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Free vibration analysis of axially moving beam under non-ideal conditions

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Abstract. In this study, linear vibrations of an axially moving beam under non-ideal support conditions have been investigated. The main difference of this study from the other studies; the non-ideal clamped support allow minimal rotations and non-ideal simple support carry moment in minimal orders. Axially moving Euler-Bernoulli beam has simple and clamped support conditions that are discussed as combination of ideal and non-ideal boundary with weighting factor (k). Equations of the motion and boundary conditions have been obtained using Hamilton's Principle. Method of Multiple Scales, a perturbation technique, has been employed for solving the linear equations of motion. Linear equations of motion are solved and effects of different parameters on natural frequencies are investigated.

Keywords: axially moving; vibration; non-ideal support; perturbation methods

1. Introduction

Axially moving beam vibrations were studied by many researchers. Some tests are performed to compare the "first order" perturbative solution with an approximate solution obtained by a finite difference scheme (Pellicano and Zirilli 1998). The transverse vibration of a simply supported beam moving with constant velocity is considered (Pakdemirli 1998). Non-linear vibrations of an axially moving beam are investigated. The non-linearity is introduced by including stretching effect of the beam. The beam is moving with a time-dependent velocity, namely a harmonically varying velocity about a constant mean velocity. (Öz *et al*. 2001). Non-ideal boundary conditions are modelled using perturbations. The idea is applied to two beam vibration problems; simply supported beam, sliding-clamped beam. Effect of non-ideal boundary conditions on the natural frequencies and mode shapes are examined for each case using the Lindstedt-Poincare technique (Pakdemirli and Boyaci 2002). The concept of non-ideal boundary conditions is applied to the beam problem. Ideal and non-ideal frequencies as well as frequency-response curves are compared (Mehmet Pakdemirli and Boyacı 2003). A non-ideal boundary condition is modelled as a linear combination of the ideal simply supported and the ideal clamped boundary conditions with the

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weighting factors respectively. When the non-ideal boundary conditions are close to the ideal simply supported boundary conditions, however, the natural frequencies hardly change as k varies (Lee 2013). The dynamic response of an axially accelerating, tensioned beam is investigated. Non-linear vibrations of an axially moving beam are investigated. Linear solutions of the vibration of axially moving beams are discussed (Ö z and Pakdemirli 1999, Ö z *et al.* 2001). The vibration of an axially moving Euler-Bernoulli beam with fixed end conditions is investigated. Natural frequencies are found depending on mean velocity. The natural frequencies for the fixed-fixed axially moving beam are higher than those of a simply supported one (Ö z 2001). The transverse vibrations of an axially moving flexible beams resting on multiple supports and effect of axial speed on first and second natural frequency of system are investigated. And obtained results are compared with older studies (Kural and Ö zkaya 2012). Nonlinear vibrations of an axially moving mid-supported and multi-supported string have been investigated. There are non-ideal supports allowing minimal deflections between ideal supports at both ends of the string (Yurddaş *et al*. 2013, Yurddaş *et al*. 2014). The method of multiple scales is directly applied to the equations of motion obtained for the general case. Natural frequency equations are presented for intermediate and multiple support cases. Results are presented to show the effects of axial speed, number of supports, and their locations (Bağdatli *et al*. 2013, Bağdatli *et al*. 2011).

In this research, linear vibrations of an axially moving beam under non-ideal support conditions have been studied. The main difference of this study from the other studies; the non-ideal supports between ideal supports at both ends of the beam is allowed minimal rotations when beam axially moving. Non-ideal boundary cases at the both ends are modeled with weighting factor. Equations of the motion and boundary conditions have been obtained using Hamilton's Principle. Method of Multiple Scales, a perturbation technique, has been employed for solving the linear equations. Equation of motion is made dimensionless using dimensionless parameters. Linear equations of motion are solved and effects on the natural frequency of the different parameters are investigated. Exact natural frequencies are compared with ideal support conditions to non-ideal boundary conditions for different weighting factor (k) parameters and axial moving values (v_0). The weighting factor which causing non-ideal effects are discussed for simple and clamped support conditions.

2. Equations of motion

Rotary inertia and shear effect are not included and cross sectional area do not change during motion. The Lagrangian can be written as follows

$$\begin{aligned} \pounds = {} & \frac{1}{2}\int_0^L \rho A \left\{ (\dot{\hat{u}} + \hat{u}'\hat{v} + \hat{v})^2 + (\dot{\hat{w}} + \hat{w}'\hat{v})^2 \right\} d\hat{x} \\ & - \left\{ \frac{1}{2} EA \int_0^L \left(\hat{u}' + \frac{1}{2}\hat{w}'^2 \right)^2 d\hat{x} + \frac{1}{2} EI \int_0^L \hat{w}''^2 d\hat{x} + \frac{1}{2}\int_0^L P\left(\hat{u}' + \frac{1}{2}\hat{w}'^2 \right) d\hat{x} \right\} \end{aligned} \tag{1}$$

ρA is the mass, $\hat{u}$ and $\hat{w}$ is axial and longitudinal displacement, $\hat{v}$ is the axial moving of the beam, L is the length of the beam, EA is longitudinal rigidity, EI is flexural rigidity, P is the axial force, $\hat{t}$ is the time. Dot denotes derivative with respect to $\hat{t}$ and prime symbol derivative with respect to $\hat{x}$ in Eq. (1). The first integral is the kinetic energy, the second integral is the elastic energy in bending, the third integral is the elastic energy in extension due to stretching of the neutral axis and the last one is the elastic energy due to axial tension. Applying Hamilton's principle and performing the necessary algebra, Eq. (1) instead of Lagrangian and after that general case for the beam is obtained as follows

$$\left(\ddot{w}+2\dot{w}'v+w'\dot{v}+w''v^2\right)-\left(w'\left(1+v_b^2\left(u'+\frac{1}{2}w'^2\right)\right)\right)'+v_f^2w^{iv}=0 \tag{2}$$

The dimensionless quantities are regards with the following relations

$$w=\frac{\hat{w}}{L},\ \ u=\frac{\hat{u}}{L},\ \ x=\frac{\hat{x}}{L},\ \ t=\sqrt{\frac{P}{\rho AL^2}}\hat{t},\ \ v=\frac{\hat{v}}{\sqrt{P/\rho A}},\ \ v_b^2=\frac{EA}{P},\ \ v_f^2=\frac{EI}{PL^2} \tag{3}$$

where axial velocity (v) is made non-dimensional by dividing with critical velocity. v_b is longitudinal rigidity and v_f flexural rigidity. The explanation for $v_b^2 >> 1$ is given in reference, (Chakraborty *et al*. 1998, Thurman and Mote 1969). The following equation of motion in the non-dimensional form is written using the Eq. (3)

$$\left(\ddot{w}+2\dot{w}'v+w'\dot{v}\right)+\left(v^2-1\right)w''+v_f^2w^{iv}=\frac{1}{2}v_b^2w''\int_0^1 w'^2dx \tag{4}$$

The method of multiple scales will be applied to the equations directly in the next section. The amplitudes of vibration are assumed small to guarantee that the nonlinear terms stay in the higher orders of perturbation, is in the second order since it is multiplied by a nonlinear term. Axial velocity of the beam is determined as shown in Eq. (5)

$$v=v_0+\varepsilon v_1\sin\Omega t \tag{5}$$

where of the beam about an arbitrary constant value with small amplitude fluctuations and the fluctuation frequency is again arbitrary. Eq. (6) can be showed after inserting velocity as follows

$$\begin{aligned}&\ddot{w}+2\dot{w}'v_0+2\varepsilon\,\dot{w}'v_1\sin\Omega t+\varepsilon\,w'v_1\Omega\cos\Omega t+v_f^2w^{iv}\\&+\left(v_0^2+\varepsilon^2v_1^2\sin^2\Omega t+2\varepsilon\,v_0v_1\sin\Omega t-1\right)w''=\frac{1}{2}v_b^2\left(\int_0^1 w'^2dx\right)w''\end{aligned} \tag{6}$$

In order to obtain a weak nonlinear system deflection w is transformed $w=\sqrt{\varepsilon}\,y$

$$\begin{aligned}&\ddot{y}+2\dot{y}'v_0+\left(v_0^2-1\right)y''+v_f^2y^{iv}+\varepsilon\left(2\,\dot{y}'v_1\sin\Omega t+y'v_1\Omega\cos\Omega t+2y''v_0v_1\sin\Omega t\right)\\&+\varepsilon^2\left(v_1^2\sin^2\Omega t\right)y''=\frac{1}{2}v_b^2\,\varepsilon\left(\int_0^1 y'^2dx\right)y''\end{aligned} \tag{7}$$

The method of multiple scales will be applied to the partial differential equation system directly (Nayfeh 1981). The displacement functions for the moving beam can be expanded as shown below

$$y(x,t;\varepsilon)=y_0(x,T_0;T_1)+\varepsilon y_1(x,T_0;T_1) \tag{8}$$

where ε is a small book-keeping parameter representing that the deflections are small. This procedure models a weak non-linear system $T_0=t$ is the usual fast time scale and $T_1=\varepsilon t$ is the slow time scales in the method of multiple scales. The time derivatives are expressed in terms of the new time variables $\partial/\partial t=D_0+\varepsilon D_1$, $\partial^2/\partial t^2=D_0{}^2+2\varepsilon D_0D_1+...$, where $D_n=\partial/\partial T_n$. After expansion, one obtains equations of motion as follows

Order (1)

$$D_0^2 y_0 + 2v_0 D_0 y_0' + \left(v_0^2 - 1\right) y_0'' + v_f^2 y_0^{iv} = 0 \tag{9}$$

Order (ε)

$$\begin{aligned} &D_0^2 y_1 + 2v_0 D_0 y_1' + v_f^2 y_1^{iv} + \left(v_0^2 - 1\right) y_1'' \\ &\quad = -2D_0 D_1 y_0 - 2v_0 D_1 y_0' - 2v_1 \sin \Omega t D_0 y_0' \\ &\quad\quad - 2y_1'' v_0 v_1 \sin \Omega t - y_0' v_1 \Omega \cos \Omega t + \frac{1}{2} v_b^2 \left(\int_0^1 y_0'^2 dx \right) y_0'' \end{aligned} \tag{10}$$

3. Numerical analysis

This study aimed to examine only the linear part of the problem. The effects of the natural frequency of the linear problem of the non-ideal boundary conditions were investigated. Nonlinear effects of the natural frequencies and stability analysis is a continuation of the work. Solution of the first order of expansion gives natural frequency values and a solvability condition is obtained from the second order of expansion. The first order of perturbation is linear given in Eq. (10); the solution may be represented by

$$y_0(x, T_0, T_1) = A(T_1) e^{i\omega T_0} Y(x) + cc \tag{11}$$

where A stands for complex amplitude of the preceding terms. Substituting Eq. (11) into Eq. (8), one has

$$v_f^2 Y^{iv} + \left(v_0^2 - 1\right) Y'' + 2iv_0 \omega Y' - \omega^2 Y = 0 \tag{12}$$

The non-dimensional form of general form showed in Eq. (13) (Lee 2013).

General Form

$$Y(x) = 0\,, \qquad kY''(x) \pm (1-k)Y'(x) = 0 \qquad 0 \le k \le 1 \tag{13}$$

When the weighting factor (k) is taken as small value that closed zero, non-ideal clamped support is obtained. Similarly, non-ideal simple support is reached when the weighting factor (k) is taken closed to 1. The sign in Eq. (13) becomes minus (−) for left end of the beam and plus (+) for right end of the beam.

General boundary cases can be found in matrix form in Eq. (14)

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ e^{i\beta_1} & e^{i\beta_2} & e^{i\beta_3} & e^{i\beta_4} \\ \left((1-k)i\beta_1 + k\beta_1{}^2\right) & \left((1-k)i\beta_2 + k\beta_2{}^2\right) & \left((1-k)i\beta_3 + k\beta_3{}^2\right) & \left((1-k)i\beta_4 + k\beta_4{}^2\right) \\ \left((1-k)i\beta_1 + k\beta_1{}^2\right)e^{i\beta_1} & \left((1-k)i\beta_2 + k\beta_2{}^2\right)e^{i\beta_2} & \left((1-k)i\beta_3 + k\beta_3{}^2\right)e^{i\beta_3} & \left((1-k)i\beta_4 + k\beta_4{}^2\right)e^{i\beta_4} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{14}$$

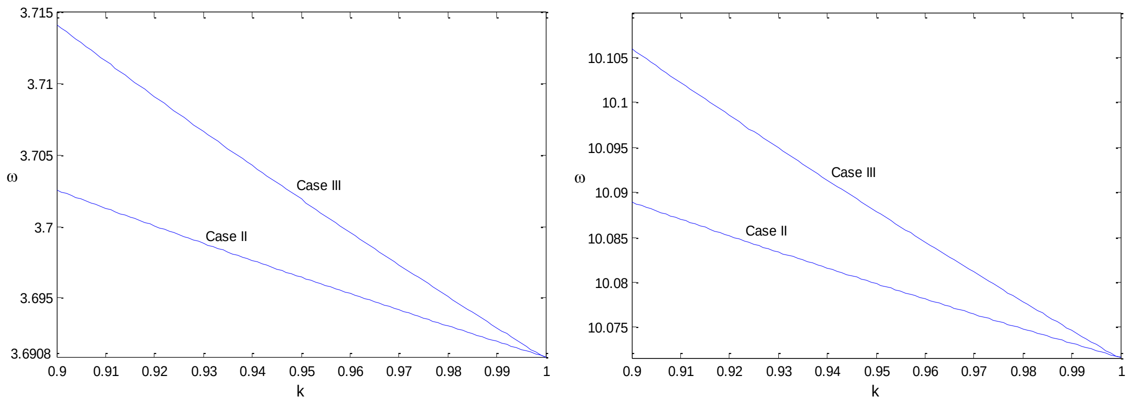
Boundary conditions for different cases given in Eqs. (15)-(20); non-ideal boundary conditions for Simple and Clamped support are determined by the variation of the parameter *k*.

CaseI: $Y(0)=0\,,Y''(0)=0$ $Y(1)=0\,,Y''(1)=0$	$k=1$	İdeal Simple İdeal Simple	(15)
CaseII: $Y(0)=0\,,Y''(0)=0$ $Y(1)=0\,,(1-k)Y'(1)+kY''(1)=0$	$0.9\le k\le 1$	Ideal Simple Non-ideal Simple	(16)
CaseIII: $Y(0)=0,\ (1-k)Y'(0)-kY''(0)=0$ $Y(1)=0,\ (1-k)Y'(1)+kY''(1)=0$	$0.9\le k\le 1$	Non-ideal Simple Non-ideal Simple	(17)
CaseIV: $Y(0)=0,\ Y'(0)=0$ $Y(1)=0,\ Y'(1)=0$	$k=0$	Ideal Clamped Ideal Clamped	(18)
CaseV: $Y(0)=0\,,Y'(0)=0$ $Y(1)=0\,,(1-k)Y'(1)+kY''(1)=0$	$0\le k\le 0.1$	Ideal Clamped Non-ideal Clamped	(19)
CaseVI: $Y(0)=0\,,(1-k)Y'(0)-kY''(0)=0$ $Y(1)=0\,,(1-k)Y'(1)+kY''(1)=0$	$0\le k\le 0.1$	Non-ideal Clamped Non-ideal Clamped	(20)

The following functions can be proposed for the solutions of Eq. (21)

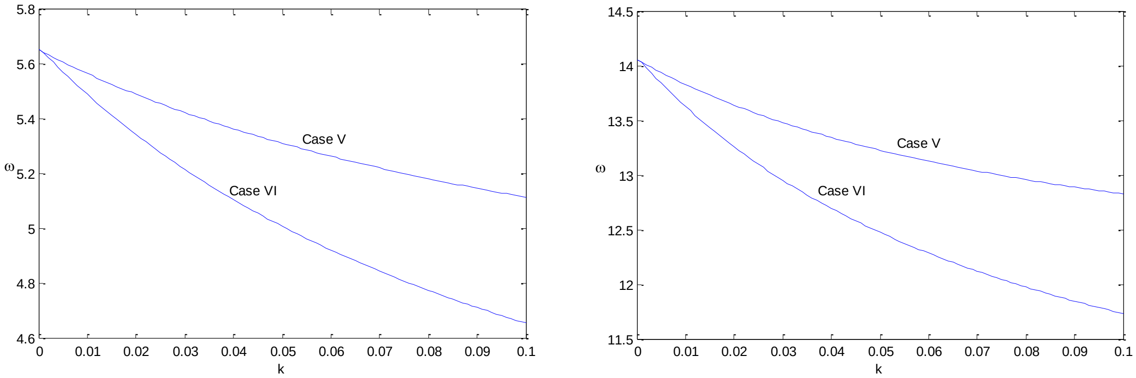
$$Y(x)=c_1e^{i\beta_1x}+c_2e^{i\beta_2x}+c_3e^{i\beta_3x}+c_4e^{i\beta_4x} \tag{21}$$

Frequency equations can be obtained when the boundary conditions are applied. Natural frequencies are plotted in Figs. 1-4 for different cases.



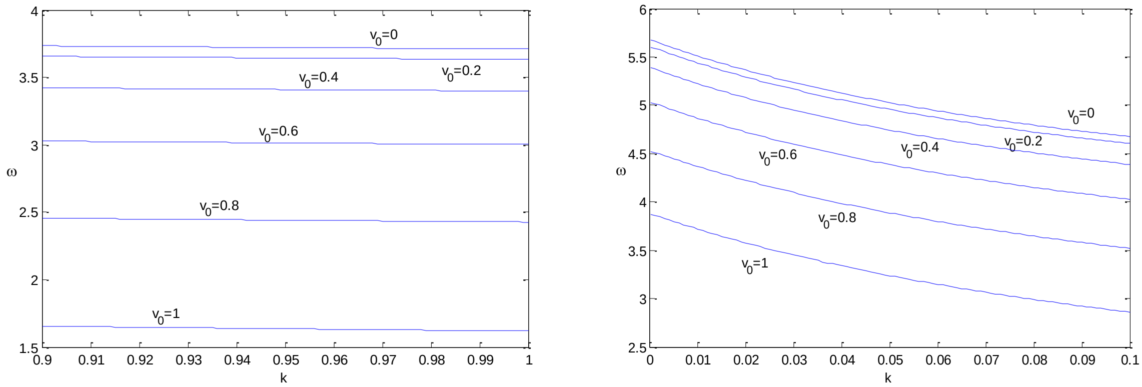
(a) first mode (b) second mode

Fig. 1 Natural frequency value according to changing parameter of *k* (v_o=0.1)



(a) first mode (b) second mode

Fig. 2 Natural frequency value according to changing parameter of k (v_o=0.1)

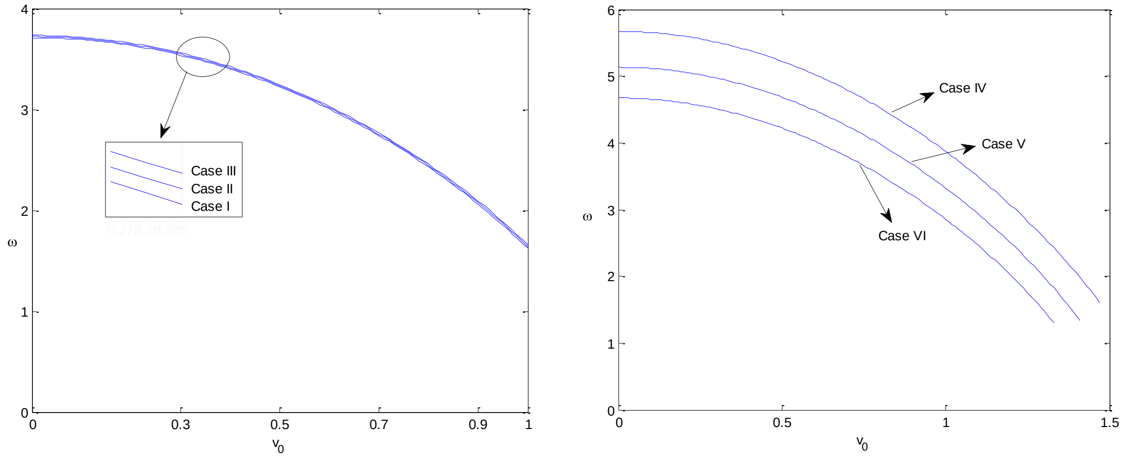


(a) Case III (b) Case VI

Fig. 3 Natural frequencies versus weighting factor for different v_o values (First mode)

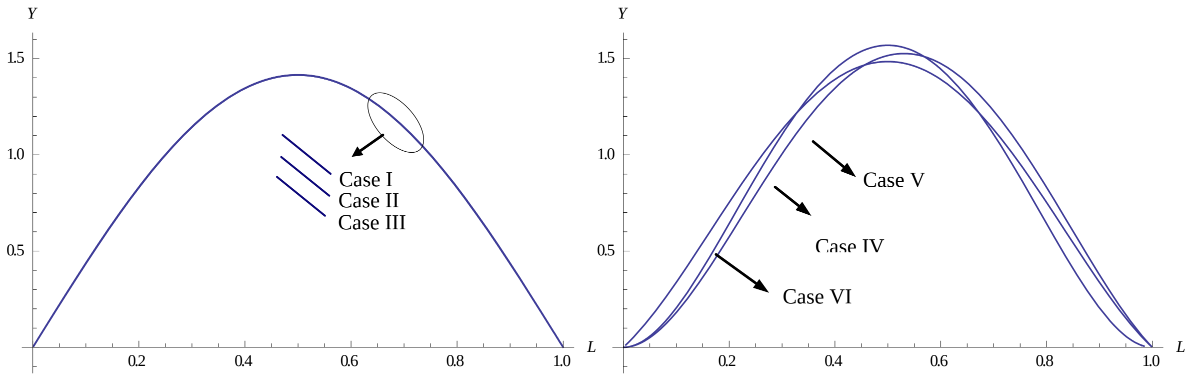
In Fig. 1(a), (b), varying natural frequency values according to weighting factor (k) are given for first and second mode vibrations of Case II and Case III. Velocity of axially moving beam is taken as v_0=0.1. Natural frequencies for ideal simple supported boundary condition are obtained when k=1. These values are ω_1=3.6908, ω_2=10.0716 for first and second mode vibrations. Natural frequencies of Case I are compared with Case II and Case III. It is seen that frequency values of Case III increased 0.63% for first mode and increased 0.34% for second mode while the weighting factor (k) varying from 1 to 0.9. Similarly, frequency values of Case II increased 0.32% for first mode and 0.17% for second mode while the weighting factor (k) varying from 1 to 0.9. Namely, natural frequencies increase when the simple supported boundary condition deviate from its ideal form.

In Fig. 2(a), (b), varying natural frequency values according to weighting factor (k) are given for first and second mode vibrations of Case V and Case VI. Velocity of axially moving beam is taken as v_0=0.1. Natural frequencies for ideal clamped boundary condition are obtained when k=0.1. These values are ω_1=5.6528, ω_2=14.0538 for first and second mode vibrations. Natural frequencies of Case IV are compared with Case V and Case VI. It is seen that frequency values of



(a) (b)

Fig. 4 Natural frequencies depending on v_o for first mode



(a) (b)

Fig. 5 Mode shape of first mode vibrations with v_0=0.1

Case VI decreased 21.39% for first mode and 19.80 % for second mode while the weighting factor (k) varying from 0 to 0.1. Similarly, frequency values of Case V decreased 10.49% for first mode and 9.58% for second mode while the weighting factor (k) varying from 0 to 0.1. Namely, natural frequencies decrease when the clamped boundary condition deviates from its ideal form. The effects of non-ideal boundary conditions on natural frequencies of clamped beam are more significant than simple supported beam.

According to changing v_o, natural frequency values are given depending on the weighting factor (k) in Fig. 3(a), (b). First mode vibrations are shown for simple and clamped with both ends non-ideal conditions using the v_0=0, 0.2, 0.4, 0.6, 0.8 and 1 values in Fig. 3(a), (b). Natural frequency values are decreased with increasing value v_0 at Fig. 3(a) and Fig. 3(b). First mode natural frequencies decreased while axial velocity v_0 is increasing. The change of natural frequency of clamped beam is more explicit than simple supported beam.

Natural frequency changes depending on v_o are given in Fig. 4(a), (b). Natural frequencies of simple supported and clamped beams decreased while the axial velocity v_o increasing.

In Fig. 5(a), (b), mode shapes of axially moving beams are given. The effects of non-ideal boundary conditions on mode shape for clamped beam are seen explicitly in Fig. 5(b). In Fig. 5(a), the effects of non-ideal boundary condition on natural frequencies for simple supported beam stayed limited.

4. Conclusions

In this study, non-ideal boundary conditions were applied to simple and clamped support with one end and both ends non-ideal. Effects of non-ideal boundary conditions and axial velocity on natural frequencies are investigated. The weighting factor (*k*) is expressed as a combination of ideal and non-ideal boundary and determines the ratio of non-ideal boundary conditions. Six different cases are created to see the effects of non-ideal boundary conditions. First tree cases taken as simple supported and the others are taken as clamped boundary conditions. Case I and Case IV are ideal simple and clamped supported beams at both ends. Case II and V are non-ideal simple supported and clamped beams at one end and ideal supported at other end. Case III and VI are non-ideal simple supported and clamped beams at both ends of the beam. Natural frequencies of Case I, Case II and Case III are compared. The weighting factor k is chosen as varying from 1 to 0.9 for simple supported cases. In Case III, frequency values increased 0.63% for first mode and increased 0.34% for second mode. Similarly, in Case II, frequency values increased 0.32% for first mode and 0.17% for second mode while the weighting factor (*k*) varying from 1 to 0.9. Natural frequencies of Case IV, Case V and Case VI are compared. The weighting factor (*k*) varying from 0 to 0.1 for clamped supported cases. In Case VI, frequency values decreased 21.39% for first mode and 19.80 % for second mode. Similarly, in Case V, frequency values decreased 10.49% for first mode and 9.58% for second mode. Namely, natural frequencies increase in simple supported boundary condition and decrease in clamped boundary condition when the boundary conditions deviate from its ideal form. The effect of non-ideal boundary conditions on natural frequencies of clamped beam is more significant than simple supported beam. Natural frequencies of simple supported and clamped beams decreased while the axial velocity v_0 increasing. The change of natural frequency of clamped beam is more explicit than simple supported beam. The changes of mode shapes of beams under non-ideal boundary conditions are shown. The variations on mode shapes stayed limited simple supported beam.

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