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New damage localization indicator based on curvature for single-span beams

Yi-lin Wang*

School of Civil Engineering, Shandong Jianzhu University, Jinan Shandong Province, P.R. China

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Abstract. Most existing damage detection methods based on curvature should investigate the indicator value changes between the intact and damaged state. However, the 'footprint' in the intact state is usually not available for most actual beams. Therefore, a new damage localization indicator called Difference of Nearby Difference Curvature Indicator (DNDCI) was introduced for single-span beams. This indicator does not require prior information of the intact beam and is sensitive to the damage occurs on or nearby the measuring points. Practical and detailed application method of DNDCI has been proposed. Besides the idealized simply supported beams, it was validated by example that DNDCI is also applicable to the actual single-span beams supported by bearings.

Keywords: beam; damage; localization; curvature; displacement

1. Introduction

Beams are widely used in practice. Damage localization of existing beams is essential for decision making in regular structural maintenance to ensure their safety. Damage is considered as weakening of the beam that negatively affects its working performance. That is, damage can be defined as any deviation in the geometric or material properties that may cause undesirable displacements, stresses or vibrations. For the beams, damage may be cracks, corrosion, broken welds, etc. Obviously, damage can severely affect the serviceability and safety of the beams.

The ability to detect damage before it endangers the structure has been of interest to engineers for many years. A lot of work has been published in this area, where various methods have been proposed (Buezas *et al*. 2011, Domaneschi *et al*. 2013, Homaei *et al*. 2014, Liu *et al*. 2011, Wu *et al*. 2011, Xu *et al*. 2010). A phenomenon was shown firstly in 1991 (Pandey *et al*. 1991): that is, the modal curvatures are sensitive to damage and can be used to localize it. By plotting the difference in modal curvature between the intact and damaged case, a peak appears at the damaged element indicating the presence of a fault. From then on, it has been noted that there are increasing studies on the application of modal curvature indicator which is the second spatial derivative of the modal shape. As a local performance index of structure, it is also applicable to multiple damage case. For example, Wahab *et al*. (1999) applied this indicator to a prestressed concrete bridge and introduced another indicator called "curvature damage factor", in which the difference in curvature

*Corresponding author, Ph.D., E-mail: xgwang_wang@163.com

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modal for all modes can be summarized in one number for each measuring point. In order to enhance the effect, Ratcliffe and Bagaria (1998) applied gapped smoothing method to the modal curvature yielding a damage index, which was used for locating a delamination in a composite beam; Dutta *et al*. (2004) adopted adaptive finite element to improve the accuracy of modal parameters used. Besides, Sahin and Shenoi (2003) presented an algorithm using a combination of global (changes in natural frequencies) and local (modal curvature) vibration-based analysis data as input in artificial neural networks (ANNs) for location and severity prediction of damage in beam-like structures. Hamey *et al*. (2004) concluded about superiority of modal curvature-based methods over other methods by evaluating various damage detection techniques in carbon/epoxy composite beams with several possible damage configurations. Recently, the uncertainties associated with the structural model and measured vibration data were considered: the influence of modal shape errors measured in situ was studied (Tomaszewska 2010). In order to distinguish between true and false damage detection results, the Absolute Damage Index was proposed; Chandrashekhar *et al*. (2009) brought together the disparate areas of probabilistic analysis and fuzzy logic to address uncertainty.

These methods usually derive the modal curvature from the displacement modal shapes by means of central difference approximation. The drawback mainly lies in the requirement of comprehensive, accurate modal shape data. In addition, when more than one fault exists in the structure, it is not possible to locate damage in all positions from the results of only one mode. Several modal shapes are required. Besides the ineluctability of obvious errors in vibration measurements, this requirement is difficult to be met because it is not practical to have a very fine sensor mesh in a real experiment. The second possibility of determining modal curvature is to directly measure the curvature by strain sensors. The use of optical fiber strain sensors made it possible to accurately measure the very small axial strains and to identify the modal curvatures at measuring sections (Reynders *et al*. 2007, Jörg *et al*. 2005). However, the strain measurements for higher modes seemed to be inaccurate. Moreover, the deduced indicator is not very sensitive to local damage occurs at a point other than the measuring points, which means the number of strain sensors impossibly can be sufficient.

Therefore, further study is needed for this topic. It seems to be a promising way to combine the curvature indicator with the static response of bridges. The difference curvature of bridges under static load, which is obtained from the static displacement data and central difference approximation, can also reflect the change of local stiffness induced by damage and be applicable to multiple damage case. This indicator allows for quick, reliable and convenient damage detection at a relatively low cost. Simplicity and no need of performing modal analysis are also its outstanding advantages. However, like most methods based on modal curvature, the ‘footprint’, or baseline data set, of the intact structure is required for comparison to inspect the change in curvature due to damage. The damage location cannot be exactly validated when intact structure curvature is absent. The ‘footprint’ is usually obtained either from measurements of the undamaged bridge, or from a finite element model of the intact beam structure. However, most suspected damaged bridges were constructed several decades ago, and the ‘footprint’ of the structures in the intact state is not available. An inaccurate finite element model can bring in large errors, and degrade or even lead to incorrect result in the damage detection. To overcome this difficulty, the ‘gapped-smoothing’ technique, which allows the damage detection in a bridge without prior knowledge on the undamaged state, was proposed. The technique was then extended and applied to bi-dimensional uniform load surface (ULS) curvature (Wu *et al*. 2004). However, it is no more than a mathematic disposal means and cannot avoid the disadvantages of indicator itself.

In this paper, aiming at single-span beams, a new damage localization indicators based on static curvature, which does not require any prior information of the intact beam, is selected as the research object. The application method and result of this new indicator will be analyzed. Special treatments for robustness also have been made. Numerical examples are given to illustrate and validate it. The first paragraph will start with an introduction of the underlying theory.

2. Underlying theory

2.1 Curvature Indicator

For beams, bend is the primary deformation form. Curvature is calculated by using the following expression

$$\frac{1}{\rho} = \frac{M}{EI} \tag{1}$$

Where M is bending moment of the section. E and I are the modulus of elasticity and the moment inertia, respectively.

The changes in the curvature are local in nature and hence can be used to detect and locate damage in the bridges. If a crack or other damage exists in a bridge, it reduces the flexural stiffness of the structure at the cracked section or in the damaged region, which will then increase the magnitude of local curvature. Hence, damage can be found through the change in local curvature between the intact and damaged state.

However, this curvature indicator is not very suitable for actual damage detection. It only can detect the damages occur on the sections with measuring points. Obviously, infinite measurement point is impossible in practice. The spacing of measurement sites is dictated by the structural configuration and availability of equipment. Therefore, it is impractical to detect all the damages using this indicator and a more proper indicator has to be formulated.

2.2 Difference curvature indicator

The central difference approximation can be used to compute the local curvature of measurement location i

$$\frac{1}{\rho_i} = \frac{M_i}{K_i} = \frac{d^2y}{dx^2} = \frac{y_{i+1} - 2y_i + y_{i-1}}{\Delta^2} \tag{2}$$

Where Δ is the distance between two successive measured points. M_i, K_i and y_i is the bending moment, flexural stiffness and vertical displacement of section i, respectively. It should be noted that different from modal curvature indicator, y_i is not the displacement in modal shapes but the directly measured vertical displacement under static load. The curvature deduced from Eq. (2) can be called 'Difference curvature'.

Obviously, this difference curvature indicator can also indicate the local damage like common curvature indicator. The absolute difference between the difference curvatures of the intact and undamaged structure should be computed and shown by curve shape firstly. Then the local peaks on the curve can indicate the damage positions.

It can be seen from Eq. (2) that the difference curvature value of a certain measuring point

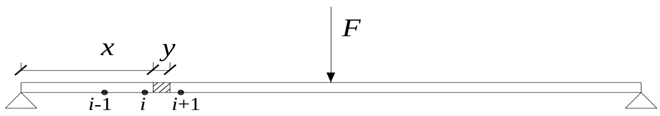

Fig. 1 Model of damaged simply supported bridge

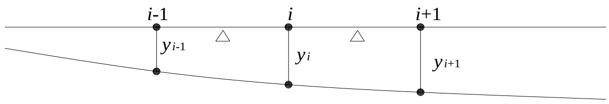

Fig. 2 Sketch of three measuring points

contains information such as the vertical displacement of itself and two successive points, which means it can reflect the damage condition within the range as 2Δ.

3. Difference of nearby difference curvature indicator

Though difference curvature indicator has many advantages, it still relies on the comparison between the difference curvatures of the intact and undamaged structure. Here, a thoroughly new indicator, which is called *Difference of Nearby Difference Curvature Indicator* will be introduced.

3.1 Methodology

Most single-span beams can be seen as simply supported beams during the analysis process. Therefore, a simply supported bridge with whole length as l is considered (Fig. 1).

It is supposed that the length of damage region is y and x is the distance between damage region and the left end. EI, of the damage region, upon the occurrence of damage, becomes zEI. Middle position of span is subjected to concentrated load F.

m is assumed as the distance between measuring point i-1 and the left end of bridge. The sketch of three measuring points is shown in Fig. 2.

According to the principle of virtual work, vertical displacement expression of three points can be obtained as below:

$$EIy_{i-1} = \frac{-Fm[24xy(x+y)(1-z)-3l^3z-12ly^2(1-z)+8y^3(1-z)-24lxy(1-z)+4lm^2z]}{48lz} \tag{3}$$

$$EIy_i = \frac{-F(\Delta+m)\left[24xy(x+y)-3l^3z-12ly^2(1-z)+8y^3(1-z)-24lxy(1-z)+4lz(\Delta^2+m^2)-24xyz(x+y)+8\Delta lmz\right]}{48lz} \tag{4}$$

$$EIy_{i+1} = \frac{\begin{array}{c}-F[16\Delta y^3(1-z)-8y^3(l-m)-6\Delta l^3z+32\Delta^3lz+4lm^3z-3l^3mz+24xy(x+y)\\(2\Delta+m-l-2z+zl-zm)+8y^3z(l-m)+24\Delta lm^2z(m+2\Delta)]\end{array}}{48lz} \tag{5}$$

The difference curvature of measuring point i can be expressed as

$$\frac{1}{\rho_i}=\frac{-F[6\Delta y^2(1-z)+6\Delta^2 z(\Delta+m)+3my^2(1-z)-6xy(x+y(1-z)-2y^3(1-z)+12\Delta xy(1-z)+6mxy(1-z)]}{12z\Delta^2 EI} \tag{6}$$

We know from the above expressions that the vertical displacements and difference curvature values of measuring points contain the information of static load, bridge length, point location and damage parameters (x, y, and z). It can be seen from Eq. (6) that the difference curvature value of point i contains the information of nearby damage (x, y, and z). Obviously, this fact brings much convenience for actual application. We don't need to locate measuring points very closely, which means huge spending and labor.

However, it also should be noted that if multiple faults occur in the space between two successive points, it is hard to differentiate them. Therefore, the interval of measuring points cannot be too large. When the damaged interval is detected, visual inspection or advanced monitoring devices can be employed to find the real damage phenomena within this small region.

For point i+1, the difference curvature under damaged state can be obtained similarly

$$\frac{1}{\rho_{i+1}}=\frac{-F\begin{bmatrix}24\Delta y(1-z)\left(ly-xy-x^2+2lx\right)+\left(3l^3z-8y^3+8y^3z\right)(m+\Delta)+68\Delta^3 lz+12lm\left(y^2-\Delta mz-y^2z\right)\\-4lm^3z-24mxy(x+y-yz-xz-l-lz)+36\Delta^2 lmz\end{bmatrix}}{12z\Delta^2 EI} \tag{7}$$

For point i+2

$$\frac{1}{\rho_{i+2}}=\frac{-F(m+3\Delta)}{2EI} \tag{8}$$

For point i+3

$$\frac{1}{\rho_{i+3}}=\frac{-F(m+4\Delta)}{2EI} \tag{9}$$

$$\frac{1}{\rho_{i+3}}-\frac{1}{\rho_{i+2}}=\frac{1}{\rho_{i+4}}-\frac{1}{\rho_{i+3}}=\cdots\cdots=\frac{-F\Delta}{2EI} \tag{10}$$

For the left side of damage:

For point i-1

$$\frac{1}{\rho_{i-1}}=\frac{-Fm}{2EI} \tag{11}$$

For point i-2

$$\frac{1}{\rho_{i-2}}=\frac{-F(m-\Delta)}{2EI} \tag{12}$$

$$\frac{1}{\rho_{i-1}}-\frac{1}{\rho_{i-2}}=\frac{1}{\rho_{i-2}}-\frac{1}{\rho_{i-3}}=\cdots\cdots=\frac{-F\Delta}{2EI} \tag{13}$$

Then a new indicator Q, which is the absolute difference of difference curvatures between two nearby points, can be defined according to the above results

$$Q_i=\frac{1}{\rho_i}-\frac{1}{\rho_{i-1}} \tag{14}$$

This indicator can be called *Difference of Nearby Difference Curvature Indicator* (*DNDCI*).

A rule can be found as below

$$Q_{i-1} = Q_{i-2} = \cdots\cdots = \frac{-F\Delta}{2EI} \qquad Q_{i+3} = Q_{i+4} = \cdots\cdots = \frac{-F\Delta}{2EI} \tag{15}$$

For point i, which is the left end point of damaged interval

$$Q_i = \frac{1}{\rho_i} - \frac{1}{\rho_{i-1}} = \frac{-F\left[6\Delta y^2(1-z)+6\Delta^3 z+3my^2(1-z)-6xy(x+y)(1-z)-2y^3(1-z)+12\Delta xy(1-z)+6mxy(1-z)\right]}{12z\Delta^2 EI} \tag{16}$$

Q_{i+2} and Q_{i+3} also cannot be constant with the rule as Eq. (15).

Then it is known that when damage occurs between measuring points i and i+1, *DNDCI* values of three points (i, i+1, i+2) are different from the nearby values distinctly. The indicator values of other points are constant.

Therefore, after the *DNDCI* values of measuring points are computed and shown by curve shape, the local peak on the curve can indicate abnormal stiffness changes at that position, i.e., it means damages occur at the corresponding positions.

The extent of peak is considered

$$|Q_i - Q_{i-1}| = \left|\frac{1}{\rho_i} - \frac{2}{\rho_{i-1}} + \frac{1}{\rho_{i-2}}\right| = \frac{F\left[6\Delta y^2(1-z)+3my^2(1-z)-6xy(x+y)(1-z)+6\Delta^2 mz-2y^3(1-z)+12\Delta xy(1-z)+6mxy(1-z)\right]}{12z\Delta^2 EI} \tag{17}$$

Obviously, the larger the value of above expression, the localization of damage will be more clear and stable.

It is stressed that this new indicator *DNDCI* has no practical difficulty even if only the measurement data for the damaged state is given.

3.2 Application process

(1) Arranging measuring points with proper interval Δ (e.g., 1 m).

(2) Putting a concentrated load F on the mid-span of beam under damaged state and measuring the vertical displacement of each point.

(3) Difference curvature of each point then can be derived according to Eq. (2).

(4) The values of *DNDCI* can be obtained according to Eq. (14) and then plotted. The local peaks on this curve can indicate the damage positions.

3.3 Example

A concrete simply supported beam is employed as example. The whole length is 10 m and the cross-section with 6 m width, 0.7 m highness. The modulus of elasticity is taken as 3.15×10^4 N/mm^2.

(1) Damage scenario

One damage scenario is marked: x=3 m, y=0.2 m, the effective height of section is reduced to 0.65 m. The finite element model of damaged beam can be seen as Fig. 3.

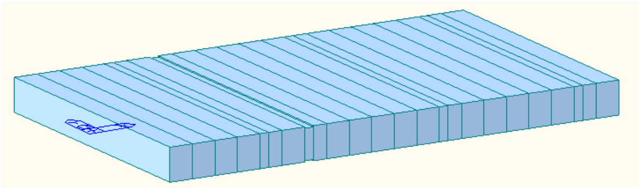

Fig. 3 model of simply supported beam

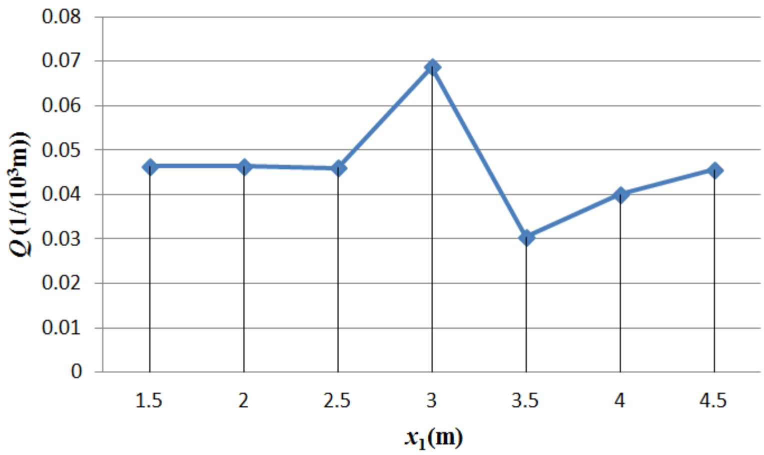

Fig. 4 Graph of Q values

(2) Loading and measurement

Then the new method based on *DNDCI* will be applied. The interval of measuring points is selected as Δ=0.5 m. A concentrated force F=1000 kN is loaded at the middle of the whole span. Based on the displacement data of finite element analysis results, Q value of each measuring section can be obtained and plotted as Fig. 4.

The abscissa is x_1 and ordinate is Q (Unit: 1/(10^3m)). It can be found that the peak occurs at x_1=3 m, which means that damage occurs within the interval [3m, 3.5m]. This result coincides with the damage scenario.

From this example, Q is found to be a sensitive indicator even when information on the intact beam is not available.

3.4 Considering the actual support condition

The above analysis is idealized. It is known that most actual single-span beams are supported by bearings. The boundary condition is between simply supported and fixed. This section will deal with this kind of single-span beams.

Bearings can provide certain stiffness at all the six directions. Considering that beams usually can be seen as one-dimension structures, we can only consider the vertical translation stiffness and rotation stiffness along the beam herein.

The values of these two kinds of stiffness depend on the character of bearings under beam. For example, rubber bearing is used widely in practice, which has E as 7.84 MPa, μ as 0.47 and G as 2.667 MPa. Calculation formulas of supporting stiffness are available: (1) vertical translation

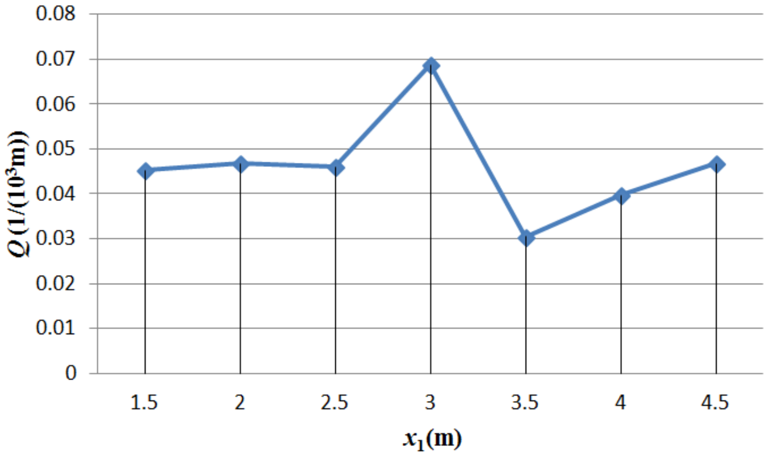

Fig. 5 Graph of Q values

stiffness: EA/L; (2) rotation stiffness along the beam: EI_y/L. Here, E is the modulus of elasticity of rubber bearing; A is the area of bearing; I_y is the bending moment of inertia of bearing section; L is the height of bearing.

When the stiffness of bearing is considered, explicit analysis is too complicated to be presented here. An example will be provided directly to show that *DNDCI* is still applicable.

Parameters and damage scenario of the selected single-span beam are identical to the above example. A rectangle bearing has section as 0.4 m×0.4 m and height as 0.18 m is employed. Vertical translation stiffness and rotation stiffness along the beam are calculated as 6.9689×10^3 kN/m, 92.9185 kNm/rad, respectively. The interval of measuring points is also selected as Δ=0.5 m. Loading force is 1000 kN, too. Based on the displacement data of finite element analysis results, Q values of each measuring section can be obtained and plotted as Fig. 5.

It can be found that the peak also occurs at x_1=3 m, which means that damage occurs within the interval [3 m, 3.5 m]. This result coincides with the damage scenario.

4. Suggestions for improvement of robustness

Some valuable suggestions can be proposed to improve the robustness of *DNDCI* and make this indicator more suitable for actual application:

(1) According to Eq. (17), the value of load F should be as large as possible under the circumstances that the structure is intact. Not only the vertical displacement and difference curvature values of measuring points, but also the magnitude of local peak will enlarge with the increase of F value. Thus the negative effect of resolution limitation and noise pollution can be decreased.

(2) According to Eq. (17), the interval of measuring points within crucial regions (Δ) should be as small as possible if only the cost is acceptable. With a reduction in Δ, the magnitude of local peak also tends to grow larger and the damage detection result becomes relatively stable.

(3) Techniques for improving the resolution and quality of displacement measurement data are also highly recommended.

There are several commonly used displacement measurement devices. However, for the need of damage detection, especially for tiny faults, advanced devices which can provide higher resolution is still needed. A thoroughly new design for displacement measurement has been proposed by authors and authorized patent in China (Patent number: 201020188621.4). This new device can provide very high resolution of displacement.

5. Conclusions

A new indicator, *Difference of Nearby Difference Curvature Indicator* (*DNDCI*), has been introduced and investigated for single-span beams damage localization in this paper. It is innovative that *DNDCI* allows the absence of prior knowledge under intact state. *DNDCI* is sensitive to local damage occurs on or nearby the measuring points. When there is limited number of spatial points that can be measured, which is often the case during field applications, *DNDCI* is effective. Another significant attraction of *DNDCI* is that it can be applicable to most actual beams supported by bearings.

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