_0a (nm)														
		0			0.5			1			1.5			2		
		TBT[(a)]	SBT[(a)]	Present	TBT[(a)]	SBT[(a)]	Present	TBT[(a)]	SBT[(a)]	Present	TBT[(a)]	SBT[(a)]	Present	TBT[(a)]	SBT[(a)]	Present
10	0	2.4056	2.4052	2.4057	2.3477	2.3473	2.3478	2.1895	2.1892	2.1896	1.9685	1.9682	1.9685	1.7247	1.7244	1.7248
	0.3	3.9921	3.9906	3.9906	3.8959	3.8945	3.8945	3.6335	3.6322	3.6321	3.2667	3.2655	3.2654	2.8621	2.8611	2.8611
	1	5.3084	5.3086	5.3084	5.1805	5.1808	5.1806	4.8315	4.8317	4.8316	4.3437	4.3440	4.3438	3.8059	3.8060	3.8059
	3	6.6720	6.6780	6.6776	6.5113	6.5172	6.5168	6.0727	6.0781	6.0778	5.4596	5.4645	5.4642	4.7835	4.7878	4.7876
	10	8.1289	8.1338	8.1337	7.9332	7.9379	7.9378	7.3987	7.4031	7.4030	6.6518	6.6558	6.6557	5.8281	5.8316	5.8315
30	0	2.4603	2.4604	2.4604	2.4536	2.4537	2.4537	2.4336	2.4337	2.4337	2.4011	2.4011	2.4011	2.3570	2.3570	2.3570
	0.3	4.0811	4.0826	4.0826	4.0699	4.0714	4.0714	4.0368	4.0383	4.0383	3.9828	3.9843	3.9843	3.9096	3.9110	3.9110
	1	5.4146	5.4147	5.4147	5.3998	5.3999	5.3999	5.3559	5.3560	5.3560	5.2843	5.2843	5.2843	5.1871	5.1872	5.1872
	3	6.8011	6.8018	6.8018	6.7825	6.7832	6.7832	6.7273	6.7280	6.7280	6.6373	6.6380	6.6380	6.5153	6.5160	6.5160
	10	8.2962	8.2968	8.2968	8.2735	8.2741	8.2741	8.2062	8.2068	8.2068	8.0964	8.0970	8.0970	7.9476	7.9481	7.9481
100	0	2.4667	2.4668	2.4668	2.4661	2.4662	2.4662	2.4643	2.4643	2.4643	2.4613	2.4613	2.4613	2.4570	2.4571	2.4571
	0.3	4.0915	4.0933	4.0933	4.0905	4.0923	4.0923	4.0874	4.0893	4.0893	4.0824	4.0842	4.0842	4.0754	4.0772	4.0772
	1	5.4270	5.4271	5.4271	5.4257	5.4257	5.4257	5.4217	5.4217	5.4217	5.4150	5.4150	5.4150	5.4057	5.4057	5.4057
	3	6.8161	6.8162	6.8162	6.8144	6.8145	6.8145	6.8094	6.8095	6.8095	6.8010	6.8011	6.8011	6.7893	6.7894	6.7894
	10	8.3157	8.3158	8.3158	8.3136	8.3137	8.3137	8.3075	8.3076	8.3076	8.2972	8.2973	8.2973	8.2830	8.2831	8.2831

[(a)]Taken from Larbi Chaht *et al*. (2015)

Table 1 shows the nondimensional maximum deflections $\overline{w}$ of a simply supported FG nanobeam subjected to uniform load. The calculated values are obtained using 100 terms in series in Eqs. (18) and (19). It should be noted that e_0a=0 corresponds to local beam theory. The obtained

Table 3 Dimensionless fundamental frequency (ϖ) of the FG nanobeam

L/h	*k*	Nonlocal parameter, e_0a (nm)														
		0			0.5			1			1.5			2		
		EBT	TBT	Present	EBT	TBT	Present	EBT	TBT	Present	EBT	TBT	Present	EBT	TBT	Present
10	0	9.8293	9.7075	9.7075	9.7102	9.5899	9.5899	9.3774	9.2612	9.2612	8.8915	8.7813	8.7813	8.3228	8.2196	8.2197
	0.3	8.2694	8.1700	8.1709	8.1692	8.0711	8.0719	7.8892	7.7944	7.7952	7.4804	7.3905	7.3913	7.0019	6.9178	6.9185
	1	6.9650	6.8814	6.8814	6.8807	6.7981	6.7981	6.6448	6.5651	6.5651	6.3005	6.2249	6.2249	5.8975	5.8267	5.8267
	3	6.1575	6.0784	6.0755	6.0829	6.0048	6.0019	5.8744	5.7990	5.7962	5.5700	5.4985	5.4959	5.2137	5.1468	5.1443
	10	5.6544	5.5794	5.5768	5.5859	5.5118	5.5092	5.3945	5.3229	5.3204	5.1150	5.0470	5.0447	4.7878	4.7242	4.7221
30	0	9.8651	9.8511	9.8511	9.8516	9.8376	9.8376	9.8114	9.7975	9.7975	9.7456	9.7318	9.7318	9.6556	9.6419	9.6419
	0.3	8.3015	8.2901	8.2902	8.2902	8.2787	8.2788	8.2564	8.2450	8.2451	8.2010	8.1897	8.1898	8.1252	8.1140	8.1141
	1	6.9929	6.9832	6.9832	6.9833	6.9737	6.9737	6.9548	6.9453	6.9452	6.9082	6.8987	6.8987	6.8444	6.8349	6.8349
	3	6.1806	6.1715	6.1712	6.1722	6.1631	6.1627	6.1470	6.1380	6.1376	6.1058	6.0968	6.0964	6.0494	6.0405	6.0401
	10	5.6744	5.6658	5.6655	5.6667	5.6581	5.6578	5.6436	5.6350	5.6347	5.6057	5.5972	5.5969	5.5540	5.5455	5.5452
100	0	9.8692	9.8679	9.8679	9.8680	9.8667	9.8667	9.8643	9.8631	9.8631	9.8583	9.8570	9.8570	9.8498	9.8485	9.8485
	0.3	8.3052	8.3042	8.3042	8.3042	8.3031	8.3032	8.3011	8.3001	8.3001	8.2960	8.2950	8.2950	8.2889	8.2878	8.2878
	1	6.9961	6.9952	6.9952	6.9952	6.9943	6.9943	6.9926	6.9917	6.9917	6.9883	6.9874	6.9874	6.9823	6.9814	6.9814
	3	6.1833	6.1825	6.1824	6.1825	6.1817	6.1817	6.1802	6.1794	6.1794	6.1764	6.1756	6.1756	6.1711	6.1703	6.1703
	10	5.6767	5.6760	5.6759	5.6761	5.6753	5.6752	5.6740	5.6732	5.6731	5.6705	5.6697	5.6697	5.6656	5.6648	5.6648

results are compared with those reported by Larbi Chaht *et al*. (2015) based on both nonlocal Timoshenko beam theory (TBT) and sinusoidal beam theory (SBT) for a wide range of nonlocal parameter (e_0a), power law index (k) and length-to-depth ratio (L/h). It can be seen that the results of present theory are in excellent agreement with those predicted by both TBT and SBT (Larbi Chaht *et al.* 2015) for all values of nonlocal parameter, power law index and length-to-depth ratio. The deflections $\overline{w}$ decrease as the power law index k increases. However, the increase of the nonlocal parameter leads to an increase of deflections.

The nondimensional critical buckling loads are presented in Table 2. Present results are compared with results of Larbi Chaht *et al*. (2015) and good agreement is observed. According to this table buckling loads decrease with increasing nonlocal parameter (e_0a). However, the increase of power law index k leads to an increase of critical buckling loads.

The fundamental nondimensional frequencies for different nonlocal parameter e_0a are presented in Table 3. The material properties of the FG nanobeam are according to those used by Eltaher *et al*. (2012). The present results are compared with those computed using both Euler-Bernoulli beam theory (EBT) and TBT and an excellent agreement is observed with TBT. From this table, it can be seen that the fundamental nondimensional frequency is reduced with the increase of the nonlocal parameter and the power law index.

In general, the effect of transverse shear deformations and the nonlocal parameter e_0a is to increase the deflections and reduce the buckling loads as well as natural frequencies, as can be seen from the results presented in Tables 1-3. The increase of power law index leads to a decrease of both the dimensionless deflections and fundamental frequencies contrary to the dimensionless buckling load. This is due to the fact that an increase in the power law index yields an increase in the stiffness of the FG nanobeam.

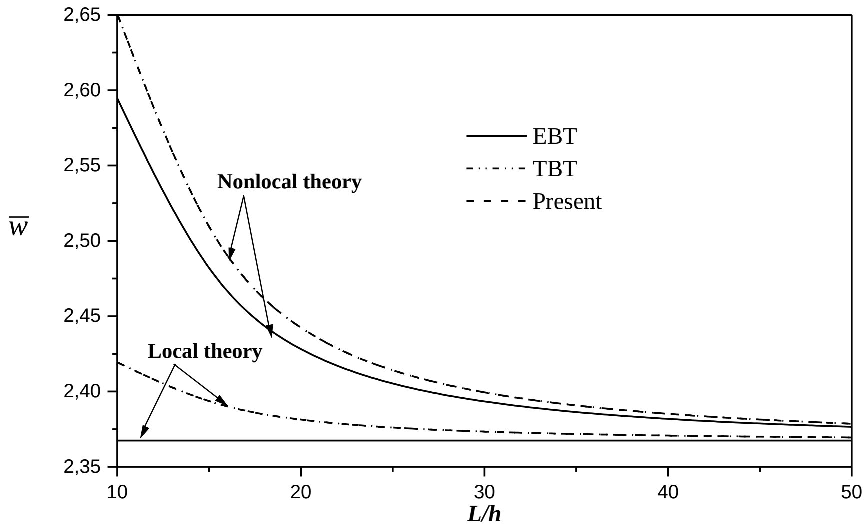

Fig. 2 Effect of the aspect ratio on dimensionless deflection ($\overline{w}$) for uniform load with k=1 and e_0a=1 nm

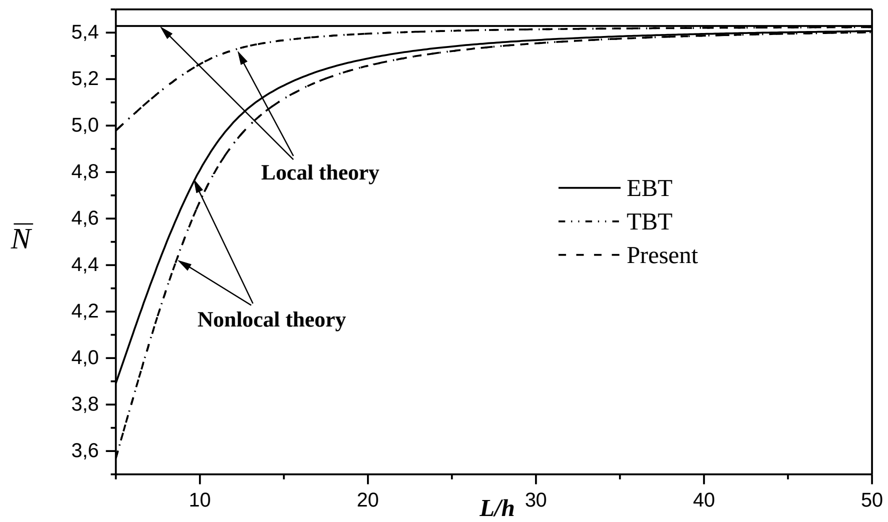

Fig. 3 Effect of the aspect ratio on dimensionless buckling load ($\overline{N}$) with k=1 and e_0a=1 nm

4.2 Parametric investigations

The bending and buckling responses of FG nanobeam are studied here by assuming the material properties used by Larbi Chaht *et al*. (2015). However, the dynamic response of FG nanobeam is investigated by assuming the material properties used by Eltaher *et al*. (2012).

Figs. 2 to 4 show the effect of the aspect ratio on static, buckling and dynamic responses of FG nanobeam, respectively. The local and nonlocal results are given for e_0a=0 and e_0a=1 nm, respectively. The power law index is assumed to be constant, k=1. It is observed from these figures that deflections predicted by the nonlocal theory are larger than those of the local results whereas

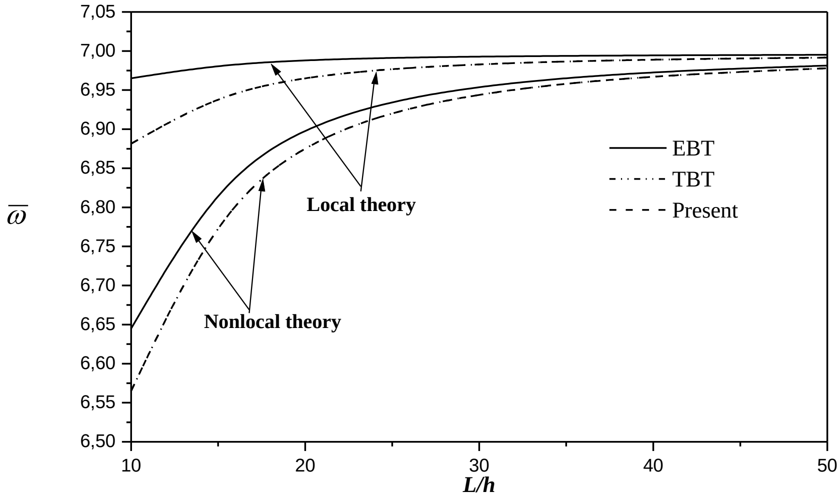

Fig. 4 Effect of the aspect ratio on dimensionless fundamental frequency ($\overline{\varpi}$) with k=1 and e_0a=1 nm

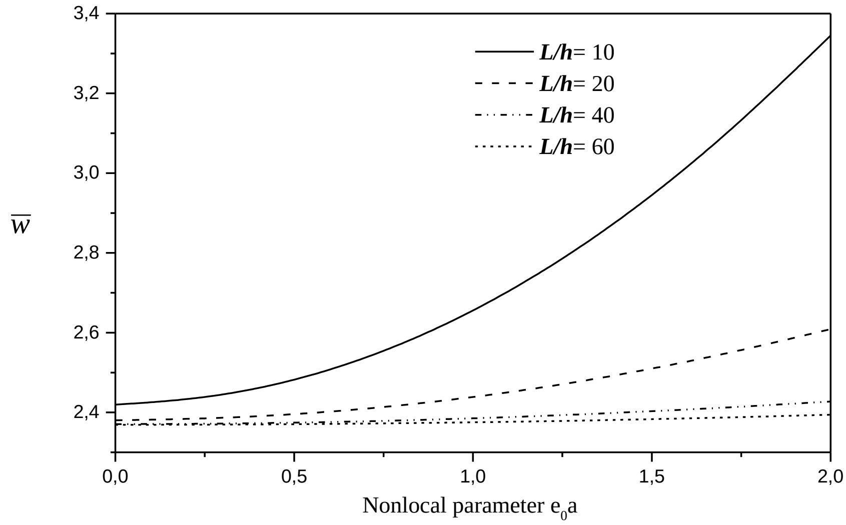

Fig. 5 Effect of nonlocal parameter on dimensionless deflection ($\overline{w}$) for uniform load with k=1

the nonlocal solution of both the buckling load and the fundamental frequency is smaller than the local one due to the small scale effects. This result indicates that the effect of nonlocal parameter softens the nanobeam. Furthermore, it can be observed that when the aspect ratio is small, the scale effects are significant. However, the scale effects on the deflection, buckling load and fundamental frequency will diminish with the ratio (i.e., L/h) increasing. It implies that the scale effects on the static, buckling and dynamic properties are not obvious for slender FG nanobeam but should be taken into account for short FG nanobeam.

In order to shown the influences of the nonlocal parameter, the dimensionless deflections, critical buckling loads and the dimensionless fundamental frequencies computed using the present nonlocal shear deformation beam theory with different aspect ratios (L/h) are presented in Figs. 5

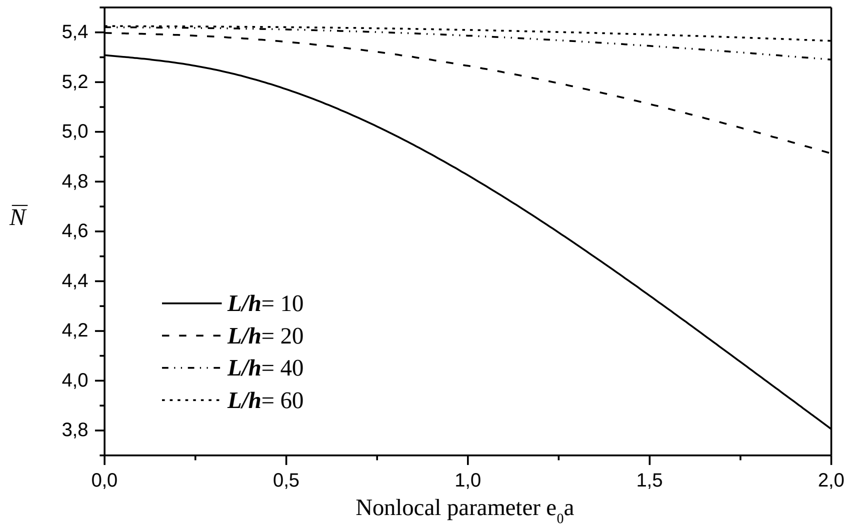

Fig. 6 Effect of nonlocal parameter on dimensionless buckling load ($\overline{N}$) with k=1

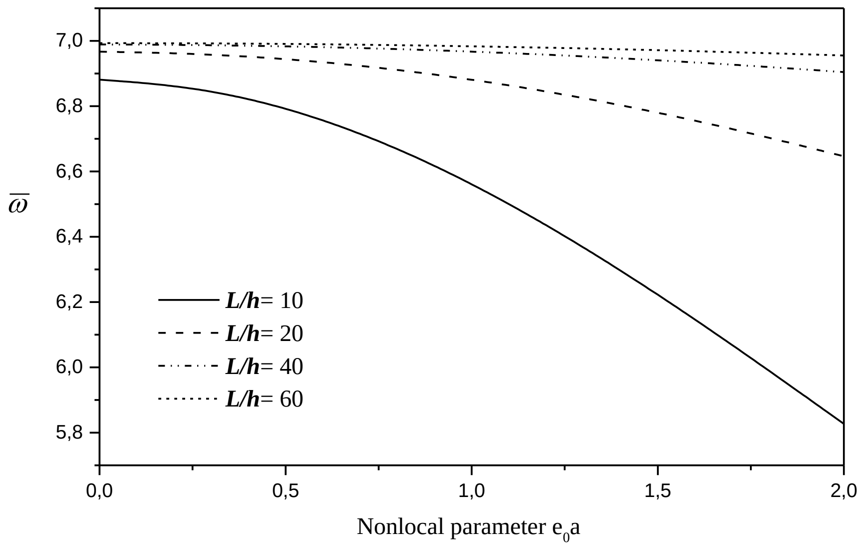

Fig. 7 Effect of nonlocal parameter on dimensionless fundamental frequency ($\overline{\varpi}$) with k=1

to 7, respectively. The power law index is assumed to be constant, k=1. These figures show that the responses vary nonlinearly with the nonlocal parameter. It can be seen that the effect of nonlocal parameter e_0a on deflections, critical buckling loads and the dimensionless fundamental frequencies of FG nanobeams is significant, especially at relatively higher aspect ratios. Therefore, it can be concluded that FG nanobeams responses are aspect ratio dependent based on nonlocal elasticity.

The effect of the power law index on the dimensionless deflection, buckling load and fundamental frequency of FG nanobeam is presented in Figs. 8 to 10 for various values of the

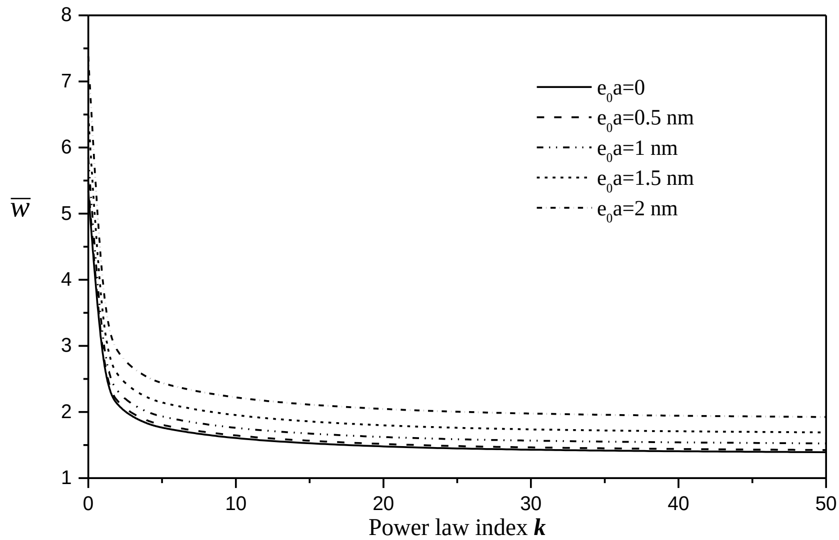

Fig. 8 Effect of the power law index on dimensionless deflection ($\overline{w}$) for uniform load with L/h=10

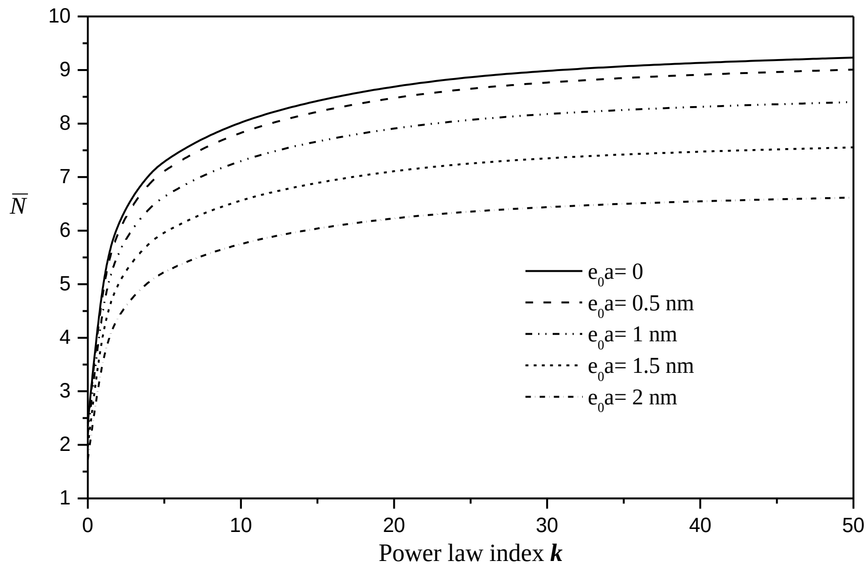

Fig. 9 Effect of the power law index on dimensionless buckling load ($\overline{N}$) with L/h=10

nonlocal parameter with L/h=10. It can be observed that both the dimensionless deflections and fundamental frequencies decrease whereas the dimensionless buckling load increases as the power law index increases. It is noted that this observation is also seen in Tables 1 to3 and this is due to the fact that an increase in the power law index yields an increase in the stiffness of the FG nanobeam.

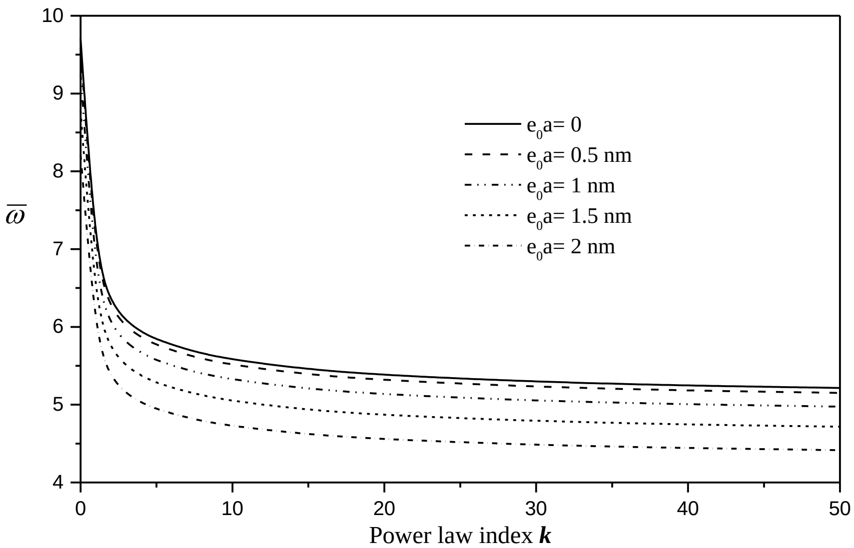

Fig. 10 Effect of the power law index on dimensionless fundamental frequency ($\overline{\omega}$) with *L/h*=10

5. Conclusions

A nonlocal shear deformation beam theory is used to study bending, buckling, and free vibration of FG nanobeams. The present model is capable of capturing both small scale and shear deformation effects of FG nanobeams, and does not require shear correction factors. Numerical examples show that the present theory gives solutions which are almost identical with those generated by TBT. Effect of nonlocal parameter, aspect ratio and various material compositions are investigated in detail.

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