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Numerical investigation on multi-degree-freedom nonlinear chaotic vibration isolation

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Abstract. A chaotic vibration isolation system is designed according to the chaotic vibration theory in this paper. The strong nonlinearity is generated by the system. Line spectra in the radiated noise maybe easily detected caused by marine vessels. It is Important to reduce the line spectra by improving the acoustic stealth of marine vessels. A multi-degree-freedom (MDF) nonlinear vibration isolation system (NVIS) system is setup by the experiment and finite element method. The model is established with finite element method. The results show that the behavior of the device gradually varies from period bifurcation into chaotic state and the line spectrum is changed from single spectral structure into broadband spectral structure . It is concluded that chaotic vibration isolation is preferable contrasted on line spectra isolation.

Keywords: chaos; nonlinear system; vibration isolation; multi-degree-freedom

1. Introduction

In recent years, the nonlinear vibration isolation system is being widely used in various fields (Hino *et al*. 2008, Liu *et al*. 2011, Sayed and Robert 2010). Nonlinear vibration isolation system has many special properties different from linear systems, such as resonance curve shift and kick, under certain parameter can present chaotic motion characteristics, internal resonance, attractor coexistence and so on (Benedettini and Salvatori 1992, Rega *et al*. 1992). These properties can be used to implement a linear system in which some function cannot be achieved. Therefore, the nonlinear vibration isolation system can be widely used in the power equipment. There are a lot of study (Lou *et al*. 2005, He *et al*. 2006, Yu and Zhu 2007) is carried out on the nonlinear vibration isolation system. However, most study was limited mostly to a few degrees of freedom system (low-dimensional systems). The vibration isolation system exhibited certain geometry, asymmetric, mass distribution, and so on in practical engineering applications. A single degree of freedom or a few degrees of freedom maybe causes error in its engineering applications. It is important to setup a multi-degree-freedom dynamics of nonlinear vibration isolation system. Yu *et al.* (2007, 2008) established the model of multi-DOF NVIS, although the decoupling condition is not satisfied in nonlinear system, he thought the motion on the symmetry plane and other motions are coupled weakly. Hence the weak coupling can be neglected in the practical analysis to reduce the complexity, and the NVIS is reduced to a planar VIS, which is a 6-dof system. And make a

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further analysis, the system can be simplified to a 4-dof system. Therefore, so far, higher-dimensional system modeling and dynamics characteristic research also rarely reported in the chaotic vibration isolation domain.

In this paper, a chaotic vibration isolation system is setup whose dimension is closer to the actual. The dynamic characteristics are analyzed by calculating and compared the effect of chaotic vibration isolation.

2. Design of the nonlinear vibration isolation system

The nonlinear vibration isolation system is presented for this study in Fig. 1.The structure of which is similar to sandwich. We call it sandwich isolation model. It is consists of three parts: (1) sandwich plate, (2) concave board, (3) plane board. All parts have its specific role as follows:

(1) The plane board: Its function is to change the top of the random load into surface load and apply it to sandwich panels. The stiffness of it is very large.

(2) Sandwich panels: Its function is when the top of the surface loads is carried out, sandwich panels has compression deformation. As a result, The deformation of sandwich plate is very small, and it is a linear deformation. The contact area is increasing with the overall force continuously rise, so its special structure can carry out nonlinear. Sandwich panels is consist of two cuboids and a middle cuboid.

(3) The concave board: its main function is by changing the pressure to achieve contact area between the concave and sandwich panels.

Fig. 2(a), (b) show two plans of the model. The arc curve which has been designed is symmetrical with *y*-axis, it's satisfy the equation $z = \alpha y^{\frac{1}{2}}$, where α is the equation coefficients, $1/n$ is the index. The both side cuboid length, width and height respectively are l_1, D_2, h_2, the middle cuboid length, width and height respectively are l_2, D_1, h_2. The back plate's length, width and height respectively are l, D_0, H. The total horizontal length of the arc curve is l_2, the width is D_1, and the height is h_2. The length of the flat plate is $2l_1+l_2$, the width is D, and the height is h_1. The elastic model of the sandwich plate is E. The flat plate and the back plate are considered as rigid body.

1. Flat plate; 2. Sandwich plate; 3. Back plate

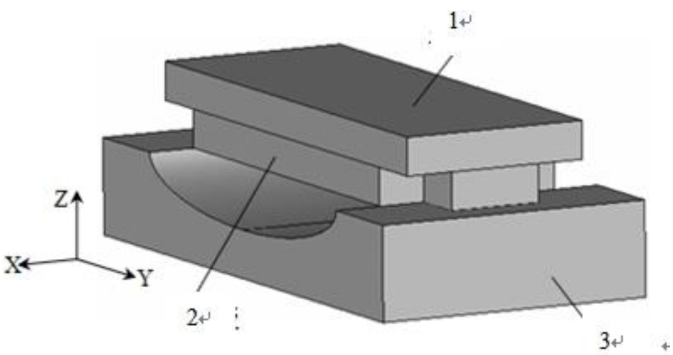
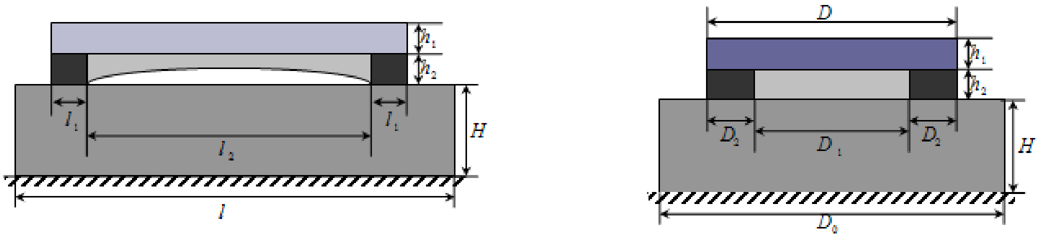


Fig.1 Vibration isolation system set-up



(a) (b)

Fig. 2 VIS plan (a) XY plan (b) YZ plan

When the load is in the vertical direction in the middle of the flat plate, the vibration isolation system is simplified to a single degree of freedom system. We can get

$$F = k_1 X + k_{n+1} X^3 \qquad (0 \le X \le z_{\max}) \tag{1}$$

$$F = k_2 X - P \qquad (X > z_{\max}) \tag{2}$$

Where $k_1 = 2E\dfrac{D_1}{h_2} l_1$, $k_3 = E\dfrac{D}{h_2}\dfrac{2}{n+1}\alpha^{-2}$, $P = 2ED\dfrac{\alpha}{h_2}\dfrac{2}{3}(\dfrac{l_2}{2})^{\frac{4}{3}}$, $k_2 = 2E(\dfrac{D_1}{h_2} l_1 + \dfrac{D}{h_2}\dfrac{l_2}{2})$

According to Eqs. (1), (2), we can get

$$\frac{k_3}{k_1} = \frac{\alpha^{-2}}{3}\frac{1}{l_1}\frac{D}{D_1} \qquad (n = 1, 2, 3, \cdots) \tag{3}$$

From Eq. (3), we can see that the stiffness of vibration isolation system as a whole can achieve to arbitrary value through the regulation of the above parameters, and the proportion of relations can be adjusted by the change of the parameters.

The parameters of our two models are presented,

Model1: E=5e6 Pa, D=0.3 m, l_1=0.1 m, D_1=0.1 m, l_2=0.6 m, h_2=0.01 m, n=2, α=0.01, l=1 m, H=0.03 m

Model 2: h_1=0.02 m. And E=5e6 Pa, D=0.3 m, l_1=0.1 m, D_1=0.1 m, l_2=0.3 m, h_2=0.01 m, α=0.01, l=0.8 m, H=0.03 m, h_1=0.02 m

On the condition of simple harmonic excitation function, the dynamics equation according to sandwich vibration isolation model is

$$M\frac{d^2X}{dT^2} + C\frac{dX}{dT} + k_1 X + k_3 X^3 = F\cos(\Omega T) + G \qquad 9 - z_{\max} < X < z_{\max}) \tag{4}$$

$$M\frac{d^2X}{dT^2} + C\frac{dX}{dT} + k_2 X - P = F\cos(\Omega T) + G \qquad (X \ge z_{\max} \text{ Or } X \le -z_{\max}) \tag{5}$$

Dimensionless of Eqs. (5), (6) are

$$\frac{d^2x}{dt^2} + \xi\frac{dx}{dt} + x + x^3 = f\cos(\omega t) + g\,(-(\frac{k_3}{k_1})^{\frac{1}{2}} z_{\max} < x < (\frac{k_3}{k_1})^{\frac{1}{2}} z_{\max}\ ) \tag{6}$$

$$\frac{d^2x}{dt^2}+\xi\frac{dx}{dt}+x-p=f\cos(\omega t)+g \quad (x\geq(\frac{k_{n+1}}{k_1})^{\frac{1}{2}}z_{\max} \text{ Or } x\leq-(\frac{k_{n+1}}{k_1})^{\frac{1}{2}}z_{\max}) \tag{7}$$

Where $x=(\frac{k_3}{k_1})^{\frac{1}{2}}X$, $\xi=C/\sqrt{Mk_1}$, $\Omega_n=\sqrt{\frac{k_1}{M}}$, $f=\frac{F}{k_1}(\frac{k_3}{k_1})^{\frac{1}{2}}$, $\omega=\Omega/\Omega_n$, $t=\Omega_n T$, $g=\frac{G}{k_1}(\frac{k_3}{k_1})^{\frac{1}{2}}$, $p=\frac{P}{k_1}(\frac{k_3}{k_1})^{\frac{1}{2}}$.

According to finite element theory, nonlinear vibration isolation system is discrete with space. Since only discrete space domain, so the displacement interpolation of discrete unit *u*, *v*, *w*, respectively, as

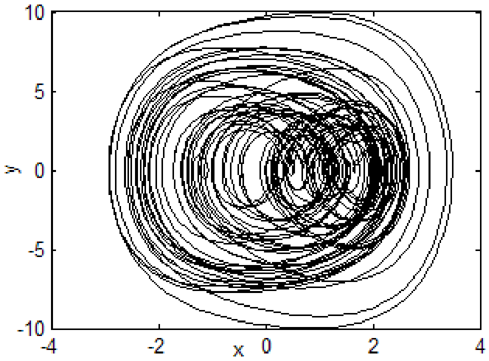
$$\begin{cases} u(x,y,z,t)=\sum_{i=1}^{n}N_i(x,y,z)u_i(t) \\ v(x,y,z,t)=\sum_{i=1}^{n}N_i(x,y,z)v_i(t) \\ w(x,y,z,t)=\sum_{i=1}^{n}N_i(x,y,z)w_i(t) \end{cases} \tag{8}$$

3. Numerical investigation

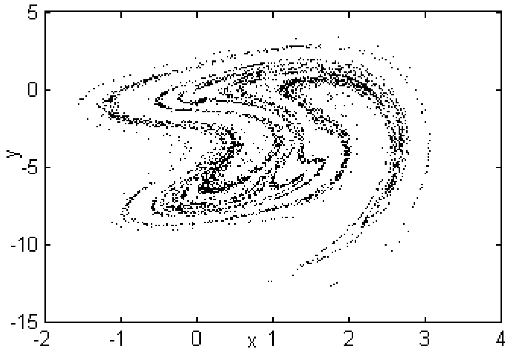
The four order runge-kutta method is adopted to carried out chaotic dynamics simulation research. The initial condition is (0.002, 0). Integral step is 1/100 of exciting force cycle. For model 1, exciting force is f=4, exciting force cycle frequency is ω=3.9, static load g=2.2, ξ=0.1. The system come into a state of chaos, calculating the maximum Lyapunov index is 0.1631. The calculation results can be seen in Fig. 4.

The same method can be used to model 2. The calculation results can be seen in Fig. 3. It can be seen that the acceleration response power spectrum of chaotic motion. The line spectrum reduction with the excitation frequency of 15.0 Hz is 90.5-65.8=24.7dB. Thus it can be seen that the system go to chaotic state, the line spectrum is reduced more than the state of non-chaotic. When the load is eccentric and the eccentricity is 0.1m. There are six points that have been chosen as the observation points; the schematic diagram is shown in Fig. 5. Its coordinates are A(0, 0), B(0.2, 0), C(0.4, 0), D(0.6, 0), E(0.7, 0), F(0.8, 0), respectively. The phase-plane trajectories of six points are presented in Fig. 4(c), (d) which shows the acceleration response power spectrum of chaotic motion. The line spectrum reduction at the excitation frequency of 15.0 Hz is 90.5-70.8=19.7dB.When the load is eccentric and the eccentricity is 0.2m. The phase-plane trajectories of six points are presented in Fig. 4(e), (f)which shows the acceleration response power spectrum of chaotic motion. The line spectrum reduction with the excitation frequency of 15.0 Hz is 90.5-75.3=15.2dB.

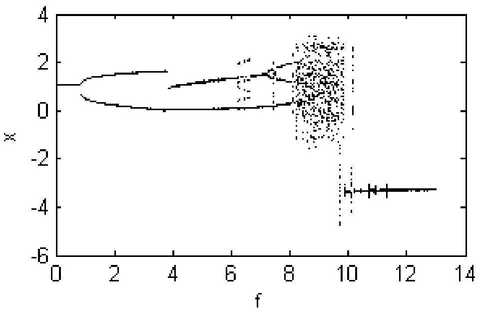
Comparing the above three cases which the eccentricity is 0.0 m, 0.1 m and 0.2 m, respectively. It can be find that the line spectrum reduction at the excitation frequency of 15.0 Hz getting lower and lower with the increasing eccentricity. And the differences are also growing between the



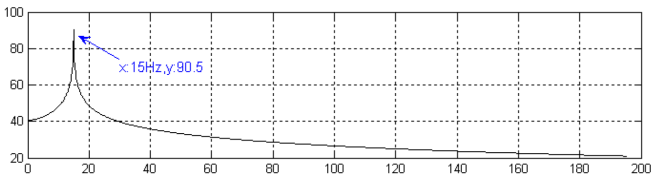
(a) Chaotic state of the phase diagram



(b) Chaotic state of the Poincaré map

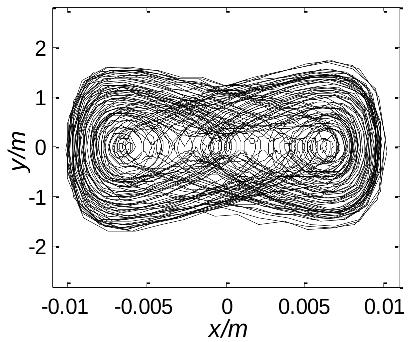


(c) Exciting force amplitude of the global bifurcation diagram

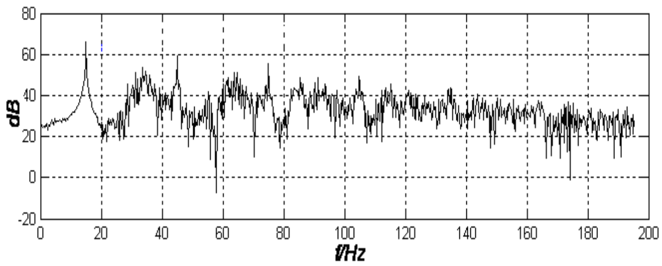


(d) Acceleration input power spectrum of chaotic state

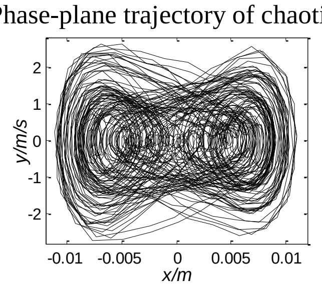
Fig. 3 The calculation results of model 1



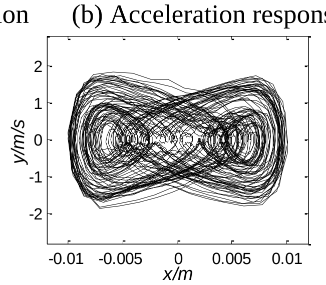
(a) Phase-plane trajectory of chaotic vibration



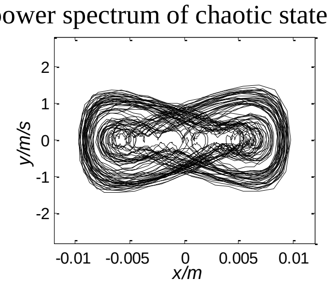
(b) Acceleration response power spectrum of chaotic state



(1)

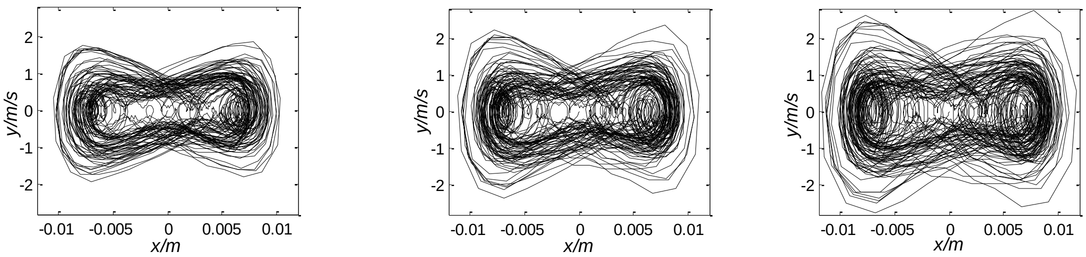


(2)



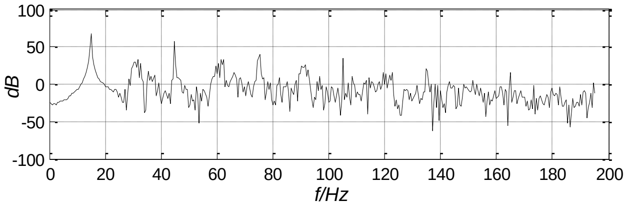
(3)

Fig. 4 The calculation results of model 2

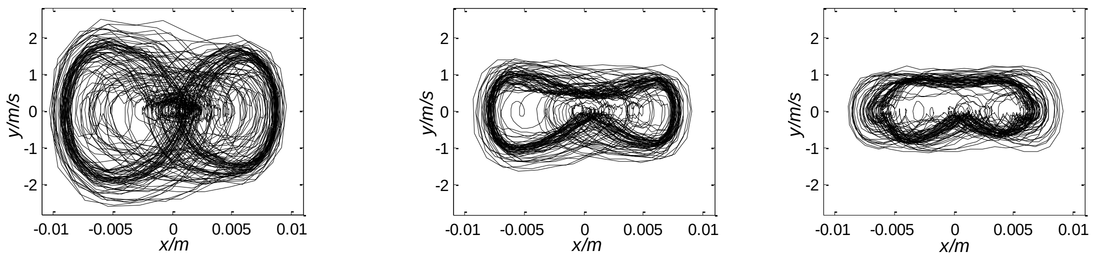


(4) (5) (6)

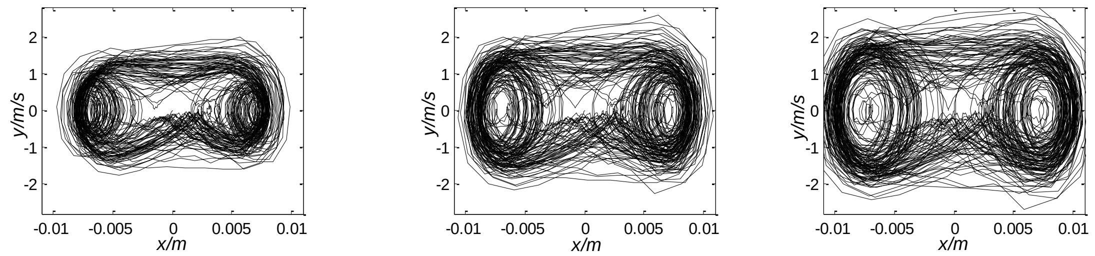
(c) The phase-plane trajectories of six points



(d)Acceleration response power spectrum of chaotic state

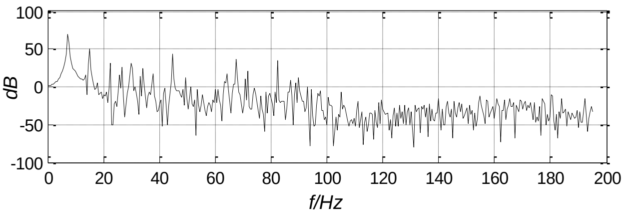


(1) (2) (3)



(4) (5) (6)

(e) The phase-plane trajectories of six points



(f) Acceleration response power spectrum of chaotic state

Fig. 4 Continued

phase-plane trajectories of six points. The displacement and velocity of the middle are smaller than these on both sides. So the vibration isolation system will enter the period state if the eccentricity continues to increase.

4. Conclusions

A chaotic vibration isolation system was designed according to chaotic vibration theory. The device can generate strong non-linear. The linear part and nonlinear part of the are completely separated. The overall stiffness, ratio of linear and nonlinear terms can be easily adjusted. The engineering applications of the device had greatly increasing. The numerical simulation had been carried out with specific parameters. A multi-degree-freedom nonlinear vibration isolation system model is established with finite element method. The line spectrum reduction at the excitation frequency is getting lower and lower.

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