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Analysis of thermally induced vibration of cable-beam structures

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Abstract. Cable-beam structures characterized by variable stiffness nonlinearities are widely found in various structural engineering applications, for example in space deployable structures. Space deployable structures in orbit experience both high temperature caused by sun's radiation and low temperature by Earth's umbral shadow. The space temperature difference is above 300K at the moment of exiting or entering Earth's umbral shadow, which results in structural thermally induced vibration. To understand the thermally induced oscillations, the analytical expression of Boley parameter of cable-beam structures is firstly deduced. Then, the thermally induced vibration of cable-beam structures is analyzed using finite element method to verify the effectiveness of Boley parameter. Finally, by analyzing the obtained numerical results and the corresponding Boley parameters, it can be concluded that the derived expression of Boley parameter is valid to evaluate the occurrence conditions of thermally induced vibration of cable-beam structures and the key parameters influencing structural thermal flutter are the cable stiffness and thickness of beams.

Keywords: thermally induced vibration; cable-beam structure; thermal flutter; Boley parameter; finite element method; vibration analysis

1. Introduction

In the field of engineering applications, many structures are typical cable–beam coupled systems, such as cable-stayed bridges in construction engineering, and space structures in aerospace field (Li and Ma 2011). The cable–beam coupled system makes full use of the tensile ability of high strength cables and the compression and bending performances of beams (Li and Wang 2011). Meanwhile, the system overcomes the possibility of beam instability by the prestressed cables. As the cables cannot be subjected to pressures and have strong geometric nonlinearity, the cable-beam structures belong to variable stiffness nonlinear systems.

Due to the large flexibility and low damping of cable-beam structures, they have complicated and dense low-frequency modes. This characteristic makes cable-beam structures easily suffer from thermally induced vibration owing to sudden temperature changes. The generated thermally induced vibration is difficult to decay. Sudden heat changes may induce time-dependent bending moments in cable-beam structures to cause structural thermal deformation (Thornton and Kim

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1993). The thermally induced vibration of cable-beam structures can be typically resolved into three ways: thermal deformation, stable and unstable thermal vibration (An 2001). The thermal deformation is a quasistatic deformation caused by a slowly changed temperature field. The unstable thermal vibration, also called thermal flutter, results from the interaction between structural deformations and incident heating. It will increase the vibration amplitude. The cable-beam structures experiencing large amplitude of unstable vibrations are unable to complete their intended missions (Johnston and Thornton 1998). Therefore, the analysis of thermally induced vibration of cable-beam structures is becoming vital for their future engineering applications.

As early as 1956, Boley (1956) was the first to analyze the thermally induced vibration of a simply supported beam under step heat flux and came up with the Boley parameter by Boley (1972). Then, Jones (1966) introduced the shear effect, torsion moment of inertia and axial force into the thermally induced vibration of beams. Mason (1968) studied the thermally induced vibration with the finite element method. Thornton and Foster (1992) traced the history of thermally induced spacecraft vibrations and illustrated stable and unstable vibrations. Ding and Xue (2005), Cheng *et al.* (2004) researched the thermal structural dynamics of complicated space structures with the Fourier elemental analysis. It is found that the Fourier elemental analysis can predict the change in focal length due to solar heat flux more accurately than the conventional finite element analysis. Kawamura *et al.* (2008), Kumar *et al.* (2008) solved the temperature field under temperature boundary conditions with sinusoidal form and obtained the thermal response of cylindrical shell. Narasimha *et al.* (2010) proved Boley's previous research work with the finite element method. All the studies on thermally induced vibration are about linear structures like bars, beams and plates, while the researches on nonlinear structures are barely considered about.

The purpose of this paper is to investigate the thermally induced vibration and flutter of nonlinear cable-beam structures. The nonlinear model of this work is a cantilever beam supported by a cable at the free end. The cable force is treated as the boundary condition of the cantilever beam to linearize the nonlinear cable-beam structure. The analytical expression of Boley parameter for cable-beam structures relating to structural and thermal parameters is firstly deduced. The derived expression of Boley parameter provides a way to analysis the thermally induced vibration of high-dimensional nonlinear cable-beam structures, which can avoid huge amounts of computation resulting from traditional numerical methods. Then the numerical analysis of thermally induced vibration of cable-beam structures is conducted using finite element method to validate the expression of Boley parameter. Finally, some conclusions are presented.

2. Nonlinear model of cable-beam structures

The typical cable-beam structure, a cantilever beam supported by a cable at the free end, is shown in Fig. 1. The cable-beam structure subjected heating on the top of the beam and insulated on the bottom. The applied incident heat flux Q is a step function varied with time t: for $t<0$, $Q=0$, and for $t\geq 0$, Q is the constant. The length, width and thickness of the cantilever beam are denoted by l, b and h respectively. L_c is the length of cable. Since the heat flux does not vary along the tube length, the temperature distribution at every cross section is the same. Thus the temperature T varies only with the thickness h and time t.

The variable stiffness of cable-beam structures results from the cables carrying only tension and strong geometric nonlinearity (Li *et al.* 2013). In this paper, the variable stiffness caused by the cables carrying only tension is considered, which can be piecewise nonlinear. Therefore, the whole

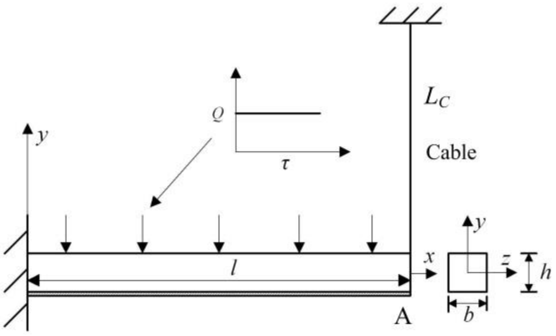
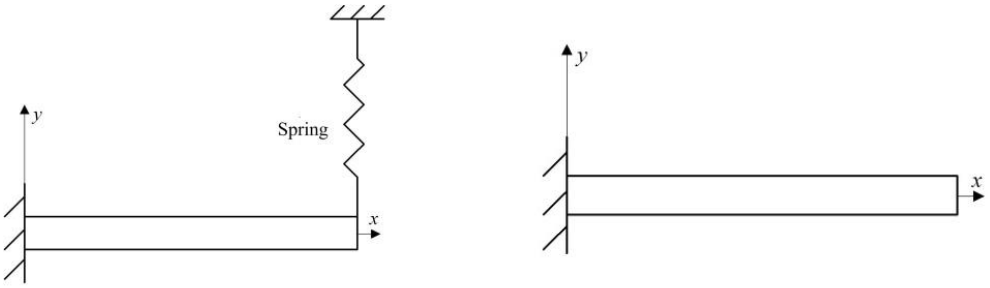


Fig. 1 Description of the cable-beam structure



(a) Cantilever beam with a spring (b) Cantilever beam

Fig. 2 Equivalent structures of the cable-beam structure

cable-beam structure can be equivalent to two types of structures: a cantilever beam supported by a spring at the free end when the cable is tightened, as shown in Fig. 2(a); a cantilever beam when the cable is loosen, as shown in Fig. 2(b). The cable force, *F*, can be expressed in piecewise form as follows.

$$F = \begin{cases} 0 & v(l,t) \geq 0 \\ kv(l,t) - \alpha_c \Delta T k L_c & v(l,t) < 0 \end{cases} \tag{1}$$

where *k* is the cable stiffness, *v* is the deflection curve of the cantilever beam, so $v(l,t)$ is deflection of point A. α_c is the coefficient of thermal expansion of cable, ΔT is the temperature change of cable.

3. Analytical expression of Boley parameter

In 1970s, a key parameter for assessing the potential of a thermally induced vibration is given by Boley (1972). The nondimensional Boley parameter is defined as the square root of the ratio of the characteristic thermal and structural response times of the system

$$B = \sqrt{t_T / t_M} \tag{2}$$

where t_T is the characteristic thermal time and t_M is the characteristic structural response time corresponding to the fundamental mode of structural vibration. Linear systems will undergo a quasistatic response or a stable thermally induced vibration when B>>1.0 or B<<1.0. While they experience a thermally flutter when B≈1.0 (Boley 1956), that is, a thermally induced vibration is more prone to be unstable when its thermal response time approximates its structural response time.

As the thermal excitation is only applied to the beam structure, the characteristic thermal time of the beam can be regarded as the thermal time of the whole cable-beam structure. It can be given as follows (Boely and Bruno 1954).

$$t_T = c\rho h^2 / \lambda \tag{3}$$

where λ is the coefficient of heat conductivity, ρ is the density, and c is the specific heat.

The fundamental frequency of the cable-beam structure can be deduced from Lagrange's equation (Fujino and Warnitchai 1992).

$$f^2 = \frac{1}{4\pi} \frac{\int_0^l EI\left(\frac{d^2\phi}{dx^2}\right)dx + \left(\frac{u_0 k}{L_c} + k\sin^2\theta\right)\phi^2(x_c)}{M} \tag{4}$$

and

$$\phi(x) = \frac{1}{2}\left[ch\beta_1 x - \cos\beta_1 x - \frac{sh\beta_1 l - \sin\beta_1 l}{ch\beta_1 l + \cos\beta_1 l}(sh\beta_1 x - \sin\beta_1 x)\right] \tag{5}$$

$$M = \int_0^l \rho A\phi^2(x)dx + \frac{1}{3}m_c\phi^2(x_c) \tag{6}$$

where, E and I are the Young's modulus and the cross-section moment of inertia of the beam, respectively. u_0 is the initial elongation of the cable, and can be assumed to be zero in this work. θ is the inclination of cable to the beam, thus θ=90°. M is the generalized mass of global vertical vibration of the beam. ϕ is the normalized global mode of the beam satisfying $\phi(l)$=1 and $\beta_1 l$=1.8751.

Substituting Eqs. (3)-(6) into Eq. (2), the analytical expression of Boley parameter of the cable-beam structure can be written as follows.

$$B = \sqrt{\frac{c\rho h^2}{\lambda}} \sqrt[4]{\frac{1}{4\pi}\frac{EIC + k}{\rho AD + \frac{1}{3}m_c}} \tag{7}$$

where

$$C = \int_0^l \left(\frac{d^2\phi}{dx^2}\right)dx = \frac{\left[ch\beta_1 l\sin\beta_1 l + \cos\beta_1 l sh\beta_1 l\right]\beta_1}{\cos\beta_1 l + ch\beta_1 l} \tag{8}$$

$$D = \int_0^l \phi^2(x)dx = \frac{1}{16}\left[\frac{8(sh\beta_1 l - \sin\beta_1 l)}{ch\beta_1 l + \cos\beta_1 l} + 4l + \frac{2(sh\beta_1 l - \sin\beta_1 l)}{ch\beta_1 l + \cos\beta_1 l}\cos 2l \right.$$
$$- \left(\frac{sh\beta_1 l - \sin\beta_1 l}{ch\beta_1 l + \cos\beta_1 l} \right)^2 \sin 2l + \sin 2l$$
$$+ 4\sin l \left(\left(\frac{sh\beta_1 l - \sin\beta_1 l}{ch\beta_1 l + \cos\beta_1 l} \right)^2 chl - chl + ashl \right) \tag{9}$$
$$- 4\cos l \left(3\left(\frac{sh\beta_1 l - \sin\beta_1 l}{ch\beta_1 l + \cos\beta_1 l} \right) chl + \left(\frac{sh\beta_1 l - \sin\beta_1 l}{ch\beta_1 l + \cos\beta_1 l} \right)^2 shl + shl \right)$$
$$\left. + sh2l + \left(\frac{sh\beta_1 l - \sin\beta_1 l}{ch\beta_1 l + \cos\beta_1 l} \right)^2 sh2l \right]$$

For given structural parameters, the Boley parameter can be obtained by solving Eq. (7) numerically. It can be noted from Eq. (7) that the Boley parameter of cable-beam structures is determined by the structural and thermal parameters of beams, as well as the cable stiffness. The mass of the cable is neglected because it is much smaller compared to the beam.

By the finite element method, the numerical analysis of thermally induced vibration of cable-beam structures is developed in the following in order to validate the effectiveness of the derived expression of Boley parameter.

4. Analysis of thermally induced vibration using finite element method

For the analysis of thermally induced vibration of the cable-beam structure, the transient temperature field of the cable-beam structure is calculated firstly to obtain thermal moment caused by temperature variations. The temperature distribution in the beam is assumed to be independent of structural deformations. Then the structural vibration equations considering the thermal moment are solved using finite element method. The applied cable force is treated as an external force, so as the thermal load.

4.1 Thermal analysis

According to Fourier's law of heat conduction, the differential heat balance equation of the cable-beam structure along the *y* axis yields

$$\lambda\left(\frac{\partial^2 T}{\partial x^2} \right) + \dot{q} = \rho c \frac{\partial T}{\partial t} \tag{10}$$

where, $\dot{q}$ is the internal heat generation rate per unit volume, thus $\dot{q} = 0$.

With the initial and boundary conditions, Eq. (8) can be rewritten using the finite element method as follows.

$$\left\{\frac{kA}{h}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}+\begin{bmatrix}h_cA & 0\\ 0 & 0\end{bmatrix}\right\}\begin{Bmatrix}T_1\\ T_2\end{Bmatrix}+\frac{\rho cAh}{6}\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix}\begin{Bmatrix}\dot{T}_1\\ \dot{T}_2\end{Bmatrix}=AQ\begin{Bmatrix}1\\ 0\end{Bmatrix}+\frac{A\dot{q}}{2}\begin{Bmatrix}1\\ 1\end{Bmatrix}+hAT_\infty\begin{Bmatrix}1\\ 0\end{Bmatrix} \quad (11)$$

Eq. (9) can be given in matrix form as

$$K^eT+M^e\dot{T}=Q^e \quad (12)$$

where, K^e is the element conduction or convection matrix, M^e is the element capacitance matrix and Q^e is the element equivalent force vector.

The parabolic equation (Eq. (10)) is solved by means of the Crank-Nicolson method (Manish 2008). As a result, the transient temperature field of the cable-beam structure is obtained. For the cantilever beam subjected to heat load, as shown in Fig. 1, a temperature gradient is formed along the thickness direction of the beam which is divided into several layers. It results in the thermal moment defined by

$$M_T=\int_A E\alpha\Delta Ty dA=b\int_y E\alpha\Delta Ty dy=\sum_i^n M_{Ti}=\sum_i^n E\alpha b\left(\frac{y_{i+1}^2-y_i^2}{2}\right) \quad (13)$$

where, α is the thermal expansion coefficient, ΔT denotes the cross-sectional temperature gradient of the beam, A is the cross sectional area of the beam. y_i and M_{Ti} indicate the coordinate of ith layer along y axis and the corresponding thermal moment, respectively.

4.2 Dynamic response

Considering the influence of temperature, the Euler-Bernoulli bending theory (Boley and Bruno 1954, Timoshenko and Googier 1951) can be written as

$$EI\left(\frac{\partial^2 v}{\partial x^2}\right)=M+M_T \quad (14)$$

where, M is the bending moment due to applied forces, v is the transverse deformation along the y axis. After differentiating Eq. (12) with respect to x twice, the governing differential equation under thermal loads is given by

$$EI\left(\frac{\partial^4 v}{\partial x^4}\right)+\rho A\left(\frac{\partial^2 v}{\partial t^2}\right)-\frac{\partial^2 M_T}{\partial x^2}=0 \quad (15)$$

The structure boundary conditions are $x=0$: $v(0,t)=0$, $v'(0,t)=0$; $x=l$:$v''(l,t)=M_T$, $v'''(l,t)=F_x$, where F_x is a variable boundary condition, that is

$$F_x=\begin{cases}0 & v(l,t)\geq 0\\ kv(l,t)-\alpha_c\Delta TkL_c & v(l,t)<0\end{cases}$$

Given boundary conditions, the equation of thermally induced vibration of each element can be expressed in the following form through the Galerkin method.

$$M^e\ddot{v}+K^ev=F_T^e \quad (16)$$

Table 1 Structural properties of the cable-beam structure

Parameter	Value
Heat flux Q	2×10^6 W/m^2
Width of beam b	0.05 m
Thickness of beam h	0.05 m
Length of beam l	1 m
Coefficient of thermal expansion α	0.05/K
Coefficient of heat conductivity λ	74.5 W/(m·k)
Specific heat c	460 J/(kg·K)
Density ρ	1650 kg/m^3
Young's modulus E	2×10^9 Pa
Cross-section moment of inertia I	5.2083×10^{-7} m^4

Table 2 Permutations of structural parameters

Type	h (m)	k (N/m)	Boley parameter
I	0.05	1000	11.0185
II	0.05	10	9.67
III	0.01	1000	2.6457
IV	0.01	10	0.9901

where element mass matrix $M^e = \dfrac{\rho A}{420}\begin{bmatrix} 156 & -22L \\ -22L & 4L^2 \end{bmatrix} - \begin{bmatrix} k & 0 \\ 0 & 0 \end{bmatrix}$, element stiffness matrix $K^e = \dfrac{EI}{L^3}\begin{bmatrix} 12 & -6L \\ -6L & 4L^2 \end{bmatrix}$, element equivalent force vector $F_T^e = \begin{bmatrix} 0 \\ \alpha_c \Delta T k L_c - M_T \end{bmatrix}$.

By assembling all element equations, the global finite element equations of thermally induced vibration are

$$M\ddot{v} + Kv = F \tag{17}$$

where M is the mass matrix, K is the stiffness matrix, and F is the force vector. The above dynamic equations are calculated by the Newmark method (Reddy 2005).

5. Numerical results

The structural properties of the cable-beam structure for numerical analysis are given in Table 1. The incident heat flux Q lasts for 1s to simulate the alternating thermal loading resulting from a spacecraft exiting or entering Earth's umbral shadow. The cable stiffness is 1000 N/m and the coefficient of thermal expansion of cable is 0.05/K.

Four types of cable-beam structures varied in thicknesses of beams and the cable stiffness, as shown in Table 2, are analyzed to investigate the influence of structural parameters on the stability of thermally induced vibration. Their corresponding Boley parameters are computed using Eq. (7).

The coupling point A is chosen to observe the vibration of whole cable-beam structure. The vibration curve of type I is shown in Fig. 3. The displacement response develops smoothly with

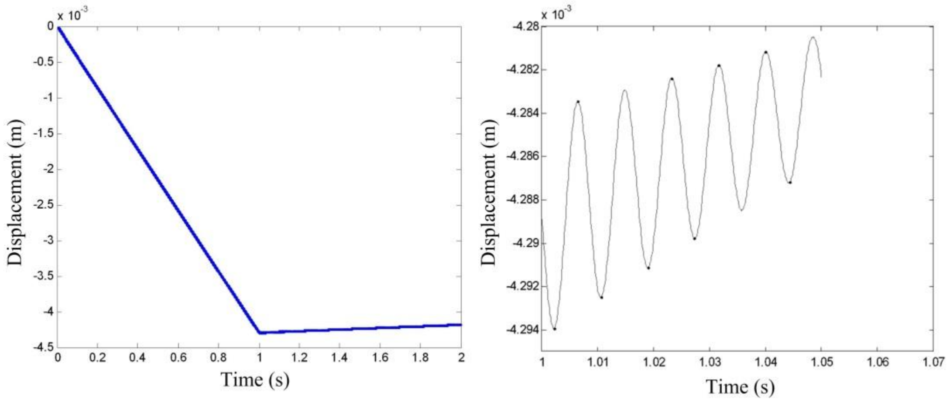


Fig. 3 Vibration curve of point A of type *I* and its local magnification

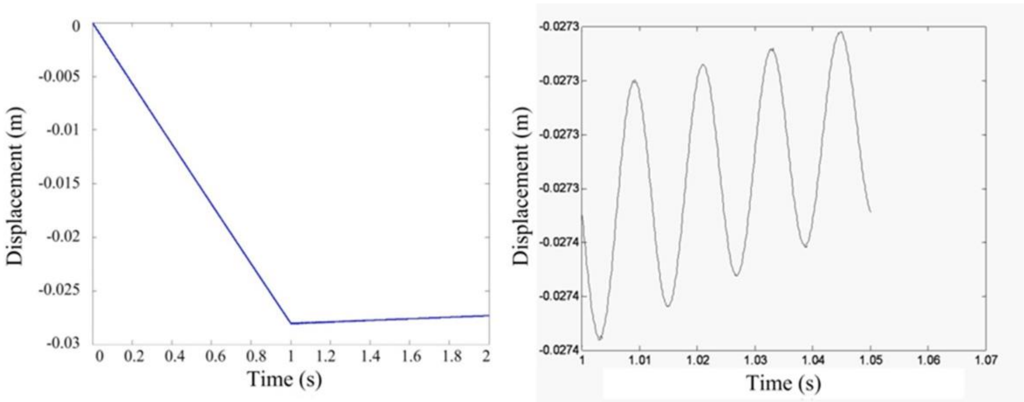


Fig. 4 Vibration curve of point A of type *II* and its local magnification

time and approaches a maximum deflection in about 1s as the heat-up period ends. The vibration amplitude of type II shown in Fig. 4 is larger than that of type I as the cable stiffness of type II is much smaller than that of type I. Both the vibrations of types I and II are practically stable vibrations and their Boley parameters are far away from 1.

The vibration curves of types III and IV are shown in Fig. 5 and Fig. 6, respectively. Their Boley parameters are much closer to 1, and the vibrations of types III and IV are unstable because their vibration amplitudes grow with time. In Fig. 6, the structure vibrates so violently that the heat-up and cool-down periods cannot be figured out.

It can be noted from Fig. 5 and Fig. 6 or Fig. 3 and Fig. 4 that the vibration amplitude of the cable-beam structure increases as the cable stiffness decreases. This is because the cable stiffness related to the first mode frequency of the cable-beam structure become lower to make the system unstable. Furthermore, the increment of the vibration amplitude between Fig. 5 and Fig. 6 is larger than that between Fig. 3 and Fig. 4. The reason is that the thickness of the beam decreases. In Figs. 3 and 5, the vibration amplitude of the cable-beam structure increases when the thickness of the beam decreases. Thus, the thickness of the beam is a critical factor for the thermally induced vibration of the cable-beam structure.

It can be seen from Figs. 3-6 that when Boley parameter approximates 1, the thermally induced

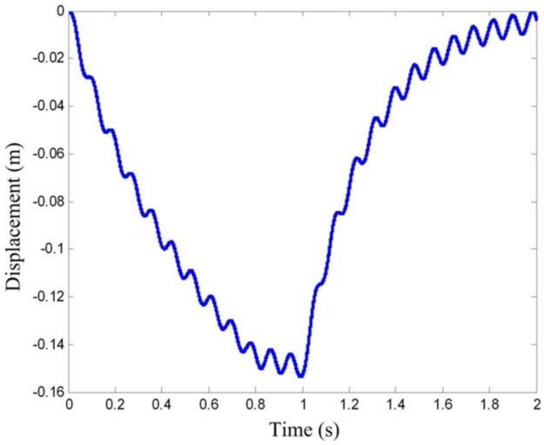


Fig. 5 Vibration curve of point A of type *III*

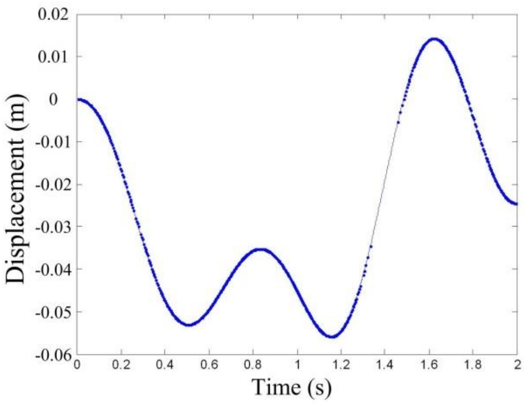


Fig. 6 Vibration curve of point A of type *IV*

vibration of the cable-beam structure is more prone to be unstable, that is, the thermal flutter is more likely to occur. It demonstrates that the proposed analytic expression of Boley parameter is valid to evaluate the thermally induced vibration of nonlinear cable-beam structures.

6. Conclusions

This article firstly comes up with the piecewise nonlinearity of the stiffness matrix of cable-beam structures. Then, an analytical approach is developed for determining the Boley parameter of the cable-beam structure. The analysis of thermally induced vibration of the cable-beam structure is solved using finite element method. By analyzing numerical results with their corresponding Bolay parameters, it is demonstrated that the stability criterion given in Boley parameter is valid to evaluate the occurrence conditions of thermal flutter of cable-beam structures. Some conclusions are

• When Boley parameter approximates 1, the thermal flutter is more likely to occur, that is, the thermally induced vibration of cable-beam structures is more prone to be unstable when its thermal response time approximates its structural response time.

• Key parameters influencing the thermal flutter related to the Boley parameter include the support condition, thermal and structural parameters. Among them, the occurrence of the unstable oscillations or thermal flutter is strongly dependent on the cable stiffness and thickness of beams.

• The proposed expression of Boley parameter provides a simple way to evaluate the thermally induced vibration of high-dimensional nonlinear cable-beam structures in order to avoid massive computing.

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