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# Free vibration of functionally graded thin elliptic plates with various edge supports

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**Abstract.** In this article, free vibration of functionally graded (FG) elliptic plates subjected to various classical boundary conditions has been investigated. Literature review reveals no study has been performed based on functionally graded elliptic plates till date. The mechanical kinematic relations are considered based on classical plate theory. Rayleigh-Ritz technique is used to obtain the generalized eigenvalue problem. The material properties of the FG plate are assumed to vary along thickness direction of the constituents according to power-law form. Trial functions denoting the displacement components are expressed in simple algebraic polynomial forms which can handle any edge support. The objective is to study the effect of geometric configurations and gradation of constituent volume fractions on the natural frequencies. New results for frequency parameters are incorporated after performing a test of convergence. A comparison study is carried out with existing literature for validation in special cases. Three-dimensional mode shapes for circular and elliptic FG plates are also presented with various boundary conditions at the edges.

**Keywords:** vibration; elliptic functionally graded plate; Rayleigh-Ritz method; classical plate theory; mode shapes

## 1. Introduction

The functionally graded materials (FGMs) are advanced composite materials which are designed to achieve functional behavior with the variation of mechanical properties continuously from one surface to the other. Use of FG structural members can be observed in various engineering applications and in manufacturing industries viz. aerospace, biomedical, nuclear, automobile, space-plane project and steel industries. The concept of FGMs was first introduced in 1984 by a group of material scientists in Japan (Loy *et al.* 1999) while preparing a space–plane project as an exceptional material to withstand a very intense temperature deviation through comparatively less thickness. The primary constituents for these materials are metal with ceramic or a combination of materials. The study of dynamic characteristics of FGMs has also been gained considerable attention in research sectors during the past decades.

As present work is based on flexural vibration of circular and elliptic plates, one may follow various sources that are available viz. (Wang *et al.* 2000, Chakraverty 2009, Leissa 1969, Rao

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2004). Dynamic characteristics of isotropic circular and elliptic plates are quite well presented by different researchers throughout the globe using various computational techniques. Natural frequencies of simply supported elliptic plates are evaluated by Leissa (1967) by means of Rayleigh-Ritz technique. Mazumdar (1971) has computed fundamental frequencies of elliptic plates, for both clamped and simply supported edge supports, by the method of constant deflection lines. Leissa and Narita (1980) have analyzed natural frequencies of simply supported circular plates using Classical plate theory with ordinary and modified Bessel functions of the first kind. Galerkin method and Bolotin's method is used by Chen and Hwang (1988) to study dynamic stability of isotropic Mindlin circular plates subjected to periodic radial loads. The spline-finite-strip method with subparametric mapping concept is employed by Cheung *et al*. (1988) in static and free vibration analysis of arbitrary shaped plates. Transverse vibration of circular and elliptic plates are studied by Singh and Chakraverty (1991, 1992b, c) with three respective boundary conditions viz. completely free, simply supported and clamped. Axisymmetric vibration of circular and its analogous elliptic plates are examined by Rajalingham and Bhat (1993) using characteristic orthogonal polynomials. Rajalingham *et al*. (1994) have analyzed vibration of clamped elliptic plates using exact circular plate modes as shape functions in Rayleigh-Ritz method. Circular and elliptic plates with variable thickness is being investigated in Singh and Chakraverty (1994) with all three boundary conditions. Natural frequencies for free vibration of nonhomogeneous circular and elliptic plates using two dimensional orthogonal polynomials is studied in (Chakraverty and Petyt 1997). Liew *et al*. (1997) used differential quadrature method and linear shear deformation Mindlin theory to analyze axisymmetric free vibration characteristics of moderately thick circular plates. Three-dimensional vibration of circular and annular plates are analyzed in (Liu and Lee 2000) using finite element method and in Zhao *et al*. (2003) applying Chebyshev-Ritz method. Free vibration of solid circular plates is studied in (Wu and Liu 2001) and (Wu *et al.* 2002) applying generalized differential quadrature rule. Chakraverty *et al*. (2007) have provided vibration behavior of plates by using the effects of non-homogeneity.

While considering functionally graded circular and elliptic plates, one may find very few literature available for such analysis. Reddy *et al*. (1999) used first-order shear deformation Mindlin plate theory to study axisymmetric bending and stretching of functionally graded solid and annular circular plates. Buckling analysis is presented in (Najafizadeh and Eslami 2002) for radially loaded functionally graded solid circular plate subject to either clamped or simply supported edge conditions. Ma and Wang (2003) investigated axisymmetric large deflection bending and thermal post-buckling behavior of functionally graded circular plate under mechanical, thermal and combined thermo-mechanical loadings based on classical nonlinear von Karman plate theory. An inverse problem of a functionally graded elliptic plate is developed by Hsieh and Lee (2006) with large deflection and distributed boundary under uniform load. Prakash and Ganpathi (2006) have found free vibration characteristics and thermoelastic stability of functionally graded circular plates using finite element procedure. Semi–analytical solution methods are used in (Nie and Zhong 2007) to analyze three-dimensional free and forced vibration of functionally graded circular plates and in (Allahverdizadeh *et al.* 2008) to examine nonlinear free and forced vibration of thin circular functionally graded plate respectively. Saidi *et al*. (2009) have investigated axisymmetric bending and buckling of perfect functionally graded solid circular plates based on unconstrained third-order shear deformation plate theory. Benachour *et al*. (2011) have used a four variable refined plate theory for free vibration analysis of FG plates and a new hyperbolic theory is presented by El Meiche *et al*. (2011) for the buckling and free vibration analysis of thick FG sandwich plates. Zhang (2013) has presented nonlinear bending analysis for functionally graded elliptical plates resting on two-parameter elastic foundations based on Reddy's higher-order shear deformation plate theory. A unified nonlocal shear deformation theory is proposed by Tounsi *et al*. (2013) to study bending, buckling and free vibration of nanobeams. Liu *et al*. (2013) have studied the free vibration problems of uniform Euler-Bernoulli beam using a modified differential transform method. Thermoelastic bending analysis of functionally graded sandwich plates has been developed respectively by Tounsi *et al*. (2013) using a refined trigonometric shear deformation theory and by Houari *et al*. (2013) using a new higher order shear and normal deformation theory. On the other hand, Bouderba *et al*. (2013) have implemented a refined trigonometric shear deformation theory in finding the thermomechanical bending response of FGM thick plates resting on Winkler–Pasternak elastic foundations. Nonlinear behavior of FG plates under thermal loads has been investigated by Bachir Bouiadjra *et al*. (2013) using an efficient sinusoidal shear deformation theory. Bessaim *et al*. (2013) have developed a new higher-order shear and normal deformation theory for the bending and free vibration analysis of sandwich plates with functionally graded isotropic face sheets. Free vibration of a sandwich curved beam with FG core is investigated by Fard (2014) based on 2D refined higher order beam theory. The bending response of FG plate resting on elastic foundation and subjected to hygro-thermo-mechanical loading has been studied by Zidi *et al*. (2014) by the use of four varibale refined plate theory. Belabed *et al*. (2014) have proposed an efficient and simple higher order shear and normal deformation theory to obtain analytical results for the bending and free vibration of simply supported FG plates and Hebali *et al*. (2014) have given a new quasi-3D hyperbolic shear deformation theory for the static and free vibration analysis of functionally graded plates. To the best of present authors' knowledge, no work is yet done on free vibration characteristics of functionally graded circular and elliptic plates.

Whence the objective of present work is to evaluate natural frequencies and mode shapes of functionally graded circular and elliptic plates within the framework of classical plate theory. Material properties are assumed to vary along thickness direction of FG constituents in simple power-law exponent form. Rayleigh-Ritz method is used in mathematical formulation to obtain the generalized eigenfrequency equation. Trial functions denoting the displacement components are expressed in simple algebraic polynomial forms which can handle any edge support. A test of convergence of present results is performed with a comparison study with existing literature for validation. New results for frequencies and three-dimensional mode shapes are also presented under various boundary conditions at the edges of elliptic FG plates.

## 2. Elliptic FG plate

Let us consider a functionally graded elliptic plate with semi-major axis $a$, semi-minor axis $b$ and thickness $h$ as shown in Fig. 1. Circular FG plate is a special case of elliptic FG plate while equating semi-major and semi-minor axes that is $a=b$.

Material properties of FG plate are assumed to vary along thickness direction according to power-law form as

$$\wp(z) = (\wp_c - \wp_m)\left(\frac{z}{h} + \frac{1}{2}\right)^k + \wp_m \tag{1}$$

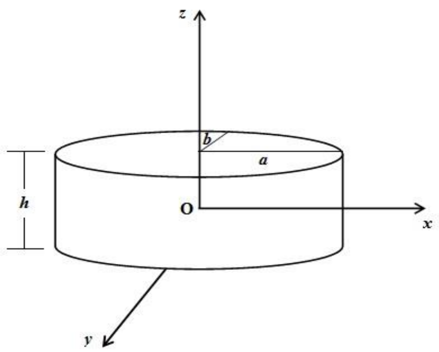
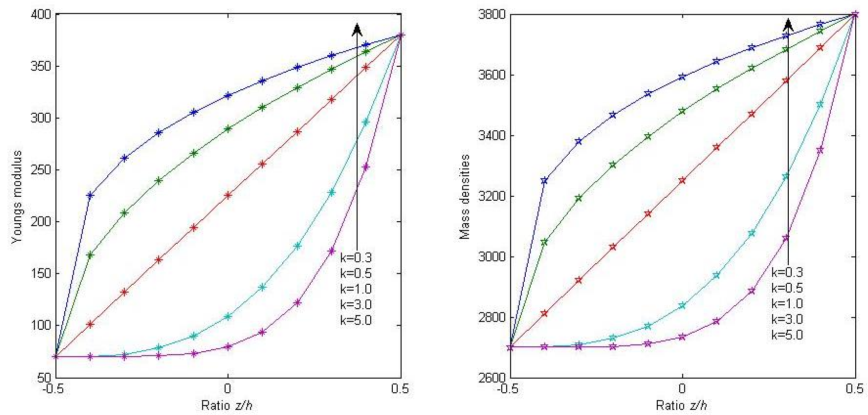



Fig. 1 A typical functionally graded elliptic plate element with Cartesian coordinates




(a) Variation of Young's modulus (b) Variation of mass densities

Fig. 2 Power-law variation of (a) Young's modulus and (b) mass densities of FG plate

where $\wp_c$ and $\wp_m$ denote the values of the material properties of the ceramic and metal constituents of the FG plate respectively. $k$ (power-law exponent) is a non-negative variable parameter. According to this distribution, the bottom surface ($z=-h/2$) of FG plate is pure metal, whereas the top surface ($z=h/2$) is pure ceramic and for different values of $k$ one can obtain different volume fractions of material plate. For our present formulations, Young's modulus ($E$) and mass densities ($\rho$) are taken into consideration as in Eq. (1) while other properties will remain constant through the thickness of the plate. It may be noted in the above Eq. (1) that $\wp = \wp_m$ at $z=-h/2$ and $\wp = \wp_c$ at $z=h/2$. Fig. 2 indicates the power-law variation of material properties of constituents of elliptic (or circular) FG plate.

## 3. Mathematical modelling

In this section, classical plate theory, mechanical kinematic relations and Rayleigh-Ritz method are used to obtain the generalized eigenvalue problem for free vibration of elliptic (or circular) functionally graded plates. An overview of these basic components are described as follows.

### *3.1 Classical plate theory*

Classical or Kirchoff's plate theory (CPT) for the free vibration of functionally graded plate is based on the displacement field (Wang *et al.* 2000)

$$\begin{aligned} u_x(x,y,z) &= -z\frac{\partial w}{\partial x} \\ u_y(x,y,z) &= -z\frac{\partial w}{\partial y} \\ u_z(x,y,z) &= w(x,y) \end{aligned} \tag{2}$$

where $u_x$, $u_y$ and $u_z$ are the displacement components along $x$, $y$ and $z$ coordinate directions respectively and $w$ is the transverse deflection of a point on the mid-plane ($x$–$y$ plane). Transverse shear deformation is neglected in case of Kirchoff assumption that is deformation is due to bending and in-plane stretching.

### *3.2 Mechanical kinematic relations*

In mechanics, the non-zero linear strains associated with the displacement field can be expressed as

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u_x}{\partial x} \\ \dfrac{\partial u_y}{\partial y} \\ \dfrac{\partial u_x}{\partial y} + \dfrac{\partial u_y}{\partial x} \end{Bmatrix} = \begin{Bmatrix} -z\dfrac{\partial^2 w}{\partial x^2} \\ -z\dfrac{\partial^2 w}{\partial y^2} \\ -2z\dfrac{\partial^2 w}{\partial x \partial y} \end{Bmatrix} \tag{3}$$

where $\varepsilon_{xx}$ and $\varepsilon_{yy}$ are the normal strains in $x-$ and $y-$ directions respectively and $\gamma_{xy}$ is the shear strain. By assuming the material constituents of FG plate to obey the generalized Hooke's law, the constitutive or stress-strain relationships can be expressed in matrix form as

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{Bmatrix} = \begin{pmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{pmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} \tag{4}$$

where $\sigma_{xx}$, $\sigma_{yy}$ are the normal stresses and $\tau_{xy}$ is the shear stress and the reduced stiffness components, $Q_{ij}$ are given by

$$Q_{11} = Q_{22} = \frac{E(z)}{1-\nu^2},\ Q_{12} = Q_{21} = \frac{\nu E(z)}{1-\nu^2},\ Q_{66} = \frac{E(z)}{2(1+\nu)}$$

Here, $E$ and $\nu$ are Young's modulus and Poisson's ratio of the material constituents respectively. The strain energy $U$ and kinetic energy $T$ of the plate at any instant in cartesian co-ordinates may be written as

$$U = \frac{1}{2}\int_{\Omega}\left[\int_{-h/2}^{h/2}\left(\sigma_{xx}\varepsilon_{xx} + \sigma_{yy}\varepsilon_{yy} + \tau_{xy}\gamma_{xy}\right)\mathrm{d}z\right]\mathrm{d}x\mathrm{d}y \tag{5}$$

$$T = \frac{1}{2}\int_{\Omega}\left[\int_{-h/2}^{h/2}\rho(z)\left(\frac{\partial u_z}{\partial t}\right)^2 \mathrm{d}z\right]\mathrm{d}x\mathrm{d}y \tag{6}$$

where Ω denotes the midplane (domain) of the elliptic FG plate.

Using Eqs. (2), (3) and (4) in Eqs. (5) and (6) lead to

$$U = \frac{1}{2}\int_{\Omega}\left[D_{11}\left\{\left(\frac{\partial^2 w}{\partial x^2}\right)^2 + \left(\frac{\partial^2 w}{\partial y^2}\right)^2\right\} + 2D_{12}\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} + 4D_{66}\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2\right]\mathrm{d}x\mathrm{d}y \tag{7}$$

$$T = \frac{1}{2}\int_{\Omega} I_0\left(\frac{\partial w}{\partial t}\right)^2 \mathrm{d}x\mathrm{d}y \tag{8}$$

where the stiffness coefficients in Eq. (7) are

$$\left(D_{11}, D_{12}, D_{66}\right) = \int_{-h/2}^{h/2}(Q_{11}, Q_{12}, Q_{66})z^2\,\mathrm{d}z$$

and inertial coefficient, $I_0$ in Eq. (8) is

$$I_0 = \int_{-h/2}^{h/2}\rho(z)\,\mathrm{d}z.$$

The displacement component can be assumed harmonic type as $w(x,y,t)=W(x,y)\cos\omega t$ with $W(x,y)$ is the maximum deflection and $\omega$ is the natural frequency of free vibration. Using the above harmonic motion, Eqs. (7) and (8) may be transformed into maximum strain ($U_{\max}$) and kinetic energies ($T_{\max}$) respectively as follows

$$U_{max} = \frac{1}{2}\int_{\Omega}\left[D_{11}\left\{\left(\frac{\partial^2 W}{\partial x^2}\right)^2 + \left(\frac{\partial^2 W}{\partial y^2}\right)^2\right\} + 2D_{12}\frac{\partial^2 W}{\partial x^2}\frac{\partial^2 W}{\partial y^2} + 4D_{66}\left(\frac{\partial^2 W}{\partial x \partial y}\right)^2\right]\mathrm{d}x\mathrm{d}y \tag{9}$$

$$T_{max} = \frac{\omega^2}{2}\int_{\Omega} I_0 W^2\,\mathrm{d}x\mathrm{d}y \tag{10}$$

### *3.3 Application of Rayleigh-Ritz method*

One may express the transverse displacement ($W(x,y)$) as the sum of simple algebraic polynomials involving both $x$ and $y$.

$$W(x,y) = \sum_{i=1}^{n} c_i\varphi_i(x,y)$$

Table 1 Ten algebraic polynomials obtained from Pascal's triangle

| $i$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| $\psi_i$ | 1 | $x$ | $y$ | $x^2$ | $xy$ | $y^2$ | $x^3$ | $x^2y$ | $xy^2$ | $y^3$ |

where $c_i$ are unknown constants to be determined and $\varphi_i$ are the admissible functions, which satisfy the essential boundary conditions and can be represented as

$$\varphi_i(x,y) = f\psi_i(x,y), \quad i = 0,1,2,\cdots,n$$

Here, $n$ is the number of polynomials involved in the admissible functions. The function $f = \left(1 - \frac{x^2}{a^2} - \frac{y^2}{b^2}\right)^p$ with the exponent $p$ controls various boundary conditions (BCs). The parameter $p$=0,1 or 2 according as the elliptic (or circular) FG plate is free (F), simply supported (S) or clamped (C). Table 1 involves the components of $\psi_i$ generated from Pascal's triangle.

Assuming constant Poisson's ratio ($v$), the Rayleigh quotient can be obtained by equating $U_{\max}$ and $T_{\max}$ as

$$\omega^2 = \frac{\int_\Omega D_{11}\left[\left\{\left(\frac{\partial^2 W}{\partial x^2}\right)^2 + \left(\frac{\partial^2 W}{\partial y^2}\right)^2\right\} + 2v\frac{\partial^2 W}{\partial x^2}\frac{\partial^2 W}{\partial y^2} + 2(1-v)\left(\frac{\partial^2 W}{\partial x \partial y}\right)^2\right]\mathrm{d}x\mathrm{d}y}{\int_\Omega I_0 W^2\,\mathrm{d}x\mathrm{d}y} \tag{11}$$

The stiffness coefficient ($D_{11}$) and inertial coefficient ($I_0$) in Eq. (11) may be expressed as

$$\begin{aligned}
D_{11} &= \int_{-h/2}^{h/2} Q_{11} z^2\,\mathrm{d}z \\
&= \frac{1}{1-v^2}\int_{-h/2}^{h/2} E(z) z^2\,\mathrm{d}z \\
&= \frac{1}{1-v^2}\int_{-h/2}^{h/2}\left\{(E_c - E_m)\left(\frac{z}{h}+\frac{1}{2}\right)^k + E_m\right\} z^2\,\mathrm{d}z \\
&= \frac{1}{1-v^2}\left[\int_{-h/2}^{h/2}\left\{(E_c - E_m)\left(\frac{z}{h}+\frac{1}{2}\right)^k\right\} z^2\,\mathrm{d}z + \int_{-h/2}^{h/2} E_m z^2\,\mathrm{d}z\right] \\
&= \frac{(E_c - E_m)h^3}{1-v^2}\left\{\frac{1}{k+3} - \frac{1}{k+2} + \frac{1}{4(k+1)}\right\} + \frac{E_m h^3}{12(1-v^2)},
\end{aligned}$$

$$\begin{aligned}
I_0 &= \int_{-h/2}^{h/2} \rho(z)\,\mathrm{d}z \\
&= \int_{-h/2}^{h/2}\left\{(\rho_c - \rho_m)\left(\frac{z}{h}+\frac{1}{2}\right)^k + \rho_m\right\}\mathrm{d}z \\
&= \int_{-h/2}^{h/2}\left\{(\rho_c - \rho_m)\left(\frac{z}{h}+\frac{1}{2}\right)^k\right\}\mathrm{d}z + \int_{-h/2}^{h/2}\rho_m\,\mathrm{d}z \\
&= \frac{(\rho_c - \rho_m)h}{k+1} + \rho_m h.
\end{aligned}$$

Accordingly, taking partial derivative of $\omega^2$ with respect to unknown constants as follows

$$\frac{\partial \omega^2}{\partial c_i} = 0, \quad i = 1,2,3,\cdots,n \tag{12}$$

Further manipulation of Eq. (12) yields the generalized eigenvalue problem of the form

$$\left([K]_{n\times n} - \lambda^2 [M]_{n\times n}\right)\{\Delta\} = 0 \tag{13}$$

where $[K]_{n\times n}$ and $[M]_{n\times n}$ are symmetric stiffness and inertia matrices respectively and $\{\Delta\}$ is the column vector of unknown generalized coefficients. Solutions of the eigenvalue problem, Eq. (13) gives the vibration characteristics viz. frequency parameters and mode shapes, for free vibration of elliptic (or circular) FG plate. As such, frequency parameters and mode shapes are incorporated in the next sections based on CPT taking various BCs. Test of convergence and validation of present results with existing literature are also performed.

## 4. Convergence study

In this part, convergence studies for frequency parameters of isotropic elliptic (or circular) plates (assuming $k$=0 in case of FG plates) are reported with respect to number of polynomials involved in the displacement component. The material properties of FG constituents are considered in Table 2.

Non-dimensional frequency parameters of elliptic FG plate may be written as

$$\lambda = \omega a^2 \sqrt{\frac{\rho_c h}{D_c}} \tag{14}$$

where $D_c\,(\text{flexural rigidity}) = \dfrac{E_c h^3}{12(1-\nu^2)}$ of the FG plate due to the deformation effect.

In Tables 3 to 4, convergence of first six frequency parameters of isotropic circular and elliptic plates are incorporated. Isotropic circular plate is considered in Table 3 and elliptic plate in Table 4 with *a/b*=2 taking clamped (C) and simply supported (S) edge supports. Rather than taking frequency parameters for combination of symmetric and antisymmetric modes separately, present study computes frequencies for all the modes at a time. This means that earlier authors used deflection function of odd-odd, even-odd, odd-even and even-even polynomials of $x$ and $y$ separately. Present authors have taken all the powers of $x$ and $y$ for the ease in computation in a single run. So the number of approximations may be seen less in previous works. It is interesting to note here that increase in number of polynomials in displacement component plays a crucial role

Table 2 Material properties of the FGM constituents

| Properties | Unit | Aluminium (Al) | Alumina ($Al_2O_3$) |
|---|---|---|---|
| $E$ | GPa | 70 | 380 |
| $\rho$ | kg/m$^3$ | 2700 | 3800 |
| $\nu$ | - | 0.3 | 0.3 |

Table 3 Convergence of frequency parameters for isotropic circular plate

| BCs | Sources | $\lambda_1$ | $\lambda_2$ | $\lambda_3$ | $\lambda_4$ | $\lambda_5$ | $\lambda_6$ |
|---|---|---|---|---|---|---|---|
| C | 10×10 | 10.217 | 21.275 | 36.661 | 43.058 | 54.650 | 69.202 |
| | 13×13 | 10.216 | 21.275 | 35.609 | 41.210 | 54.650 | 69.202 |
| | 16×16 | 10.216 | 21.266 | 34.941 | 39.921 | 52.479 | 64.682 |
| | 19×19 | 10.216 | 21.263 | 34.941 | 39.921 | 51.914 | 62.439 |
| | 20 ×20 | 10.216 | 21.261 | 34.941 | 39.921 | 51.209 | 61.407 |
| S | 10×10 | 4.941 | 13.987 | 35.665 | 46.706 | 59.195 | 88.161 |
| | 13×13 | 4.938 | 13.987 | 30.391 | 39.456 | 59.195 | 88.161 |
| | 16×16 | 4.935 | 13.941 | 25.986 | 30.503 | 46.102 | 59.195 |
| | 19×19 | 4.935 | 13.915 | 25.986 | 30.503 | 42.362 | 45.976 |
| | 20 ×20 | 4.935 | 13.899 | 25.986 | 30.503 | 40.915 | 42.362 |

Table 4 Convergence of frequency parameters for isotropic elliptic plate with *a/b*=2

| BCs | Sources | $\lambda_1$ | $\lambda_2$ | $\lambda_3$ | $\lambda_4$ | $\lambda_5$ | $\lambda_6$ |
|---|---|---|---|---|---|---|---|
| C | 10×10 | 27.395 | 39.594 | 61.455 | 70.023 | 88.595 | 95.687 |
| | 13×13 | 27.378 | 39.594 | 56.329 | 70.023 | 88.595 | 89.812 |
| | 16×16 | 27.378 | 39.503 | 56.328 | 70.023 | 78.029 | 88.665 |
| | 19×19 | 27.378 | 39.500 | 56.328 | 69.884 | 78.025 | 88.665 |
| | 20 ×20 | 27.378 | 39.499 | 56.328 | 69.884 | 78.020 | 88.665 |
| S | 10×10 | 13.258 | 23.910 | 46.747 | 55.136 | 91.258 | 93.936 |
| | 13×13 | 13.218 | 23.910 | 39.388 | 46.747 | 76.705 | 93.936 |
| | 16×16 | 13.214 | 23.696 | 39.366 | 46.747 | 62.309 | 64.636 |
| | 19×19 | 13.214 | 23.653 | 39.366 | 46.341 | 60.605 | 64.636 |
| | 20 ×20 | 13.214 | 23.645 | 39.366 | 46.341 | 60.497 | 64.636 |

in the conv ergence of frequency parameters irrespective of geometric configuration and edge support of the plate.

## 5. Verification of results

After satisfactory test of convergence of frequency parameters, we have performed a comparison study for frequency parameters of elliptic (or circular) plates with the existing literature. As there are not much articles related to free vibration of elliptic FG plates, it is worth taking isotropic plates to compare the results. First five frequency parameters of isotropic circular (*a/b*=1.0) and elliptic plate with different boundary conditions are compared in Table 5 assuming Poisson's ratio as $v$=0.3. It may be concluded that frequencies related to present study are in excellent agreement with the existing literature.

## 6. New results and discussion

In view of the above verification, new results for frequency parameters of elliptic FG plates

Table 5 Comparison of first five frequency parameters of isotropic elliptic plate

| *a/b* | BCs | Sources | $\lambda_1$ | $\lambda_2$ | $\lambda_3$ | $\lambda_4$ | $\lambda_5$ |
|---|---|---|---|---|---|---|---|
| 1.0 | C | Present | 10.2158 | 21.261 | 34.878 | 39.773 | 51.209 |
| | C | Exact | 10.216 | 21.260 | 34.878 | 39.773 | - |
| | C | Leissa (1969) | 10.2158 | 21.26 | 34.88 | 39.771 | 51.04 |
| | C | Mazumdar (1971) | 10.2151 | - | - | - | - |
| | C | Cheung *et al*. (1988) | 10.2062 | 21.27 | 34.94 | 40.21 | 52.05 |
| | C | Singh and Chakraverty (1992c, 1994) | 10.216 | 21.260 | 34.878 | 39.773 | - |
| | C | Rajalingham *et al*. (1994) | 10.2158 | 21.2604 | 34.8770 | 39.7711 | 51.0300 |
| | C | Chakrverty and Petyt (1997) | 10.216 | 21.260 | 34.878 | 39.773 | 51.030 |
| | C | Wu and Liu (2001), Wu *et al*. (2002) | 10.216 | 21.260 | 34.877 | 39.771 | 51.030 |
| | C | Prakash and Ganpathi (2006) | 10.213 | 21.259 | 34.849 | - | 50.974 |
| | C | Chakraverty *et al*. (2007) | 10.2158 | 21.2604 | 34.8770 | 39.7712 | - |
| | S | Present | 4.9351 | 13.899 | 25.619 | 29.737 | 40.915 |
| | S | Exact | 4.935 | 13.898 | 25.613 | 29.720 | - |
| | S | Leissa (1969) | 4.9351 | 13.8982 | 25.6173 | 29.7200 | 39.9573 |
| | S | Leissa and Narita (1980) | 4.93515 | 13.8982 | 25.6133 | 29.7200 | 39.9573 |
| | S | Cheung *et al*. (1988) | 4.927 | 13.88 | 25.54 | 29.84 | 40.30 |
| | S | Singh and Chakraverty (1992b,1994) | 4.9351 | 13.898 | 25.613 | 29.720 | - |
| | S | Chakraverty and Petyt (1997) | 4.9351 | 13.898 | 25.613 | 29.720 | 39.957 |
| | S | Wu and Liu (2001), Wu *et al*. (2002) | 4.935 | 13.898 | 25.613 | 29.720 | 39.957 |
| | S | Prakash and Ganpathi (2006) | 4.935 | 13.898 | 25.613 | - | 39.957 |
| | S | Chakraverty *et al*. (2007) | 4.9351 | 13.8982 | 25.6133 | 29.7201 | - |
| | F | Present | 5.3583 | 9.0035 | 12.5645 | 21.2331 | 22.1935 |
| | F | Exact | 5.3583 | 9.0031 | 12.439 | 20.475 | - |
| | F | Leissa (1969) | 5.253 | 9.084 | 12.23 | 20.52 | 21.6 |
| | F | Singh and Chakraverty (1991,1994) | 5.3583 | 9.0031 | 12.439 | 20.475 | - |
| | F | Chakraverty and Petyt (1997) | 5.3583 | 9.0031 | 12.439 | 20.475 | 21.835 |
| | F | Wu and Liu (2001), Wu *et al*. (2002) | 5.358 | 9.003 | 12.439 | 20.475 | 21.835 |
| | F | Chakraverty *et al*. (2007) | 5.3583 | 9.0031 | 12.4390 | 20.4746 | - |
| 2.0 | C | Present | 27.377 | 39.499 | 55.985 | 69.863 | 78.020 |
| | C | Leissa (1969) | 27.378 | - | - | - | - |
| | C | Mazumdar (1971) | 27.471 | - | - | - | - |
| | C | Singh and Chakraverty (1992c) | 27.377 | 39.497 | 55.985 | 69.858 | - |
| | C | Singh and Chakraverty (1994) | 27.377 | 39.497 | 55.985 | 69.858 | 77.037 |
| | C | Chakraverty *et al*. (2007) | 27.3774 | 39.4974 | 55.9758 | 69.8580 | - |
| | S | Present | 13.213 | 23.645 | 38.354 | 46.165 | 60.497 |
| | S | Singh and Chakraverty (1992b) | 13.213 | 23.641 | 38.354 | 46.151 | 57.625 |
| | S | Singh and Chakraverty (1994) | 13.213 | 23.641 | 38.354 | 46.151 | - |
| | S | Chakraverty *et al*. (2007) | 13.2135 | 23.6410 | 38.3259 | 46.1504 | - |
| | F | Present | 6.6706 | 10.548 | 17.213 | 22.353 | 32.696 |
| | F | Singh and Chakraverty (1994) | 6.6706 | 10.548 | 16.923 | 22.019 | - |
| | F | Chakraverty *et al*. (2007) | 6.6705 | 10.5476 | 16.9212 | 22.0149 | - |

Table 6 Effect of aspect ratios (*a/b*) on frequency parameters of clamped elliptic FG plate

| $a/b$ | $k$ | $\lambda_1$ | $\lambda_2$ | $\lambda_3$ | $\lambda_4$ | $\lambda_5$ | $\lambda_6$ |
|---|---|---|---|---|---|---|---|
| 1.0 | 0.0 | 10.216 | 21.261 | 34.878 | 39.773 | 51.209 | 61.407 |
| | 0.1 | 9.851 | 20.501 | 33.631 | 38.352 | 49.379 | 59.213 |
| | 0.5 | 8.973 | 18.673 | 30.633 | 34.933 | 44.978 | 53.935 |
| | 1.0 | 8.500 | 17.689 | 29.020 | 33.093 | 42.609 | 51.094 |
| | 2.0 | 8.125 | 16.909 | 27.741 | 31.634 | 40.730 | 48.841 |
| | 5.0 | 7.576 | 15.767 | 25.865 | 29.496 | 37.976 | 45.539 |
| 1.5 | 0.0 | 17.129 | 28.472 | 41.487 | 44.392 | 57.038 | 65.369 |
| | 0.1 | 16.517 | 27.454 | 40.005 | 42.806 | 54.999 | 63.032 |
| | 0.5 | 15.045 | 25.007 | 36.439 | 38.990 | 50.097 | 57.414 |
| | 1.0 | 14.253 | 23.690 | 34.519 | 36.936 | 47.458 | 54.389 |
| | 2.0 | 13.624 | 22.646 | 32.998 | 35.308 | 45.366 | 51.992 |
| | 5.0 | 12.703 | 21.115 | 30.767 | 32.921 | 42.299 | 48.477 |
| 2.0 | 0.0 | 27.377 | 39.499 | 55.985 | 69.863 | 78.020 | 88.074 |
| | 0.1 | 26.399 | 38.087 | 53.984 | 67.366 | 75.232 | 84.926 |
| | 0.5 | 24.046 | 34.692 | 49.172 | 61.361 | 68.526 | 77.356 |
| | 1.0 | 22.779 | 32.865 | 46.582 | 58.129 | 64.917 | 73.282 |
| | 2.0 | 21.775 | 31.416 | 44.529 | 55.566 | 62.055 | 70.051 |
| | 5.0 | 20.303 | 29.292 | 41.518 | 51.809 | 57.859 | 65.315 |
| 3.0 | 0.0 | 56.801 | 71.626 | 90.350 | 116.81 | 147.34 | 150.18 |
| | 0.1 | 54.771 | 69.066 | 87.121 | 112.64 | 142.08 | 144.81 |
| | 0.5 | 49.888 | 62.909 | 79.356 | 102.59 | 129.41 | 131.90 |
| | 1.0 | 47.261 | 59.596 | 75.176 | 97.195 | 122.59 | 124.95 |
| | 2.0 | 45.177 | 56.969 | 71.862 | 92.909 | 117.19 | 119.44 |
| | 5.0 | 42.122 | 53.117 | 67.003 | 86.628 | 109.27 | 111.37 |

may be evaluated. Effect of aspect ratios (*a/b*) on first six frequency parameters are discussed in Tables 6 to 8 with various boundary conditions and gradation of properties in FG constituents. Clamped edge support is considered in Table 6 with different power-law exponents ($k$) to evaluate the first seven frequency parameters of elliptic FG plates. In a similar fashion, frequencies are computed for simply supported and completely free edge supports in Tables 7 and 8 respectively. In case of clamped and simply supported edge supports, it can be observed that frequencies are increasing with increase in aspect ratios and act in a reverse order with increase in power-law indices, whereas frequencies follow peculiar behavior in case of completely free FG elliptic plates that is frequencies are decreasing with increase in $k$ and are showing fluctuations with increase in *a/b* at higher modes.

In Tables 9 to 10, effect of variation of Poisson's ratio ($v$) on natural frequencies of elliptic (or circular) FG plates supported by various edge conditions with fixed aspect ratio (*a/b*) and power-law index ($k$) are summarized. For both isotropic ($k$=0) and FG plates, it can be easily seen that frequencies remain constant for different values of $v$ in case of clamped plates. Assuming simply supported, frequencies are increasing with increase in $v$, whereas frequencies are showing fluctuating order while considering free edge condition, keeping both *a/b* and $k$ fixed. One may conclude that effects of edge supports and Poisson's ratio on free vibration response of FG elliptic plates is also quite similar to that of isotropic elliptic plates.

Table 7 Effect of aspect ratios (*a/b*) on frequency parameters of simply supported elliptic FG plate

| *a/b* | *k* | $\lambda_1$ | $\lambda_2$ | $\lambda_3$ | $\lambda_4$ | $\lambda_5$ | $\lambda_6$ |
|---|---|---|---|---|---|---|---|
| 1.0 | 0.0 | 4.935 | 13.899 | 25.619 | 29.736 | 40.915 | 51.072 |
| | 0.1 | 4.759 | 13.402 | 24.703 | 28.674 | 39.453 | 49.247 |
| | 0.5 | 4.335 | 12.207 | 22.501 | 26.118 | 35.936 | 44.857 |
| | 1.0 | 4.106 | 11.564 | 21.316 | 24.742 | 34.043 | 42.495 |
| | 2.0 | 3.925 | 11.055 | 20.376 | 23.651 | 32.542 | 40.621 |
| | 5.0 | 3.659 | 10.307 | 18.999 | 22.052 | 30.342 | 37.875 |
| 1.5 | 0.0 | 8.282 | 17.835 | 27.452 | 31.805 | 41.409 | 52.246 |
| | 0.1 | 7.986 | 17.198 | 26.471 | 30.668 | 39.929 | 50.379 |
| | 0.5 | 7.274 | 15.665 | 24.112 | 27.935 | 36.369 | 45.889 |
| | 1.0 | 6.891 | 14.839 | 22.842 | 26.463 | 34.454 | 43.472 |
| | 2.0 | 6.587 | 14.186 | 21.835 | 25.297 | 32.935 | 41.555 |
| | 5.0 | 6.142 | 13.227 | 20.358 | 23.586 | 30.708 | 38.746 |
| 2.0 | 0.0 | 13.213 | 23.645 | 38.354 | 46.165 | 60.497 | 62.848 |
| | 0.1 | 12.741 | 22.799 | 36.984 | 44.515 | 58.335 | 60.602 |
| | 0.5 | 11.606 | 20.767 | 33.687 | 40.547 | 53.135 | 55.199 |
| | 1.0 | 10.994 | 19.673 | 31.913 | 38.412 | 50.337 | 52.292 |
| | 2.0 | 10.509 | 18.806 | 30.506 | 36.718 | 48.118 | 49.987 |
| | 5.0 | 9.799 | 17.535 | 28.443 | 34.236 | 44.864 | 46.607 |
| 3.0 | 0.0 | 27.081 | 40.146 | 57.050 | 84.282 | 98.673 | 116.06 |
| | 0.1 | 26.114 | 38.711 | 55.011 | 81.269 | 95.146 | 111.91 |
| | 0.5 | 23.786 | 35.260 | 50.108 | 74.026 | 86.665 | 101.94 |
| | 1.0 | 22.533 | 33.403 | 47.469 | 70.127 | 82.101 | 96.569 |
| | 2.0 | 21.539 | 31.931 | 45.376 | 67.035 | 78.481 | 92.312 |
| | 5.0 | 20.083 | 29.772 | 42.308 | 62.503 | 73.175 | 86.070 |

Table 8 Effect of aspect ratios (*a/b*) on frequency parameters of completely free elliptic FG plate

| *a/b* | *k* | $\lambda_1$ | $\lambda_2$ | $\lambda_3$ | $\lambda_4$ | $\lambda_5$ | $\lambda_6$ |
|---|---|---|---|---|---|---|---|
| 1.0 | 0.0 | 5.358 | 9.003 | 12.564 | 21.233 | 22.193 | 37.564 |
| | 0.1 | 5.167 | 8.682 | 12.115 | 20.474 | 21.400 | 36.221 |
| | 0.5 | 4.706 | 7.908 | 11.035 | 18.649 | 19.493 | 32.993 |
| | 1.0 | 4.458 | 7.491 | 10.454 | 17.667 | 18.466 | 31.255 |
| | 2.0 | 4.262 | 7.161 | 9.993 | 16.888 | 17.652 | 29.877 |
| | 5.0 | 3.974 | 6.677 | 9.318 | 15.746 | 16.459 | 27.857 |
| 1.5 | 0.0 | 6.477 | 7.986 | 16.309 | 16.510 | 17.767 | 29.971 |
| | 0.1 | 6.245 | 7.701 | 15.726 | 15.920 | 17.132 | 28.899 |
| | 0.5 | 5.689 | 7.014 | 14.324 | 14.501 | 15.605 | 26.323 |
| | 1.0 | 5.389 | 6.645 | 13.569 | 13.737 | 14.783 | 24.937 |
| | 2.0 | 5.151 | 6.352 | 12.971 | 13.132 | 14.131 | 23.838 |
| | 5.0 | 4.803 | 5.922 | 12.094 | 12.244 | 13.176 | 22.226 |
| 2.0 | 0.0 | 6.671 | 10.548 | 17.212 | 22.353 | 27.773 | 32.696 |
| | 0.1 | 6.432 | 10.171 | 16.596 | 21.554 | 26.780 | 31.527 |
| | 0.5 | 5.859 | 9.264 | 15.117 | 19.633 | 24.393 | 28.717 |
| | 1.0 | 5.550 | 8.776 | 14.321 | 18.599 | 23.108 | 27.205 |

Table 8 Continued

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 2.0 | 2.0 | 5.306 | 8.389 | 13.689 | 17.779 | 22.089 | 26.005 |
| | 5.0 | 4.947 | 7.822 | 12.764 | 16.577 | 20.596 | 24.247 |
| 3.0 | 0.0 | 6.757 | 15.615 | 17.618 | 31.415 | 33.961 | 51.246 |
| | 0.1 | 6.516 | 15.057 | 16.988 | 30.292 | 32.748 | 49.414 |
| | 0.5 | 5.935 | 13.714 | 15.474 | 27.592 | 29.829 | 45.009 |
| | 1.0 | 5.622 | 12.992 | 14.659 | 26.139 | 28.258 | 42.639 |
| | 2.0 | 5.374 | 12.419 | 14.013 | 24.986 | 27.012 | 40.759 |
| | 5.0 | 5.011 | 11.579 | 13.065 | 23.297 | 25.185 | 38.003 |

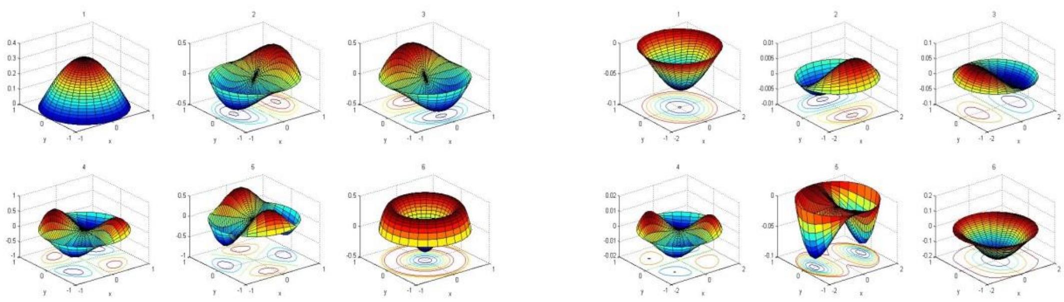
Table 9 Effect of Poisson's ratio ($v$) on frequency parameters of elliptic FG plate with various BCs for $k$=0

| $a/b$ | BC | $v$ | $\lambda_1$ | $\lambda_2$ | $\lambda_3$ | $\lambda_4$ | $\lambda_5$ | $\lambda_6$ |
|---|---|---|---|---|---|---|---|---|
| 1.0 | C | 0.00 | 10.216 | 21.260 | 34.878 | 39.773 | 51.209 | 61.407 |
| | | 0.25 | 10.216 | 21.260 | 34.878 | 39.773 | 51.209 | 61.407 |
| | | 0.33 | 10.216 | 21.260 | 34.878 | 39.773 | 51.209 | 61.407 |
| | | 0.50 | 10.216 | 21.260 | 34.878 | 39.773 | 51.209 | 61.407 |
| | S | 0.00 | 4.4436 | 13.502 | 25.249 | 29.379 | 40.519 | 50.644 |
| | | 0.25 | 4.8601 | 13.835 | 25.559 | 29.678 | 40.851 | 51.002 |
| | | 0.33 | 4.9790 | 13.936 | 25.654 | 29.771 | 40.953 | 51.114 |
| | | 0.50 | 5.2127 | 14.141 | 25.849 | 29.963 | 41.166 | 51.347 |
| | F | 0.00 | 6.1531 | 8.2441 | 14.148 | 20.758 | 24.659 | 37.779 |
| | | 0.25 | 5.5112 | 8.8902 | 12.881 | 21.158 | 22.701 | 37.599 |
| | | 0.33 | 5.2620 | 9.0692 | 12.363 | 21.277 | 21.867 | 37.543 |
| | | 0.50 | 4.6404 | 9.4141 | 11.021 | 19.648 | 21.519 | 34.641 |
| 2.0 | C | 0.00 | 27.377 | 39.499 | 55.985 | 69.862 | 78.019 | 88.074 |
| | | 0.25 | 27.377 | 39.499 | 55.985 | 69.862 | 78.019 | 88.074 |
| | | 0.33 | 27.377 | 39.499 | 55.985 | 69.862 | 78.019 | 88.074 |
| | | 0.50 | 27.377 | 39.499 | 55.985 | 69.862 | 78.019 | 88.074 |
| | S | 0.00 | 12.646 | 22.829 | 37.367 | 45.803 | 59.139 | 62.416 |
| | | 0.25 | 13.125 | 23.519 | 38.201 | 46.106 | 60.283 | 62.777 |
| | | 0.33 | 13.265 | 23.718 | 38.444 | 46.201 | 60.624 | 62.889 |
| | | 0.50 | 13.546 | 24.115 | 38.930 | 46.399 | 61.314 | 63.126 |
| | F | 0.00 | 7.060 | 12.269 | 18.235 | 25.329 | 27.675 | 34.809 |
| | | 0.25 | 6.778 | 10.870 | 17.473 | 22.938 | 27.775 | 33.196 |
| | | 0.33 | 6.597 | 10.346 | 17.034 | 21.981 | 27.768 | 32.359 |
| | | 0.50 | 6.037 | 9.0676 | 15.683 | 19.540 | 27.710 | 29.828 |

Three-dimensional mode shapes of FG circular and elliptic plates are plotted in Figs. 3 to 5 taking various BCs with $k$=1. Fig. 3 is meant for clamped FG plates. In a similar fashion, Fig. 4 is for simply supported and Fig. 5 is for completely free FG plates respectively. Looking into the deflected shapes of mode shapes, it is easy to predict the edge support to be clamped, simply supported or free, irrespective of the geometry and power-law exponent used in gradation.

Table 10 Effect of Poisson's ratio ($v$) on frequency parameters of elliptic FG plate with various BCs for $k$=1

| $a/b$ | BC | $v$ | $\lambda_1$ | $\lambda_2$ | $\lambda_3$ | $\lambda_4$ | $\lambda_5$ | $\lambda_6$ |
|---|---|---|---|---|---|---|---|---|
| 1.0 | C | 0.00 | 8.5001 | 17.689 | 29.019 | 33.093 | 42.609 | 51.094 |
| | | 0.25 | 8.5001 | 17.689 | 29.019 | 33.093 | 42.609 | 51.094 |
| | | 0.33 | 8.5001 | 17.689 | 29.019 | 33.093 | 42.609 | 51.094 |
| | | 0.50 | 8.5001 | 17.689 | 29.019 | 33.093 | 42.609 | 51.094 |
| | S | 0.00 | 3.6973 | 11.234 | 21.009 | 24.445 | 33.715 | 42.139 |
| | | 0.25 | 4.0439 | 11.512 | 21.266 | 24.694 | 33.989 | 42.436 |
| | | 0.33 | 4.1428 | 11.596 | 21.346 | 24.771 | 34.075 | 42.529 |
| | | 0.50 | 4.3372 | 11.766 | 21.508 | 24.931 | 34.253 | 42.724 |
| | F | 0.00 | 5.1197 | 6.8595 | 11.772 | 17.272 | 20.517 | 31.434 |
| | | 0.25 | 4.5856 | 7.3971 | 10.718 | 17.605 | 18.889 | 31.284 |
| | | 0.33 | 4.3783 | 7.5461 | 10.286 | 17.704 | 18.194 | 31.238 |
| | | 0.50 | 3.8610 | 7.8330 | 9.1701 | 16.348 | 17.9045 | 28.823 |
| 2.0 | C | 0.00 | 22.779 | 32.865 | 46.582 | 58.129 | 64.916 | 73.282 |
| | | 0.25 | 22.779 | 32.865 | 46.582 | 58.129 | 64.916 | 73.282 |
| | | 0.33 | 22.779 | 32.865 | 46.582 | 58.129 | 64.916 | 73.282 |
| | | 0.50 | 22.779 | 32.865 | 46.582 | 58.129 | 64.916 | 73.282 |
| | S | 0.00 | 10.522 | 18.995 | 31.092 | 38.111 | 49.207 | 51.933 |
| | | 0.25 | 10.921 | 19.569 | 31.785 | 38.362 | 50.158 | 52.234 |
| | | 0.33 | 11.038 | 19.735 | 31.988 | 38.441 | 50.442 | 52.328 |
| | | 0.50 | 11.271 | 20.065 | 32.392 | 38.607 | 51.017 | 52.524 |
| | | 0.00 | 5.874 | 10.208 | 15.173 | 21.075 | 23.027 | 28.962 |
| | | 0.25 | 5.639 | 9.0445 | 14.538 | 19.085 | 23.111 | 27.621 |
| | | 0.33 | 5.489 | 8.6086 | 14.173 | 18.289 | 23.105 | 26.925 |
| | | 0.50 | 5.023 | 7.5447 | 13.049 | 16.258 | 23.056 | 24.819 |

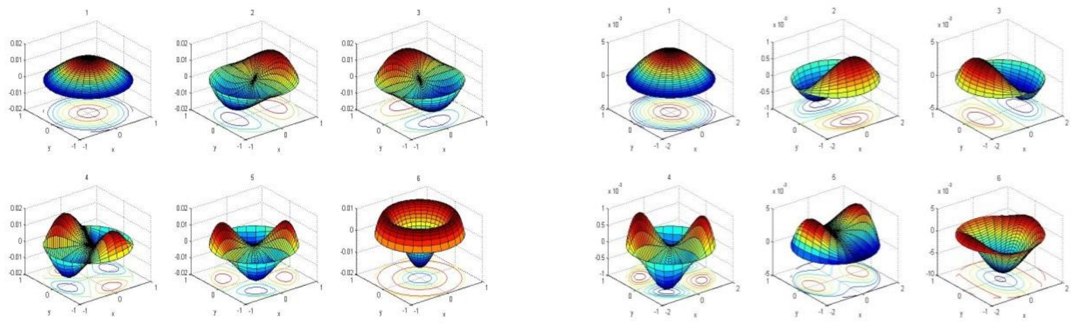


(a) FG circular plate (b) FG elliptic plate

Fig. 3 Three-dimensional mode shapes of clamped (a) circular and (b) elliptic FG plate with $k$ = 1

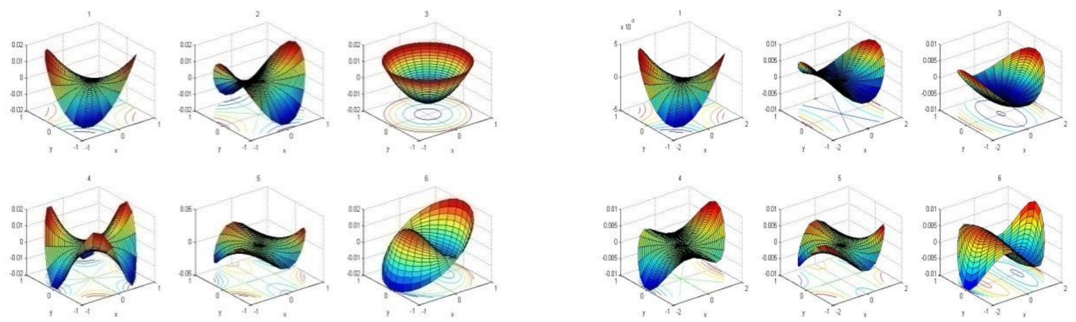
## 7. Conclusions

The analysis of natural frequencies and mode shapes of functionally graded elliptic and circular plates with various boundary conditions based on classical plate theory is proposed in the present



(a) FG circular plate (b) FG elliptic plate

Fig. 4 Three-dimensional mode shapes of simply-supported (a) circular and (b) elliptic FG plate with $k$=1



(a) FG circular plate (b) FG elliptic plate

Fig. 5Three-dimensional mode shapes of free (a) circular and (b) elliptic FG plate with $k$=1

investigation. Generalized eigenfrequency equation for free vibration can be obtained by means of Rayleigh-Ritz method. Trial functions denoting displacement component may handle any boundary condition very easily. Looking into the present modelling and results, one may conclude as follows.

• The aspect ratios (*a/b*), power-law indices ($k$) and different material distributions play key factors to study free vibration characteristics of FG elliptic (or circular) plates.

• In Rayleigh-Ritz method, increase in the number of polynomials ($n$) play a crucial role in the convergence of frequency parameters.

• From Tables and Figers, it is observed that the frequencies are increasing with increase in aspect ratios for a fixed power-law index and are decreasing with increase in power-law exponents for a fixed aspect ratio while assuming clamped and simply supported BCs, whereas a fluctuating order of frequencies can be seen in case of free FG elliptic plates.

• Assuming effect of Poisson's ratio ($v$), one may see that frequencies are independent of $v$ for clamped elliptic plates. But frequencies are increasing with increase in $v$ in case of simply supported and are showing fluctuating behavior while considering free edge condition, keeping both *a/b* and $k$ fixed.

• Other shear deformation plate theories can also be extended easily following the above analysis.

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