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Edge wave propagation in an Electro-Magneto-Thermoelastic homogeneous plate subjected to stress

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Abstract. This paper introduces the combined effect of electric field, magnetic field and thermal field on edge wave propagating in a homogeneous isotropic prestressed plate of finite thickness and infinite length. The dispersion relation of edge wave has been obtained by using classical dynamical theory of thermoelasticity. The phase velocity has been computed and shown graphically for various initial stress parameter, electro-magneto parameter, electric parameter and thermoelastic coupling parameter.

Keywords: edge waves; electric field; magnetic field; temperature; initial stress; homogeneous

1. Introduction

Elastic waves are a complex form of vibratory movement transmitted through a medium because when they involve boundaries. This happens due to practical mode of conversion of waves after reflection from the boundary and the multiple reflections of so created waves between the neighbouring boundaries. Due to this wave velocity will be different towards the core than at the edge. The waves propagates in the plate of finite thickness are known as edge waves. The edge waves become surface waves if plate is of infinite thickness. The interaction of elastic and electromagnetic fields has numerous applications in various field of science such as detection of mechanical explosions in the interior of the earth. The mutual interactions between an externally applied electric field, magnetic field, thermal field and the elastic deformation in the solid body, give rise to the coupled field of electro-magneto-thermoelasticity. The interaction of elastic and electromagnetic fields has numerous applications in various field of science such as detection of mechanical explosions in the interior of the earth. The electro-magneto-elastic materials are used as magnetic field probes, electric packing, acoustic, hydrophones, medical, ultrasonic image processing, sensors and actuators with the responsibility of magnetic-electro-mechanical energy conversion. Inspite of the fact that the Maxwell equations governing the electro-magnetic field have been known for quite a long time, the interest in the coupled fields of electro-magneto-thermoelasticity is of recent origin. This is due to the fact only recently has been recognized the possibility of applying these coupled theories in such practical situations as optics, acoustics,

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geophysics, plasma physics and earthquake science.

Niraula and Wang (2006) studied an effect of temperature on magneto-electro-elastic material with a penny-shaped crack. Dey and De (2009) investigated edge wave in non-homogeneous plate of finite thickness. Zhu and Shi (2008) discussed magneto-electro-elastic wave propagation in non-homogeneous hollow cylinders. Jiangong *et al*. (2008) investigated non-homogeneous magneto-electro-elastic wave propagation in plates. Othman (2010) studied electro-magneto-thermo shock plane waves for a finite conducting half-space with two relaxation times. Dai and Coman (2010) discussed edge-wave buckling phenomenon. Kumar and Partap (2011) discussed vibration analysis of wave motion in micropolar thermoviscoelastic plate. Rao (2011) investigated interaction of functionally graded piezoelectric hollow spheres in the presence of electric, magnetic and thermal fields. Ponnusamy and Selvamani (2012), discussed waves in thermo elastic plate embedded in elastic medium. Das and Kanoria (2012) investigated thermo-magneto-elastic interactions in an unbounded in perfectly conducting elastic medium with three-phase-lag effect. Abd-Alla and Mahmoud (2012) studied radial vibrations in non-homogeneous isotropic cylinder subjected to initial stress and magnetic field. Yu *et al.* (2013) developed a theory (FOGEMTE) for linearly and anisotropic thermo-electro-magneto-elastic medium by introducing the dynamic magnetic-electro fields. Selvamani and Ponnusamy (2013) studied wave propagation in a generalized thermo elastic circular plate immersed in fluid. Kocaturk and Akbaş (2013) discussed wave propagation in a microbeam based on the modified couple stress theory. Alashti and Pashaei (2014) studied functionally graded conical shell subjected to temperature and magnetic field. Shankar and Ganesan (2012, 1013) studied pyroelectric and pyromagnetic materials in presence of electric and magnetic fields and electro-magneto-thermo elastic cantilever beam. Zhen-Bang Kuang (2013, 2014) discussed inertial entropy, the Cattaneo-Vernotte's and Mindlin-type plate bending theories. Kakar (2014) investigated magneto-electro-viscoelastic torsional waves in aeolotropic tube under initial compression stress.

In this work, we have investigated the combined effect of electric field, magnetic field, thermal field and initial stress on edge waves propagated in a homogeneous, isotopic plate of finite thickness. Biot's equations are modified in context of classical dynamical theory with uniform magnetic field and electric field. The dispersion equation for the edge waves has been obtained. Further, the dispersion equation is approximated and analyzed numerically for copper and stainless steel plate to study the effect of electro-magneto pressure number, initial stress parameter and thermoelastic coupling parameter, on the phase velocity of edge waves with the help of MATLAB. This study is useful to solve the problems where the factors like elastic field, thermal field, electric field, magnetic field and initial stress coexist such as the seismic wave propagation inside the earth, geophysics, nuclear devices etc.

2. Governing equations

The governing equations of linear, isotropic and homogenous electro-magneto-thermoelastic solid with hydrostatic initial stress are

i. The stress-strain-temperature relation

$$s_{ij} = -P(\delta_{ij} + \omega_{ij}) + \bar{\lambda}\, \mathrm{e}_{PP}\, \delta_{ij} + 2\bar{\mu} e_{ij} - \frac{\alpha}{k_T}(T + \alpha \dot{T})\delta_{ij}, \tag{1}$$

where, s_{ij} are the components of stress tensor, P is initial pressure, δ_{ij} is the Kronecker delta, ω_{ij} are the components of small rotation tensor, $\bar{\lambda}$, $\bar{\mu}$ are the counterparts of Lame parameters, e_{ij} are the components of the strain tensor, α is the volume coefficient of thermal expansion, k_T is the isothermal compressibility, $T=\Theta-T_0$ is small temperature increment, Θ is the absolute temperature of the medium, T_0 is the reference uniform temperature of the body chosen such that $\left|\frac{T}{T_0}\right| << 1$

ii. The displacement-strain relation

$$e_{ij} = \frac{1}{2}(u_{i.j} + u_{j,i}), \tag{2}$$

where, u_i are the components of the displacement vector

iii. The small rotation-displacement relation

$$\omega_{ij} = \frac{1}{2}(u_{i.j} - u_{j,i}), \tag{3}$$

where, u_i are the components of the displacement vector

iv. The modified Fourier's law

$$h_i + a^* \dot{h}_i = K \frac{\partial T}{\partial x_i}, \tag{4}$$

where, K is the thermal conductivity, a, $a^* \geq 0$ are the thermal relaxation times

v. The heat conduction equation

$$KT_{,ii} = \rho c_p(\dot{T} + \tau \ddot{T}) + \gamma T_0(\dot{u}_{i,i} + \tau_0 \delta_{ij} \ddot{u}_{i,i}) \tag{5}$$

where, K is the thermal conductivity, c_p is specific heat per unit mass at constant strain, τ_0 is the first relaxation time, δ_{ij} is the Kronecker delta, ρ is density and T is the incremental change of temperature from the initial state of the solid half space. In the above equations a dot denotes differential with respect to time, and a comma in subscript denotes partial differential w. r. t. the corresponding coordinates. Moreover the use of the relaxation times τ, τ_0 and a parameter δ_{ij} marks the aforementioned fundamental equations possible for the three different theories:

(1) Classical Dynamical theory: $\tau=\tau_0=0$, $\delta_{ij}=0$.

(2) Lord and Shulman's theory: $\tau=0$, $\tau_0>0$, $\delta_{ij}=1$.

(3) Green and Lindsay's theory: $\tau \geq \tau_0 > 0$, $\delta_{ij}=0$.

vi. Maxwell's equations

$$\vec{\nabla} \cdot \vec{E} = 0, \vec{\nabla} \cdot \vec{B} = 0, \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}, \vec{\nabla} \times \vec{B} = \mu_e \varepsilon_e \frac{\partial \vec{E}}{\partial t} \tag{6}$$

where, $\vec{E}$, $\vec{B}$, μ_e and ε_e are electric field, magnetic field, permeability and permittivity of the medium.

vii. The components of electric and magnetic field

$$\vec{E}(0,0,E) = \vec{E}_0 + \vec{e}, \quad \vec{H}(0,0,H) = \vec{H}_0 + \vec{h} \tag{7}$$

where, $\vec{h}$ is the perturbed magnetic field over $\vec{H}_0$ and $\vec{e}$ is the perturbed electric field over $\vec{E}_0$.

viii. Maxwell stress components

$$T_{ij} = \mu_e \left[H_i e_i + H_j e_j - \left(H_k e_k \right) \delta_{ij} \right] + \varepsilon_e [E_i e_i + E_j e_j - \left(E_k e_k \right) \delta_{ij} \right] \text{ (where } i, j, k\text{=1, 2,3)} \quad (8)$$

where, H_i, H_j, H_k are the components of primary magnetic field, E_i, E_j, E_k are the components of primary electric field, e_i, e_j, e_k are the stress components acting along X-axis, Y-axis, Z-axis respectively and δ_{ij} is the Kronecker delta.

Using Eq. (8), we get

$$T_{22} = \mu_e H_0^2 \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) + \varepsilon_e E_0^2 \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \text{ and } T_{12} = 0 \quad (9)$$

The dynamical equations of motion for the propagation of wave have been derived by Biot (1965) and in two dimensions these are given by

$$\frac{\partial s_{11}}{\partial x} + \frac{\partial s_{12}}{\partial y} - P \frac{\partial \omega}{\partial y} + B_x = \rho \frac{\partial^2 u}{\partial t^2} \quad (10)$$

$$\frac{\partial s_{12}}{\partial x} + \frac{\partial s_{22}}{\partial y} - P \frac{\partial \omega}{\partial x} + B_y = \rho \frac{\partial^2 v}{\partial t^2} \quad (11)$$

where, s_{11}, s_{22} and s_{12} are incremental thermal stress components. The first two are principal stress components along x- and y-axes, respectively and last one is shear stress component in the x-y plane, ρ is the density of the medium and u, v are the displacement components along x and y directions respectively, B is body force and its components along x and y axis are B_x and B_y respectively. ω is the rotational component i.e., $\omega = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right)$ and $P = s_{22} - s_{11}$.

We consider a homogeneous isotropic prestressed plate of finite thickness and infinite length, under constant primary magnetic field H_0 and electric field E_0 parallel to z-axis. Therefore the body forces along x and y axis are given by

$$B_x = \mu_e H_0^2 \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial x \partial y} \right) + \varepsilon_e E_0^2 \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial x \partial y} \right) \quad (12)$$

$$B_y = \mu_e H_0^2 \left(\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 v}{\partial x^2} \right) + \varepsilon_e E_0^2 \left(\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 v}{\partial x^2} \right) \quad (13)$$

where, μ_e and ε_e are permeability and permittivity of the medium.

Following Biot (1965), the stress-strain relations with incremental isotropy are

$$s_{11} = \lambda(e_{xx} + e_{yy}) + 2\mu e_{xx} - \gamma \left(T + \tau \frac{\partial T}{\partial x} \right) \quad (14)$$

$$s_{22} = \lambda(e_{xx} + e_{yy}) + 2\mu e_{yy} - \gamma \left(T + \tau \frac{\partial T}{\partial x} \right) \quad (15)$$

$$s_{12} = 2\mu e_{xy} \quad (16)$$

where

$$e_{xx} = \frac{\partial u}{\partial x}, \qquad e_{yy} = \frac{\partial v}{\partial x}, \qquad e_{xy} = \frac{1}{2}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right) \tag{17}$$

where, e_{xx} and e_{yy} are the principle strain components and e_{xy} is the shear strain component, γ=(3λ+2μ)α_t, α_t is the coefficient of linear expansion of the material, λ μ are Lame's constants, T is the incremental change of temperature from the initial state and τ is second relaxation time.

3. Formulation of the problem

We consider a homogeneous isotropic prestressed elastic plate of finite thickness 2h composed of incompressible elastic medium. Let the origin of the co-ordinate system be located in the middle of the layer, the x-axis is taken in the direction of wave propagation and y axis be taken positive vertically upwards as shown in the Fig. 1.

4. Solution of the problem

From Eq. (12), Eq. (13), Eq. (14), Eq. (15), Eq. (16) and Eq. (17), we get

$$(\lambda + 2\mu)\frac{\partial^2 u}{\partial x^2} + (\lambda + \mu)\frac{\partial^2 v}{\partial x \partial y} + \mu\frac{\partial^2 u}{\partial^2 y} + (\mu_e H_0^2 + \varepsilon_e E_0^2)\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial x \partial y}\right) = \rho\frac{\partial^2 u}{\partial t^2} + \gamma\left(\frac{\partial T}{\partial x} + \tau\frac{\partial^2 T}{\partial t \partial x}\right) \tag{18}$$

$$(\lambda + 2\mu)\frac{\partial^2 v}{\partial y^2} + (\lambda + \mu)\frac{\partial^2 u}{\partial x \partial y} + \mu\frac{\partial^2 v}{\partial^2 x} + (\mu_e H_0^2 + \varepsilon_e E_0^2)\left(\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 v}{\partial y^2}\right) = \rho\frac{\partial^2 v}{\partial t^2} + \gamma\left(\frac{\partial T}{\partial y} + \tau\frac{\partial^2 T}{\partial t \partial y}\right) \tag{19}$$

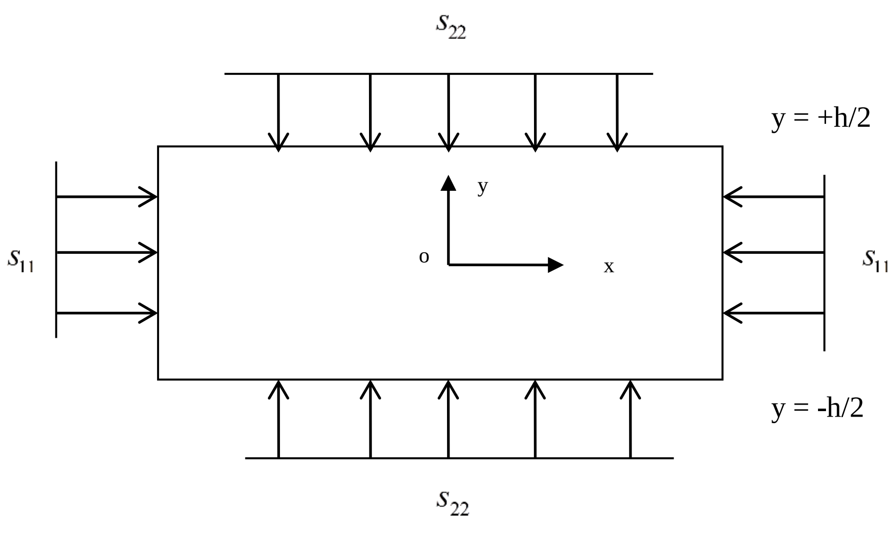

Fig. 1 Geometry of the problem

From Eq. (18) and (19) by using classical dynamical theory (1956): $\tau=\tau_0=0$, $\delta_{ij}=0$, we get

$$(\lambda+2\mu)\frac{\partial^2 u}{\partial x^2}+(\lambda+\mu)\frac{\partial^2 v}{\partial x\partial y}+\mu\frac{\partial^2 u}{\partial^2 y}+(\mu_e H_0^2+\varepsilon_e E_0^2)\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 v}{\partial x\partial y}\right)=\rho\frac{\partial^2 u}{\partial t^2}+\frac{\partial}{\partial x}(\gamma T) \tag{20}$$

$$(\lambda+2\mu)\frac{\partial^2 v}{\partial y^2}+(\lambda+\mu)\frac{\partial^2 u}{\partial x\partial y}+\mu\frac{\partial^2 v}{\partial^2 x}+(\mu_e H_0^2+\varepsilon_e E_0^2)\left(\frac{\partial^2 u}{\partial x\partial y}+\frac{\partial^2 v}{\partial y^2}\right)=\rho\frac{\partial^2 v}{\partial t^2}+\frac{\partial}{\partial y}(\gamma T) \tag{21}$$

Eq. (20) and Eq. (21) can be solved by choosing potential functions ϕ and ψ as

$$u=\frac{\partial\phi}{\partial x}-\frac{\partial\psi}{\partial y}\ \text{ and }\ v=\frac{\partial\phi}{\partial x}+\frac{\partial\psi}{\partial y} \tag{22}$$

From Eqs. (20), (21) and (22), we get

$$\nabla^2\phi=\frac{\rho}{(\lambda+2\mu+\mu_e H_0^2+\varepsilon_e E_0^2)}\frac{\partial^2\phi}{\partial t^2}+\frac{\gamma T}{(\lambda+2\mu+\mu_e H_0^2+\varepsilon_e E_0^2)} \tag{23}$$

$$\nabla^2\psi=\frac{\rho}{\mu}\frac{\partial^2\psi}{\partial t^2} \tag{24}$$

where, $\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$

By using classical dynamical theory: $\tau=\tau_0=0$, $\delta_{ij}=0$, Eq. (5) reduces to

$$K\nabla^2 T=\rho c_p\frac{\partial T}{\partial t}\ +\ \gamma T_0\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right) \tag{25}$$

Introduce Eq. (22) in Eq. (25), we get

$$\nabla^2 T-\frac{c_p\rho}{K}\frac{\partial T}{\partial t}-\frac{\lambda T_0}{c_p}\nabla^2\frac{\partial\phi}{\partial t}=0 \tag{26}$$

From Eq. (23) and Eq. (25), eliminating T, we get

$$\left(\nabla^2-\frac{1}{\beta_1^2}\frac{\partial^2}{\partial t^2}\right)\left(\nabla^2-\frac{c_p\rho}{K}\frac{\partial}{\partial t}\right)\phi-\chi\xi\nabla^2\left(\frac{\partial\phi}{\partial t}\right)=0 \tag{27}$$

where, $\beta_1^2=\frac{(\lambda+2\mu+\mu_e H_0^2+\varepsilon_e E_0^2)}{\rho}$, $\chi=\frac{\gamma}{(\lambda+2\mu+\mu_e H_0^2+\varepsilon_e E_0^2)}$ and $\xi=\frac{\gamma T_0}{K}$

From Eq. (24) and Eq. (25), eliminating T, we get

$$\left(\nabla^2-\frac{1}{\beta_2^2}\frac{\partial^2}{\partial t^2}\right)\psi=0 \tag{28}$$

where, $\beta_2^2=\frac{\mu}{\rho}$

Eq. (27) and Eq. (28) can further be solved by plane harmonic waves travelling along *x*-axis as

$$\phi(\mathrm{x},\mathrm{y},\mathrm{t})=\frac{1}{k^2}f(k\,\mathrm{y})\,\mathrm{e}^{i(\mathrm{ax}-\omega\mathrm{t})} \tag{29}$$

$$\psi(\mathrm{x},\mathrm{y},\mathrm{t})=\frac{1}{k^2}g(k\,\mathrm{y})\,\mathrm{e}^{i(\mathrm{ax}-\omega\mathrm{t})} \tag{30}$$

where, k is wave number and ω is frequency of oscillation of the harmonic wave.

From Eq. (27) and Eq. (29), we get

$$\left(\frac{\partial^2}{\partial(ky)^2}-\lambda_1^2\right)\left(\frac{\partial^2}{\partial(ky)^2}-\lambda_2^2\right)k^4 f(ky)=0 \tag{31}$$

$$\left(\frac{\partial^2}{\partial(ky)^2}-\nu_1^2\right)k^2 g(ky)=0 \tag{32}$$

where, $k^2\lambda_1^2=k^2-\alpha^2, k^2\lambda_2^2=k^2-\beta^2$ and $k^2\nu_1^2=k^2-\delta^2$.

Here $\delta^2=\dfrac{\omega^2}{\beta_2^2}$ and α^2, β^2 are the roots of following biquadratic equation

$$\Lambda^4-\Lambda^2[\sigma^2+\mathrm{q}(1+\varepsilon)]+\sigma^2\,\mathrm{q}=0 \tag{33}$$

where, $\Lambda^2=-\nabla^2$ and the roots α^2, β^2 are

$$\alpha^2=\mathrm{q}\left[1-\frac{q\varepsilon}{\sigma^2-q}\right] \text{ and } \beta^2=\sigma^2\left[1+\frac{q\varepsilon}{\sigma^2-q}\right] \tag{34}$$

Here, $\sigma^2=\dfrac{\omega}{\beta_1^2}, q=\dfrac{i\omega c_p\rho}{K}$ and $\varepsilon=\dfrac{\gamma^2 T_0}{K\rho(\lambda+2\mu+\mu_e H_0^2+\varepsilon_e E_0^2)}$ are electro-magneto-thermoelastic coupling parameters.

The requirement that the stresses and hence the functions ϕ and ψ vanish as $(x^2+y^2)\to\infty$ leads to the following solutions of Eq. (31) and Eq. (32)

$$f(ky)=A\cosh\lambda_1(ky)+B\cosh\lambda_2(ky) \tag{35}$$

$$g(ky)=C\sinh\nu_1(ky) \tag{36}$$

Introducing Eq. (35) and Eq. (36) in Eq. (29) and Eq. (30), we get

$$\phi(x,y,t)=\frac{1}{k^2}[A\cosh\lambda_1(ky)+B\cosh\lambda_2(ky)]\,\mathrm{e}^{i(kx-\omega t)} \tag{37}$$

$$\psi(x,y,t)=\frac{1}{k^2}[C\sinh\nu_1(ky)]\,\mathrm{e}^{i(kx-\omega t)} \tag{38}$$

Eq. (23) gives

$$T=\frac{(\lambda+2\mu+\mu_e H_0^2+\varepsilon_e E_0^2)}{\gamma}\left[\nabla^2\phi-\frac{1}{\beta_1^2}\frac{\partial^2\phi}{\partial t}\right] \tag{39}$$

Eq. (37) and Eq. (39), we get

$$T = \frac{(\lambda + 2\mu + \mu_e H_0^2 + \varepsilon_e E_0^2)}{\gamma} \frac{1}{k^2} [(\sigma^2 - \alpha^2)\cosh \lambda_1(ky) + (\sigma^2 - \beta^2) B \cosh \lambda_2(ky)] \mathrm{e}^{i(kx-\omega t)} \quad (40)$$

5. Boundary conditions and dispersion equation

The boundary conditions on plane plate $y=\pm h/2$ are

$$\nabla f_x = s_{12} - P\frac{\partial v}{\partial x} + T_{12} = 0 \quad (41)$$

$$\nabla f_y = s_{22} - P\frac{\partial u}{\partial x} + T_{22} = 0 \quad (42)$$

$$\frac{\partial T}{\partial y} + h'T = 0 \quad (43)$$

where ∇f_x and ∇f_y are incremental boundary forces per unit initial area and is h' is the ratio of heat transfer coefficient and thermal conductivity.

From Eqs. (14), (15), (16), (9), (37) and (38), the first boundary condition (41) becomes

$$2i(1-\zeta)\lambda_1 \sin h(\lambda_1\Xi)A + 2i(1-\zeta)\lambda_2 \sin h(\lambda_2\Xi)B + \left(\frac{\delta^2}{k^2} - 2(1-\zeta)\right)\sin h(\nu_1\Xi)C = 0 \quad (44)$$

From Eqs. (14), (15), (16), (9), (37), (38) and (40), the second boundary condition (42) becomes

$$\left(2(1-\zeta) - \frac{\omega^2}{k^2}\frac{\rho}{\mu}\right)\cos h(\lambda_1\Xi)A + \left(2(1-\zeta) - \frac{\omega^2}{k^2}\frac{\rho}{\mu}\right)\cos h(\lambda_2\Xi)B + 2i\nu_1(1-\zeta)\cos h(\nu_1\Xi)C = 0 \quad (45)$$

From Eq. (40) and $h' = 0$ (thermal insulation), the third boundary condition (43) becomes

$$\lambda_1\eta_1 \sin h(\lambda_1\Xi)A + \lambda_2\eta_2 \sin h(\lambda_2\Xi)B = 0 \quad (46)$$

where, $\zeta = \dfrac{P}{2\mu}$, is dimensionless initial parameter and $\Xi = \dfrac{kh}{2}$

Now eliminating *A*, *B*, *C* from Eq. (44), Eq. (45) and Eq. (46), we get

$$\begin{vmatrix} 2i(1-\zeta)\lambda_1 & 2i(1-\zeta)\lambda_2 & \left(\frac{\delta^2}{k^2} - 2(1-\zeta)\right) \\ \left(2(1-\zeta) - \frac{\omega^2}{k^2}\frac{\rho}{\mu}\right)\cot h(\lambda_1\Xi) & \left(2(1-\zeta) - \frac{\omega^2}{k^2}\frac{\rho}{\mu}\right)\cot h(\lambda_2\Xi) & 2i\nu_1(1-\zeta)\cot h \\ \lambda_1\eta_1 & \lambda_2\eta_2 & 0 \end{vmatrix} = 0 \quad (47)$$

Solving Eq. (47), we get

$$4(1-\zeta)^2\lambda_1\lambda_2\nu_1(\beta-\alpha)\cot h(\lambda_1\Xi) + \left(2(1-\zeta)-\frac{\omega^2}{k^2}\frac{\rho}{\mu}\right)\left(\frac{\delta^2}{k^2}-2(1-\zeta)\right)[\lambda_2\beta\cot h(\lambda_1\Xi)-\lambda_1\alpha\cot h(\lambda_2\Xi)]=0 \tag{48}$$

$$\text{Let } 1-\frac{\alpha^2}{k^2}=\alpha_1^2 \text{ and } 1-\frac{\beta^2}{k^2}=\alpha_2^2 \tag{49}$$

$$\text{If } h\to 0, \Xi\to 0 \text{ then } \cot h(\lambda_1\Xi)\to\frac{1}{\lambda_1\Xi}, \cot h(\lambda_2\Xi)\to\frac{1}{\lambda_2\Xi} \text{ and } \cot h(\nu_1\Xi)\to\frac{1}{\nu_1\Xi} \tag{50}$$

With the help of Eq. (49) and Eq. (50), Eq. (48) reduces to

$$4(1-\zeta)^2\left[\frac{c^2}{\beta_1^2}+\alpha_1^2+\alpha_2^2-\alpha_1^2\alpha_2^2-1\right]=\left[2(1-\zeta)\left(\frac{\delta^2}{k^2}+\frac{c^2}{\beta_2^2}\right)-\frac{c^2}{\beta_2^2}\frac{\delta^2}{k^2}\right]\left(\frac{c^2}{\beta_1^2}+\alpha_1^2+\alpha_2^2-1\right)=0 \tag{51}$$

where, $c^2=\dfrac{\omega^2}{k^2}$, $\beta_2^2=\dfrac{\mu}{\rho}$, $\zeta=\dfrac{P}{2\mu}$, and $\delta^2=\dfrac{\omega^2}{\beta_2^2}$.

From Eq. (33), we get

$$\alpha^2+\beta^2=\sigma^2+\mathrm{q}(1+\varepsilon) \text{ and } \alpha^2\beta^2=\sigma^2\,\mathrm{q} \tag{52}$$

From Eq. (49) and Eq. (52), we get

$$\alpha_1^2+\alpha_2^2=2-\frac{c^2}{\beta_1^2}-\frac{ic^2}{\varpi\beta_1^2}(1+\varepsilon) \text{ and } \alpha_1^2\alpha_2^2=1-\frac{c^2}{\beta_1^2}-\frac{ic^2}{\varpi\beta_1^2}\left(1+\varepsilon-\frac{c^2}{\beta_1^2}\right) \tag{53}$$

where, $\varpi=\dfrac{K\omega}{c_p\rho\beta_1^2}$, is reduced frequency.

From Eq. (51) and Eq. (53), we get

$$4(1-\zeta)^2\frac{c^2}{\beta_1^2}-4(1-\zeta)^2\frac{ic^4}{\varpi\beta_1^4}=\left(4(1-\zeta)-\frac{c^2}{\beta_2^2}\right)\frac{c^2}{\beta_2^2}-\left(4(1-\zeta)-\frac{c^2}{\beta_2^2}\right)\frac{ic^4}{\varpi\beta_1^2\beta_2^2}(1+\varepsilon) \tag{54}$$

The Eq. (54) is the expression for frequency equations of edge waves.

Equating imaginary parts of Eq. (54), we get

$$4(1-\zeta)^2\frac{\beta_2^2}{\beta_1^2(1+\varepsilon)}=4(1-\zeta)-\frac{c^2}{\beta_2^2} \tag{55}$$

Let $\dfrac{\beta_2^2}{\beta_1^2(1+\varepsilon)}=\Omega$ and $v_p=\dfrac{c^2}{\beta_2^2}$, then Eq. (55) becomes

$$4(1-\zeta)^2\Omega=4(1-\zeta)-v_p \tag{56}$$

Table 1 Material properties (Copper)

λ	μ	α_t	K	c_p	ρ
9.5×10^{10}N/m^2	4.5×10^{10}N/m^2	16.6×10^{-6}K^{-1}	401 W/(m.K)	0.39 KJ/Kg K	8746 Kg/m^3

Table 2 Material properties (Stainless Steel)

λ	μ	α_t	K	c_p	ρ
11.2×10^{10}N/m^2	8.1×10^{10}N/m^2	17.3×10^{-6}K^{-1}	16 W/(m.K)	0.49 KJ/Kg K	7800 Kg/m^3

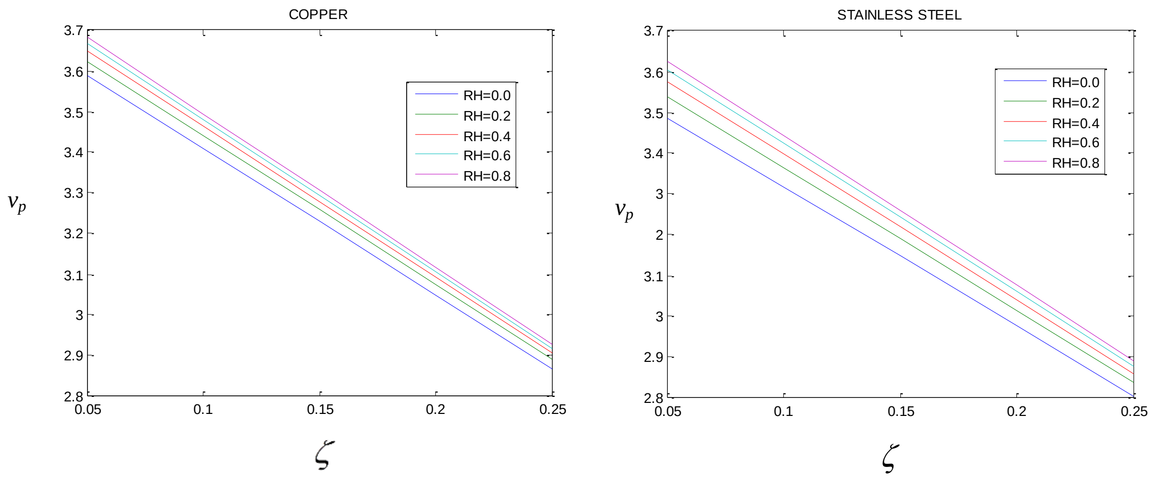

Fig. 2 Variation of $v_p = \dfrac{c^2}{\beta_2^2}$ (phase velocity) of edge waves with $\zeta = \dfrac{P}{2\mu}$ (initial stress parameter) for different values of $R_H = \dfrac{c_a^2}{c_0^2}$ (electro-magneto pressure number) at constant ε (thermoelastic coupling parameter) for copper and stainless steel

$$\text{Also, } \Omega = \frac{\beta_2^2}{\beta_1^2(1+\varepsilon)} = \frac{\beta_2^2}{\beta_0^2(1+R_H)(1+\varepsilon)} \tag{57}$$

where, $R_H = \dfrac{c_a^2}{c_0^2}$ is electro-magneto pressure number, $c_a^2 = \dfrac{\mu_e H_0^2 + \varepsilon_e E_0^2}{\rho}$ is electro-magneto wave velocity, $v_p = \dfrac{c^2}{\beta_2^2}$ is edge phase velocity, $c_0^2 = \dfrac{\lambda + 2\mu}{\rho}$ is isothermal dilatational, ε is thermoelastic coupling parameter and $\beta_2^2 = \dfrac{\mu}{\rho}$ is rotational wave velocity.

6. Numerical analysis

We consider copper and stainless steel material and as a numerical example because it has wide applications in industry. The parameters for the copper material are taken in Table 1 and parameters for the stainless steel material are taken in Table 2 respectively. Numerical results have been obtained graphically for both copper and stainless steel materials. The effect of electric field, magnetic field, temperature, thermoelastic coupling and initial stress on phase velocity of edge wave is shown.

Fig. 2 shows the variation of $v_p = \frac{c^2}{\beta_2^2}$ (phase velocity) of edge waves with $\zeta = \frac{P}{2\mu}$ (initial stress parameter) for different values of $R_H = \frac{c_a^2}{c_0^2}$ (electro-magneto pressure number) at constant ε (thermoelastic coupling parameter) for copper and stainless steel. The value of ε=0.005 is taken to draw Fig. 2. It is observed that in both the materials, the phase velocity of edge waves decreases sharply with the increase in the initial stress parameter. Also, the curves show that the phase velocity increases as electro-magneto stress parameter increases for both copper and stainless steel materials for all R_H's, but the behavior of the curve remains the same.

Fig. 3 represents the variation of $v_p = \frac{c^2}{\beta_2^2}$ (phase velocity) of edge waves with $R_H = \frac{c_a^2}{c_0^2}$ (electro-magneto pressure number) for different values of $G = \zeta = \frac{P}{2\mu}$ (initial stress parameter) at constant ε (thermoelastic coupling parameter) for stainless steel and copper. The value of ε=0.005 is taken to draw Fig. 3. The phase velocity of edge waves increases slowly with the increase in the electro-magneto pressure number. The magnitude of phase velocity decreases as the initial stress

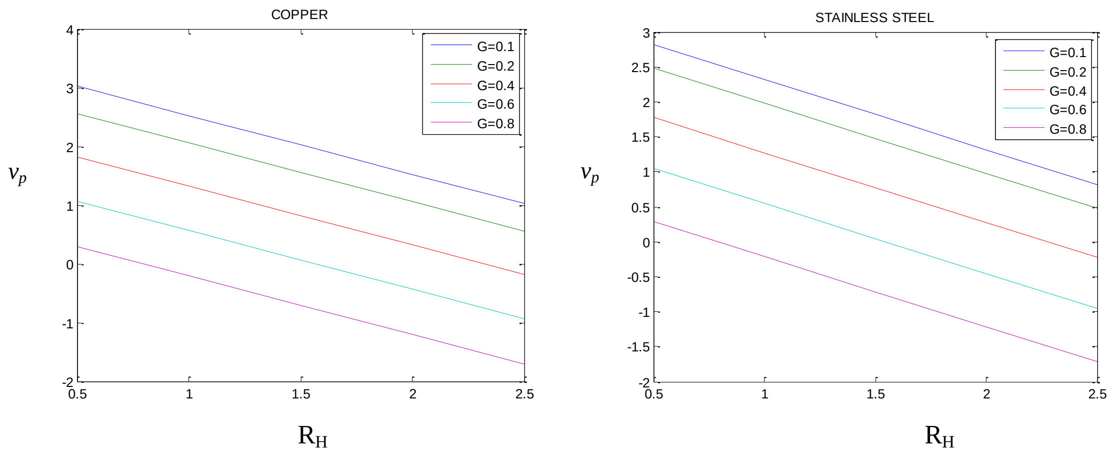

Fig. 3 Variation of $v_p = \frac{c^2}{\beta_2^2}$ (phase velocity) of edge waves with $R_H = \frac{c_a^2}{c_0^2}$ (electro-magneto pressure number) for different values of $G = \zeta = \frac{P}{2\mu}$ (initial stress parameter) at constant ε (thermoelastic coupling parameter) for stainless steel and copper

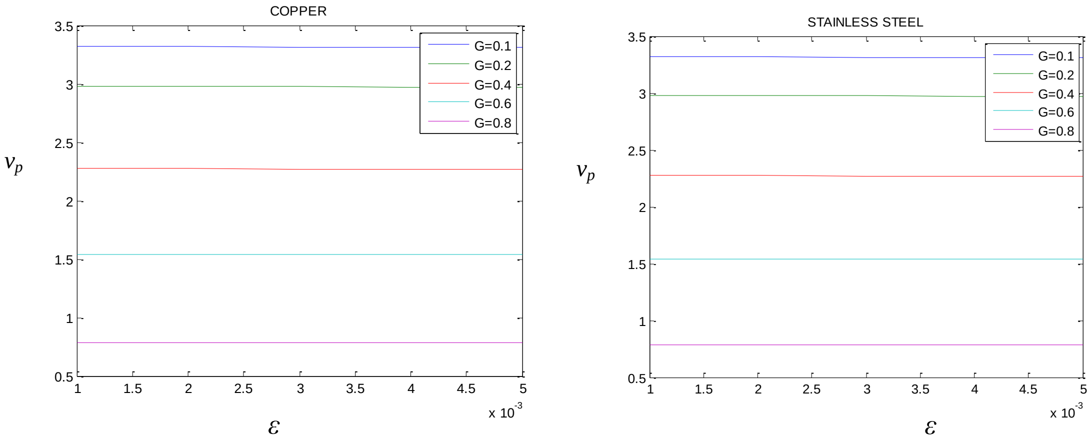

Fig. 4 Variation of $v_p = \dfrac{c^2}{\beta_2^2}$ (phase velocity) of edge waves with ε (thermoelastic coupling parameter) for different values of $G = \zeta = \dfrac{P}{2\mu}$ (initial stress parameter) at constant $R_H = \dfrac{c_a^2}{c_0^2} = 0.7$ (electro-magneto pressure number) for stainless steel and copper

parameter increases, but the behavior of the curve remains the same.

Fig. 4 shows that the $v_p = \dfrac{c^2}{\beta_2^2}$ (phase velocity) of edge waves practically remains constant with the increase in the ε (thermoelastic coupling parameter) for copper and stainless steel, but the behavior of the curve remains the same. Moreover the magnitude of phase velocity is less when initial stress is more. The curves show the variation of $v_p = \dfrac{c^2}{\beta_2^2}$ (phase velocity) of edge waves with ε (thermoelastic coupling parameter) for different values of $G = \zeta = \dfrac{P}{2\mu}$ (initial stress parameter) at constant $R_H = \dfrac{c_a^2}{c_0^2} = 0.7$ (electro-magneto pressure number).

Fig. 5 shows the variation of $v_p = \dfrac{c^2}{\beta_2^2}$ (phase velocity) of edge waves with $\zeta = \dfrac{P}{2\mu}$ (initial stress parameter) for different values of ε (thermoelastic coupling parameter) keeping $R_H = \dfrac{c_a^2}{c_0^2} = 0.7$ (electro-magneto pressure number) constant for both the materials. It is observed that the phase velocity of edge waves decreases sharply with the increase in the initial stress parameter and effect of thermal variation on the phase velocity is negligible, but the behavior of the curve remains the same.

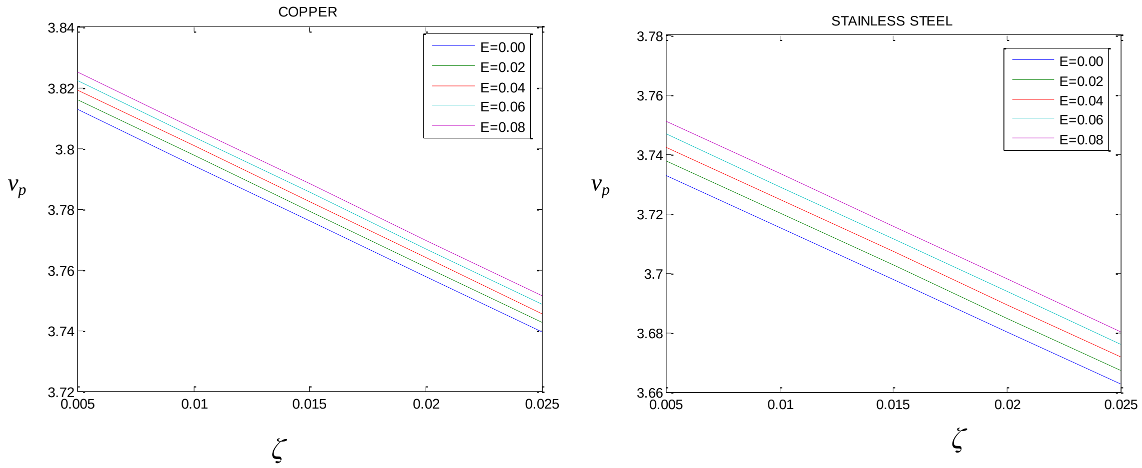

Fig. 5 Variation of $v_p = \dfrac{c^2}{\beta_2^2}$ (phase velocity) of edge waves with $\zeta = \dfrac{P}{2\mu}$ (initial stress parameter) for different values of ε (thermoelastic coupling parameter) keeping $R_H = \dfrac{c_a^2}{c_0^2}$ (electro-magneto pressure number) constant for stainless steel and copper.

7. Conclusions

It can be concluded that the magnetic field, electric field, temperature as well as initial compressive hydrostatic stress have significant influence on the phase velocity of edge waves. This study also shows that the magnitude of phase velocity decreases as the initial stress parameter increases. Also, thermoelastic coupling parameter has a small influence on the phase velocity of edge waves. The edge wave phase velocity is higher for higher electro-magnetic stress parameter.

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