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Flexure of cross-ply laminated plates using equivalent single layer trigonometric shear deformation theory

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Abstract. An equivalent single layer trigonometric shear deformation theory taking into account transverse shear deformation effect as well as transverse normal strain effect is presented for static flexure of cross-ply laminated composite and sandwich plates. The inplane displacement field uses sinusoidal function in terms of thickness coordinate to include the transverse shear deformation effect. The cosine function in thickness coordinate is used in transverse displacement to include the effect of transverse normal strain. The kinematics of the present theory is much richer than those of the other higher order shear deformation theories, because if the trigonometric term (involving thickness coordinate z) is expanded in power series, the kinematics of higher order theories (which are usually obtained by power series in thickness coordinate z) are implicitly taken into account to good deal of extent. Governing equations and boundary conditions of the theory are obtained using the principle of virtual work. The closed-form solutions of simply supported cross-ply laminated composite and sandwich plates have been obtained. The results of present theory are compared with those of the classical plate theory (CPT), first order shear deformation theory (FSDT), higher order shear deformation theory (HSDT) of Reddy and exact three dimensional elasticity theory wherever applicable. The results predicted by present theory are in good agreement with those of higher order shear deformation theory and the elasticity theory.

Keywords: shear deformation; transverse normal strain; static flexure; cross-ply laminated plate; sandwich plate; transverse shear stress

1. Introduction

Advances in the technology of composite materials has led to the use of composite plates as structural components in various engineering applications due to superior mechanical properties of these materials. However, shear deformation effects become more pronounced in such structures due to low transverse shear moduli as compared to high inplane tensile moduli, when subjected to transverse loads. This necessitates the accurate structural analysis of composite plates.

Classical plate theory (CPT) is based on the assumption that straight lines which are normal to

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the neutral surface before deformation remain straight and normal to the neutral surface after deformation. Since the transverse shear deformation is neglected, it cannot be applied to thick plate wherein shear deformation effects are more significant. The errors in deflection and stresses are quite significant for plate made out of advanced composite material when obtained using classical plate theory.

Mindlin (1951) has developed displacement based first order shear deformation theory (FSDT) which is based on the assumption that straight lines which are normal to neutral surface before deformation remain straight but not necessarily normal to the deformed neutral surface. Reissner (1944, 1945) was the first to provide consistent stress-based plate theory which incorporates the effect of shear deformation. In these theories, the transverse shear strain distribution is assumed to be constant through the plate thickness and therefore problem dependent shear correction factor is required to account for the shear deformation.

The limitations of classical plate theory and first order shear deformation theory stimulated the development of higher order or equivalent shear deformation theories to avoid the use of shear correction factors, to include effect of cross sectional warping and to get the realistic variation of the transverse shear strains and stresses through the thickness of plates. Krishna Murty (1986, 1987) used such theory for the analysis of beams and plates. Lo *et al*. (1977a, b) presented a generalized displacement function in which the in-plane displacements were represented up to cubic polynomials in thickness coordinate and out of plane displacement up to second order polynomial. This theory contains eleven unknowns. Savithri and Varadan (1992) used Krishna Murty's displacement function for the plate analysis. Reddy (1984, 1985) simplified Lo's theory by assuming constant transverse displacement and satisfying transverse shear stress boundary conditions at top and bottom surfaces of the plate, which was not satisfied in the Lo's theory. Doong *et al.* (1991) utilized Lo's displacement function for vibration and buckling analysis of the plates. In all above theories the heterogeneous plate is treated as a equivalent single layer plate. Soldatos (1988) developed hyperbolic shear deformation theory for the bending analysis of laminated composite plates. Kant and Swaminathan (2002) presented an analytical solutions for the static analysis of laminated composite and sandwich plates based on a higher order refined theory. Leung *et al*. (2003) proposed a new unconstrained third-order plate theory for the symmetrically laminated composite plates. Metin (2006) presented comparative study of various shear deformation theories for the bending, buckling and free vibration analysis of symmetrically laminated composite plates. Akavci (2007) proposed new hyperbolic theory in-terms of tangent and secant functions for the analysis of plates. Karama *et al*. (2009) have proposed an exponential shear deformation plate theory which is modified form of a new shear deformation theory developed by Metin (2009). Bending analysis of unsymmetrically laminated sandwich flat panels with a soft core have been presented by Brischetto *et al*. (2009). Zhen and Wanji (2010) developed C^0-type higher-order theory for bending analysis of laminated composite and sandwich plates subjected to thermal/mechanical loads. Finite element models based on an improved higher order zigzag plate theory are developed by Pandit *et al*. (2010) and Chalak *et al*. (2012) for the bending and vibration analysis of soft core sandwich plates. Nik and Tahani (2010) presented a semi-analytical method for the free vibration analysis of laminated composite plates with arbitrary boundary conditions. Kapuria and Nath (2013) proposed global–local theories for bending and vibration of laminated and sandwich plates. Grover *et al*. (2013), Sahoo and Singh (2013a) proposed a new inverse hyperbolic shear deformation theory for the laminated composite and sandwich plates. A layer-wise stress model for the bending analysis of laminated and sandwich plates have been presented by Thai *et al*. (2013). A new set of models in the framework of

Carrera's Unified Formulation for the static analysis of sandwich plates have been presented by Dehkordi *et al*. (2013). Comprehensive reviews of higher order theories have been given by Noor and Burton (1989), Ghugal and Shimpi (2002), Wanji and Zhen (2008), Kreja (2011).

A class of refined shear deformation theories in which trigonometric functions are used in-terms of thickness coordinate are designated as trigonometric shear deformation theories. Levy (1877) developed a refined theory for thick plate for the first time using trigonometric functions in the displacement field. However, efficiency of this particular plate theory was not assessed for more than a century. The discussion on Levy's theory can be found in Todhunter and Pearson (1893). Use of trigonometric functions to describe the plate behavior in thickness direction was also proposed by Stein (1986) and developed refined theories for laminated beams, plates and shells.

Shimpi and Ghugal (2000) presented layerwise trigonometric shear deformation theory for the flexural analysis of two layered cross-ply laminated plates whereas Shimpi and Ainapure (2004) implemented it for the free vibration analysis of two layered cross-ply laminated plates. However, effect of transverse normal strain is not included in the theory and the theory is only applicable to two layered cross-ply laminated plates. Ghugal and Kulkarni (2011) applied trigonometric shear deformation theory without considering effect of transverse normal strain for the thermal stress analysis of cross-ply laminated plates. Mantari *et al*. (2012) developed a new trigonometric shear deformation theory for isotropic, laminated composite and sandwich plates. Sahoo and Singh (2013b) also proposed a new inverse trigonometric zig-zag theory for the static analysis of laminated composite and sandwich plates.

Ghugal and Sayyad (2010) developed a new trigonometric shear deformation theory which includes effects of transverse shear deformation and transverse normal strain. The theory differs from trigonometric shear deformation theory of Stein (1986) in which shear stress free conditions at top and bottom surfaces of plates are not satisfied, whereas these conditions are fulfilled in the proposed theory. The present theory also differs from other higher order theories; because, in the present theory effect of transverse normal strain is included which is not assessed by the other researchers. The theory is initially applied for the static and free vibration analysis of isotropic plates (Ghugal and Sayyad 2010, 2011a) which is then successfully extended to static and free vibration analysis of orthotropic plates (Ghugal and Sayyad 2011b, 2013a). Sayyad and Ghugal (2011) also applied this theory for the bending analysis of cross-ply laminated beams subjected to various loading cases and further applied to laminated composite and sandwich plates (Sayyad and Ghugal 2014a). Sayyad and Ghugal (2014b) also extended this theory for the buckling analysis of laminated rectangular plates. The effect of transverse normal strain is more important to assess the effect of local stress concentration due to concentrated load and at built-in or clamped edges. This local effect can be effectively assessed by the present theory (Ghugal and Sayyad 2013b, Sayyad and Ghugal 2013). In the present study, this theory is applied for the static flexural analysis of laminated composite and sandwich plates under uniformly distributed, uniformly varying and concentrated loads. The results of present theory are compared with those of other higher order and lower order shear deformation theories and exact solution given by Pagano (1970) wherever applicable.

1.1 Plate under consideration

Consider a rectangular plate of length a, width b, and thickness h made up of linearly elastic and orthotropic material as shown in Fig. 1. The plate consists of N number of homogenous layers

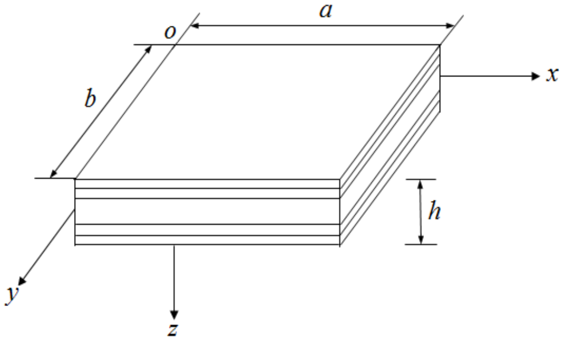

Fig. 1 Plate geometry and coordinate system

which are perfectly bounded. The plate occupies (in *O–x–y–z* right-handed Cartesian coordinate system) a region

$$0 \le x \le a; \qquad 0 \le y \le b; \qquad -h/2 \le z \le h/2; \tag{1}$$

1.2 Assumptions made in theoretical formulation

1. The displacement components *u* and *v* are the inplane displacements in *x* and *y* –directions, respectively and *w* is the transverse displacement in *z*-direction. These displacements are small in comparison with the plate thickness.

2. The in-plane displacement *u* in *x* -direction and *v* in *y* -direction each consists of three parts (extension, bending and shear):

a) The extension components are middle surface components in *x* and *y* directions.

b) The bending components analogous to displacement in classical plate theory.

c) Shear component is assumed to be sinusoidal in nature with respect to thickness coordinate.

3. The transverse displacement *w* in *z* -direction is assumed to be a function of *x, y* and *z* coordinates.

4. The body forces are ignored in the analysis (body forces, if required, can be considered, without much loss of accuracy, as external forces).

5. The plate is subjected to transverse load only.

1.3 The displacement field

Based upon the before mentioned assumptions, the displacement field of the present plate theory is

$$\begin{aligned} u(x,y,z) &= u_0(x,y) - z\frac{\partial w(x,y)}{\partial x} + f(z)\phi(x,y) \\ v(x,y,z) &= v_0(x,y) - z\frac{\partial w(x,y)}{\partial y} + f(z)\psi(x,y) \\ w(x,y,z) &= w(x,y) + g(z)\xi(x,y) \end{aligned} \tag{2}$$

where $f(z)=(h/\pi)\sin(\pi z/h)$ and $g(z)=(h/\pi)\cos(\pi z/h)$ and 'u', 'v' and 'w' are the displacements in x, y and z-directions, respectively. u_0 and v_0 are midplane displacements and are functions of x and y. The ϕ, ψ and ξ represents rotations of the plate at neutral surface. The normal strains ε_x, ε_y, ε_z and shear strains γ_{xy}, γ_{xz}, γ_{yz} are obtained within the framework of linear theory of elasticity using displacement field given by Eq. (2).

$$\begin{aligned}
\varepsilon_x &= \frac{\partial u}{\partial x} = \frac{\partial u_0}{\partial x} - z\frac{\partial^2 w}{\partial x^2} + \frac{h}{\pi}\sin\frac{\pi z}{h}\frac{\partial \phi}{\partial x} \\
\varepsilon_y &= \frac{\partial v}{\partial y} = \frac{\partial v_0}{\partial y} - z\frac{\partial^2 w}{\partial y^2} + \frac{h}{\pi}\sin\frac{\pi z}{h}\frac{\partial \psi}{\partial y} \\
\varepsilon_z &= \frac{\partial w}{\partial z} = -\xi\sin\frac{\pi z}{h}
\end{aligned} \tag{3}$$

$$\begin{aligned}
\gamma_{xy} &= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} - 2z\frac{\partial^2 w}{\partial x \partial y} + \frac{h}{\pi}\sin\frac{\pi z}{h}\left(\frac{\partial \phi}{\partial y} + \frac{\partial \psi}{\partial x}\right) \\
\gamma_{zx} &= \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = \cos\frac{\pi z}{h}\left(\frac{h}{\pi}\frac{\partial \xi}{\partial x} + \phi\right) \\
\gamma_{yz} &= \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} = \cos\frac{\pi z}{h}\left(\frac{h}{\pi}\frac{\partial \xi}{\partial y} + \psi\right)
\end{aligned} \tag{4}$$

It can be noted that transverse shear strains are zero at top and bottom of the plate. Since the laminate is made of several orthotropic layers, the constitutive relations in the k^{th} layer are given as

$$\begin{Bmatrix} \sigma_x^k \\ \sigma_y^k \\ \sigma_z^k \\ \tau_{xy}^k \\ \tau_{yz}^k \\ \tau_{zx}^k \end{Bmatrix} = \begin{bmatrix} Q_{11}^k & Q_{12}^k & Q_{13}^k & 0 & 0 & 0 \\ Q_{21}^k & Q_{22}^k & Q_{23}^k & 0 & 0 & 0 \\ Q_{31}^k & Q_{32}^k & Q_{33}^k & 0 & 0 & 0 \\ 0 & 0 & 0 & Q_{66}^k & 0 & 0 \\ 0 & 0 & 0 & 0 & Q_{44}^k & 0 \\ 0 & 0 & 0 & 0 & 0 & Q_{55}^k \end{bmatrix} \begin{Bmatrix} \varepsilon_x^k \\ \varepsilon_y^k \\ \varepsilon_z^k \\ \gamma_{xy}^k \\ \gamma_{yz}^k \\ \gamma_{zx}^k \end{Bmatrix} \tag{5}$$

$[Q]_{ij}^k$ are reduced stiffness coefficients given by Jones (1975).

2. Derivation of governing equations and boundary conditions

Using Eq. (3) through (5) and principle of virtual work, variationally consistent differential equations and boundary conditions for the plate under consideration are obtained. The principle of virtual work applied is

$$\int_{z=-h/2}^{z=h/2}\int_{y=0}^{y=b}\int_{x=0}^{x=a}\left[\sigma_x\,\delta\varepsilon_x+\sigma_y\,\delta\varepsilon_y+\sigma_z\,\delta\varepsilon_z+\tau_{yz}\,\delta\gamma_{yz}+\tau_{zx}\,\delta\gamma_{zx}+\tau_{xy}\,\delta\gamma_{xy}\right]dx\,dy\,dz$$
$$-\int_{y=0}^{y=b}\int_{x=0}^{x=a} q\left(x,y\right)\delta w\,dx\,dy=0 \tag{6}$$

Integrating the Eq. (6) by parts and collecting the coefficients of $\delta u_0, \delta v_0, \delta w, \delta\phi, \delta\psi$ and $\delta\xi$ the following governing equations and boundary conditions are obtained. The governing equations in terms of stress resultants are as follows

$$\delta u_0:\quad \frac{\partial N_x}{\partial x}+\frac{\partial N_{xy}}{\partial y}=0,\quad \delta v_0:\quad \frac{\partial N_{xy}}{\partial x}+\frac{\partial N_y}{\partial y}=0,$$
$$\delta w:\quad \frac{\partial^2 M_x}{\partial x^2}+2\frac{\partial^2 M_{xy}}{\partial x\partial y}+\frac{\partial^2 M_y}{\partial y^2}+q=0,$$
$$\delta\phi:\quad \frac{\partial M_x^s}{\partial x}+\frac{\partial M_{xy}^s}{\partial y}-V_x^s=0,\quad \delta\psi:\quad \frac{\partial M_y^s}{\partial y}+\frac{\partial M_{xy}^s}{\partial x}-V_{yz}^s=0, \tag{7}$$
$$\delta\xi:\quad \frac{\partial V_{xz}^s}{\partial x}+\frac{\partial V_{yz}^s}{\partial y}-\frac{\pi}{h}V_{zz}^s=0$$

The boundary conditions along edges x=0 and x=a obtained are of the following form

$N_x=0$	or	u_0 is specified
$N_{xy}=0$	or	v_0 is specified
$V_x=\partial M_x/\partial x+2\partial M_{xy}/\partial y=0$	or	w is specified
$M_x=0$	or	$\partial w/\partial x$ is specified
$M_x^s=0$	or	ϕ is specified
$M_{xy}^s=0$	or	ψ is specified
$V_{xz}^s=0$	or	ξ is specified

(8)

and along y=0 and y=b edges, the boundary conditions are as follows

$N_y=0$	or	v_0 is specified
$N_{xy}=0$	or	u_0 is specified

$$\begin{aligned}
&V_y = \partial M_y/\partial y + 2\partial M_{xy}/\partial x = 0 && \text{or} && w \text{ is specified} \\
&M_y = 0 && \text{or} && \partial w/\partial y \text{ is specified} \\
&M_y^s = 0 && \text{or} && \psi \text{ is specified} \\
&M_{xy}^s = 0 && \text{or} && \phi \text{ is specified} \\
&V_{yz}^s = 0 && \text{or} && \xi \text{ is specified}
\end{aligned} \tag{9}$$

At corners (x=0 y=0), (x=a y=0), (x=0 y=b) and (x=a y=b) boundary condition is

$$M_{xy} = 0 \quad \text{or} \quad w \text{ is specified.} \tag{10}$$

The stress resultants appeared in the governing equations and boundary conditions are defined as follows

$$\begin{aligned}
\left(M_x, M_y, M_{xy}\right) &= \sum_{k=1}^{N} \int_{h_k}^{h_{k+1}} \left(\sigma_x, \sigma_x, \tau_{xy}\right) z\, dz \\
\left(M_x^s, M_y^s, M_{xy}^s\right) &= \sum_{k=1}^{N} \int_{h_k}^{h_{k+1}} \left(\sigma_x, \sigma_x, \tau_{xy}\right) f(z)\, dz \\
\left(V_{xz}^s, V_{yz}^s\right) &= \sum_{k=1}^{N} \int_{h_k}^{h_{k+1}} \left(\tau_{xz}, \tau_{yz}\right) f'(z)\, dz \\
V_{zz}^s &= \sum_{k=1}^{N} \int_{h_k}^{h_{k+1}} \sigma_{zz} g'(z)\, dz
\end{aligned} \tag{11}$$

where M_x, M_y, M_{xy} are the bending and twisting moment resultants or the stress couples analogous to classical plate theory, M_x^s, M_y^s, M_{xy}^s are refined moments or stress couples due to transverse shear deformation effects and V_{xz}^s, V_{yz}^s, V_{zz}^s are the transverse shear and transverse normal stress resultants and the prime ( )′ indicates the differentiation of function with respect to z. The governing equations in-terms of unknown variables in the displacement field are of the form

$$\delta u_0:\quad L_1 \frac{\partial^2 u_0}{\partial x^2} + L_2 \frac{\partial^2 u_0}{\partial y^2} + L_3 \frac{\partial^2 v_0}{\partial x \partial y} - L_4 \frac{\partial^3 w}{\partial x^3} - L_5 \frac{\partial^3 w}{\partial x \partial y^2} + L_6 \frac{\partial^2 \phi}{\partial x^2} + L_7 \frac{\partial^2 \phi}{\partial y^2} + L_8 \frac{\partial^2 \psi}{\partial x \partial y} - L_9 \frac{\partial \xi}{\partial x} = 0 \tag{12}$$

$$\delta v_0:\quad L_3 \frac{\partial^2 u_0}{\partial x \partial y} + L_2 \frac{\partial^2 v_0}{\partial x^2} + L_{10} \frac{\partial^2 v_0}{\partial y^2} - L_{11} \frac{\partial^3 w}{\partial y^3} - L_5 \frac{\partial^3 w}{\partial x^2 \partial y} + L_8 \frac{\partial^2 \phi}{\partial x \partial y} + L_7 \frac{\partial^2 \psi}{\partial x^2} + L_{12} \frac{\partial^2 \psi}{\partial y^2} - L_{13} \frac{\partial \xi}{\partial y} = 0 \tag{13}$$

$$\delta w: \quad -L_4\frac{\partial^3 u_0}{\partial x^3}-L_5\frac{\partial^3 u_0}{\partial x\partial y^2}-L_{11}\frac{\partial^3 v_0}{\partial y^3}-L_5\frac{\partial^3 v_0}{\partial x^2\partial y}+L_{14}\frac{\partial^4 w}{\partial x^4}+L_{15}\frac{\partial^4 w}{\partial x^2\partial y^2}+L_{16}\frac{\partial^4 w}{\partial y^4}$$
$$-L_{17}\frac{\partial^3\phi}{\partial x^3}-L_{18}\frac{\partial^3\phi}{\partial x\partial y^2}-L_{19}\frac{\partial^3\psi}{\partial y^3}-L_{18}\frac{\partial^3\psi}{\partial x^2\partial y}+L_{20}\frac{\partial^2\xi}{\partial x^2}+L_{21}\frac{\partial^2\xi}{\partial y^2}=q \tag{14}$$

$$\delta\phi: \quad L_6\frac{\partial^2 u_0}{\partial x^2}+L_7\frac{\partial^2 u_0}{\partial y^2}+L_8\frac{\partial^2 v_0}{\partial x\partial y}-L_{17}\frac{\partial^3 w}{\partial x^3}-L_{18}\frac{\partial^3 w}{\partial x\partial y^2}+L_{22}\frac{\partial^2\phi}{\partial x^2}+L_{23}\frac{\partial^2\phi}{\partial y^2}$$
$$-L_{24}\phi+L_{25}\frac{\partial^2\psi}{\partial x\partial y}-L_{26}\frac{\partial\xi}{\partial x}=0 \tag{15}$$

$$\delta\psi: \quad L_8\frac{\partial^2 u_0}{\partial x\partial y}+L_7\frac{\partial^2 v_0}{\partial x^2}+L_{12}\frac{\partial^2 v_0}{\partial y^2}-L_{19}\frac{\partial^3 w}{\partial y^3}-L_{18}\frac{\partial^3 w}{\partial x^2\partial y}+L_{25}\frac{\partial^2\phi}{\partial x\partial y}+L_{23}\frac{\partial^2\psi}{\partial x^2}$$
$$+L_{27}\frac{\partial^2\psi}{\partial y^2}-L_{28}\psi-L_{29}\frac{\partial\xi}{\partial y}=0 \tag{16}$$

$$\delta\xi: \quad L_9\frac{\partial u_0}{\partial x}+L_{13}\frac{\partial v_0}{\partial y}-L_{20}\frac{\partial^2 w}{\partial x^2}-L_{21}\frac{\partial^2 w}{\partial y^2}+L_{26}\frac{\partial\phi}{\partial x}+L_{29}\frac{\partial\psi}{\partial y}+L_{30}\frac{\partial^2\xi}{\partial y^2}+L_{31}\frac{\partial^2\xi}{\partial x^2}-L_{32}\xi=0 \tag{17}$$

The associated boundary conditions are as follows:
On edges x=0 and x=a, the following conditions hold

$$L_1\frac{\partial u_0}{\partial x}+L_{33}\frac{\partial v_0}{\partial y}-L_4\frac{\partial^2 w}{\partial x^2}-L_{34}\frac{\partial^2 w}{\partial y^2}+L_6\frac{\partial\phi}{\partial x}+L_{35}\frac{\partial\psi}{\partial y}-L_9\xi=0 \quad \text{or } u_0 \text{ is prescribed} \tag{18}$$

$$L_2\left(\frac{\partial u_0}{\partial y}+\frac{\partial v_0}{\partial x}\right)-2L_{36}\frac{\partial^2 w}{\partial x\partial y}+L_7\left(\frac{\partial\phi}{\partial y}+\frac{\partial\psi}{\partial x}\right)=0 \quad \text{or } v_0 \text{ is prescribed} \tag{19}$$

$$L_4\frac{\partial^2 u_0}{\partial x^2}+2L_{36}\frac{\partial^2 u_0}{\partial y^2}+L_5\frac{\partial^2 v_0}{\partial x\partial y}-L_{14}\frac{\partial^3 w}{\partial x^3}-L_{37}\frac{\partial^3 w}{\partial x\partial y^2}$$
$$+L_{17}\frac{\partial^2\phi}{\partial x^2}+2L_{38}\frac{\partial^2\phi}{\partial y^2}+L_{18}\frac{\partial^2\psi}{\partial x\partial y}-L_{20}\frac{\partial\xi}{\partial x}=0 \quad \text{or } w \text{ is prescribed} \tag{20}$$

$$L_4\frac{\partial u_0}{\partial x}+L_{34}\frac{\partial v_0}{\partial y}-L_{14}\frac{\partial^2 w}{\partial x^2}-L_{39}\frac{\partial^2 w}{\partial y^2}+L_{17}\frac{\partial\phi}{\partial x}+L_{40}\frac{\partial\psi}{\partial y}-L_{20}\xi=0 \quad \text{or } \frac{\partial w}{\partial x} \text{ is prescribed} \tag{21}$$

$$L_6\frac{\partial u_0}{\partial x}+L_{35}\frac{\partial v_0}{\partial y}-L_{17}\frac{\partial^2 w}{\partial x^2}-L_{40}\frac{\partial^2 w}{\partial y^2}+L_{22}\frac{\partial\phi}{\partial x}+L_{41}\frac{\partial\psi}{\partial y}-L_{42}\xi=0 \quad \text{or } \phi \text{ is prescribed} \tag{22}$$

$$L_7\left(\frac{\partial u_0}{\partial y}+\frac{\partial v_0}{\partial x}\right)-2L_{38}\frac{\partial^2 w}{\partial x\partial y}+L_{23}\left(\frac{\partial\phi}{\partial y}+\frac{\partial\psi}{\partial x}\right)=0 \quad \text{or } \psi \text{ is prescribed} \tag{23}$$

$$L_{43}\,\phi + L_{31}\frac{\partial \xi}{\partial x} = 0 \qquad \text{or } \xi \text{ is prescribed} \tag{24}$$

On edges y=0 and y=b, the following conditions hold

$$L_2\left(\frac{\partial u_0}{\partial y}+\frac{\partial v_0}{\partial x}\right)-2L_{36}\frac{\partial^2 w}{\partial x \partial y}+L_7\left(\frac{\partial \phi}{\partial y}+\frac{\partial \psi}{\partial x}\right)=0 \qquad \text{or } u_0 \text{ is prescribed} \tag{25}$$

$$L_{33}\frac{\partial u_0}{\partial x}+L_{10}\frac{\partial v_0}{\partial y}-L_{34}\frac{\partial^2 w}{\partial x^2}-L_{11}\frac{\partial^2 w}{\partial y^2}+L_{35}\frac{\partial \phi}{\partial x}+L_{12}\frac{\partial \psi}{\partial y}-L_{13}\xi = 0 \quad \text{or } v_0 \text{ is prescribed} \tag{26}$$

$$L_5\frac{\partial^2 u_0}{\partial x \partial y}+2L_{36}\frac{\partial^2 v_0}{\partial x^2}+L_{11}\frac{\partial^2 v_0}{\partial y^2}-L_{16}\frac{\partial^3 w}{\partial y^3}-L_{37}\frac{\partial^3 w}{\partial x^2 \partial y}$$

$$+L_{18}\frac{\partial^2 \phi}{\partial x \partial y}+2L_{38}\frac{\partial^2 \psi}{\partial x^2}+L_{19}\frac{\partial^2 \psi}{\partial y^2}-L_{21}\frac{\partial \xi}{\partial y}=0 \qquad \text{or } w \text{ is prescribed} \tag{27}$$

$$L_{34}\frac{\partial u_0}{\partial x}+L_{11}\frac{\partial v_0}{\partial y}-L_{39}\frac{\partial^2 w}{\partial x^2}-L_{16}\frac{\partial^2 w}{\partial y^2}+L_{40}\frac{\partial \phi}{\partial x}+L_{19}\frac{\partial \psi}{\partial y}-L_{21}\xi = 0 \quad \text{or } \frac{\partial w}{\partial y} \text{ is prescribed} \tag{28}$$

$$L_7\left(\frac{\partial u_0}{\partial y}+\frac{\partial v_0}{\partial x}\right)-2L_{38}\frac{\partial^2 w}{\partial x \partial y}+L_{23}\left(\frac{\partial \phi}{\partial y}+\frac{\partial \psi}{\partial x}\right)=0 \qquad \text{or } \phi \text{ is prescribed} \tag{29}$$

$$L_{35}\frac{\partial u_0}{\partial x}+L_{12}\frac{\partial v_0}{\partial y}-L_{40}\frac{\partial^2 w}{\partial x^2}-L_{19}\frac{\partial^2 w}{\partial y^2}+L_{41}\frac{\partial \phi}{\partial x}+L_{27}\frac{\partial \psi}{\partial y}-L_{44}\xi = 0 \quad \text{or } \psi \text{ is prescribed} \tag{30}$$

$$L_{45}\,\psi + L_{30}\frac{\partial \xi}{\partial y} = 0 \qquad \text{or } \xi \text{ is prescribed} \tag{31}$$

At corners (x=0, y=0), (x=0, y=b), (x=a, y=0) and (x=a, y=b) the following condition hold

$$L_{36}\left(\frac{\partial u_0}{\partial y}+\frac{\partial v_0}{\partial x}\right)-2L_{46}\frac{\partial^2 w}{\partial x \partial y}+L_{38}\left(\frac{\partial \phi}{\partial y}+\frac{\partial \psi}{\partial x}\right)=0 \qquad \text{or } w \text{ is prescribed} \tag{32}$$

where L_{ij} are the stiffness constants appears in the governing equations and boundary conditions are given in Appendix.

3. Illustrative examples

In order to prove the reliability/accuracy of the present theory, the following numerical examples on laminated composites plates drawn from literature are described and discussed. Some problems of sandwich plates with unavailable results are also presented.

Example 1: The square laminated composite square plates with simply supported boundary conditions and subjected to sinusoidal loading q=$q_0$sin($\pi x/a$)sin($\pi y/b$) on the top surface of the plate are considered where 'q_0' is the magnitude of the sinusoidal loading at the centre. The laminate configuration considered in this example is shown in Fig. 2.

Example 2: The square laminated composite plates with simply supported boundary conditions and subjected to uniformly distributed transverse loading are considered. The loading is represented by $q(x,y)=\sum_{m=1}^{m=\infty}\sum_{n=1}^{n=\infty} q_{mn}\sin(m\pi x/a)\sin(n\pi y/b)$ on the top surface of the plate where m and n are positive integers and q_{mn} is the coefficient of Fourier expansion of load as given below

$$q_{mn} = \frac{16 q_0}{mn\pi^2}. \tag{33}$$

where q_0 is the intensity of uniformly distributed load. The laminate configurations considered in this example are shown in Fig. 3.

Example 3: The laminated composite and sandwich square plates with simply supported boundary conditions and subjected to linearly varying load on the top surface of the plate are considered. The load is given by $q(x,y)=\sum_{m=1}^{m=\infty}\sum_{n=1}^{n=\infty} q_{mn}\sin(m\pi x/a)\sin(n\pi y/b)$ with the coefficient of Fourier expansion q_{mn} of the load as follows

$$q_{mn} = -\frac{8 q_0}{mn\pi^2}\cos(m\pi). \tag{34}$$

The laminate configurations considered in this example are shown in Fig. 4.

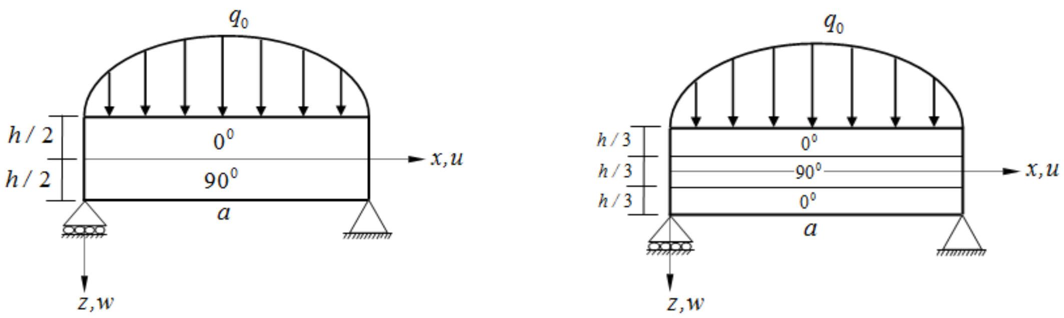

Fig. 2 Simply supported laminated plates under sinusoidal loading

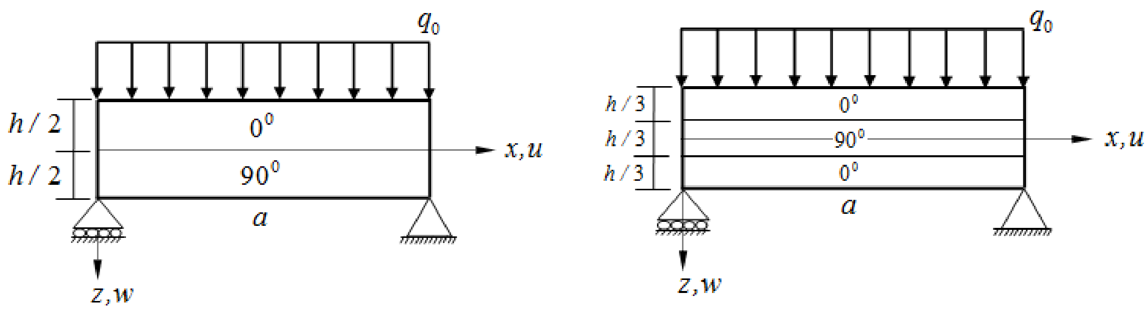

Fig. 3 Simply supported laminated plates under uniformly distributed loading

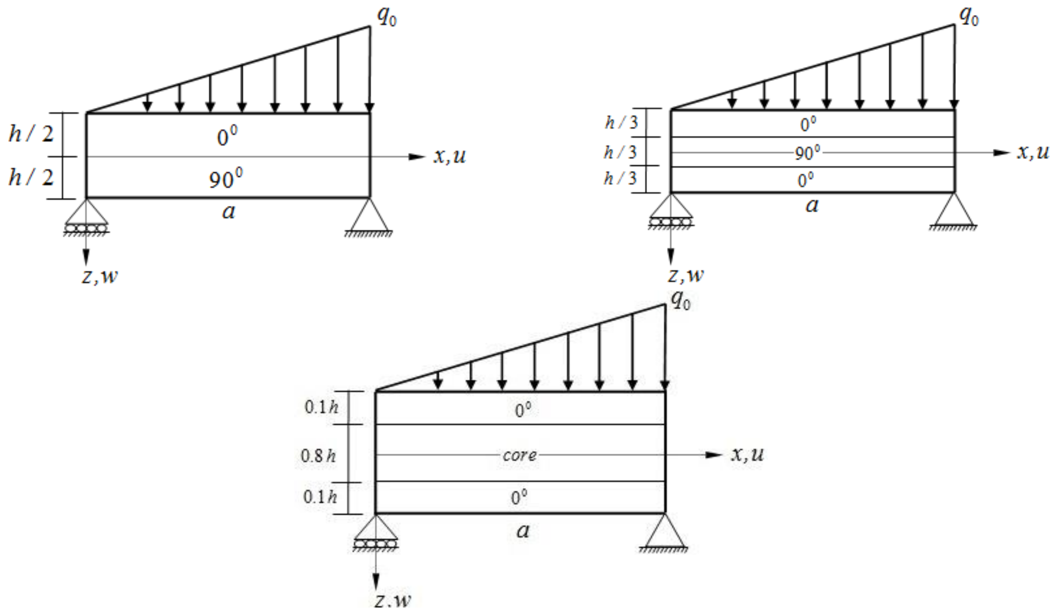

Fig. 4 Simply supported laminated and sandwich plates under linearly varying load

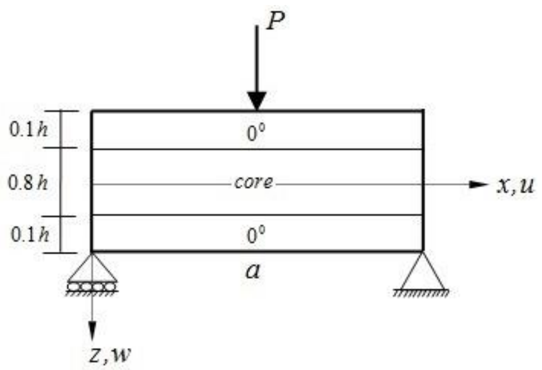

Fig. 5 Simply supported Sandwich plate under central concentrated load

Example 4: The simply supported three layered sandwich plates subjected to concentrated load $q(x,y) = \sum_{m=1}^{m=\infty} \sum_{n=1}^{n=\infty} q_{mn} \sin(m\pi x/a) \sin(n\pi y/b)$ on the top surface of the plate are considered (see Fig. 5). The coefficient of Fourier expansion q_{mn} of the load is given as

$$q_{mn} = \frac{4\mathrm{P}}{a\,b} \sin\left(\frac{m\pi x_0}{a}\right) \sin\left(\frac{m\pi y_0}{b}\right). \tag{35}$$

where x_0 and y_0 are the position coordinates of concentrated load from origin, i.e. for central concentrated load $x_0 = a/2$ and $y_0 = b/2$. The laminate configuration considered in this example is shown in Fig. 4.

The following boundary conditions are imposed at the simply supported edges

$$w = \psi = \xi = M_x = M_x^s = 0 \quad \text{at } x = 0, x = a \tag{36}$$

$$w = \phi = \xi = M_y = M_y^s = 0 \quad \text{at } y = 0, y = b \tag{37}$$

The following is the solution form of $u_0(x,y)$, $v_0(x,y)$, $w(x,y)$, $\phi(x,y)$, $\psi(x,y)$ and $\xi(x,y)$ is assumed for the above examples, which satisfies boundary conditions exactly

$$\begin{aligned}
u_0(x,y) &= \sum_{m=1,3,5}^{m=\infty} \sum_{n=1,3,5}^{n=\infty} u_{mn} \cos\frac{m\pi x}{a} \sin\frac{n\pi y}{b}, \\
v_0(x,y) &= \sum_{m=1,3,5}^{m=\infty} \sum_{n=1,3,5}^{n=\infty} v_{mn} \sin\frac{m\pi x}{a} \cos\frac{n\pi y}{b}, \\
w(x,y) &= \sum_{m=1,3,5}^{m=\infty} \sum_{n=1,3,5}^{n=\infty} w_{mn} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b}, \\
\phi(x,y) &= \sum_{m=1,3,5}^{m=\infty} \sum_{n=1,3,5}^{n=\infty} \phi_{mn} \cos\frac{m\pi x}{a} \sin\frac{n\pi y}{b}, \\
\psi(x,y) &= \sum_{m=1,3,5}^{m=\infty} \sum_{n=1,3,5}^{n=\infty} \psi_{mn} \sin\frac{m\pi x}{a} \cos\frac{n\pi y}{b}, \\
\xi(x,y) &= \sum_{m=1,3,5}^{m=\infty} \sum_{n=1,3,5}^{n=\infty} \xi_{mn} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b}
\end{aligned} \tag{38}$$

where u_{mn}, v_{mn}, w_{mn}, ϕ_{mn}, ψ_{mn} and ξ_{mn} are the unknown coefficients of the respective Fourier expansions and m, n are positive integers**.** In case of single sine load m=1 and n=1. Substituting this form of solution and the load $q(x,y)$ into the governing Eqs. (12) through (17) yields the six algebraic simultaneous equations from which the unknowns u_{mn}, v_{mn}, w_{mn}, ϕ_{mn}, ψ_{mn} and ξ_{mn} can be readily determined. Having obtained values of these unknown coefficients one can then calculate all the displacement and stress components within the plate. Transverse shear stresses (τ_{zx}, τ_{yz}) are obtained by using constitutive relations and integrating equations of equilibrium of theory of elasticity to ascertain the continuity at layer interface.

$$\tau_{zx}^k = \int_{-h/2}^{h_k} \left(\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} \right) dz, \quad \tau_{yz}^k = \int_{-h/2}^{h_k} \left(\frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} \right) dz \tag{39}$$

It may be noted that τ_{zx} obtained by constitutive relations is indicated by τ_{zx}^{CR} and it is indicated by τ_{zx}^{EE} when obtained by using equilibrium equations. Similar notations are also used for τ_{yz}. The following plate material properties given by Pagano (1970) are used for the analysis of laminated composite and sandwich plates in this paper.

Material 1: For laminated composite plate and face sheet of sandwich plate

$$E_1 = 25E_2,\ E_3 = E_2,\ G_{12} = G_{13} = 0.5E_2,\ G_{23} = 0.2E_2 \quad \text{and} \quad \mu_{12} = \mu_{13} = \mu_{23} = 0.25 \tag{40}$$

Material 2: For core of sandwich plate.

$$E_1 = E_2 = 0.04, \quad E_3 = 0.5, \quad G_{13} = G_{23} = 0.06, \quad G_{12} = 0.016, \quad \mu_{12} = \mu_{32} = \mu_{31} = 0.25 \tag{41}$$

4. Numerical results and discussion

4.1 Numerical results

The results obtained for displacements, and stresses are presented using following non-dimensional forms.

$$\bar{u}\left(0, \frac{b}{2}, \frac{z}{h}\right) = \frac{u\, h^2 E_3}{q\, a^3}, \quad \bar{w}\left(\frac{a}{2}, \frac{b}{2}, \frac{z}{h}\right) = \frac{w 100 h^3 E_3}{q\, a^4}, \quad \left(\bar{\sigma}_x, \bar{\sigma}_y\right)\left(\frac{a}{2}, \frac{b}{2}, \frac{z}{h}\right) = \frac{\left(\sigma_x, \sigma_y\right) h^2}{q a^2},$$
$$\left(\bar{\tau}_{xy}\right)\left(\frac{a}{2}, \frac{b}{2}, \frac{z}{h}\right) = \frac{\left(\tau_{xy}\right) h^2}{q a^2}, \quad \left(\bar{\tau}_{zx}\right)\left(0, \frac{b}{2}, \frac{z}{h}\right) = \frac{\left(\tau_{zx}\right) h}{q\, a}, \quad \left(\bar{\tau}_{yz}\right)\left(\frac{a}{2}, 0, \frac{z}{h}\right) = \frac{\left(\tau_{zx}, \tau_{yz}\right) h}{q\, a}. \tag{42}$$

Table 1 Comparison of inplane displacement ($\bar{u}$), transverse displacement ($\bar{w}$), normal stresses ($\bar{\sigma}_x$ and $\bar{\sigma}_y$), inplane shear stress $\bar{\tau}_{xy}$ and transverse shear stresses ($\bar{\tau}_{xz}$ and $\bar{\tau}_{yz}$) in simply supported square laminated plate subjected to single sine load

Layer	h/a	Theory	Model	$\bar{u}$ (h/2)	$\bar{w}$ (z = 0)	$\bar{\sigma}_x$ (-h/2)	$\bar{\sigma}_y$ (-h/2)	$\bar{\tau}_{xy}$ (-h/2)	$\bar{\tau}_{xz}^{CR}$ (0)	$\bar{\tau}_{xz}^{EE}$ (*)	$\bar{\tau}_{yz}^{CR}$ (0)	$\bar{\tau}_{yz}^{EE}$ (*)
$0^0/90^0$	0.25	Present	TSDT	0.0111	1.9424	0.9063	0.0964	0.0562	0.3189	0.3370	0.3189	0.3370
		Reddy	HSDT	0.0113	1.9985	0.9060	0.0891	0.0577	0.3128	0.3396	0.3128	0.3396
		Mindlin	FSDT	0.0088	1.9682	0.7157	0.0843	0.0525	0.2274	0.3356	0.2274	0.3356
		Kirchhoff	CPT	0.0088	1.0636	0.7157	0.0843	0.0525	---	0.3356	---	0.3356
		Pagano	Elasticity	---	2.0670	0.8410	0.1090	0.0591	0.3210	---	0.3130	---
	0.1	Present	TSDT	0.0092	1.2089	0.7471	0.0876	0.0530	0.3261	0.3352	0.3261	0.3352
		Reddy	HSDT	0.0092	1.2161	0.7468	0.0851	0.0533	0.3190	0.3357	0.3190	0.3357
		Mindlin	FSDT	0.0088	1.2083	0.7157	0.0843	0.0525	0.2274	0.3356	0.2274	0.3356
		Kirchhoff	CPT	0.0088	1.0636	0.7157	0.0843	0.0525	---	0.3356	---	0.3356
		Pagano	Elasticity	---	1.2250	0.7302	0.0886	0.0535	0.3310	---	0.3310	---
$0^0/90^0/0^0$	0.25	Present	TSDT	0.0092	1.9015	0.7535	0.0880	0.0496	0.2092	0.2768	0.1914	0.2088
		Reddy	HSDT	0.0091	1.9218	0.7345	0.0782	0.0497	0.2024	0.2855	0.1832	0.2086
		Mindlin	FSDT	0.0055	1.5681	0.4370	0.0614	0.0369	0.1201	0.3368	0.1301	0.1968
		Kirchhoff	CPT	0.0068	0.4312	0.5387	0.0267	0.0213	---	0.3951	---	0.0823
		Pagano	Elasticity	---	2.0046	0.7984	0.0949	0.0505	0.2550	---	0.2170	---
	0.1	Present	TSDT	0.0071	0.7155	0.5720	0.0411	0.0278	0.2577	0.3670	0.1070	0.1179
		Reddy	HSDT	0.0071	0.7125	0.5684	0.0387	0.0277	0.2447	0.3693	0.1033	0.1167
		Mindlin	FSDT	0.0065	0.6306	0.5134	0.0353	0.0252	0.1363	0.3806	0.0762	0.1108
		Kirchhoff	CPT	0.0068	0.4312	0.5387	0.0267	0.0213	---	0.3951	---	0.0823
		Pagano	Elasticity	---	0.7528	0.5898	0.0418	0.0289	0.3570	---	0.1200	---

(*) indicates maximum value of transverse shear stress.

where E_3 is elastic modulus of the middle layer. The results obtained for displacement and stresses are presented in Tables 1 through 5 and graphically in Figs. 6 through 9. The percentage error in result of a particular theory with respect to the result of exact elasticity solution is calculated as follows

$$\%\text{error} = \frac{\text{value by a particular model} - \text{value by exact elasticity solution}}{\text{value by exact elasticity solution}} \times 100$$

Table 2 Comparison of inplane displacement ($\bar{u}$), transverse displacement ($\bar{w}$), normal stresses ($\bar{\sigma}_x$ and $\bar{\sigma}_y$), inplane shear stress $\bar{\tau}_{xy}$ and transverse shear stresses ($\bar{\tau}_{xz}$ and $\bar{\tau}_{yz}$) in simply supported square laminated plate subjected to uniformly distributed load

Layer	h/a	Theory	Model	$\bar{u}$ ($h/2$)	$\bar{w}$ (0)	$\bar{\sigma}_x$ ($-h/2$)	$\bar{\sigma}_y$ ($-h/2$)	$\bar{\tau}_{xy}$ ($-h/2$)	$\bar{\tau}_{xz}^{CR}$ (0)	$\bar{\tau}_{xz}^{EE}$ (*)	$\bar{\tau}_{yz}^{CR}$ (0)	$\bar{\tau}_{yz}^{EE}$ (*)
$0^0/90^0$	0.25	Present	TSDT	0.0189	2.9983	1.2603	0.1394	0.1104	0.5966	0.8945	0.5966	0.8945
		Reddy	HSDT	0.0190	3.0706	1.2691	0.1314	0.1070	0.6034	0.8648	0.6034	0.8648
		Mindlin	FSDT	0.0144	3.0082	1.0636	0.1258	0.0992	0.4775	0.7265	0.4775	0.7265
		Kirchhoff	CPT	0.0147	1.6955	1.0763	0.1269	0.0934	---	0.7415	---	0.7415
		Pagano	Elasticity	---	3.1580	1.1840	0.1590	---	0.647	---	0.591	---
	0.1	Present	TSDT	0.0153	1.9070	1.1057	0.1307	0.0978	0.6669	0.7545	0.6669	0.7545
		Reddy	HSDT	0.0154	1.9173	1.1049	0.1274	0.0977	0.6591	0.7530	0.6591	0.7530
		Mindlin	FSDT	0.0146	1.9050	1.0533	0.1265	0.0961	0.4849	0.7369	0.4849	0.7369
		Kirchhoff	CPT	0.0147	1.6955	1.0763	0.1269	0.0934	---	0.7415	---	0.7415
		Pagano	Elasticity	---	1.9320	1.0860	0.1300	---	0.702	---	0.744	---
$0^0/90^0/0^0$	0.25	Present	TSDT	0.0154	2.8934	1.0343	0.1138	0.1097	0.3575	0.3751	0.4359	0.2933
		Reddy	HSDT	0.0152	2.9091	1.0177	0.1030	0.1092	0.3530	0.4036	0.4425	0.3947
		Mindlin	FSDT	0.0088	2.3538	0.6546	0.0852	0.0736	0.2286	0.6395	0.3427	0.5528
		Kirchhoff	CPT	0.0107	0.6660	0.8076	0.0307	0.0426	---	0.7233	---	0.3859
		Pagano	Elasticity	---	3.0438	1.1229	0.1238	---	0.4428	---	0.4867	---
	0.1	Present	TSDT	0.0115	1.0954	0.8436	0.0510	0.0594	0.4607	0.6139	0.3467	0.3553
		Reddy	HSDT	0.0115	1.0900	0.8395	0.0481	0.0593	0.4409	0.6259	0.3443	0.3859
		Mindlin	FSDT	0.0102	0.9642	0.7720	0.0442	0.0515	0.2530	0.7054	0.2633	0.4230
		Kirchhoff	CPT	0.0107	0.6660	0.8076	0.0307	0.0426	---	0.7233	---	0.3859
		Pagano	Elasticity	---	1.1539	0.8708	0.0529	---	0.6279	---	0.4009	---

(*) indicates maximum value of transverse shear stress.

Table 3 Comparison of inplane displacement ($\bar{u}$), transverse displacement ($\bar{w}$), normal stresses ($\bar{\sigma}_x$ and $\bar{\sigma}_y$), inplane shear stress $\bar{\tau}_{xy}$ and transverse shear stresses ($\bar{\tau}_{xz}$ and $\bar{\tau}_{yz}$) in simply supported square laminated plate subjected to linearly varying load.

Layer	h/a	Theory	Model	$\bar{u}$ ($h/2$)	$\bar{w}$ (0)	$\bar{\sigma}_x$ ($-h/2$)	$\bar{\sigma}_y$ ($-h/2$)	$\bar{\tau}_{xy}$ ($-h/2$)	$\bar{\tau}_{xz}^{CR}$ (0)	$\bar{\tau}_{xz}^{EE}$ (*)	$\bar{\tau}_{yz}^{CR}$ (0)	$\bar{\tau}_{yz}^{EE}$ (*)
$0^0/90^0$	0.25	Present	TSDT	0.0095	1.4992	0.6301	0.0697	0.0552	0.2983	0.4472	0.2983	0.4472
		Reddy	HSDT	0.0095	1.5353	0.6345	0.0657	0.0535	0.3017	0.4324	0.3017	0.4324

Table 3 Continued

$0^0/90^0$	0.25	Mindlin	FSDT	0.0072	1.5041	0.5318	0.0629	0.0496	0.2387	0.3632	0.2387	0.3632
		Kirchhoff	CPT	0.0074	0.8478	0.5381	0.0635	0.0467	---	0.3707	---	0.3707
		Pagano	Elasticity	---	1.5790	0.5920	0.0795	---	0.3235	---	0.3235	---
	0.1	Present	TSDT	0.0077	0.9535	0.5528	0.0653	0.0489	0.3334	0.3772	0.3334	0.3772
		Reddy	HSDT	0.0077	0.9587	0.5524	0.0637	0.0488	0.3295	0.3765	0.3295	0.3765
		Mindlin	FSDT	0.0073	0.9525	0.5267	0.0632	0.0480	0.2424	0.3684	0.2424	0.3684
		Kirchhoff	CPT	0.0074	0.8478	0.5381	0.0635	0.0467	---	0.3707	---	0.3707
		Pagano	Elasticity	---	0.9660	0.5430	0.0650	---	0.3510	---	0.3510	---
$0^0/90^0/0^0$	0.25	Present	TSDT	0.0077	1.4467	0.5171	0.0569	0.0548	0.1788	0.1876	0.2180	0.1467
		Reddy	HSDT	0.0076	1.4545	0.5088	0.0515	0.0546	0.1765	0.2018	0.2213	0.1974
		Mindlin	FSDT	0.0044	1.1769	0.3273	0.0426	0.0368	0.1143	0.3197	0.1709	0.2764
		Kirchhoff	CPT	0.0054	0.3330	0.4038	0.0154	0.0213	---	0.3616	---	0.1930
		Pagano	Elasticity	---	1.5219	0.5614	0.0619	---	0.2214	---	0.2433	---
	0.1	Present	TSDT	0.0057	0.5477	0.4218	0.0255	0.0297	0.2304	0.3070	0.1734	0.1777
		Reddy	HSDT	0.0057	0.5450	0.4198	0.0241	0.0296	0.2205	0.3130	0.1722	0.1930
		Mindlin	FSDT	0.0051	0.4821	0.3860	0.0221	0.0258	0.1265	0.3527	0.1317	0.2115
		Kirchhoff	CPT	0.0054	0.3330	0.4038	0.0154	0.0213	---	0.3616	---	0.1930
		Pagano	Elasticity	---	0.5769	0.4354	0.0264	---	0.3139	---	0.2005	---

(*) indicates maximum value of transverse shear stress.

Table 4 Comparison of inplane displacement ($\bar{u}$), transverse displacement ($\bar{w}$), normal stresses ($\bar{\sigma}_x$ and $\bar{\sigma}_y$), inplane shear stress $\bar{\tau}_{xy}$ and transverse shear stresses ($\bar{\tau}_{xz}$ and $\bar{\tau}_{yz}$) in simply supported sandwich (0^0/*core*/0^0) plate subjected to linearly varying load

h/*a*	Theory	Model	$\bar{u}$ (*h*/2)	$\bar{w}$ (0)	$\bar{\sigma}_x$ (-*h*/2)	$\bar{\sigma}_y$ (-*h*/2)	$\bar{\tau}_{xy}$ (-*h*/2)	$\bar{\tau}_{xz}^{CR}$ (0)	$\bar{\tau}_{xz}^{EE}$ (0)	$\bar{\tau}_{yz}^{CR}$ (0)	$\bar{\tau}_{yz}^{EE}$ (0)
0.25	Present	TSDT	0.0073	2.6328	0.9663	0.1694	0.1476	0.2496	0.2095	0.1460	0.1137
	Reddy	HSDT	0.0073	2.6529	0.9630	0.1590	0.1496	0.2426	0.2139	0.1414	0.1164
	Mindlin	FSDT	0.0044	1.5427	0.6803	0.0954	0.0864	0.0968	0.2651	0.0607	0.1013
	Kirchhoff	CPT	0.0053	0.3314	0.8026	0.0254	0.0470	---	0.2882	---	0.0710
	Pagano	Elasticity	0.0077	2.9768	1.1100	0.1938	0.1649	0.2166	---	0.0967	---
0.1	Present	TSDT	0.0056	0.7750	0.8289	0.0643	0.0739	0.3110	0.2692	0.1029	0.0846
	Reddy	HSDT	0.0056	0.7764	0.8286	0.0602	0.0748	0.2979	0.2707	0.0989	0.0843
	Mindlin	FSDT	0.0051	0.5432	0.7800	0.0412	0.0576	0.1041	0.2848	0.0465	0.0776
	Kirchhoff	CPT	0.0053	0.3314	0.8026	0.0254	0.0470	---	0.2882	---	0.0710
	Pagano	Elasticity	0.0059	0.8768	0.9049	0.0858	0.0668	0.2726	---	0.0510	---

4.2 Discussion of results

The results obtained by present theory for displacements and stresses are compared with those of classical plate theory (CPT), first order shear deformation theory (FSDT) of Mindlin (1951), higher order shear deformation theory (HSDT) of Reddy (1984), and exact theory by Pagano (1970).

Example 1: A simply supported laminated composite square plates under sinusoidal load Table 1 shows comparison of displacements and stresses for the two layered ($0^0/90^0$) anti-symmetric and

Table 5 Comparison of inplane displacement ($\bar{u}$), transverse displacement ($\bar{w}$), normal stresses ($\bar{\sigma}_x$ and $\bar{\sigma}_y$), inplane shear stress $\bar{\tau}_{xy}$ and transverse shear stresses ($\bar{\tau}_{xz}$ and $\bar{\tau}_{yz}$) in simply supported sandwich (0^0/*core*/0^0) plate subjected to central concentrated load

h/a	Theory	Model	$\bar{u}$ (*h*/2)	$\bar{w}$ (0)	$\bar{\sigma}_x$ (-*h*/2)	$\bar{\sigma}_y$ (-*h*/2)	$\bar{\tau}_{xy}$ (-*h*/2)	$\bar{\tau}_{xz}^{CR}$ (0)	$\bar{\tau}_{xz}^{EE}$ (*)	$\bar{\tau}_{yz}^{CR}$ (0)	$\bar{\tau}_{yz}^{EE}$ (*)
0.25	Present	TSDT	0.0314	23.3483	103.802	14.2953	0.4231	0.8806	2.9810	0.2057	1.1213
	Reddy	HSDT	0.0323	26.9313	89.3693	10.4715	0.4266	0.8688	2.9086	0.2648	0.8487
	Mindlin	FSDT	0.0251	19.9594	8.54325	2.8370	0.2555	0.4485	1.2317	0.1959	0.2885
	Kirchhoff	CPT	0.0322	2.3471	11.7500	1.6441	0.0857	---	1.3366	---	0.1042
0.1	Present	TSDT	0.0297	7.2239	45.0978	5.7239	0.1887	1.1878	2.7295	0.1381	0.4766
	Reddy	HSDT	0.0301	7.4910	40.1169	4.7600	0.1877	1.1770	2.4621	0.1716	0.3563
	Mindlin	FSDT	0.0302	5.3082	9.64308	2.3860	0.1323	0.4894	1.3428	0.1843	0.1886
	Kirchhoff	CPT	0.0322	2.3471	11.7500	1.6441	0.0857	---	1.3366	---	0.1042

(*) indicates maximum value of transverse shear stress.

three layered ($0^0/90^0/0^0$) symmetric cross-ply laminated plates subjected to sinusoidal loading. Layers are of equal thickness and made up of Material 1. The inplane displacements predicted by present theory for ($0^0/90^0$) and ($0^0/90^0/0^0$) cross-ply laminated plates are more or less identical with those of HSDT of Reddy. The FSDT and CPT underestimate the inplane displacement for all aspect ratios. The maximum transverse displacements obtained by present theory are in good agreement with those of exact solution for ($0^0/90^0$) and ($0^0/90^0/0^0$) cross-ply laminated plates. The transverse displacements predicted by Reddy's theory are in tune with exact solution whereas FSDT and CPT underpredict the same for all aspect ratios. The inplane normal stress $\bar{\sigma}_x$ predicted by present theory is in excellent agreement with that of exact solution for symmetric and anti-symmetric cross-ply laminated plates whereas FSDT and CPT underestimate this stress for all aspect ratios when compared with the values of other refined theories. For both ($0^0/90^0$) and ($0^0/90^0/0^0$) cross-ply laminated plates, the inplane normal stress $\bar{\sigma}_y$ and shear stress $\bar{\tau}_{xy}$ predicted by present theory are in good agreement with those of exact solution. Table 1 also presents the comparison of transverse shear stresses for the two layered ($0^0/90^0$) anti-symmetric and three ($0^0/90^0/0^0$) layered symmetric cross-ply laminated plates subjected to sinusoidal loading. The present theory gives more accurate transverse shear stresses than those given by other refined theories as compared to exact values. For two layered anti-symmetric cross-ply laminated plates, transverse shear stresses predicted by present theory are in excellent agreement when obtained using constitutive relations as well as by the equations of equilibrium. The present theory underestimates the transverse shear stress of cross-ply laminated plates by 0.65 % and 1.48 % for *h*/*a*=0.25 and *h*/*a*=0.1 respectively when obtained using constitutive relations and overestimate it by 4.98% and 1.26% when obtained using equations of equilibrium. Reddy's theory underestimates the transverse shear stresses when obtained using constitutive relations and overestimates the same when obtained using equations of equilibrium compared to those of exact solution. For symmetric cross-ply laminated plate, present theory predicts excellent transverse shear stress by equations of equilibrium. FSDT underestimates the transverse shear stress when obtained using constitutive relations and overestimates the same when obtained using equations of equilibrium. Transverse shear stresses predicted by FSDT and CPT are identical for cross-ply laminated plates for *h*/*a*=0.25 and *h*/*a*=0.1. Through thickness variations of displacements and stresses of ($0^0/90^0$) and ($0^0/90^0/0^0$) cross-ply laminated plates under sinusoidal loading for h/a=0.25 are shown in Figs. 6 and 7, respectively.

Example 2: A simply supported laminated composite square plates under uniformly distributed load Table 2 shows the comparison of displacements and stresses for cross-ply laminated plates subjected to uniformly distributed load. Layers are of equal thickness and made up of Material 1. From Table 2 it is observed that the inplane displacements predicted by present theory and HSDT of Reddy are in excellent agreement with each other whereas FSDT and CPT underestimate the results of inplane displacement compared to those of present theory and HSDT. The present theory underpredicts the transverse displacement for ($0^0/90^0$) cross-ply laminated plates by 5.05 % for h/a=0.25 and 1.29% for h/a=0.1 and Reddy's theory underpredicts it by 2.77% for h/a=0.25 and 0.76% for h/a=0.1 as compared to the exact value. The transverse displacements predicted by present theory are in good agreement with those of exact solution for ($0^0/90^0/0^0$) cross-ply laminated plate. The inplane normal stresses obtained by present theory are in excellent agreement with those of exact solution for ($0^0/90^0$) and ($0^0/90^0/0^0$) cross-ply laminated plates. The inplane shear stresses predicted by present theory and theory of Reddy are in good agreement with each other for ($0^0/90^0$) and ($0^0/90^0/0^0$) cross-ply laminated plates. FSDT and CPT underestimate the inplane stresses for all aspect ratios as compared to the results of other theories. Results from Table 2 indicate that the present theory underestimates the transverse shear stresses when obtained using constitutive relations and overestimates the same when obtained using equations of equilibrium for h/a=0.25. Transverse shear stresses predicted by present theory for ($0^0/90^0$) and ($0^0/90^0/0^0$) cross-ply laminated plate are in excellent agreement with those of exact solution for h/a=0.1.

Example 3: A simply supported laminated composite and sandwich square plates under linearly varying load Displacements and stresses of cross-ply laminated plates subjected to linearly varying load are presented in Table 3. Layers are of equal thickness and made up of Material 1. The maximum transverse displacement and inplane normal stresses predicted by present theory are in close agreement with exact solution. Present theory and Reddy's theory overestimate the inplane normal stress $\bar{\sigma}_x$ for ($0^0/90^0$) cross-ply laminated plate and underestimates the same for ($0^0/90^0/0^0$) cross-ply laminated plate for both the aspect ratios as compared to the exact value. FSDT and CPT underestimate the transverse displacement and inplane normal stresses. Present theory overestimates the inplane normal stress $\bar{\sigma}_x$ by 1.80 % for ($0^0/90^0$) cross-ply laminated plate for h/a=0.1. The inplane shear stresses obtained by present theory and Reddy's theory are in good agreement with each other. The present theory and theory of Reddy underestimate the transverse shear stresses when obtained using constitutive relations and overestimates the same when obtained using equations of equilibrium as compared to those of exact solution. For symmetric cross-ply laminated plates, present theory predicts excellent transverse shear stresses by equations of equilibrium for h/a=0.1.

Table 4 shows the comparison of displacements and stresses for the three layered (0^0/*core*/0^0) square sandwich plate subjected to linearly varying load. The thickness of top and bottom face sheet is 0.1h whereas thickness of core is 0.8h, where 'h' is the total thickness of the plate. The face sheets are made up of Material 1 whereas core is made up of material 2 defined by Eqs. (40) and (41), respectively. From Table 4 it is observed that the inplane displacements predicted by present theory are in close agreement with those of exact solution and HSDT of Reddy. The transverse displacement obtained by present theory is in good agreement with that of exact solution and Reddy's theory. The inplane normal stress $\bar{\sigma}_x$ predicted by present theory and Reddy's theory is in excellent agreement with that of exact solution whereas FSDT and CPT

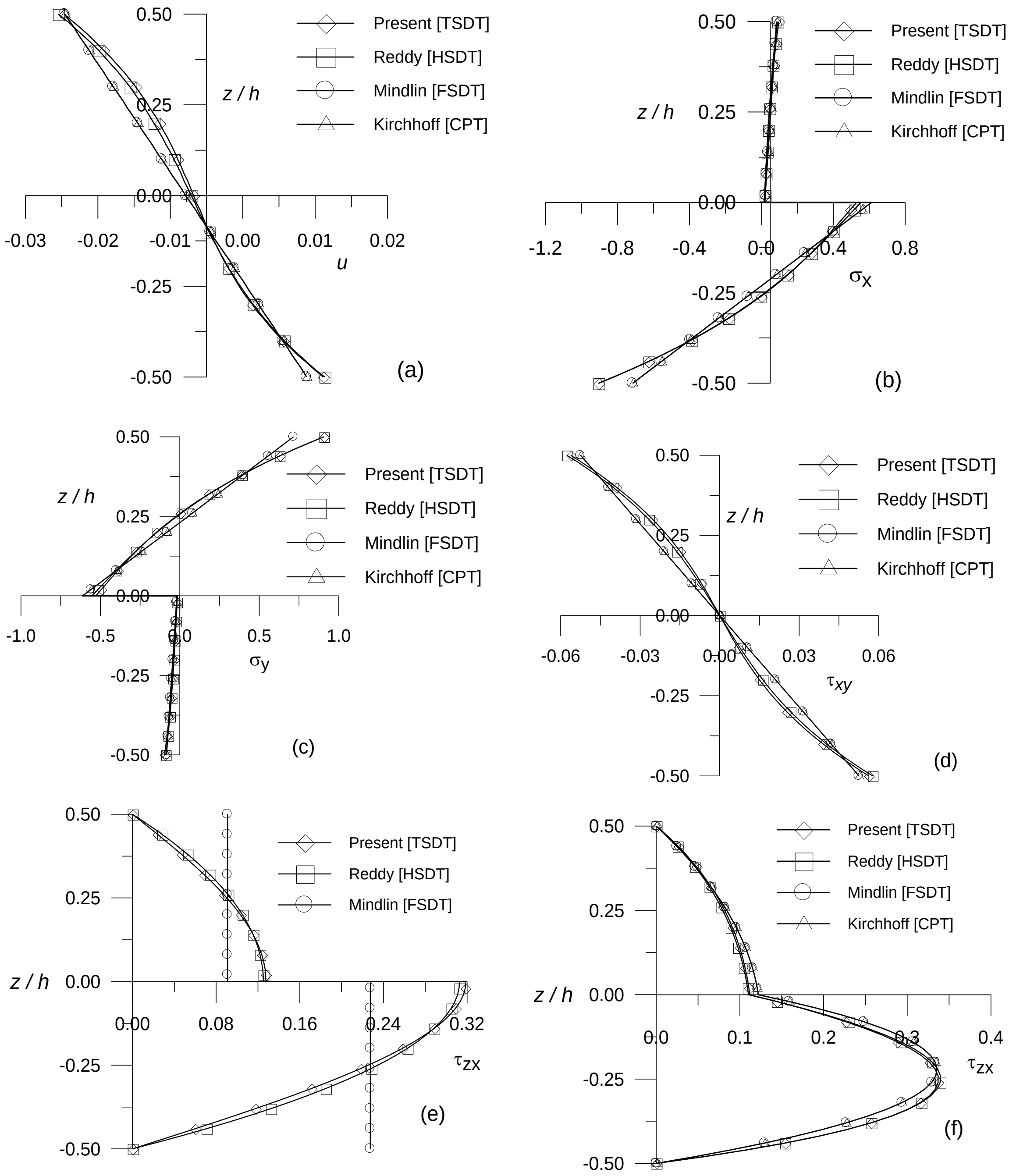

Fig. 6 Through thickness distribution of displacement and stresses of two layered ($0^0/90^0$) cross-ply laminated plate under sinusoidal loading for *h/a* = 0.25: (*a*) Inplane displacement ($\bar{u}$) (*b*) Inplane normal stress ($\bar{\sigma}_x$) (c) Inplane normal stress ($\bar{\sigma}_y$) (d) Inplane shear stress ($\bar{\tau}_{xy}$) (e) Transverse shear stress *via* constitutive relation ($\bar{\tau}_{zx}^{CR}$) (f) Transverse shear stress *via* equilibrium equation ($\bar{\tau}_{zx}^{EE}$)

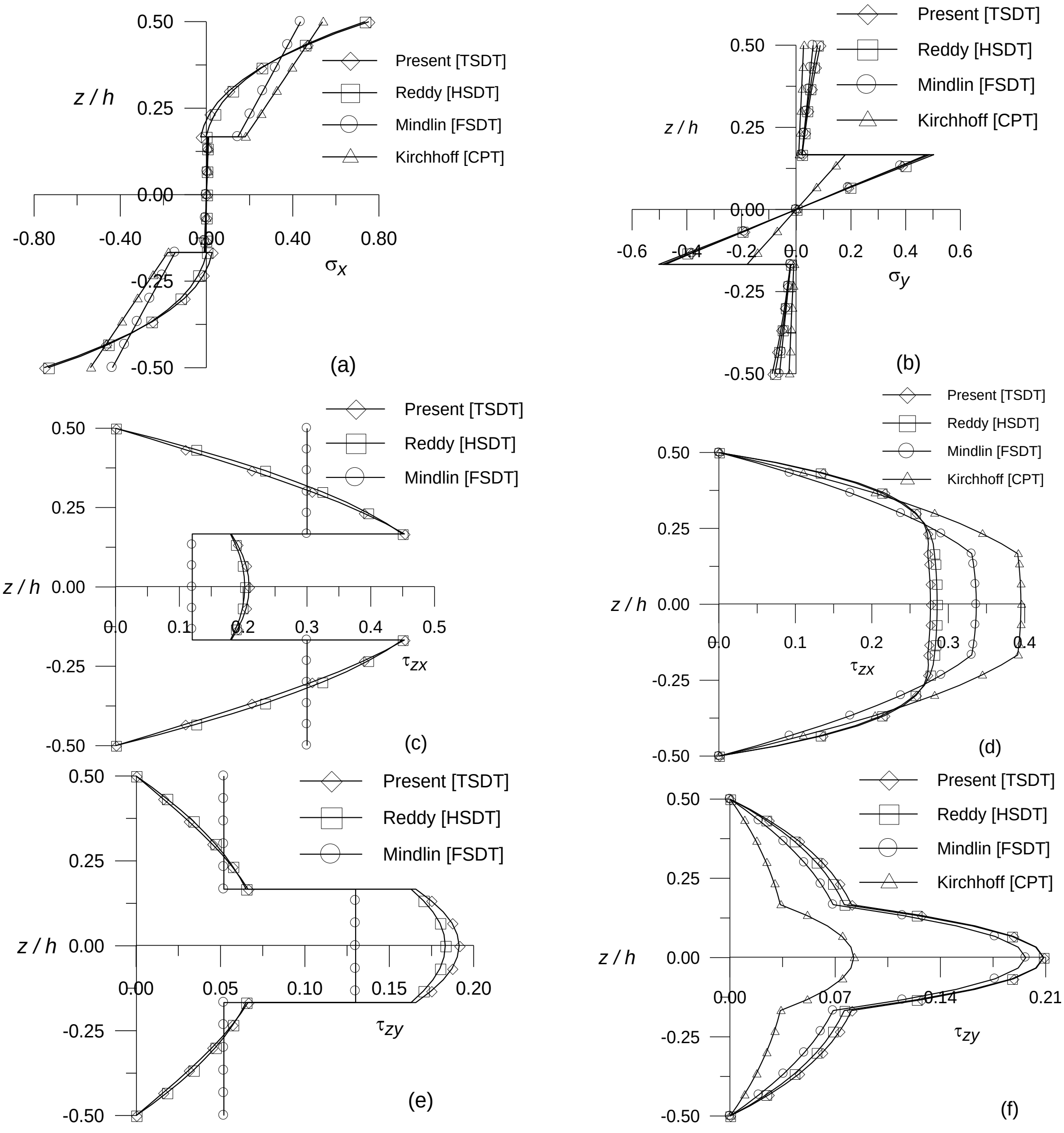

Fig. 7 Through thickness distribution of displacement and stresses of three layered ($0^0/90^0/0^0$) cross-ply laminated plate under sinusoidal loading at *h/a*=0.25: (*a*) Inplane normal stress ($\bar{\sigma}_x$) (b) Inplane normal stress ($\bar{\sigma}_y$) (c) Transverse shear stress ($\bar{\tau}_{zx}^{CR}$) *via* constitutive relation (d) Transverse shear stress ($\bar{\tau}_{zx}^{EE}$) *via* equilibrium equation (e) Transverse shear stress ($\bar{\tau}_{yz}^{CR}$) *via* constitutive relation (f) Transverse shear stress ($\bar{\tau}_{yz}^{EE}$) *via* equilibrium equation

underestimate the normal stresses compared to those of TSDT and HSDT for all aspect ratios. The inplane normal stress $\bar{\sigma}_y$ and inplane shear stress $\bar{\tau}_{xy}$ predicted by present theory are in good agreement with those of exact elasticity solution. The present theory predicts excellent values of

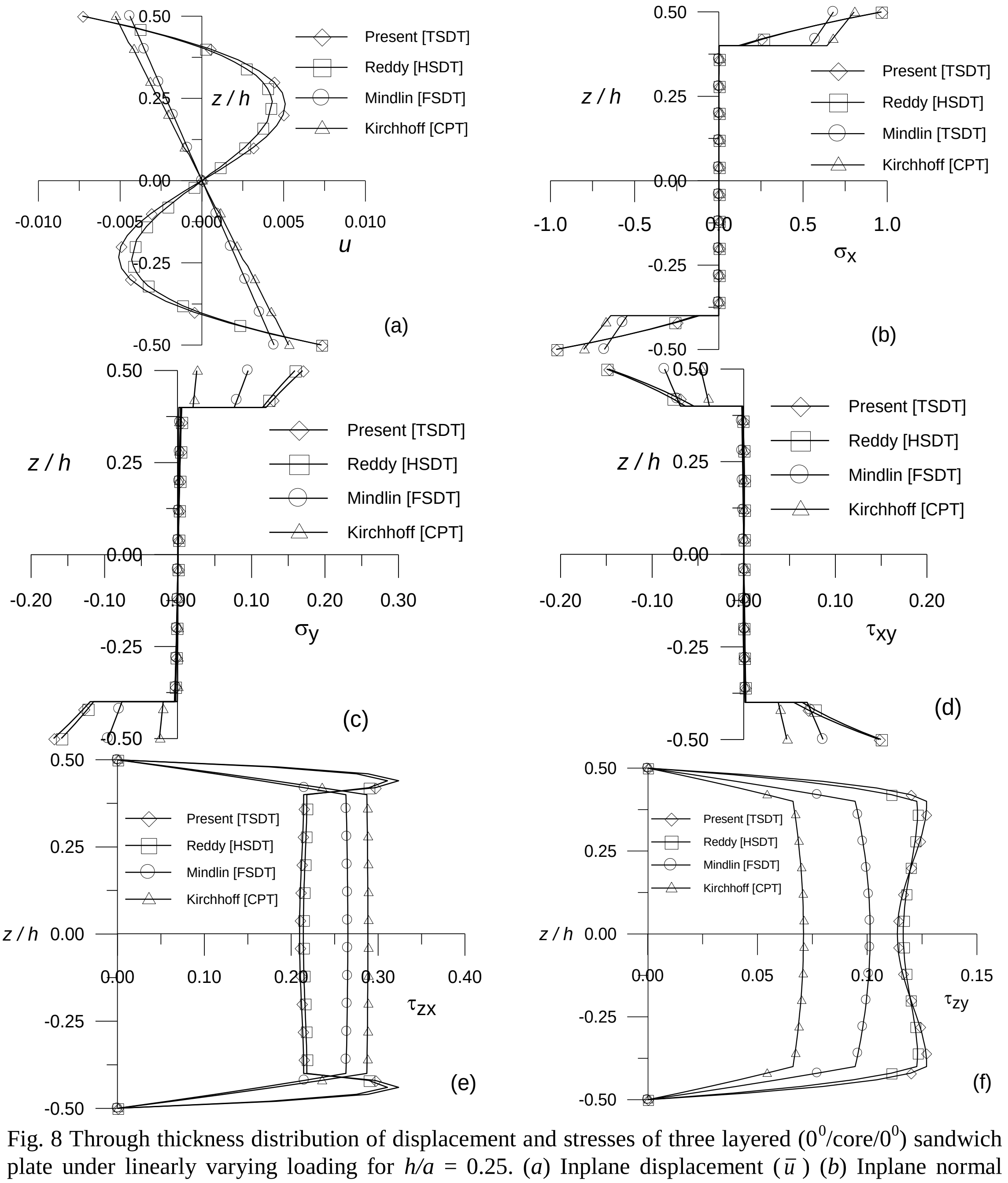

Fig. 8 Through thickness distribution of displacement and stresses of three layered (0^0/core/0^0) sandwich plate under linearly varying loading for *h/a* = 0.25. (*a*) Inplane displacement ($\bar{u}$) (*b*) Inplane normal stress ($\bar{\sigma}_x$) (c) Inplane normal stress ($\bar{\sigma}_y$) (d) Inplane shear stress ($\bar{\tau}_{xy}$) (e) Transverse shear stress ($\bar{\tau}_{zx}^{EE}$) *via* equilibrium equation (f) Transverse shear stress ($\bar{\tau}_{yz}^{EE}$) *via* equilibrium equation

transverse shear stresses when obtained using equations of equilibrium. The through the thickness variations of displacements and stresses of three layered sandwich plate are shown in Fig 8.

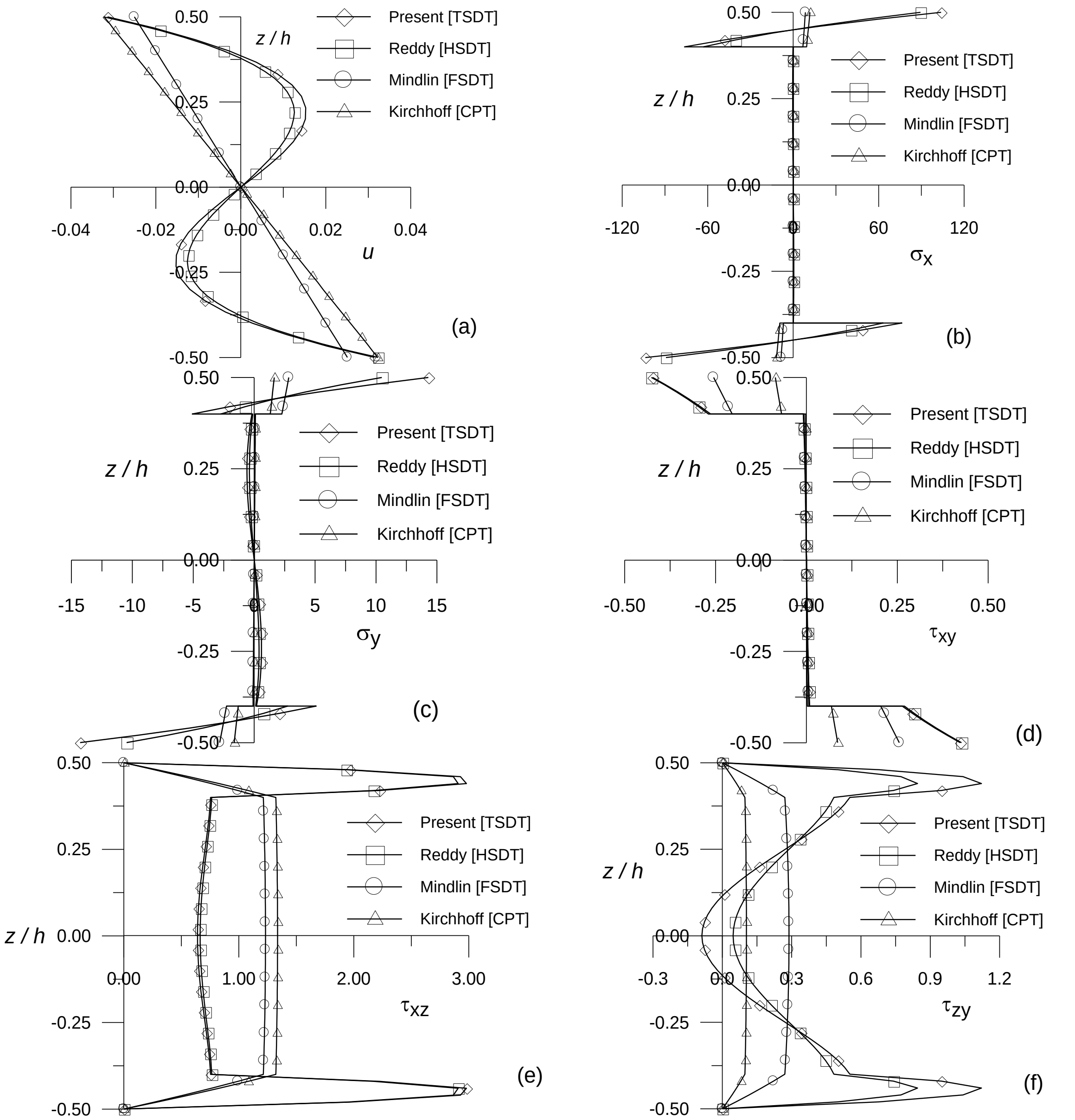

Fig. 9 Through thickness distribution of displacement and stresses of three layered (0^0/core/0^0) sandwich plate under central concentrated loading for *h/a* = 0.25. (*a*) Inplane displacement ($\bar{u}$) (*b*) Inplane normal stress ($\bar{\sigma}_x$) (c) Inplane normal stress ($\bar{\sigma}_y$) (d) Inplane shear stress ($\bar{\tau}_{xy}$) (e) Transverse shear stress ($\bar{\tau}_{zx}^{EE}$) *via* equilibrium equation (f) Transverse shear stress ($\bar{\tau}_{yz}^{EE}$) *via* equilibrium equation

Example 4: A simply supported sandwich square plates under central concentrated load When a plate is subjected to concentrated load, the effect of stress concentration becomes more pronounced, which leads to non-linear variations in displacements and stresses with increased magnitude. Table 5 presents the displacements and stresses of three layered (0^0/*core*/0^0) square sandwich plate under central concentrated load. Exact elasticity solution for this loading case is not available in the literature; therefore results are compared with those of other refined theories. Thickness of layers and material properties are same as used in previous problem. Table 5 indicates that the inplane and the transverse displacements obtained by present theory are in close agreement with those of HSDT of Reddy. From Fig. 9 it is observed that the present theory overpredicts the values of inplane normal stresses ($\bar{\sigma}_x, \bar{\sigma}_y$) and transverse shear stresses ($\bar{\tau}_{zx}, \bar{\tau}_{yz}$) compared to those of CPT and FSDT. The through-the-thickness distribution of transverse shear stress ($\bar{\tau}_{yz}^{EE}$) obtained by present theory *via* equilibrium equation showed the change in signs in core due to the effect of stress concentration. This effect cannot be captured by CPT and FSDT even with the use equilibrium equation of 3D elasticity theory as shown in Fig. 9(f). Thus the use of equilibrium equations is inevitable to assess the effect of local stress concentration on transverse stresses in conjunction with the equivalent and higher order theories. FSDT and CPT underestimate the values of transverse displacements and stresses due to the neglect of stress concentration effect. The through-the-thickness distribution of displacements and stresses given by CPT and FSDT deviates considerably from those given by present theory and higher order theory of Reddy as a consequence of local stress concentration.

5. Conclusions

In this paper, an equivalent single layer trigonometric shear deformation theory is applied to the static flexural analysis of cross-ply laminated composite and sandwich plates subjected to various loading conditions. The effect of linearly varying and central concentrated loads on the bending behavior of sandwich plates is assessed. From the discussion of results, presented numerically and graphically, following conclusions are drawn.

1. The present theory is variationally consistent and obviates the need of a shear correction factor due to the realistic variation of transverse shear stress.
2. The theory is applied to static flexure of two layered anti-symmetric and three layered symmetric cross-ply laminated and sandwich plates and shown to be superior to other existing higher order theories.
3. The present theory is shown to be capable of producing excellent results for transverse displacement and inplane normal stresses due to the inclusion of transverse normal strain in the theory.
4. The theory is capable of producing reasonably good transverse shear stresses using constitutive relations and better values of these stresses can be obtained by integration of equilibrium equations.
5. The effect of stress concentration on displacements and stresses due to concentrated load is effectively assessed by the present theory in case of sandwich plate with soft core.

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Appendix

The constants L_i appeared in governing equations (see Eqs. (12) through (17)) and boundary conditions (see Eqs. (18) through (32)) are as follows

$$L_1 = A_{11};\quad L_2 = A_{66};\quad L_3 = A_{12} + A_{66};\quad L_4 = B_{11};\quad L_5 = B_{12} + 2B_{66};\quad L_6 = As_{11};\quad L_7 = As_{66};$$

$$L_8 = As_{12} + As_{66};\quad L_9 = \frac{\pi}{h} As_{13};\quad L_{10} = A_{22};\quad L_{11} = B_{22};\quad L_{12} = As_{22};\quad L_{13} = \frac{\pi}{h} As_{23};\ L_{14} = D_{11};$$

$$L_{15} = 2\left(D_{12} + 2D_{66}\right);\quad L_{16} = D_{22};\quad L_{17} = Bs_{11};\quad L_{18} = Bs_{12} + 2Bs_{66};\ L_{19} = Bs_{22};\ L_{20} = \frac{\pi}{h} Bs_{13};$$

$$L_{21} = \frac{\pi}{h} Bs_{23};\quad L_{22} = Ass_{11};\quad L_{23} = Ass_{66};\quad L_{24} = Acc_{55};\quad L_{25} = Ass_{12} + Ass_{66};$$

$$L_{26} = \frac{\pi}{h} Ass_{13} + \frac{h}{\pi} Acc_{55};\ L_{27} = Ass_{22};\ L_{28} = Acc_{44};\ L_{29} = \frac{\pi}{h} Ass_{23} + \frac{h}{\pi} Acc_{44};\ L_{30} = \frac{h^2}{\pi^2} Acc_{44};$$

$$L_{31} = \frac{h^2}{\pi^2} Acc_{55};\ L_{32} = \frac{\pi^2}{h^2} Ass_{33};\ L_{33} = A_{12};\ L_{34} = B_{12};\ L_{35} = As_{12};\ L_{36} = B_{66};\ L_{37} = D_{12} + 4D_{66};$$

$$L_{38} = Bs_{66};\ L_{39} = D_{12};\ L_{40} = Bs_{12};\ L_{41} = Ass_{12};\ L_{42} = \frac{\pi}{h} Ass_{13};\ L_{43} = \frac{h}{\pi} Acc_{55};\ L_{44} = \frac{\pi}{h} Ass_{23};$$

$$L_{45} = \frac{h}{\pi} Acc_{44};\quad L_{46} = D_{66}$$

where A_{ij}, B_{ij} etc., are the plate stiffnesses, defined as follows

$$\left(A_{ij}, B_{ij}, D_{ij}\right) = \sum_{k=1}^{N} Q_{ij}^{k} \int_{h_k}^{h_{k+1}} \left(1,\ z,\ z^2\right) dz \quad \left(i, j = 1,2,6\right),$$

$$\left(As_{ij}, Bs_{ij}, Ass_{ij}\right) = \sum_{k=1}^{N} Q_{ij}^{k} \int_{h_k}^{h_{k+1}} \left(f(z), z.f(z),\ f^2(z)\right) dz \quad \left(i, j = 1,2,3,6\right),$$

$$Acc_{ij} = \sum_{k=1}^{N} Q_{ij}^{k} \int_{h_k}^{h_{k+1}} \left[\frac{df(z)}{dz}\right]^2 dz \qquad \left(i, j = 4,5\right)$$