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Theoretical and experimental study of robustness based design of single-layer grid structures

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Abstract. Structural robustness refers to the ability of a structure to avoid disproportionate consequences to the original cause. Currently attentions focus on the concepts of structural robustness, and discussions on methods of robustness based structural design are rare. Firstly, taking basis in robust H_∞ control theory, structural robustness is assessed by H_∞ norm of the system transfer function. Then using the SIMP material model, robustness based design of grid structures is formulated as a continuum topology optimization problem, where the relative density of each element and structural robustness are considered as the design variable and the optimization objective respectively. Generalized elitist genetic algorithm is used to solve the optimization problem. As examples, robustness configurations of plane stress model and the rectangular hyperbolic shell model were obtained by robustness based structural design. Finally, two models of single-layer grid structures were designed by conventional and robustness based method respectively. Different interference scenarios were simulated by static and impact experiments, and robustness of the models were analyzed and compared. The results show that the H_∞ structural robustness index can indicate whether the structural response is proportional to the original cause. Robustness based structural design improves structural robustness effectively, and it can provide a conceptual design in the initial stage of structural design.

Keywords: structural robustness; robustness based structural design; grid structures; topology optimization; experimental verification

1. Introduction

Since the concept of robustness was proposed in the 1960's, it has been studied and applied widely in control system and many other areas. After the progressive collapse of World Trade Center in 2001, structural robustness has been attracting increasing attention. Structural robustness can be defined as the ability of a structure to avoid disproportionate consequences to the original cause (EN 1991-1-7 2006). Robustness is usually related to scenarios including human errors, unforeseen loads and unexpected accidental actions like explosion or impact (Sørensen 2011). By now, some qualitative interpretations of structural robustness have been made, including parameter

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sensitivity (Lee *et al.* 2010), collapse resistance (Khandelwal *et al.* 2011) and so on.

Robustness based structural design is still a relatively new research field. Structural robustness is usually improved by some conceptual measures, such as enhancement of ductility, redundancy and energy dissipation capacity (Branco *et al.* 2011). Direct structural design methods based on robustness are rare.

To do evaluation and optimization, the quantification of structural robustness is needed. Regarding the type of the original cause, robustness can be assessed from two points of view.

The first considers unexpected accidental actions to be the original cause, owing to which initial local damage occurs. Then robustness is calculated by the ratio of behavior of the damaged and intact structure. From this perspective, Yan and Smith calculated structural robustness from the perspective of load-carrying capacity reserves and energy absorption respectively (Yan *et al.* 2010, Smith *et al.* 2003). Considering uncertainties, Čizmar analyzed the robustness of a wooden stadium roof using the reliability index (Čizmar *et al.* 2011); Baker assessed structural robustness by computing the ratio of direct risk and total risk (Baker *et al.* 2008).

The second focus on the structural behavior under the interference of unforeseen or fluctuating loads. In this situation, structures are treated as systems that transform external loads to deformations. According to this point of view, robustness can be assessed by the level of interference that a structure can withstand without damage (Au *et al.* 2003), or the structure performance under a certain level of interference (Kanno and Ben-Haim 2011). Also Shannon entropy was used to measure structural robustness (Beer *et al.* 2008).

According to the second point of view, structural robustness is assessed by H_∞ norm of system transfer function in this paper. Then setting robustness as objective function, reasonable distributions of the structural material are obtained though continuum topology optimization and robustness based design of single-layer grid structures is achieved. Finally the proposed evaluation and design method were verified by overloaded static experiment and impact experiment.

2. H_∞ structural robustness index

Proposed in the early 1980s, H_∞ robust control theory, which takes ∞-norm of system transfer function as the performance indicator, is a relatively successful and complete theory in the control area. Many robust performance criteria can be described by the H_∞ norm (Doyle *et al.* 1989). The following will briefly introduce the H_∞ optimal problem, and then propose a framework for quantitative assessing structural robustness based on H_∞ theory.

2.1 H_∞ optimal problem

State equation and output equation of a linear time-invariant structure system are as follows

$$\begin{cases} \dot{\boldsymbol{\eta}}(t) = \boldsymbol{A}_0\boldsymbol{\eta}(t) + \boldsymbol{B}_0\boldsymbol{u}(t) \\ \boldsymbol{y}(t) = \boldsymbol{C}_0\boldsymbol{\eta}(t) + \boldsymbol{D}_0\boldsymbol{u}(t) \end{cases} \tag{1}$$

where: $\boldsymbol{\eta}(t)$, $\boldsymbol{u}(t)$, $\boldsymbol{y}(t)$ are the state vector, input vector and output vector of a structure system respectively; $\boldsymbol{A}_0$, $\boldsymbol{B}_0$, $\boldsymbol{C}_0$, $\boldsymbol{D}_0$ denote constant system matrices with appropriate dimensions.

Transfer function matrix $\boldsymbol{G}$ can then be obtained by Laplace transformation of Eq. (1)

$$\boldsymbol{G}(s) = \boldsymbol{C}_0(s\boldsymbol{I}\text{-A}_0)^{-1}\boldsymbol{B}_0+\boldsymbol{D}_0 \tag{2}$$

Fig. 1 Input-output relationship of a structure system

According to the principle of superposition, $\boldsymbol{Y}(s)=\boldsymbol{G}(s)\boldsymbol{U}(s)$, where $\boldsymbol{U}(s)$ and $\boldsymbol{Y}(s)$ are the Laplace transformation of $\boldsymbol{u}(t)$ and $\boldsymbol{y}(t)$ respectively. Therefore, from a mathematical point, a structure is a mapping from the input function space $\boldsymbol{U}(s)$ to the output function space $\boldsymbol{Y}(s)$.

The conventional structural design is optimal only when the structural system can be described accurately by Eq. (1), but the load, geometry and material parameters of an actual structure are uncertain, which may lead to disproportionate destruction. The basic idea of the H_∞ theory is that a structure can be treated as a family of uncertain structures, and if all members of the family meet the requirements of the performance indicators, the actual structure will meet the requirements as well (Mei *et al.* 2008).

If we set the transfer function from the disturbance $\boldsymbol{w}(t)$ to the output $\Delta\boldsymbol{y}(t)$ for $\boldsymbol{G}_{w\Delta y}$, H_∞ optimal problem is to make the H_∞ norm of $\boldsymbol{G}_{w\Delta y}$ minimal to ensure the worst performance of the structural family is optimal, so that the robust performance of the system can meet requirements. H_∞ norm is defined as

$$\left\|\boldsymbol{G}(s)\right\|_\infty = \sup_{\omega\in[0,\infty)} \sigma_{\max}\left[\boldsymbol{G}(\mathrm{j}\omega)\right] \tag{3}$$

where sup donate to supremum, $\sigma_{\max}$ is the largest singular value of a matrix, j is the imaginary unit, ω is a real variable.

2.2 Robustness of a structure system

The structural robustness can be defined by whether the consequence is disproportionate to the initial causes. If we take the initial causes of a structure system as a set of input signals $\boldsymbol{w}(t)$, then the output response signals $\Delta\boldsymbol{y}(t)$ are the consequences. Fig. 1 shows the input-output relationship of a structure system, where $\boldsymbol{G}$(s) donates the transfer function matrices. Structural robustness is used to express whether the output signal is disproportionate with the input signal, so it can be measured through the system transfer function matrices. Considering uncertainties, H_∞ norm of the system transfer function takes into account the most unfavorable circumstances.

With the assumption of ideal elastic material and small deformation, the motion equation of an n-degree of freedom structure is as follow

$$\boldsymbol{M}\ddot{\boldsymbol{x}}(t)+\boldsymbol{C}\dot{\boldsymbol{x}}(t)+\boldsymbol{K}\boldsymbol{x}(t)=\boldsymbol{u}(t) \tag{4}$$

where: $\boldsymbol{M}$, $\boldsymbol{C}$, $\boldsymbol{K}$ is the mass, damping and stiffness matrix respectively; $\boldsymbol{x}(t)$ is the displacement vector, and $\boldsymbol{u}(t)$ is the load vector. According to Eq. (4), the state equation of the structure system is

$$\dot{\boldsymbol{\eta}}(t)=\boldsymbol{A}_0\boldsymbol{\eta}(t)+\boldsymbol{B}_0\boldsymbol{u}(t) \tag{5}$$

where: $\boldsymbol{\eta}(t)=\begin{bmatrix}\boldsymbol{x}(t)\\ \dot{\boldsymbol{x}}(t)\end{bmatrix}$ is the state vector, $\boldsymbol{u}$=$\boldsymbol{u}(t)$ is the input vector, $\boldsymbol{y}$=$\boldsymbol{x}(t)$ is the output vector, and system matrices $\boldsymbol{A}_0 = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{I} \\ -\boldsymbol{M}^{-1}\boldsymbol{K} & -\boldsymbol{M}^{-1}\boldsymbol{C} \end{bmatrix}$, $\boldsymbol{B}_0 = \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{M}^{-1} \end{bmatrix}$.

Let $\boldsymbol{G}_{\boldsymbol{uy}}$ donates the transfer function matrices from the input $\boldsymbol{u}(t)$ to output $\boldsymbol{y}(t)$, and $\boldsymbol{G}_{\boldsymbol{w}\Delta\boldsymbol{y}}$ donates the transfer function matrices from the interference $\boldsymbol{w}(t)$ to the output $\Delta\boldsymbol{y}(t)$. For linear systems, $\boldsymbol{G}_{\boldsymbol{uy}}$ is equal to $\boldsymbol{G}_{\boldsymbol{w}\Delta\boldsymbol{y}}$. Therefore, structural robustness can be expressed as

$$I_{\mathrm{R}} = \left\|\boldsymbol{G}_{\boldsymbol{w}\Delta\boldsymbol{y}}\right\|_{\infty} = \left\|\boldsymbol{G}_{\boldsymbol{uy}}\right\|_{\infty} \tag{6}$$

The transfer function matrices of nonlinear structural systems depend on the inputs, therefore structural robustness could not be calculated directly according to Eqs. (3) and (6). Following L_2 performance criteria (Mei *et al.* 2008), structural robustness can be measured by L_2norm of the structure system as

$$I_{\mathrm{R}} = \left\|\boldsymbol{G}_{\boldsymbol{w}\Delta\boldsymbol{y}}\right\|_{\infty} = \sup_{\|\boldsymbol{w}\|_2 \neq 0} \frac{\left\|\Delta\boldsymbol{y}(t)\right\|_2}{\left\|\boldsymbol{w}(t)\right\|_2} \tag{7}$$

where: $\left\|(\bullet)\right\|_2 = \left(\int_0^{\infty} (\bullet)^T(\bullet)dt\right)^{1/2}$. Robustness of structures subjected to determine forms of loads can also be analyzed according to Eq. (7).

From the definition of I_{R}, a structure owns a higher robustness when I_{R} is lower.

3. Robustness based structural design

Structural robustness represents the overall performance of a structure. When the structure form is determined, the most significant factor that affects robustness is the structural connectivity and the distribution of members. So it is necessary to consider structural robustness before determining the layout of structural members.

Topology optimization is the most general form of structural optimization. The purpose of topology optimization is to find the best layout of material in the design domain, which can provide a conceptual design in the initial stage of structural design. The following will transform robustness based structural design to continuum topology optimization aimed at structural robustness, and obtain robust structural configurations.

3.1 Mathematical model of robustness based structural design

Since Bendsøe proposed homogenization method in 1988 (Bendsøe *et al.* 1988), many topology optimization methods like variable density method (Rozvany *et al.* 1992, Takezawa *et al.* 2011), level set method (Wang *et al.* 2003, Xia *et al.* 2012) and evolutionary structural optimization (Yang *et al.* 2005, Zuo *et al.* 2012) have been developed, among which the Solid Isotropic Microstructure with Penalization for intermediate densities (SIMP) method is the most popular one (Rozvany 2009).In variable density method, the relative density ρ_{e} of each element is used as design variable, where ρ_{e}=0 means void region and ρ_{e}=1 means material. A more favorable topology can be obtained by the presence or absence of elements. The SIMP method transforms the general variable density method to a continuous optimization problem, and assumes there is a corresponding relationship between the density ρ_{e} and elastic modulus. Then the topology optimization problem is to be solved. The following will set structural robustness as the target, and the robustness based structural design will be transformed into a continuum topology optimization problem.

First the geometric parameters of the design domain are selected in accordance with requirements of the appearance and applications. Then the design domain is divided into elements. The elastic modulus of each element is expressed as an exponential function of relative density ρ_e using the SIMP interpolation model, i.e., (Rozvany 2009)

$$E_e = \rho_e^p E_0 \,, \quad e=1,2, \ \dots, \ n \tag{8}$$

where E_e is the equivalent elastic modulus of element e, and E_0 is the elastic modulus of the actual material. $0<\rho_e\leq 1$ is the relative density. p is a penalization factor, and $p\geq 3$. n denotes the number of elements.

Furthermore, robustness based structural design can be transformed to the following mathematical optimization model

$$\left.\begin{array}{l} \min\limits_{\boldsymbol{\rho}} \ I_{\mathrm{R}} = \left\| \boldsymbol{G}(s) \right\|_{\infty} \\ \text{s.t.} \ \ \sum\limits_{e=1}^{n} \rho_e V_e \leq f_r V_0 \\ \qquad 0 < \rho_e \leq 1 \end{array}\right\} \tag{9}$$

where the optimization objective is to make the structural robustness indicator I_{R} minimum, and the design variable is the relative density vector $\boldsymbol{\rho}=[\rho_1, \rho_2, \dots \rho_n]$. The results need to meet the constraint of the total volume of the structure. V_e and V_0 denote the volume of element e and the total volume of the design domain respectively, and f_r denotes the ratio of preserving volume.

3.2 Optimization algorithm

To solve topology optimization problems, mathematical programming method (de Kruijf *et al.* 2007), optimization criteria method (Zheng *et al.* 2012) and stochastic search methods like genetic algorithm are commonly used currently. The first two methods possess rigorous theoretical foundations and require less iterative computations. But for the robustness topology optimization, whether the objective function is calculated by Eqs. (6) or (7), its gradient is difficult to calculate, which is necessary. So they may often converge to locally optimal regions of the design space.

Genetic algorithm (GA) has been gradually recognized as a powerful method for structural topology optimization (Jakiela *et al.* 2000; Wang *et al.* 2006), which only requires zero'th order function evaluations and don't need to solve gradient. To make it more effective to converge to the global optimal solution, generalized elitist genetic algorithm (GEGA) (Soremekun *et al.* 2001) is applied here. Standard selection in GA is replaced by multiple elitist selection, which could preserve more information about elitist designs from the parent population. Certain amount of the best individuals in the parent populations are selected and placed into the new population. A schematic of the GEGA procedure is given in Fig. 2, where the fitness is $1/I_{\mathrm{R}}$.

4. Numerical examples

4.1 Plane stress model

This example describes the optimization of a cantilevered beam. The design domain is shown in Fig. 3. Because the result of topology optimization is independent of the units of parameters, during the optimization process, all parameters were dimensionless. The geometric dimensions of the beam are 16×10×0.01. One side of the beam is fixed, and a vertical load is applied on the midpoint of the other side. Elastic modulus is set to be E_0=2×10^{11}. The design domain is mapped into 160 (16×10) elements in SIMP model. The penalty factor (p=4) and ratio of preserving volume (f_r=0.5) are defined. GEGA parameters like probabilities of crossover (P_c=1.0) and mutation (P_m=0.02) are chosen according to values suggested by Soremekun (Soremekun *et al.* 2001), and the population size is chosen to be 500. The top 5% individuals of the parent generation are retained in the selection procedure. Because the form of the load is given, the robustness index

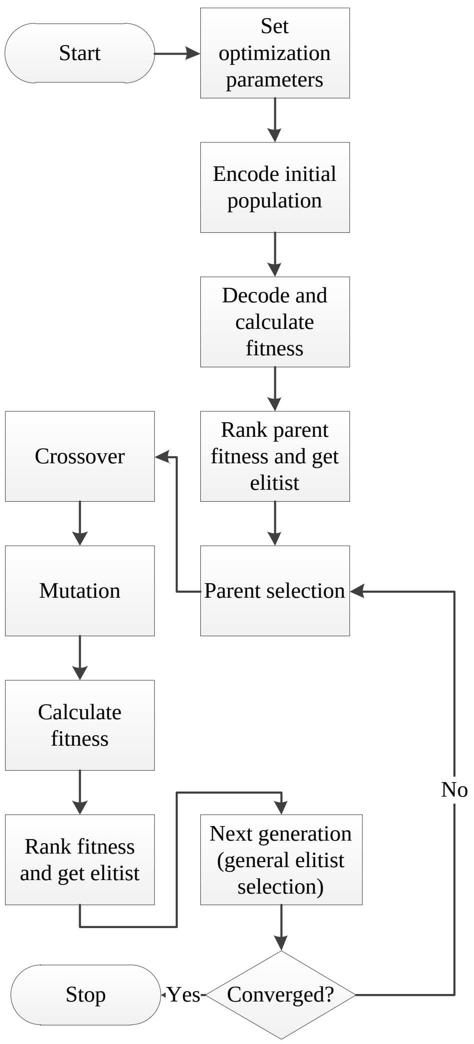


Fig. 2 GEGA procedure

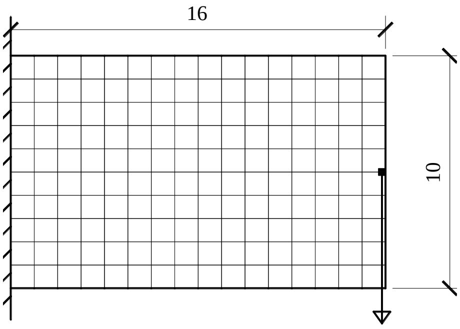


Fig. 3 Design domain of plane stress model

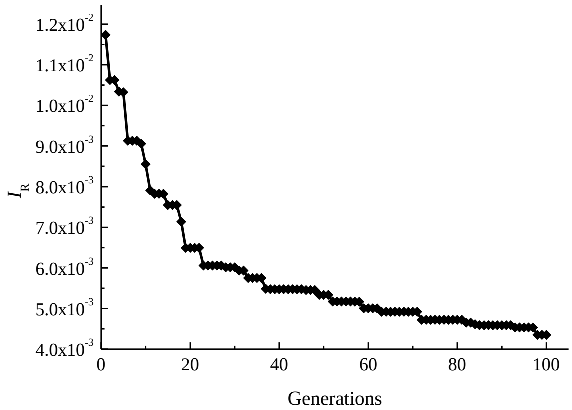


Fig. 4 Optimization convergence history of the plane stress model

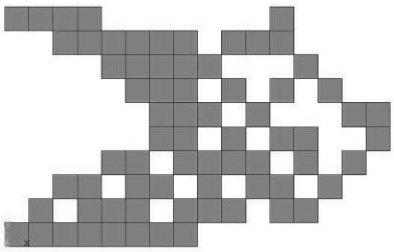


Fig. 5 Topology optimization result of the plane stress model

is calculated according to Eq. (7), in which both horizontal and vertical displacements are included in the outputs.

Robustness based topology optimization was carried out then, and connectivity analysis (Jakiela *et al.* 2000) was performed in this procedure. The optimization convergence history is shown in Fig. 4. Structural robustness is enhanced quickly during the first 40 iterations, and structural robustness becomes steady after the 70th iteration. The result could be considered to be convergent after 100 iterations, and the final topology is relatively robust. As a result, Fig. 5 shows the elements whose densities are higher than 0.5. After topology optimization, an obvious hollow was generated in the interior region at the restraint end of the beam, while elements on the upper and lower boundary of the beam connect more closely. From constraint end to cantilever end, the

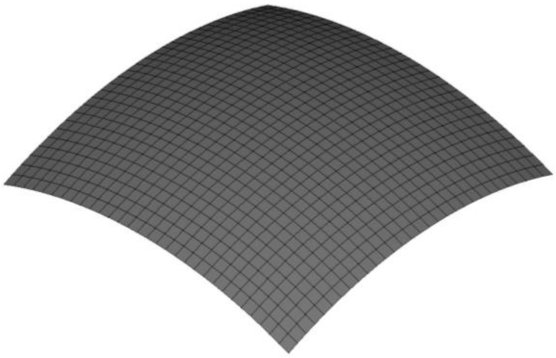


Fig. 6 Design domain of the rectangular hyperbolic shell model

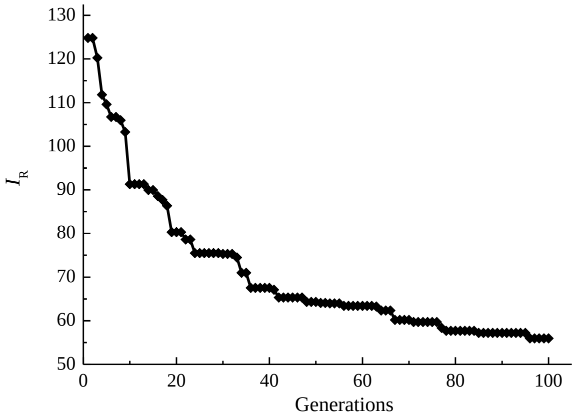


Fig. 7 Structural robustness optimization convergence history of the rectangular hyperbolic shell model

cross-section of the beam is gradually decreases, which is in line with the general design principle of beams. Targeting stiffness-to-volume ratio, Jakiela also got similar results (Jakiela *et al.* 2000).

4.2 Rectangular hyperbolic shell model

This example describes the optimization of a rectangular hyperbolic shell subjected to vertical uniform load. The design domain is shown in Fig. 6. The plane dimensions of the shell are 1×1, and the thickness and rise-span ratio are 0.01 and 1/5. Four corners are fixed. Elastic modulus is set to be $E_0=2\times10^{11}$. The design domain is mapped into 900 (30×30) elements in SIMP model, and there are 121 optimization variables considering the symmetry. The penalty factor (p=4) and ratio of preserving volume (f_r=0.4) are defined. The population size of GEGA is chosen to be 1000, and other parameters are same with example 1.

Robustness based structure topology optimization was carried out then, and the optimization convergence history is shown in Fig. 7. As a result, Fig. 8 shows the elements whose densities are higher than 0.4 after connectivity analysis. The elements on diagonals of two supports bear loads directly, so they are the most important for structural robustness. The final topology in Fig. 8 is clear to design the main frame of the structure, according to which the layout of the components can be preliminary designed.

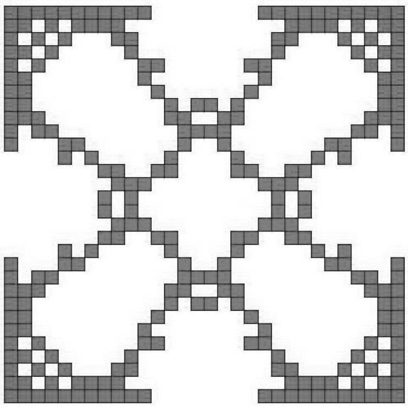
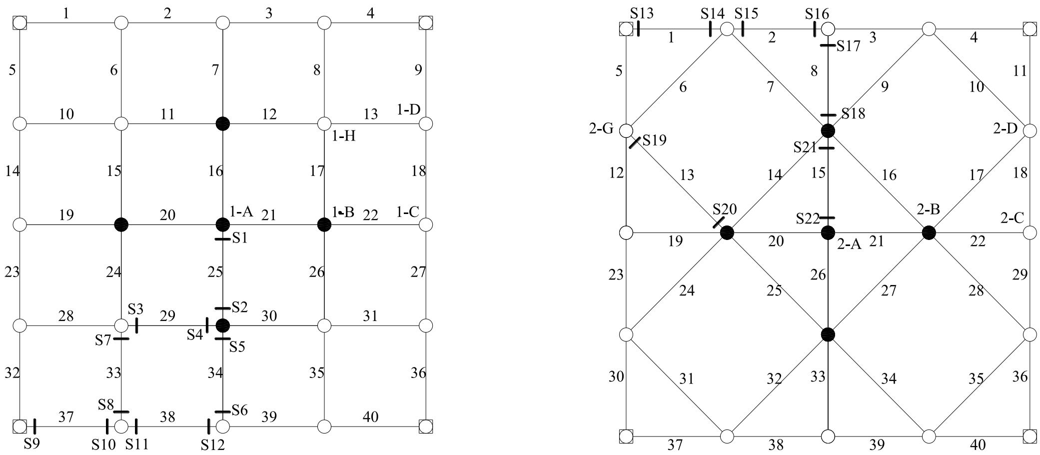


Fig. 8 Topology optimization result of the rectangular hyperbolic shell model



(a) the model designed conventionally (b) the model designed based on robustness

Fig. 9 Layout of members and strain gauges

5. Experimental verification

In actual environment, structures may be subjected to loads exceed the design value; at the same time, the impact load is usually treated as one of the factors that should be considered in robustness based design (EN 1991-1-7 2006). Structures with high robustness should have the ability to resist these interferences.

To verify the proposed evaluation and design method, interference scenarios were simulated by overloaded static experiment and impact experiment. Structural robustness was compared by the displacement and strain responses of the experimental models.

5.1 Experimental models

Based on the optimized topology in Fig. 8, a layout scheme of members based on robustness is shown in Fig. 9(b), which turns to a hyperbolic grid shell. To avoid large slenderness ratios, and taking into account the service requirements, some necessary members (e.g., No.2,3,8,15) were added. Plane dimensions of the model are 3 m×3 m, and the rise-span ratio is 1/5. The model is supported at the four corners. In contrast, another model with the same shape was designed in accordance with the conventional form, as shown in Fig. 9(a).

The design loads are S_0=1 kN, which are concentrated load applied at each solid joint in Fig. 9. Q235 steel tubes were used to build the models. All the joints of members and supports were considered to be rigid. By full stress optimized design, grouping of cross-section configurations are shown in Table 1. According to Eq. (7), robustness of the model designed conventionally and the model designed based on robustness are 27.4 and 2.56 mm/kN respectively.

The experiments were performed in the structure laboratory, Zhejiang University, China.

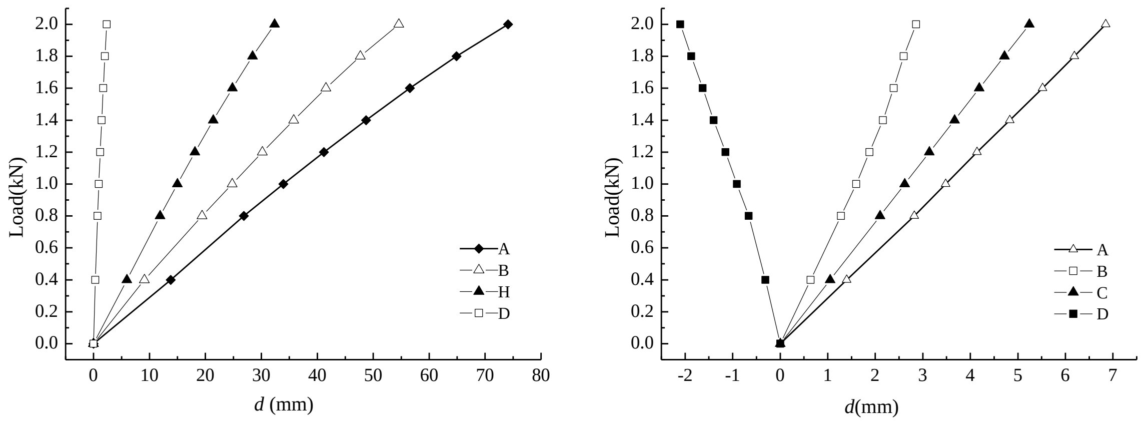
Table 1 Grouping of cross-section configurations

	Number of members	Cross-section (mm)	Self-weight (kg)
The model designed conventionally	2,3,7,14,18,19,22,23,27,34,38,39	Φ21.5×2.5	74
	1,4~6,8~13,15~17,20,21,24~26,28~33,35~37,40	Φ32×3.5	
The model designed based on robustness	1~6,8,10~12,15,18~23,26,29~31,33,35~40	Φ21.5×2.5	55
	7,9,13,14,16,17,24,25,27,28,32,34	Φ25×2.5	

(a) the model designed conventionally (b) the model designed based on robustness

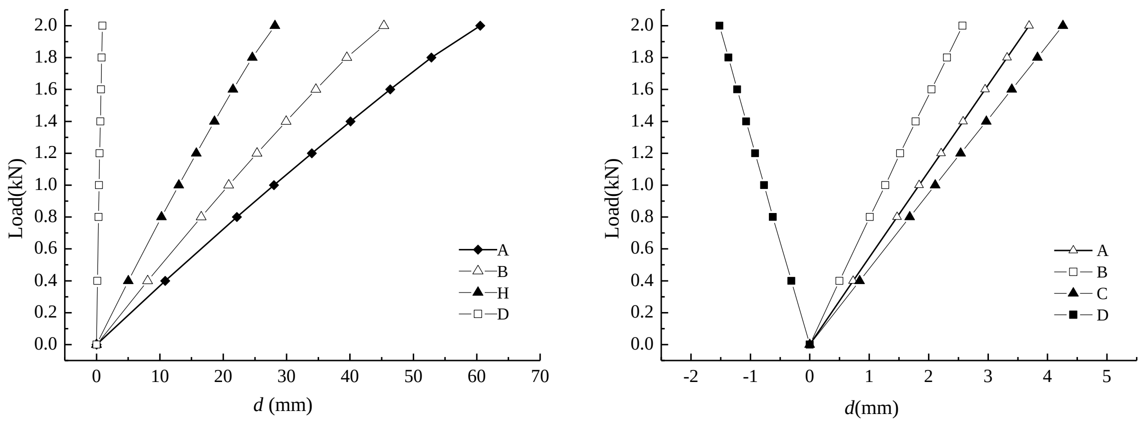
(c) support joint (d) middle joint

Fig. 10 Photos of the models



(a) the model designed conventionally (b) the model designed based on robustness

Fig. 11 The experimental displacements of the models



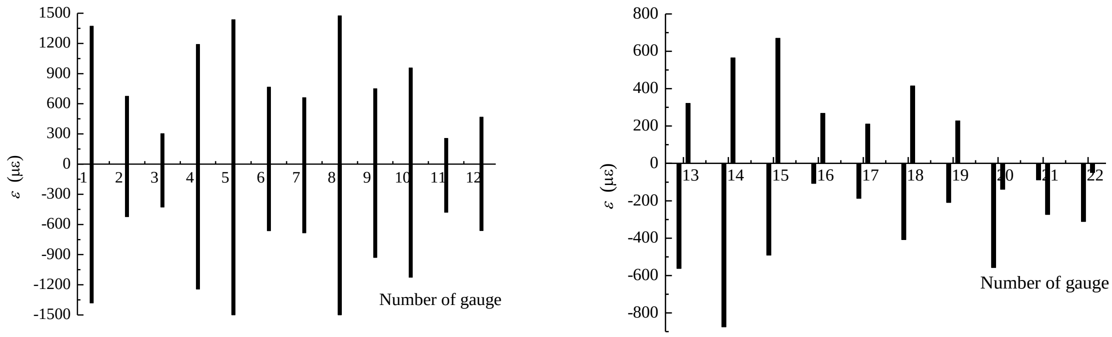
(a) the model designed conventionally (b) the model designed based on robustness

Fig. 12 The theoretical displacements of the models

Photos of the models are shown in Fig. 10. The support joints are Φ114×4 steel tubes, which were connected to laboratory's platform through a 200×200×150×10 box each. Other joints are Φ76×4 steel tubes.

5.2 Static experiment

Considering elastic perfectly-plastic material model and geometric nonlinearity, the static ultimate bearing capacities of the model designed conventionally and the model designed based on robustness were obtained by ANSYS, which were 2.3 and 6.8 kN respectively. In order to simulate the overload scenario, twice of the design loads S=2 kN were applied to the models. Considering symmetry, the layout of strain gauges S1~S22 are shown in Fig. 9. The tubes bear axial forces and moments, so there were two opposite gauges at each point. Displacements d and strains ε were surveyed respectively, and the results are shown in Fig. 11, Fig. 12 and Fig. 13.



(a) the model designed conventionally (b) the model designed based on robustness

Fig. 13 Strains of members of models subjected to load S=2kN

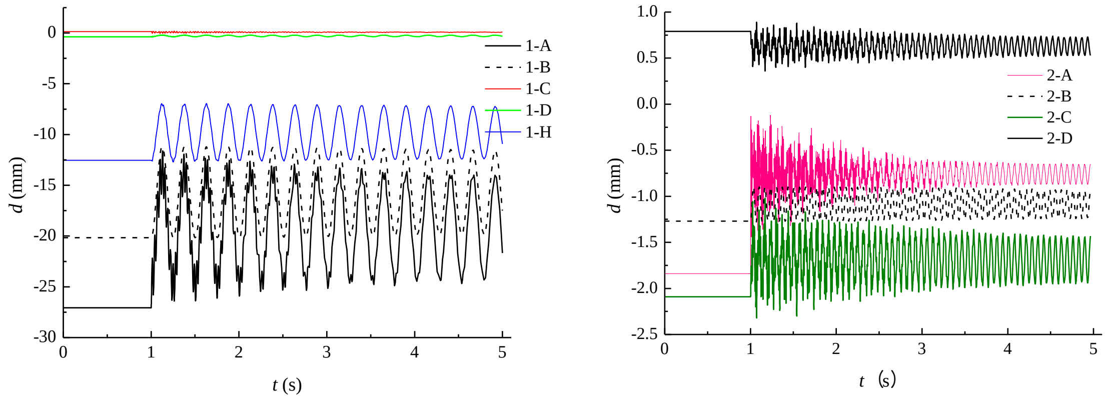
Because the tubular joints of the experimental models are semi-rigid, for both of the models, the experimental displacements were 10%-40% larger than the theoretical values. But the trends of the curves are similar. It can be seen from Fig. 11 that for the model designed conventionally, the displacement responses were basically linear when the load S was less than 1.2kN (1.2S_0). As the load increased, the slope of the load-displacement curve decreased gradually, and strains of some members like No.1,4,5,8,10 were relatively large when S=2S_0, as Fig. 13 showed. On the other hand, the load-displacement curve of the robustness based designed model remains linear when S=2S_0. Compared to the model designed conventionally, the stress level of the robustness based designed model was generally lower.

If assuming a node is failed when its displacement is more than 1/100 of the span, for the model designed conventionally under the interference of 2.0S_0, all nodes other than those on the four outside edge were failed, which means the failure area was 50% of the total projected area. In the same overload scenario, the robustness based designed model did not fail at last. So during the static experiment, the robustness based design structure was less affected by the same interference. If this consequence could be called proportionate, the conventionally designed model responded disproportionately to the initial cause.

5.3 Impact experiment

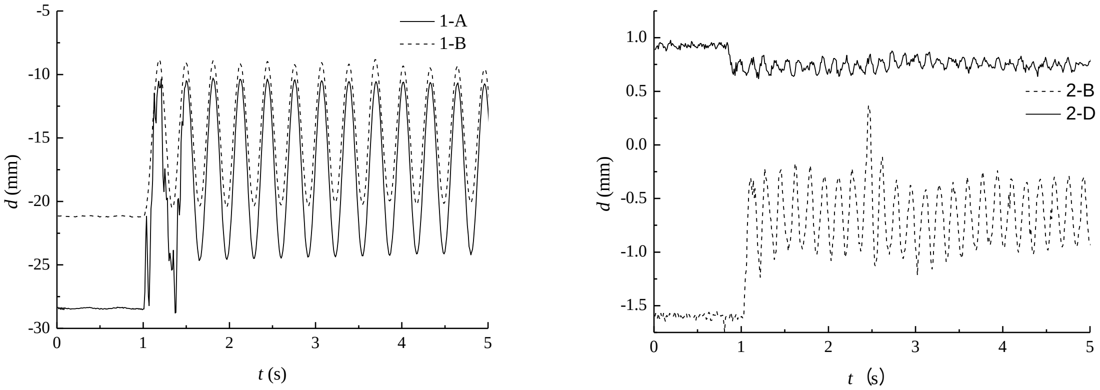
At the beginning of the impact experiment, the load of each joint maintained 1kN as they were in the static experiment, and the unexpected impact scenario was made by sudden unloading of the intermediate joints. The theoretical values were analyzed by ANSYS before the experiment, in which members of the models were simulated using BEAM188, and each member was divided into 10 finite elements to simulate possible instability. Results of numerical analysis are shown in Fig. 14. During the experiments, displacements were sampled using TEC R-series dynamic displacement sensor and NI9205 data acquisition system.

The results are shown in Fig. 15, in which the shapes of the experimental displacements are in conformity with the numerical results, and the frequency of the robustness based model is higher than the conventional one. The experimental amplitudes of the models are close to the numerical values, and amplitudes of the conventionally designed model are much bigger than the robustness based model. So the robustness based designed model can be considered not sensitive to the interference of the impact load.



(a) the model designed conventionally (b) the model designed based on robustness

Fig. 14 Numerical displacements of the models subjected to impact



(a) the model designed conventionally (b) the model designed based on robustness

Fig. 15 Experimental displacements of the models subjected to impact

6 Conclusions

• Taking basis on robust H_∞ control theory, structural robustness is assessed by H_∞ norm of the system transfer function, which can indicate whether the structural response is proportional to the initial cause.

• Robustness based structural design is achieved though continuum topology optimization. Both the SIMP material model and genetic algorithm used in the proposed method are versatile and easy for applications. Numerical examples of the plane stress model and the three-dimensional rectangular hyperbolic shell model show that the robustness optimized topology is clear to arrange main components of structures.

• The results of static and impact experiments verify the proposed evaluation and design method of structural robustness. During the experiments of two grid shells, the robustness based designed model was less affected by interferences and showed higher robustness, while the conventionally designed model responded disproportionately subjected to the same interferences.

Acknowledgements

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