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Failure analysis of composite plates under static and dynamic loading

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Abstract. The present paper deals with the first ply failure analysis of the laminated composite plates under various static and dynamic loading conditions. Static analysis has been carried out under patch load and triangular load. The dynamic failure analysis has been carried out under triangular pulse load. The formulation has been carried out using the finite element method and a computer code has been developed. The first order shear deformation theory has been applied in the present formulation. The displacement time history analysis of laminated composite plate has been carried out and the results are compared with those published in literature to validate the formulation. The first ply failure load for laminated composite plates with various lamination schemes under static and dynamic loading conditions has been calculated using various failure criteria. The failure index-time history analysis has also been carried out and presented in this paper.

Keywords: laminated composites; first ply failure; pulse load, finite element; dynamic failure

1. Introduction

The fibre reinforced composite laminates are extensively used in the fields of aerospace and marine structures, bridges and other high performance structures. These structures need high reliability assurance in their service life. For the safe design of such structures, the necessity of prediction of sustainable load by the structure is indispensable. The failure analysis of laminated composites is much more complex than that of isotropic materials. The laminate under flexural bending is subjected to non- uniform stress distribution in the laminae and the prediction of failure load becomes complicated. Failure in an anisotropic material is a gradual process and starts with the failure of the weakest ply in the laminate. Various strength-based failure criteria are already developed and extensively applied to predict the failure load of the laminated composite plates.

Extensive research work on the flexural failure of composite laminates under uniformly distributed static load has been carried out by many researchers. A finite element procedure based on the first order shear deformation theory subjected to in plane and transverse uniformly distributed load using 4 noded and 9 noded plate elements has been carried out by Reddy and

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Pandey (1987).

The dynamic response analysis of antisymmetric angle-ply laminated composite plates under arbitrary loading is presented by Khdeir and Reddy (1988). Initial and progressive failure analysis of laminated composite structures under dynamic loading has been carried out by Bogdanovich *et al*. (1994). An analytical procedure for studying the dynamic response of antisymmetric angle-ply laminated plates has been developed by Khdeir (1994). A 20 noded isoparametric parabolic solid element for laminated anisotropic plates has been proposed for the static and dynamic analysis by Meimaris and Day (1995). A method has been developed for predicting the linear transient response to dynamic loading of rectangular homogeneous and heterogeneous laminated plates by Chen and Dawe (1996). Dynamic response analysis has been carried out using modified nine noded degenerated shell by Swaddinudhipong and Liu (1997). A first ply failure analysis of laminated composite stiffened plates using eight noded isoparametric plate bending element has been carried out by Ray and Satsangi (1999). The transient dynamic analysis has been carried out using three dimensional higher order theory to study the behaviour of anti-symmetric angle ply laminates by Han *et al*. (2003). The three dimensional layer- wise mixed finite element method to perform the free vibration and first ply failure analysis of laminated composite plates has been developed by Ramtekkar *et al*. (2003, 2004). The dynamic behaviour under low velocity impact load has been investigated for nonprismatic composite folded plate by Kyong and Samuel (2005). A quasi conforming shell element has been used for the static and dynamic analysis of laminated composite plates and shells by Park *et al*. (2006). The plates with different stiffener configurations have been analysed under blast load by Kadid *et al*. (2007). Progressive failure analysis using Tai-Hill and Tsai-Wu theories have been carried out based on the micromechanical approach by Mondaca *et al*. (2012). The structural vibration based on Mindlin's plate theory and Hamiltonian formulation has been studied by Ni and Hu (2012). Although first ply failure analysis under static transverse load have been carried out by many researchers, the results on the failure analysis under patch and triangular loading has not been reported in the published literature. The studies on the first ply failure analysis of laminated composite plates under dynamic pulse loads based on various strength related failure criteria are also scarce in the published literature. The objective of the present paper is to study the failure behaviour of laminated composite plates with varying lamination sequences under various static and dynamic loading conditions.

2. Finite element formulation

The plate element has been formulated using eight noded isoparametric quadratic plate bending element. The plate element consists of five degrees of freedom per node, three translations u, v, w and two rotations θx and θy. First order shear deformation theory based on Reissner and Mindlin's assumption for laminated plate has been applied in the present formulation. A shear correction factor has been taken into account to eliminate the discrepancy between the assumption of constant shear strain as per first order shear deformation theory and the actual parabolic variation in the shear strain. The shear correction factor for a rectangular section is considered to be 5/6. The mid-surface of the laminated plate is assumed as the reference plane.

2.1 Computation of stress and strain in the laminae

The stress resultants of the plate are calculated as

$$\{F\}=[D][B]\{\delta_e\} \tag{1}$$

where $\{F\}$ is the vector representing stress resultants. [B] is the strain -displacement matrix of the plate element. $[D]$ is the rigidity matrix of the laminate. δ_e is the displacement vector to be calculated under static or dynamic loading. The rigidity matrix of the laminated plate is expressed by

$$[D]=\begin{bmatrix}[A_{ij}] & [B_{ij}] & 0\\ [B_{ij}] & [D_{ij}] & 0\\ 0 & 0 & [S_{ij}]\end{bmatrix} \tag{2}$$

where the extensional, bending-stretching and bending stiffness of a laminate are expressed in the usual form as

$$(A_{ij},B_{ij},D_{ij})=\sum_{k=1}^{n}\int_{z_{k-1}}^{z_h}(\overline{Q}_{ij})_k\,(1,z,z^2)dz,\quad i,\ j=1,2,6 \tag{3}$$

Similarly, the shear stiffness is expressed as

$$(S_{ij})=\sum_{k=1}^{n}\int_{z_{k-1}}^{z_h}\alpha\,(\overline{Q}_{ij})_k\ dz,\quad i,\ j=4,5 \tag{4}$$

α is the shear correction factor.

The strain components at the mid plane of the laminate can be expressed as

$$\varepsilon=[D]^{-1}\{F\} \tag{5}$$

The strain components at each lamina with respect to the global axis system can be given by

$$\begin{aligned}\varepsilon_x\ (z)&=\varepsilon_x^0+z\kappa_x\\ \varepsilon_y\ (z)&=\varepsilon_y^0+z\kappa_y\\ \varepsilon_s\ (z)&=\varepsilon_s^0+z\kappa_s\end{aligned} \tag{6}$$

where z is the distance of the lamina from the midplane of the laminate. The strain components with respect to the material axis system for kth lamina are expressed by

$$\begin{Bmatrix}\varepsilon_1\\ \varepsilon_2\\ \varepsilon_6\end{Bmatrix}=\begin{bmatrix}m^2 & n^2 & mn\\ n^2 & m^2 & -mn\\ -2mn & 2mn & m^2-n^2\end{bmatrix}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_s\end{Bmatrix} \tag{7}$$

The on axis stresses at k^{th} lamina are calculated from the following constitutive relationship

$$\begin{Bmatrix}\sigma_1\\ \sigma_2\\ \sigma_6\end{Bmatrix}=\begin{bmatrix}Q_{xx} & Q_{xy} & 0\\ Q_{xy} & Q_{yy} & 0\\ 0 & 0 & Q_{ss}\end{bmatrix}\begin{Bmatrix}\varepsilon_1\\ \varepsilon_2\\ \varepsilon_6\end{Bmatrix}_k \tag{8}$$

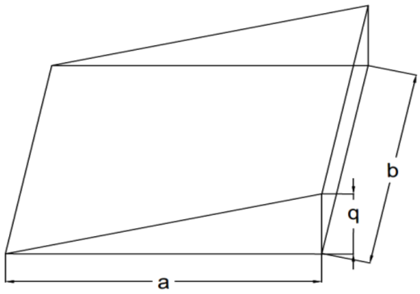
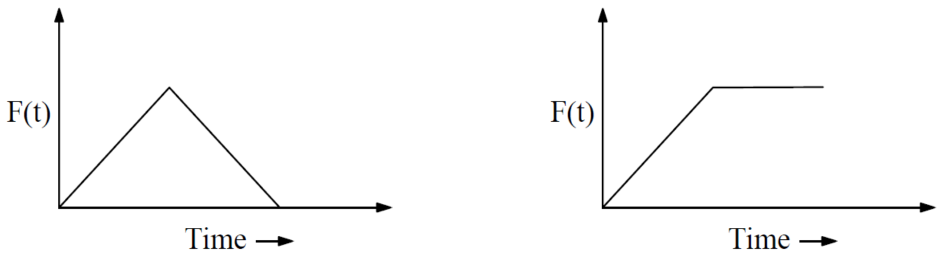


Fig. 1 Triangular load varying along x direction of the plate



(a) Triangular pulse (b) Step pulse

Fig. 2 Step and Triangular pulse

2.2 Static analysis

The basic equation to calculate the nodal displacements the plate element subjected to static loading is given by

$$[K]\{\delta\}=\{P\} \tag{9}$$

where $[K]$ is the overall stiffness matrix of the laminated plate, $\{\delta\}$ is the displacement vector and $\{P\}$ is the transversely applied triangular or patch load. The load vector is calculated as

$$\{P\}=\iint_A [N]q\,dxdy \tag{10}$$

$[N]$ is the shape function matrix of the plate element. The triangular load, $q=qx/a$ which varies only along x axis as shown in Fig.1.

2.3 Transient dynamic analysis

The dynamic problem without damping gives rise to a set of ordinary differential equations of the form

$$[M]\{\ddot{\delta}\}+[K]\{\delta\}=\{F_t\} \tag{11}$$

In which $\ddot{\delta}$ and δ are unknown acceleration vector and displacement vectors respectively. $[M]$

is the overall mass matrix of the laminated plate. The mass matrix has been computed using the lumped mass model. The dynamic loads considered in the present study are step and triangular pulse loads as shown in Fig. 2.

Newmark's time integration method has been adopted to solve the equation of motion for transient dynamic analysis. The nodal displacements at different time steps are computed. The stresses and strains at all the nodes for all the layers of the laminate at different time steps are calculated in the following manner.

2.4 First ply failure analysis method

A composite laminate consists of a number of plies and the failure occurs gradually from one ply to another with the increment of load. Failure of a composite laminate starts with that of the weakest lamina. Hence the failure of the weakest ply is the beginning of the entire failure process. Various failure theories based on macro- mechanical behavior are used to compute the load level at which initial failure occurs. The normal stresses and the shear stresses vary through the thickness of the laminates and hence the first ply failure analysis requires the computation of stresses and strains for all the element nodes at all layers of a laminate for a particular load level and then the magnitude of load is increased gradually. The nodal stresses for all the laminae are calculated for the increased load levels and maximum stresses are selected for each case. Then maximum stresses are compared with the strength of a lamina based on various existing failure criteria.

The computation of first-ply failure load is carried out using an iterative procedure. The procedure involves solving the stress problem for an initial load. This iteration is continued until the difference between any two consecutive failure loads is less than 1%.

The time history analysis of failure index (ratio of calculated stress and strength) has been carried out under dynamic pulse load and the range of load level is identified where first ply failure may occur. Failure index is calculated using Maximum stress, Tsai-Wu and Hoffman's failure criteria. The first ply failure load is then computed by an iteration process and required time has also been predicted. The following failure criteria are considered for the first ply failure analysis.

2.5 Various failure criteria

A number of existing failure theories are used to estimate of the failure load of a composite laminate. The strength based non-interactive failure criterion (maximum stress theory) and interactive criteria viz. Hoffman and Tsai-Wu have been used for the present investigation.

2.5.1 Maximum stress theory

According to this theory failure occurs when at least one of the following criteria is satisfied

$$\text{Failure index (FI)} = \frac{\sigma_1}{X},$$

$$\text{Failure index (FI)} = \frac{\sigma_2}{Y} \quad \text{and}$$

$$\text{Failure index (FI)} = \frac{\sigma_6}{S} \tag{12}$$

Where X and Y are tensile and compressive strengths according to the nature of σ_1 and σ_2 of the

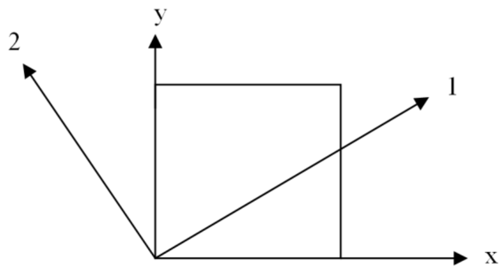


Fig. 3 Axis system of a lamina

lamina in which σ_1, σ_2 are the normal stress components and σ_6 is the in-plane shear stress and S is the shear strength. The material axis system (1, 2) in a lamina is shown in Fig. 3.

2.5.2 Hoffman's theory

The failure index (FI) according to this criterion is given by

$$FI = \frac{1}{2}\left(\frac{1}{X_T X_C} - \frac{1}{Y_T Y_C}\right)\sigma_1^2 + \frac{1}{2}\left(\frac{1}{Y_T Y_C} - \frac{1}{X_T X_C}\right)\sigma_2^2$$

$$+\frac{1}{2}\left(\frac{1}{X_T X_C} + \frac{1}{Y_T Y_C}\right)\sigma_1^2 - \sigma_2^2 + \left(\frac{1}{Y_T Y_C}\right)\sigma_1 + \left(\frac{\sigma_6}{S}\right)^2 \quad (13)$$

where X_T and X_C are the tensile and compressive strength respectively along X-axis and Y_C and Y_T are along Y-axis.

2.5.3 Tsai-Wu theory

According to this theory, the failure index is given by

$$FI = \left(\frac{1}{X_T} - \frac{1}{X_C}\right)\sigma_1 + \left(\frac{1}{Y_T} - \frac{1}{Y_C}\right)\sigma_2 + \frac{1}{X_T X_C}\sigma_1^2 + \frac{1}{Y_T Y_C}\sigma_2^2$$

$$-\frac{1}{2}\sqrt{X_T X_C Y_T Y_C}\,\sigma_1\sigma_2 + \left(\frac{\sigma_6}{S}\right)^2 \quad (14)$$

For the first ply failure, Failure index (FI) ≥ 1.

3. Numerical results

Example 1 Symmetric cross ply laminated simply supported plate subjected to udl

The rectangular simply supported plate of 9 in.×5 in. (228.6 mm×127 mm) dimensions with symmetric cross-ply (0°/90°/90°/0°) lamination scheme under uniformly distributed transverse loading of 0.1 psi (6.875×10^{-4} MPa) has been analysed in the present investigation. The graphite/epoxy composite material has been used as the constituent material of the lamina with following properties: E_1=132.38×103 MPa (19.2×106 psi), E_2=10.75×103 MPa (1.56×106 psi), G_{12}=G_{13}=5.65×103 MPa (0.82×106 psi), G_{23}=3.38×103 MPa (0.49×106 psi), ν_{12}=ν_{13}=0.24, ν_{23}=0.49, X_T=1.513×103 MPa (219.5×103 psi), X_C=1.696×103 MPa (246×103 psi), Y_T=Z_T=Y_C= Z_C=0.044×103 MPa (6.35×103 psi), ply thickness= 0.127 mm (0.005 in.). The full plate has been modeled using 8×8 mesh division. The first ply failure load is computed using the maximum stress theory and the interactive failure criteria viz. Tsai-Wu and Hoffman's failure criteria. The results in terms of the first ply failure load obtained in the present formulation are given in Table 1 and are compared with those available in Reddy and Pandey (1987). The comparisons show very good agreement.

Example 2 First ply failure analysis under static triangular load

The rectangular simply supported plate of 228.6mm×127mm dimensions with symmetric and antisymmetric cross-ply and angle-ply lamination schemes under triangular loading as shown in Fig. 1 has been analysed in the present investigation. The laminate consists of 4 layers. The graphite/epoxy composite material has been used as in Example 1. The full plate has been modeled using 8×8 mesh division. The location of failure is also indicated in Table 2. The mesh division along with node numbers are shown in Fig. 4. The first ply failure loads based on maximum stress, Hoffman and Tsai-Wu theories under static triangular loading are computed in the present investigation and presented in Table 2. Table 2 indicates that the first ply failure load is the highest for antisymmetric angle-ply laminate with 60°/-60°/60°/-60° lamination sequence and lowest for symmetric cross-ply lamination sequence.

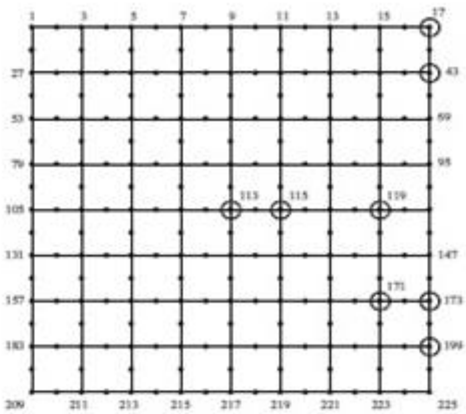


Fig. 4 Mesh division of plate with node numbers

Table 1 First ply failure load for symmetric cross-ply laminate under udl

Failure theory	First ply failure load in MPa (psi.)		
	Present (8 noded element)	Reddy and Pandey (1987) 4 noded element	Reddy and Pandey (1987) 9 noded element
Maximum stress	4.016(0.5823)	4.011(0.5816)	3.978(0.5768)
Tsai-Wu	4.077(0.5911)	4.143(0.6008)	4.121(0.5976)
Hoffman	3.984(0.5777)	3.989(0.5784)	3.967(0.5752)

Table 2 First ply failure analysis of laminated plates under static triangular loading

Lamination sequence	First ply load in (×10^{-3}MPa)					
	Max Stress		Hoffman		Tsai-Wu	
	Load value	Node no.	Load value	Node no.	Load value	Node no.
0°/90°/90°/0°	7.94275	115	7.80485	115	8.01170	115
0°/90°/0°/90°	10.0056	119	9.94223	119	10.1490	119
60°/-60°/-60°/60°	15.1132	43	14.4927	43	14.0101	43
60°/-60°/60°/-60°	16.4232	199	16.2854	199	16.2854	199
45°/-45°/-45°/45°	13.6653	17	12.8380	17	12.0796	17
45°/-45°/45°/-45°	15.8717	171	15.7338	171	15.3890	173
30°/-30°/-30°/30°	10.7696	17	10.4938	17	10.1490	17
30°/-30°/30°/-30°	12.0796	113	11.8727	113	12.2177	113

Table 3 First ply failure analysis of laminated plate under static patch load

Lamination sequence	First play failure load (×10^{-3}MPa)		
	Max Stress	Hoffman	Tsai-Wu
0°/90°/90°/0°	19.4569	19.1812	19.7327
0°/90°/0°/90°	25.1796	24.9727	25.5243
60°/-60°/-60°/60°	39.4520	37.8662	39.9346
60°/-60°/60°/-60°	41.1068	39.6588	41.5205
45°/-45°/-45°/45°	33.4534	32.4192	33.7982
45°/-45°/45°/-45°	36.0735	35.1082	36.4872
30°/-30°/-30°/30°	23.6627	23.2490	23.9385
30°/-30°/30°/-30°	25.8691	25.3864	26.2138

Example 3 First ply failure analysis under static patch load

The rectangular laminated plates of 228.6 mm×127 mm dimensions having simply supported boundary conditions are studied here. The lamination sequences considered are symmetric and antisymmetric cross-ply and angle-ply. The plates have been analysed under patch load placed at the centre. The dimension of the patch is 57.15 mm×31.75 mm. The laminate consists of 4 layers. The graphite/epoxy composite material has been used as in Example 1. The full plate has been modeled using 8×8 mesh division. The first ply failure load based on maximum stress, Hoffman and Tsai-Wu theories under the patch load has been computed in the present investigation and presented in Table 3. The values of failure load are quite close based on the three theories.

Example 4 Square simply supported cross-ply laminated plate subjected to step loading

The transient response analysis of a rectangular cross-ply laminate (0°/90°) under a uniformly distributed step load of magnitude 100 KPa is carried out in the present investigation. The laminate is simply supported along all its edges. The material properties are as follows: E_1=25.0 GPa, E_2=1.0 GPa, G_{12}=0.5 GPa, G_{23}=0.2 GPa, G_{31}=0.5GPa, υ_{12}=0.25, ρ=1 kg/m3. The same problem has been solved by Meimaris and Day (1995) using 20 node isoparametric parabolic solid element. The central deflection has been plotted at different times in Fig. 5 and compared with those available in the published literature. The drift between the time dependent displacements obtained from the present formulation and those from Meimaris and Day (1995) occurs because the present

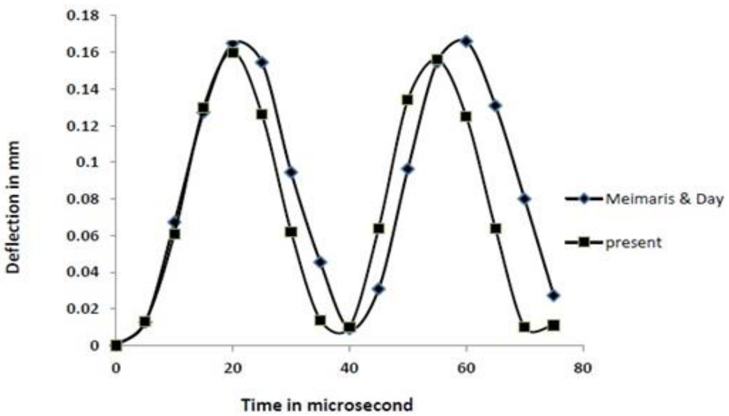


Fig. 5 Central displacement of cross-ply laminated plate subjected to step loading

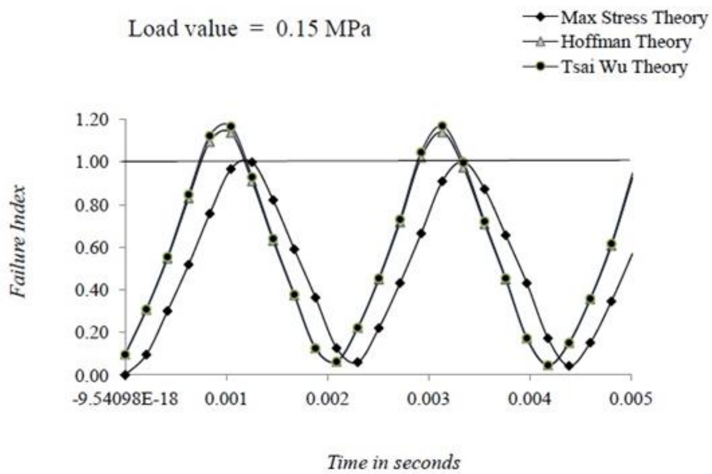


Fig. 6 Failure index-time history for antisymmetric angle-ply laminated composite plate

formulation is a two dimensional analysis whereas the published result is based on three dimensional analysis for a thick plate analysed here.

Example 5 First ply failure analysis of laminated composite plates under triangular pulse load

The square simply supported laminated plates with varying lamination sequences have been analysed under triangular pulse load. The dimension of the plate is 240 mm×240 mm and it is made of carbon-epoxy materials depicted in Example 4. The laminate consists of 4 layers. The width to thickness ratio is a/h=50. The pulse duration is considered to be 0.1 sec and the time step, $\Delta t=Tn/10$. The triangular pulse load is uniformly distributed over the plate. The time history analysis of failure index of antisymmetric angle-ply laminate has been carried out and presented in Fig. 6. The region in the curves above the horizontal line at failure index 1 represents the failed zone as shown in Fig. 6. The amplitude of triangular pulse load at the stage of first ply failure has also been calculated for all the lamination sequences. The results are presented in Table 4 which

Table 4 First ply failure analysis of laminated composite plate under triangular pulse load

Lamination sequence	Time period in sec.	First ply failure load in MPa (time in sec)		
		Max Stress	Hoffman	Tsai-Wu
0°/90°/90°/0°	2.51e-03	0.194577 (0.0271)	0.185429 (0.0271)	0.181724 (0.0193)
0°/90°/0°/90°	2.66e-03	0.187412 (0.0534)	0.177478 (0.0891)	0.162937 (0.0891)
60°/-60°/-60°/60°	2.228e-03	0.144745 (0.0837)	0.100805 (0.0608)	0.107915 (0.0331)
60°/-60°/60°/-60°	2.187e-03	0.152381 (0.0868)	0.134644 (0.0868)	0.132188 (0.0868)
45°/-45°/-45°/45°	2.14e-03	0.147181 (0.0915)	0.099566 (0.0164)	0.112106 (0.0164)
45°/-45°/45°/-45°	2.09e-03	0.149630 (0.0440)	0.133906 (0.0440)	0.131191 (0.0440)
30°/-30°/-30°/30°	2.226e-03	0.144505 (0.0331)	0.101026 (0.0331)	0.108152 (0.0331)
30°/-30°/30°/-30°	2.187e-03	0.152976 (0.0776)	0.135164 (0.0776)	0.132697 (0.0776)

indicate that the first ply failure amplitude is the highest for symmetric cross-ply laminates according to Tsai-Wu theory. The Maximum stress theory indicates slightly higher value of the first ply failure load than that by other failure theories viz. Tsai-Wu and Hoffman's theory. The probable reason may be that the maximum stress theory is a non-interactive failure theory whereas other theories are interactive.

4. Conclusions

The first ply failure analysis of various symmetric and antisymmetric cross-ply and angle-ply laminated plates has been carried out in the present investigation. The strength-based failure criteria viz. maximum stress, Tsai-Wu and Hoffman are applied to predict the first ply failure load of composite laminates under static and dynamic loads. The first ply failure load under uniformly distributed static transverse load has been predicted and compared with the result available in the published literature. The results tally very well. It is observed that the first ply failure load under dynamic loading condition differs between interactive (Tsai-Wu and Hoffman's theory) and non-interactive (maximum stress) failure theories. It is also observed that the highest value of first ply failure load under dynamic pulse loading is obtained for symmetric cross-ply laminated plates whereas the failure load value for same lamination scheme is the lowest for static load.

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