Structural Engineering and Mechanics, *Vol. 52, No. 4 (2014) 723-738*
DOI: http://dx.doi.org/10.12989/sem.2014.52.4.723

Performance of non-prismatic simply supported prestressed concrete beams

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(Received March 7, 2013, Revised May 5, 2014, Accepted May 26, 2014)

Abstract. Prestressing is the most commonly employed technique in bridges and long span beams in commercial buildings as prestressing results in slender section with higher load carrying capacities. This work is an attempt to study the performance of a minimum weight prestressed concrete beam adopting a non-prismatic section so that there will be a reduction in the volume of concrete which in turn reduces the self-weight of the structure. The effect of adopting a non-prismatic section on parameters like prestressing force, area of prestressing steel, bending stresses, shear stresses and percentage loss of prestress are established theoretically. The analysis of non-prismatic prestressed beams is based on the assumption of pure bending theory. Equations are derived for dead load bending moment, eccentricity, and depth at any required section. Based on these equations an algorithm is developed which does the stress checks for the given section for every 500 mm interval of the span. Limit state method is used for the design of beam and finite difference method is used for finding out the deflection of a non-prismatic beam. All the parameters of non-prismatic prestressed concrete beams are compared with that of the rectangular prestressed concrete members and observed that minimum weight design and economical design are not same. Minimum weight design results in the increase in required area of prestressing steel.

Keywords: prestressing; non-prismatic; deflection; finite difference method; eccentricity; loss of prestress; limit state method; economical design

1. Introduction

1.1 Importance of prestressing

Prestressed concrete is the most important of the major forms of construction to be introduced into Structural Engineering systems in 19th century. The idea of prestressing, or preloading, a structure is not new. Barrels and Bullock cart wheels were, and still are, made from separate wooden staves, kept in place by metal hoops. Within the field of building structures, most

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http://www.techno-press.org/?journal=sem&subpage=8 ISSN: 1225-4568 (Print), 1598-6217 (Online)

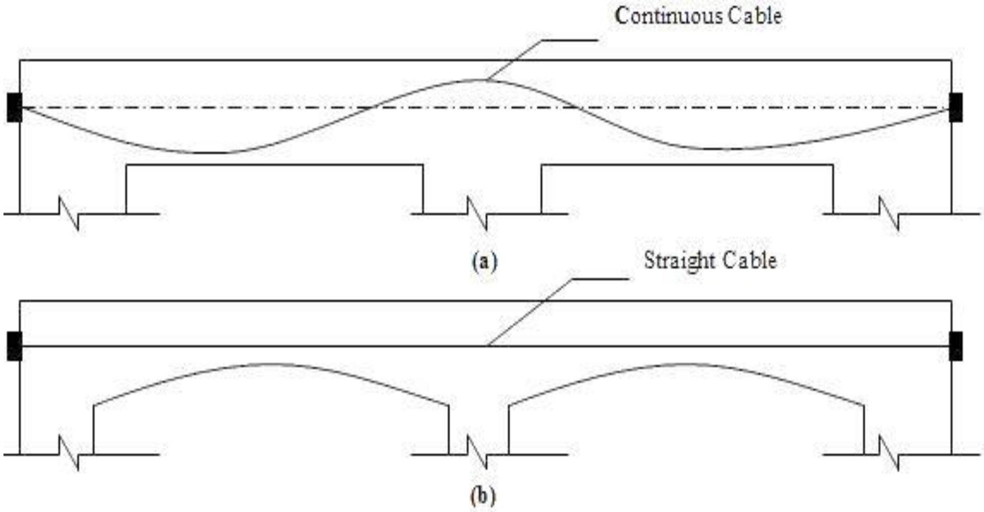

Fig. 1 Typical cable profiles of continuous beam

prestressed concrete applications are in the form of simply supported precast floors and roof beams. Where large spans are required, in situ prestressed concrete beams are sometimes used, and in situ prestressed concrete flat slab construction is also increasingly being employed. In the field of bridge engineering, the introduction of prestressed concrete has aided the construction of long-span concrete bridges. The introduction of ranges of standard beam sections has simplified the design and construction of these bridges.

1.2 Prestressed non-prismatic beams

Reinforced prismatic concrete beams are uneconomical for large spans as the cross section required to resist the load is heavier. Therefore, prestressed prismatic beams came into existence in which slender sections can be deigned to resist more load. By using non-prismatic prestressed beams, the section can be designed with varying depth, hence there is saving in the volume of concrete with controlled deflection when compared with the prismatic prestressed concrete beams. Therefore this research is an attempt to establish that the performance of a prestressed non-prismatic simply supported beam is superior to that of a rectangular PSC beam.

1.3 Prismatic and non-prismatic prestressed beams

In prismatic beams the entire section is not stressed to its design strength and hence the cross-section is not effectively utilized. Non-prismatic beams optimize the distribution of weight and strength to achieve a better distribution of internal stresses and hence reduce the dead load. Prismatic beams are not economical in case of large spans as the depth required for resisting a given load will be high. In case of non-prismatic beams we can achieve the required strength with minimum weight and material as the depth is varied throughout the section. Therefore non-prismatic beams are economical in case of large spans. Prismatic beams are susceptible to deflect more as the depth at any section is constant throughout the span.

In case of prismatic continuous beams, the tendons are to be placed in parabolic profile at center and at supports the tendon is to be placed above the centroid as there is negative bending moment at supports. To achieve continuity, the cable should be positioned as shown in Fig. 1(a). In case of non-prismatic beams, the cable can be straight. This is because as the depth of the section varies, the centroid shifts up and has a parabolic profile and thus the cable can be positioned straight throughout the continuous span as shown in Fig. 1(b). The straight tendon profile in case of Non-Prismatic prestressed concrete beam can be approximated to the case of a parabolic tendon in a prismatic prestressed concrete beam. As the depth is varied along the length of the beam, the volume of concrete in case of Non-Prismatic beams decreases and hence reduces the self-weight of the beam when compared with the rectangular prestressed concrete beam.

2. Literature review

Different approaches have been developed for the analysis and design of non-prismatic members. Ramakrishnan and Carlo, Jr. (1972) proposed a limit design theory for the design of prismatic and non-prismatic prestressed concrete girder subjected to moving loads. Taylor (1987) has shown that the number of unknown variables makes the design of non-prismatic beams inefficient and complicated. Taylor (1987) have computed the section at every point in the span as required by the loads, and adjusted in a simple manner to minimize the cost. El-Mezani *et al*. (1991) demonstrated the problems due to discontinuity of member axis and arching effect in analysing non-prismatic members. Rossow (1996), referring to more refined mathematical and experimental studies, concluded that, for practical design applications, simple bending theory can be used without incurring significant error. Khan and Al-Gahtani worked on analysis of continuous non-prismatic beams using boundary procedures. Balkaya (2001) investigated the behavior of the non-prismatic members having T-sections and computed the fixed end moments, bending stiffness coefficients and carry over factors from the three dimensional finite element models by considering thrust effects. Brojan (2007) analysed bending of non-prismatic beams made of Ludwick type material with different stress–strain relationships in tension and compression. Bahadir Yusksel (2009) proposed an approach for determining temperature effects on symmetrically haunched non-prismatic members. Miha Brojan (2011) made a study on large deflections of slender, non-prismatic cantilever beams subjected to a combined loading of non-uniformly distributed continuous load over the span and a concentrated load at the free end of the beam. Bahadir Yuksel (2012) proposed effective formulas and estimated dimensionless coefficients to predict the fixed end moments, forces and stiffness coefficients and carry-over factors with reasonable accuracy for the analysis and re-valuation of the non-prismatic beams having symmetrical parabolic haunches. He adopted a constant haunch length ratio of 0.5 using FEM. Sassari (2012) studied the vibration analysis of non-prismatic beams under variable axial forces and formulated a procedure for the analysis.

Based on the literature, it is clear that the non-prismatic beams are advantageous when compared to prismatic prestressed concrete beams. However they involve complexity in the analysis and design as the number of unknown variables increases compared to prismatic beams. However, little literature is available on performance of prestressed non-prismatic beams. In the present study, a simply supported beam with parabolic haunch and having straight tendon profile was considered for the study. A methodology is developed for the analysis and design of non-prismatic prestressed concrete beams and its performance in terms of various parameters was studied.

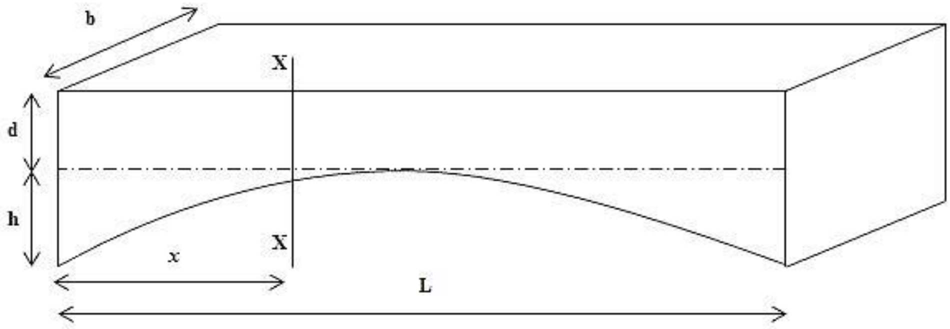

Fig. 2 Details of non-prismatic beam

3. Objective and scope

The objective of this work is to establish a procedure for design of a minimum weight prestressed beam adopting a non-prismatic section and compare its performance with the prismatic prestressed concrete beam. A program is developed for the analysis and design of this non-prismatic and prismatic prestressed simply supported beam based on the procedure. This gives the cross-section of the beam with the area of prestressing steel and the amount of eccentricity to be provided for a given span and loading. The volume of concrete obtained for non-prismatic section and equivalent prismatic section are compared for constant span with varying load and vice-versa. The variation in percentage saving in volume of concrete in each span with varying load is graphically shown.

The analysis of non-prismatic prestressed beams is based on the assumptions of pure bending theory. Limit state method is adopted for the design of both non-prismatic and prismatic sections subjected to uniformly distributed load. For finding out the deflection in non-prismatic beams, a finite difference method is used. It is assumed that the stress induced in the beam is by pre-tensioning and hence losses corresponding to pre-tensioning are only considered. Span and load are limited to 25 m and 80 kN respectively.

4. Practical application

This kind of non-prismatic prestressed beams can be applied in medium span beams in structures like commercial buildings and bridges. In case of continuous beams, this concept can be used easily as the tendons can be kept straight without any parabolic profile. It is suitable for culverts and for beams of buildings (especially multi-storied) as they enhance the aesthetics of the structure in which they are adopted.

5. Methodology for analysis

Analysis of non-prismatic prestressed beams is based on the assumptions of pure bending theory. It mainly includes the determination of the following parameters at any section along the length of the beam.

5.1 Depth, moment of inertia and section modulus

The variation of the parabolic haunch (profile) in Fig. 2 is given by

$$y_x = a_0 + a_1 x + a_2 x^2 \tag{1}$$

Here
b= Width of the beam.
d= depth of the beam
h= height of the parabola at the centre.
L= length of the beam.
Using the boundary conditions with left end as origin,

$$a_0 = 0\,,\ a_1 = \frac{4h}{L}\ ,\ a_2 = -\frac{4h}{L^2}$$

$$y_x = \frac{4hx}{L} \times \left(1 - \frac{x}{L}\right)$$

The expressions for depth, moment of inertia and section modulus at any section along the length of the beam are obtained as follows.

Depth at any section (d_x) is given by

$$d_x = d + h - y$$

$$d_x = d + h + \frac{4h}{L^2}x^2 - \frac{4h}{L}x \tag{2}$$

Moment of inertia (I_{xx}) at any section is given by

$$I_{xx} = \frac{b\left(d + h + \frac{4h}{L^2}x^2 - \frac{4h}{L}x\right)^3}{12} \tag{3}$$

Section modulus ($Z_t(x)$ and $Z_b(x)$) at any section is given by

$$Zt(x) = Zb(x) = \frac{b\left(d + h + \frac{4h}{L^2}x^2 - \frac{4h}{L}x\right)^2}{6} \tag{4}$$

Area (A_x) at any section is given by

$$A_x = b \times d_x$$

$$A_x = b\left(d + h + \frac{4h}{L^2}x^2 - \frac{4h}{L}x\right) \tag{5}$$

5.2 Dead and live load shear and bending moment

For finding out the dead load bending moment and shear force at any section, Principle of superposition is used. The bending moment due to the rectangular portion of width b and depth (d+h) minus bending moment due to parabolic haunch portion is calculated.

At any section *X-X*, the dead load bending moment is given by

MD(x) = Moment due to rectangular beam of width b, depth d and span L - Moment due to the parabolic section at *X-X*

= *Mr(x) – Mp(x)*

$$MD(x) = [b(d+h)12x)](L-x) - \left[\left(\frac{h}{3L^2}x^4 - \frac{2h}{3L}x^3 + \frac{hL}{3}x\right)24b\right] \tag{6}$$

In similar lines, the dead load shear force at any section for the parabolic portion is given by

$$VD\ (x) = Vr\ (x) - Vp\ (x)$$

$$VD(x) = [b(d+h)12x)](L-x) - \left[\left(\frac{4h}{3L^2}x^3 - \frac{4h}{2L}x^2 + \frac{hL}{3}\right)24b\right] \tag{7}$$

Bending moment due to live load is given by

$$ML(x) = 0.5Wx \times (L-x) \tag{8}$$

Where

W = Total live load per unit length

Total bending moment is (M_x) is given by

$$M_x = MD(x) + ML(x) \tag{9}$$

Shear force due to live load $VL(x)$ is given by

$$VL(x) = W \times (0.5L - x) \tag{10}$$

Total shear force is (V_x) is given by

$$V_x = VD(x) + VL(x) \tag{11}$$

5.3 Eccentricity

Eccentricity at any section is derived by considering the two possible cases (Figs. 3 and 4) for achieving the effective prestressing in the compression zone. In the derivation of eccentricity equation a non-dimensional parameter k is introduced which is defined as the ratio of the depth of the rectangular portion of the beam 'd' to the total depth of beam (d+h).

To achieve the compressive stresses in fibers below the neutral axis, the following two possible cases are evolved.

Case 1:

Tendon positioned below the neutral axis as shown in Fig. 3.

From Fig. 3, depth at any section *X-X* is given by

$$d_x = e1 + e2 + e3$$

Where

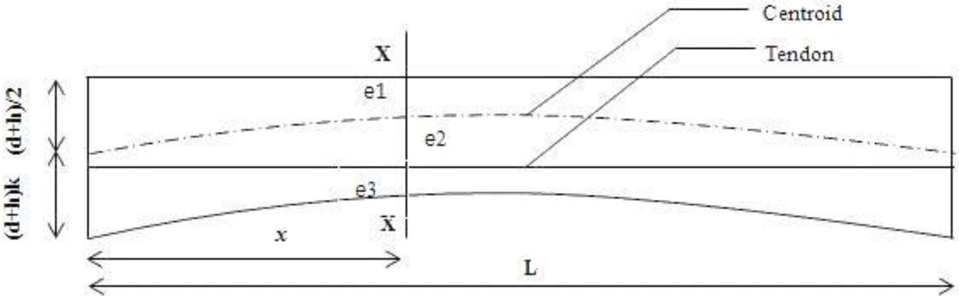

Fig. 3 Beam with tendon below centroid

$e1 = \frac{d_x}{2}$, $e2 = e_x$, $e3 = (d+h)k - y_x$

Where the non-dimensional parameter k is defined by

$$k = \frac{h}{d+h}$$

and e_x= eccentricity at section *X-X*

$$d_x = \frac{d_x}{2} + e_x + (d-h)k - y_x$$

$$e_x = \frac{d_x}{2} - (d+h)k + y_x$$

But

$$d_x = d + h + y_x$$

Substituting d_x in e_x

$$e_x = (d+h) \times (0.5 - k) + \frac{y_x}{2} \tag{12}$$

Case 2:

Tendon positioned partially above the neutral axis at supports and below the neutral axis at the center as shown in figure below.

From Fig. 3, depth at any section *X-X* is given by

$$d_x = e1 + e2 + e3$$

Where

$e1 = \frac{d_x}{2}$, $e2 = e_x$, $e3 = y_x$

$$dx = (d+h)(1-k) + e_x + \frac{d_x}{2}$$

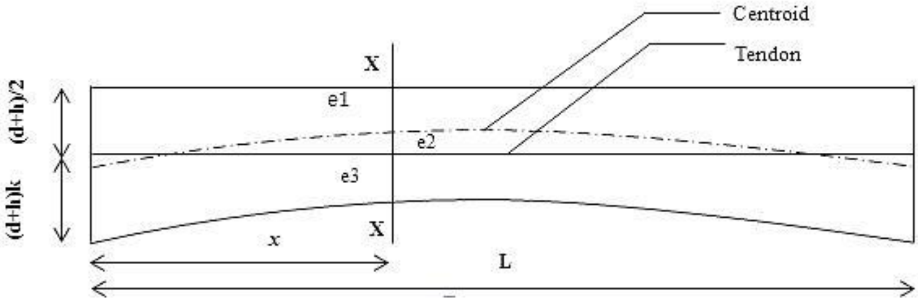

Fig. 4 Beam with tendon below the centroid

$$e_x = \frac{d+h-y_x}{2} - (d+h)(1-k)$$

Substituting d_x in e_x

$$e_x = (d+h) \times (k-0.5) - \frac{y_x}{2} \quad (13)$$

Case 2 involves positioning of the tendon above the neutral axis which leads to development of compressive stresses in fibers above the neutral axis at supports. This is of little practical applicability and hence this case is ruled out in this research work.

Range for k:

'k' value depends on the positioning of the tendon from neutral axis. As Case 2 is ruled out, k value depends on Case 1 alone and the range for k is given by

$$\frac{h}{(d+h)} < k < 0.5$$

The maximum eccentricity that can be provided at any section is given by

$$e_{\max} = \frac{(d+h)}{2} + \frac{y_x}{2} \quad (14)$$

The minimum eccentricity that can be provided at any section is given by

$$e_{\min} = \frac{y_x}{2} \quad (15)$$

5.4 Deflection

For obtaining the deflection equation for the beam double integration method and finite differences method are used. As the deflection equations obtained by double integration method involve complex numbers, it is not easy to solve them. So in this work, finite difference method has been adopted as it more suitable for irregular cross-sections. The method was adopted by discretizing the beam into 2, 4, 6 and 8 parts. The expressions for beam corresponding to discretization of 8 parts alone are presented below. The deflection equations at transfer stage is given by

$$y_t = -\frac{3L^2\left\{-bL^2\left[\frac{21}{16}d + \frac{175h}{512}\right] + P\left[(d+h)(0.5-k) + \frac{7h}{32}\right]\right\}}{16 \times E \times b \times \left(d + \frac{9h}{16}\right)^3}$$
$$-\frac{12L^2\left\{-bL^2\left[3d + \frac{h}{2}\right] + P\left[(d+h)(0.5-k) + \frac{h}{2}\right]\right\}}{32 \times E \times b \times d^3}$$
$$-\frac{9L^2\left\{-3bL^2\left[\frac{45}{16}d + \frac{255h}{512}\right] + P\left[(d+h)(0.5-k) + \frac{15h}{32}\right]\right\}}{16 \times E \times b \times \left(d + \frac{h}{16}\right)^3}$$
$$-\frac{6L^2\left\{-3bL^2\left[3d + \frac{5h}{8}\right] + 4P\left[(d+h)(0.5-k) + \frac{3h}{8}\right]\right\}}{16 \times E \times b \times \left(d + \frac{h}{4}\right)^3} \tag{16}$$

The deflection equation at working stage is given by

$$y_b = -\frac{3L^2\left\{-bL^2\left[\frac{21}{16}d + \frac{175h}{512}\right] + P\left[(d+h)(0.5-k) + \frac{7h}{32}\right] - \frac{7}{128}WL^2\right\}}{16 \times E \times b \times \left(d + \frac{9h}{16}\right)^3}$$
$$-\frac{12L^2\left\{-bL^2\left[3d + \frac{h}{2}\right] + P\left[(d+h)(0.5-k) + \frac{h}{2}\right] - \frac{1}{8}WL^2\right\}}{32 \times E \times b \times d^3}$$
$$-\frac{9L^2\left\{-3bL^2\left[\frac{45}{16}d + \frac{255h}{512}\right] + P\left[(d+h)(0.5-k) + \frac{15h}{32}\right] - \frac{15}{128}WL^2\right\}}{16 \times E \times b \times \left(d + \frac{h}{16}\right)^3}$$
$$-\frac{6L^2\left\{-3bL^2\left[3d + \frac{5h}{8}\right] + 4P\left[(d+h)(0.5-k) + \frac{3h}{8}\right] - \frac{3}{32}WL^2\right\}}{16 \times E \times b \times \left(d + \frac{h}{4}\right)^3} \tag{17}$$

Here

P=Prestressing Force

E=Young's modulus of concrete

6. Results and discussions

An algorithm and the corresponding program have been developed based on the analytical procedure and methodology discussed and developed in the previous sections. The program is helpful in quickly choosing the design parameters for arriving at an economical non-prismatic prestressed concrete section. The flow chart corresponding to this Algorithm is shown in Fig. 5.

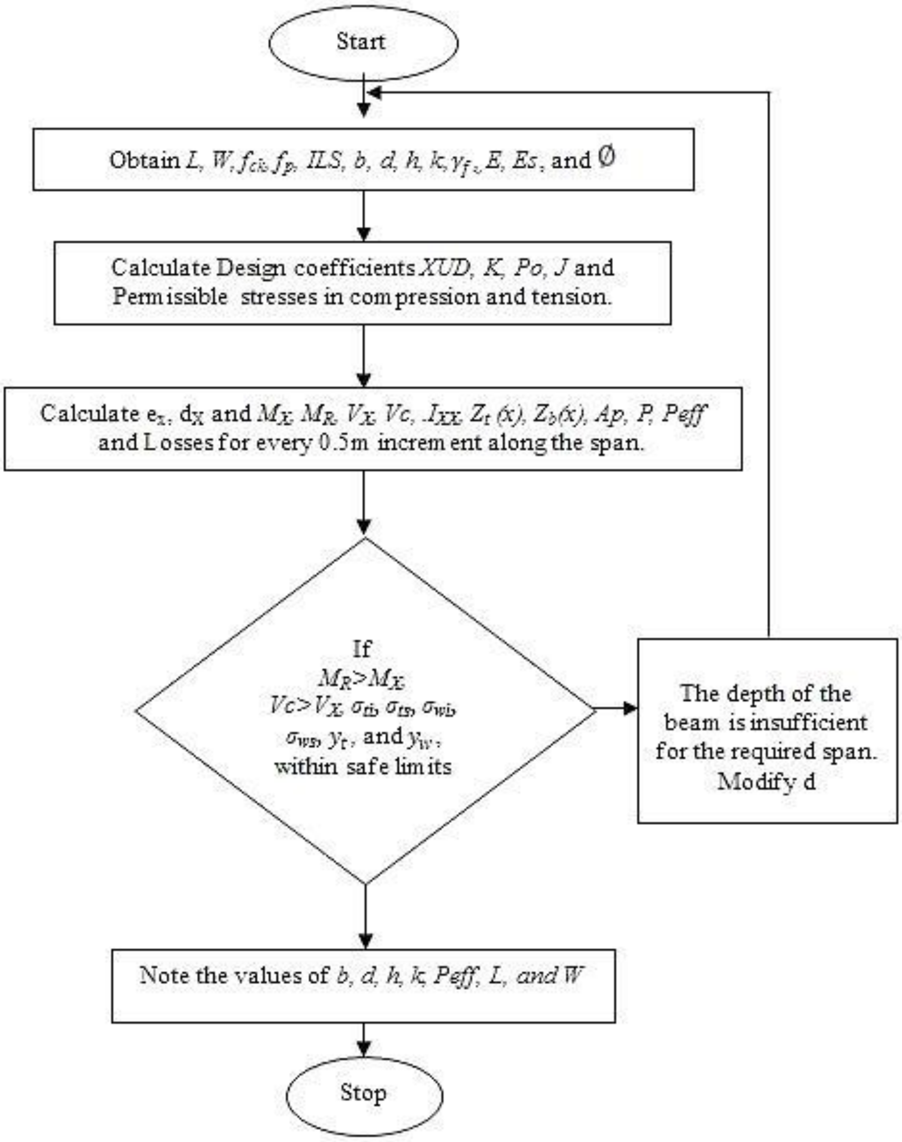

Fig. 5 Flowchart for the analytical procedure

The following notation is adopted in the flow chart.
f_{ck}= Characteristic strength of concrete = 40 N/mm^2
f_p= Initial stress in steel = 1400 N/mm^2
ILS = Initial Loss in prestress = 30 %
γ_f = Partial safety factor = 1.15
ϕ = Creep coefficient = 1.6
E_s = Young's modulus for the material of the prestressing steel
XUD, K, Po, J = Neutral axis coefficients
MR = Moment of resistance of the section
V_c = Shear capacity of the section
A_p = Area of prestressing steel
P_{eff} = Net prestressing force after considering losses.
σ_{ti} = Tensile stresses at bottom fiber at transfer state.
σ_{ts} = Tensile stresses at top fibber at transfer state.
σ_{wi}= Tensile stresses at bottom fiber at working state.
σ_{ws}= Tensile stress at top fiber at working state.

Based on program, beams of span up to 25 m are analysed with an increment of 5 m in each case and load being varied from 10 kN to 80 kN with an increment of 10 kN. Graphs are plotted for different parameters for a constant span and varying load. Parameter *k* is considered to be constant for both prismatic as well as non-prismatic prestressed concrete beams. The haunch depth h is obtained by fixing the sum (*d*+*h*) equal to the depth obtained for prismatic beam and *h* is varied to obtain the optimum non-prismatic prestressed concrete section. The variation of haunch depth with constant span and varying load and variation of haunch depth with constant load with varying span is also shown.

For a given span, the % decrease in volume decreases with increase in live load. However the % decrease is more as the span increases.

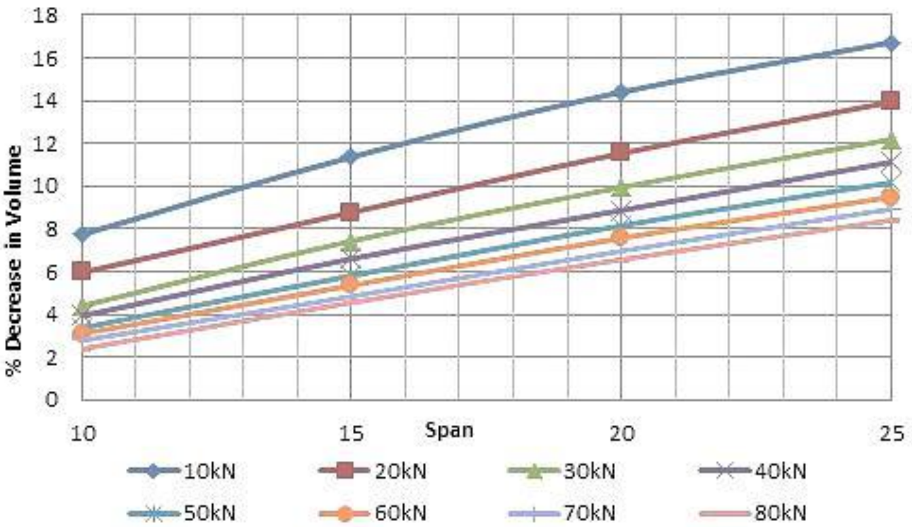

Fig. 6 Variation of % decrease in volume with varying spans and constant load

Table 1 Range for volume

Parameter	Span	Range (%)	Load (kN)	Remarks
Volume	10	2.38	80	The decrease in volume decreases for higher load carrying capacities and increase with the increase in span.
	25	16.72	10	

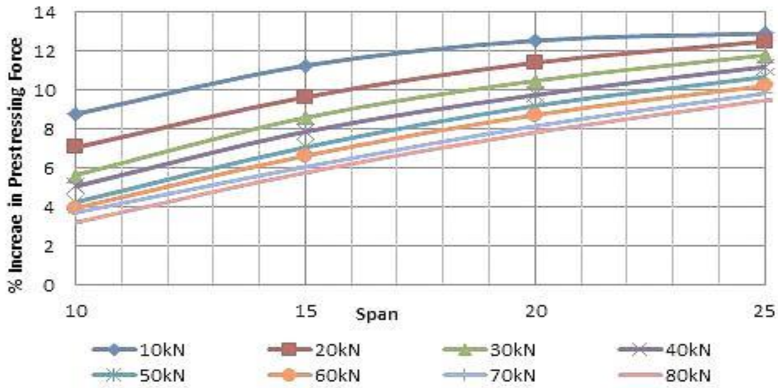

Fig. 7 Variation of % increase in prestressing force with varying span and constant load

For a given span, the % increase in prestressing force increases with increase in loads. However the % increase is more as the span increases.

Table 2 Range for prestressing force

Parameter	Span	Range (%)	Load (kN)	Remarks
Prestressing force	10	3.22	80	The increase in prestressing force decreases for higher load carrying capacities and increases with the increase in span.
	25	12.92	10	

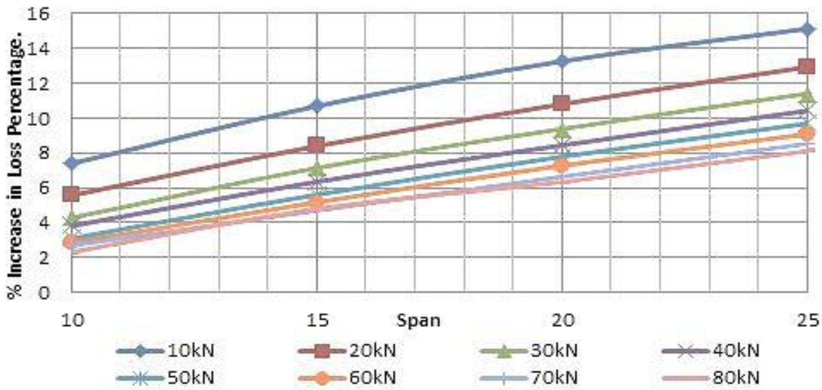

Fig. 8 Variation of % increase in loss percentage with varying span and constant load

For a given span, the % increase in loss percentage decreases with increase in loads. However the % increase is more as the span increases.

Table 3 Range for loss percentage

Parameter	Span	Range (%)	Load (kN)	Remarks
Loss percentage	10	2.31	80	The increase in loss percentage decreases for higher load carrying capacities and increases with the increase in span.
	25	15.31	10	

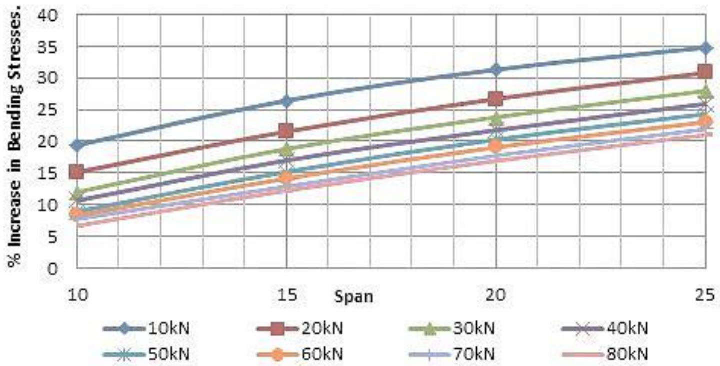

Fig. 9 Variation of % increase in bending stress with varying span and constant load

For a given span, % increase in bending stresses decreases with increase in load. Further, the %increase increases with span.

Table 4 Range for bending stress

Parameter	Span	Range (%)	Load (kN)	Remarks
Bending stress	10	6.65	80	The increase in bending stresses decreases for higher load carrying capacities and increases with the increase in span.
	25	34.77	10	

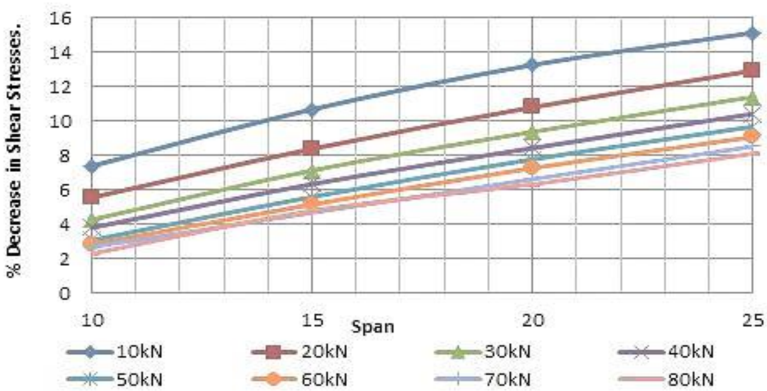

Fig. 10 Variation of % decrease in shear stress with varying span and constant load

For a given span, the % decrease in shear stresses decrease with increase in loads. However the %decrease increases with increasing spans.

Table 5 Range for shear stress

Parameter	Span	Range (%)	Load (kN)	Remarks
Shear stress	10	2.31	80	The decrease in shear stress decreases for higher load carrying capacities and increases with the increase in span.
	25	15.31	10	

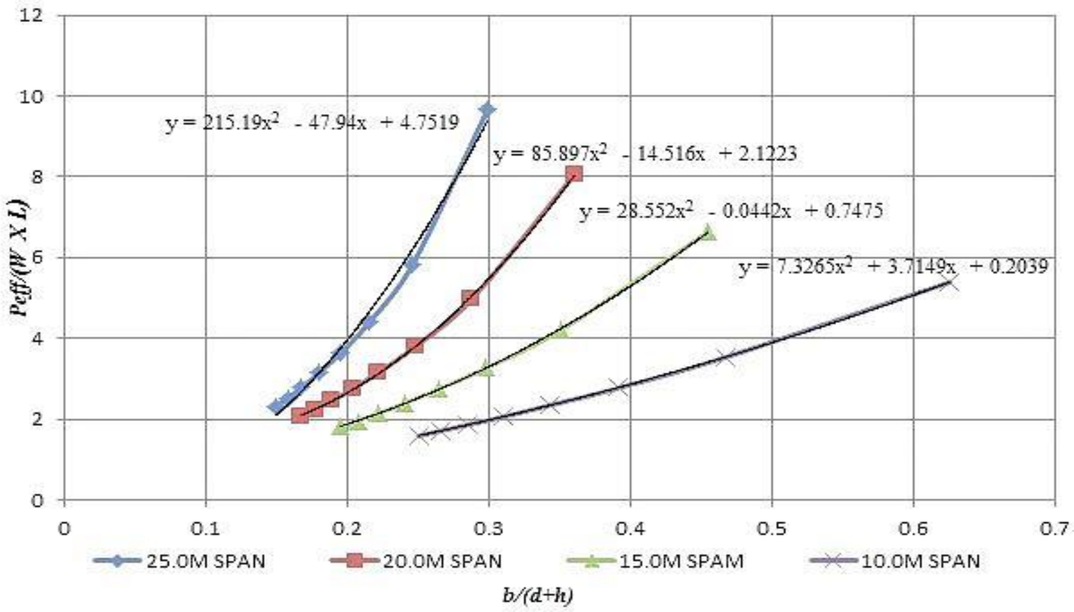

Fig. 11 Interaction curves

Interaction curves

Based on the results obtained, an interaction curve is plotted such that for a given span and known *b*, *d* & *h* values the required prestressing force can be obtained. Non-dimensional parameters *b/(d+h)* and $P_{eff}/(W \times L)$ are used for plotting along X-axis and Y-axis.

The equation of the interaction curves are in the form

$$Y = P_0 X^2 + QX + R \tag{18}$$

Where P_0, Q, R are constants and $X = \dfrac{b}{(d+h)}$, $Y = \dfrac{P_{eff}}{(W \times L)}$

To find the trend line of the constants P_O, Q and R, graphs are plotted for constants as a function of span and they are as follows. The following values where obtained from graph.

From the equations obtained from the graphs the constants are a function of span and can be modified as follows.

$$P_0 = -0.0478L^3 - 1.4273L^2 + 17.237L - 70.08 \tag{19}$$

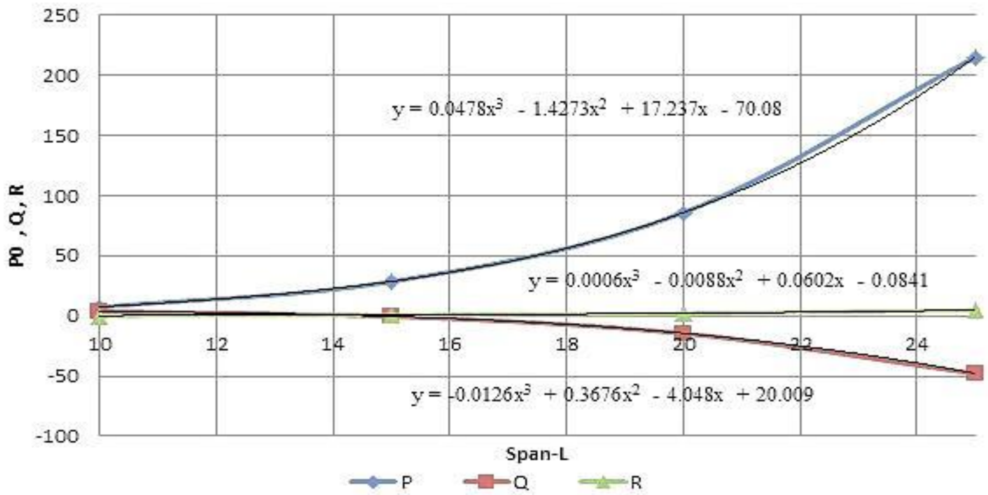

Fig. 12 Curve for constants *Po*, *Q* and *R*

$$Q = -0.0126L^3 + 0.3676L^2 - 4.048L + 20.009 \tag{20}$$

$$R = 0.0006L^3 + 0.0088L^2 + 0.0602L - 0.0841 \tag{21}$$

From Eqs. (18), (18), (19), (20) and (21)

$$\begin{aligned} \frac{P_{eff}}{(W \times L)} &= \left[0.0478 \times L^3 - 1.47273 \times L^2 + 17.237 \times L - 70.08\right] \times \left(\frac{b}{d+h}\right)^2 \\ &+ \left[0.0126 \times L^3 + 0.3676 \times L^2 + 4.048 \times L + 20.009\right] \times \left(\frac{b}{d+h}\right) \\ &+ \left[0.0006 \times L^3 - 0.0088 \times L^2 + 0.0602 \times L + 0.0841\right] \end{aligned} \tag{22}$$

7. Conclusions

It can be concluded from the results discussed in this work that Finite difference method is handy for calculating deflections for non-prismatic beams. For continuous non-prismatic beams, prestressing cables can be placed straight throughout without placing them in a parabolic profile to have superior performance. Further, it is observed that there is a better utilization of material of the cross-section for a non-prismatic simply supported prestressed concrete beam. It is also observed that decrease in the cross-section reduces volume in concrete which in turn decreases the weight of the structure, thus satisfying minimum weight design criteria. Decrease in volume increases the prestressing steel required as the section required to resist the bending moment is decreased which is not an economical design for short spans. Therefore for a non-prismatic beam, an economical design can be achieved for longer spans as the volume of concrete decreases, required prestressing force decreases and losses also decrease.

Acknowledgments

This paper is based on the work carried out as a part of M.E course of T. Raghuram Sandeep. The authors thank the department of civil engineering, Andhra University, Visakhapatnam as the work presented is the M.E thesis work submitted to it.

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