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Optimum design of axially symmetric cylindrical reinforced concrete walls

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Abstract. The main aim of this paper is to investigate the relationship between thickness and height of the axially symmetric cylindrical reinforced concrete (RC) walls by the help of a meta-heuristic optimization procedure. The material cost of the wall which includes concrete, reinforcement and formwork, was chosen as objective function of the optimization problem. The wall thickness, compressive strength of concrete and diameter of reinforcement bars were defined as design variables and tank volume, radius and height of the wall, loading condition and unit cost of material were defined as design constants. Numerical analyses of the wall were conducted by using superposition method (SPM) considering ACI 318-Building code requirements for structural concrete. The optimum wall thickness-height relationship was investigated under three main cases related with compressive strength of concrete and density of the stored liquid. According to the results, the proposed method is effective on finding the optimum design with minimum cost.

Keywords: aially symmetric cylindrical reinforced concrete walls; optimization; harmony search; optimum cost; optimum design

1. Introduction

Basically, there are two main principles in structural design. The first one is structural safety. Developing technology provides opportunity of transferring the scientific advances to software, simultaneously. Thus, accurate or close analysis results of exact structural behavior can be obtained. Optimization in the design is the second principle. Optimum design can be defined as the process of selection the best result among that providing structural safety.

Lately, in addition to mathematical methods, the metaheuristic algorithms are employed for this optimization process. Metaheuristic algorithms are optimization algorithms which are usually developed from inspiration of natural phenomena. As an example the genetic algorithm (GA) imitates a model or abstraction of biological evolution, based on Charles Darwin’s theory of natural selection, such as inheritance, mutation, selection and crossover (Holland 1975, Goldberg 1989). The simulating annealing (SA) algorithm was inspired from the annealing process in metallurgy (Kirkpatrick *et al.* 1983). The particle swarm optimization (PSO) method developed by Kennedy and Eberhart (1995), mimics the social behavior of organisms such as bird or fish school. The ant colony optimization (ACO) inspired from the behavior of ants which are seeking the best

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path between their colony and source of food (Dorigo *et al.* 1996). The natural foraging behavior of honey bees are the inspiration of Honey Bee Algorithm (HBA) (Nakrani and Tovey 2004), Virtual Bee Algorithm (VBA) (Yang 2005), Honey-Bee mating Optimization (HBMO) (Afshar and Haddad 2007) and Artificial Bee Colony (ABC) (Karaboga 2005) algorithms. Also, the Big Bang-Big Crunch method (BB-BC) (Erol and Eksin 2006) inspired from the evolution of the universe, Firefly Algorithm (FA) (Yang 2008); from flashing characteristic of fireflies and Bat Algorithm (BA) (Yang and Gandomi 2012); from the echolocation characteristic of microbats.

Harmony Search (HS), which is inspired by the improvisation process of a musician searching for a better state of harmony, is also one of these metaheuristic algorithms (Geem *et al*. 2001). The HS algorithm has been applied to solve many optimization problems including civil engineering ones. Examples for these applications are structural design (Lee and Geem 2004, Saka 2007, Degertekin 2008, Saka 2009, Hasancebi 2009, Degertekin *et al*. 2009, Erdal and Saka 2009, Hasancebi *et al*. 2010, Togan *et al*. 2011, Erdal *et al*. 2011), structural analysis (Toklu 2004), structural material problems (Lee and Yoon 2007, Suh *et al*. 2010, Mun and Lee 2011), hydraulic problems (Geem 2009, Baek *et al*. 2010, Geem and Cho 2011, Geem 2011), cost optimization and construction management (Geem 2010, Gholizadeh *et al*. 2010, Kaveh *et al*. 2010, 2011, Kaveh and Sabzi 2011), and structural vibration control (Bekdaş and Nigdeli 2011, 2012, Nigdeli and Bekdaş 2012).

Axially symmetric cylindrical walls, commonly used in the water tanks, are another application field of optimization in civil engineering. The studies in this field, based on different optimization techniques, and are generally concentrated on cost optimization of reinforced concrete (RC) water tanks (Adidam and Subramanyam 1982, Saxena *et al*. 1987, Thevendran and Thambiratnam 1988, Thevendran 1993, Tan *et al*. 1993, Barakat and Altoubat 2009, Ansary *et al*. 2011).

This paper presents an optimization process based on harmony search algorithm to find relationship between wall thickness-height of the axially symmetric cylindrical RC walls. The thickness-height relationship was investigated under three main cases. In the first case, the effect of the compressive strength of concrete between 20 MPa-50 MPa for different tank volumes were investigated. In the second one, the effects of density of liquid between 5 kN/m^3-10 kN/m^3 were investigated. For the third example, optimum thickness of the wall and compressive strength of the concrete were optimized.

2. Cost optimization of cylindrical walls

The model of axially symmetrical cylindrical wall used in optimization can be seen in Fig. 1. h represent thickness of the wall. This value and compressive strength of concrete are the design variables of the optimization process. H and r are height and radius of the wall, respectively. These are the dimension parameters of the wall which are constant in the optimization. The distributed load on the wall is a user defined variable which is constant. Also, the material properties elasticity modulus (E) and Poisson's ratio (v), tank volume, compressive strength of concrete and unit cost of material are constant.

The objective function of the optimization is the total cost of the wall including concrete, reinforced steel and formwork. It can be written as

$$\min C(x) = C_c V_c(x) + C_s W_s(x) + C_{fw} A_{fw}(x) \tag{1}$$

where C_c, C_s, and C_{fw} are the unit costs for concrete, steel and formwork, respectively. Usually the

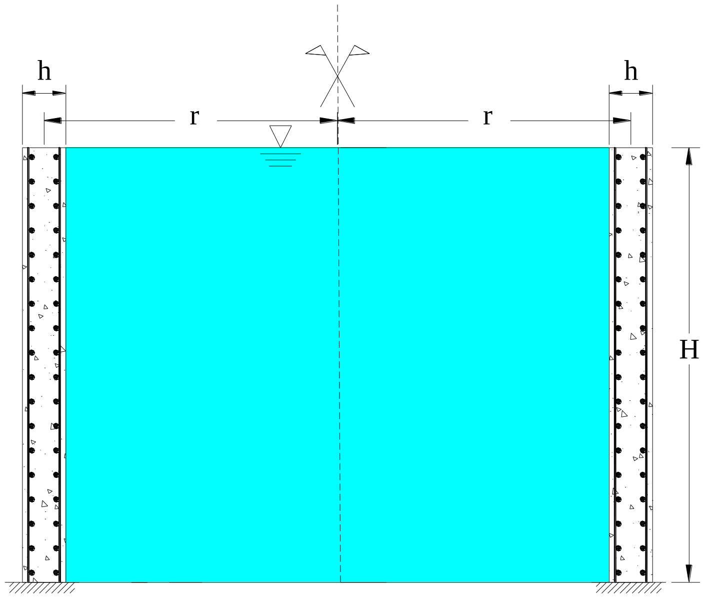

Fig. 1 Model of cylindrical wall used in optimization process

concrete cost is described by volume, the steel by weight and formwork by the surface area. Thus, in the Eq. (1), V_c, W_s, and A_{fw} are the concrete volume of the wall, total weight of the reinforcement and surface area of the wall. The W_s terms includes steel weight in vertical and horizontal directions calculated according to the moments and hoop tensions. These effects, shear forces and displacements are calculated by using superposition method during the optimization process. The detail of the method is explained at the following section.

2.1 Superposition Method (SPM) for analyses of wall

The superposition method (Hetenyi 1936) is actually developed for the analyses of beams on elastic foundation. But, this method can also be for cylindrical wall analyses by taking advantage of the similarity between beams on elastic foundations and axially symmetric cylindrical walls (Timoshenko and Young 1962, Timoshenko and Woinowsky-Krieger 1984). In several studies, analyses of walls are conducted by using this similarity (Billington 1965, Ghali 1979, Calladine 1983, Timoshenko and Woinowsky-Krieger 1984, Kelkar and Sewell 1987, Ventsel and Krauthammer 2001).

Analysis by the superposition method is conducted in three steps.

First step: All internal forces are calculated by assuming wall as infinite long

Fig. 2(a) illustrates both ends free wall with length l subject to various loads. In Fig. 2(b), an infinite-length wall with the same loading condition and properties with Fig. 2(a) is given. For the infinite length wall, bending moment, shear forces and displacements approach to zero when getting away from application point of loads; and at points A and B there occurs moment M_A, shear force Q_A, moment M_B and shear force Q_B as shown in Fig. 2(b). However, free ends moments and shear forces at points A and B of the wall in Fig. 2(a) must be zero.

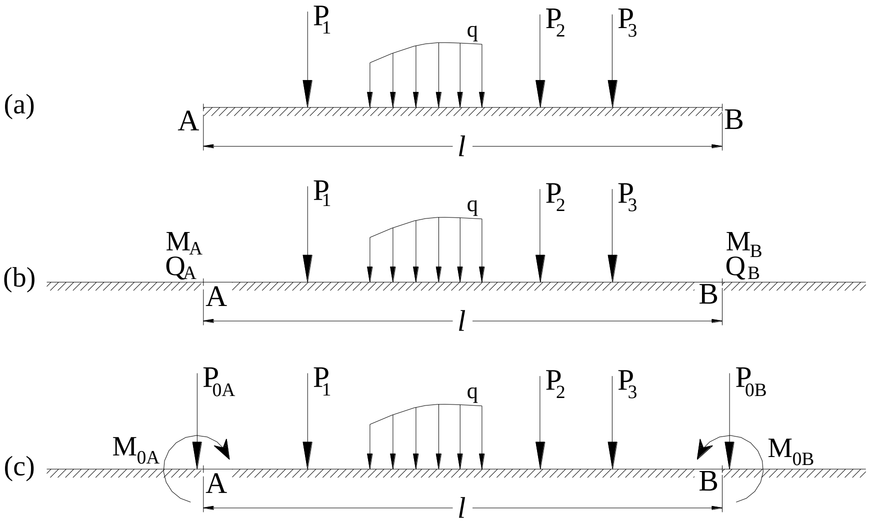

Fig. 2 Wall subjected to different loadings

The SPM provides these boundary conditions at a wall's ends using P_{0A}, M_{0A}, P_{0B} and M_{0B}, called end-conditioning forces. In other words, by the help of these forces the displacements and influences of infinite length-wall (Fig. 2(c)) become same as the wall of length *l* with both ends free (Fig. 2(a)).

Second step: Determining the end-conditioning forces

These forces can be determined using four additional equations that are written based on the boundary conditions of the wall. For the walls with free ends, these additional equations are as follows (Hetenyi 1967)

$$M_A + \frac{P_{0A}}{4\beta} + \frac{P_{0B}}{4\beta} C_{\beta l} + \frac{M_{0A}}{2} + \frac{M_{0B}}{2} D_{\beta l} = 0 \tag{2}$$

$$Q_A - \frac{P_{0A}}{2} + \frac{P_{0B}}{2} D_{\beta l} - \frac{\beta M_{0A}}{2} + \frac{\beta M_{0B}}{2} A_{\beta l} = 0 \tag{3}$$

$$M_B + \frac{P_{0A}}{4\beta} C_{\beta l} + \frac{P_{0B}}{4\beta} + \frac{M_{0A}}{2} D_{\beta l} + \frac{M_{0B}}{2} = 0 \tag{4}$$

$$Q_B - \frac{P_{0A}}{2} D_{\beta l} + \frac{P_{0B}}{2} - \frac{\beta M_{0A}}{2} A_{\beta l} - \frac{\beta M_{0B}}{2} = 0 \tag{5}$$

In the equations the $A_{\beta l}$, $B_{\beta l}$, $C_{\beta l}$ and $D_{\beta l}$ functions can be written as

$$\left.\begin{array}{ll} A_{\beta l} = e^{-\beta l}(\cos\beta l + \sin\beta l) & B_{\beta l} = e^{-\beta l}\sin\beta l \\ C_{\beta l} = e^{-\beta l}(\cos\beta l - \sin\beta l) & D_{\beta l} = e^{-\beta l}\cos\beta l \end{array}\right\} \tag{6}$$

where the term of β is

Table 1 Constraints on strength and dimensions of wall

Description	Constraints
Flexural strength capacity	$M_d \geq M_u$
Shear strength capacity	$V_d \geq V_u$
The axial tension, T	$\phi A_s^{hoop} f_y \geq \mathrm{T_u}$
Minimum steel ratio, $\rho_{\min}$	$A_s \geq A_{s\min}$
Maximum crack width, $w_{\max}$	$w_{\max} \leq 0.1\ mm$
Maximum steel bars spacing, $S_{\max}$	$S \leq S_{\max}$
Minimum steel bars spacing, $S_{\min}$	$S \geq S_{\min}$
Minimum concrete cover, $c_{c\min}$	$c_{c\min} \geq 40$ mm

$$\beta^4 = \frac{3(1-\nu^2)}{r^2h^2} \tag{7}$$

Third step: Superposition process

At this step the effects of applied loads and the end-conditioning forces are superimposed. Eqs. (2)-(5) are written for the wall defined in Fig. 2. If the boundary conditions of the wall are different, then the equations must be rearranged properly.

2. Harmony Search (HS) approach for optimization process

Optimization procedure by using harmony search (HS) algorithm can be summarized in five steps.

- In the first step, the cylindrical wall properties including height and radius, material properties of the wall, loading condition, harmony search algorithm parameters, termination convergence ratio (TCR), and solution range for wall thickness and diameter of reinforcement bars are fixed.
- Then, the initial harmony memory (HM) matrix is generated by using harmony vectors (HV) containing randomly created values for wall thickness, horizontal and vertical reinforcements (bar size and spacing) and cost of the wall. The size of HV is six. The number of stored HVs is equal to the harmony memory size (HMS).
- After that, analyses of internal forces, design of reinforcement bars and cost calculation of the wall are applied, respectively. The internal forces of the wall including longitudinal moment, transverse moment, shear force, hoop tension force and displacements are calculated by using superposition method. And then, the RC design of the wall is done according to the rules described in ACI318 code (Table 1). After that total cost of the wall is calculated. These operations are made for each HV and the cost values are stored in relevant vector to use for comparison of objective function when the stopping criterion is controlled.
- After generation of initial HM matrix, the stopping criterion (Eq. (8)) is checked.

$$TCR \geq \frac{P_{\max} - P_{\min}}{P_{\max}} \tag{8}$$

$P_{\min}$ and $P_{\max}$ represents total cost values of the best and worst vector existing vectors defined

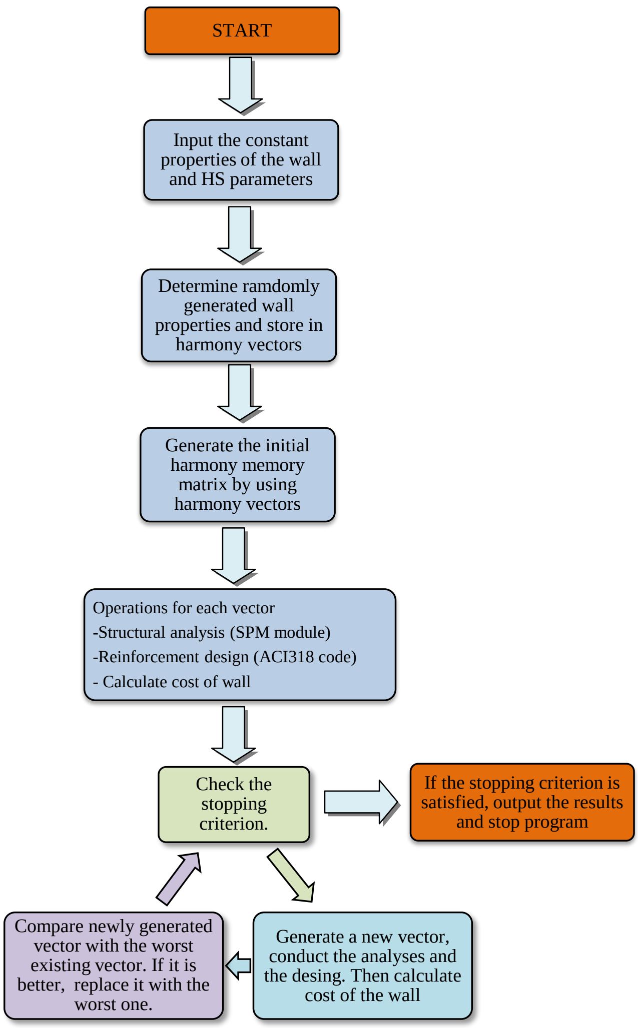

Fig. 3 Flowchart of program

according to objective function in HM matrix, respectively. If the stopping criteria are not satisfied, a new vector is generated.

- A new vector can be generated randomly from whole solution range or from smaller range around the one of the existing vectors in the HM matrix. The possibility of generating a new vector from existing one is equal to harmony memory considering rate (HMCR) and the ratio of smaller and whole range is called pitch adjacent range (PAR). Then, newly generated vector is compared with the worst existing vector in HM matrix. If the solution of new vector is better than the worst existing vector in HM matrix, it is replaced with the worst one. This process is repeated until the stopping criterion is satisfied.

The optimization process is summarized in flowchart given in Fig. 3.

3. Numerical examples

Three different examples were presented by using the developed optimization algorithm. In the first example, the relationship between optimum cost-radius of wall (or height) and optimum thickness-radius of wall were investigated by using three different walls with constant volume. In the second analysis, the effect of loading condition to optimum cost and thickness were investigated. Load condition was changed by using different density of storage. The third example mainly searches optimum wall thickness and compressive strength of concrete. This example is done for the verification of the optimization method.

For all analysis, the solution range of wall thickness and reinforcement bars were defined as 0.05-2.0 m and 8-50 mm, respectively. The wall was assumed as fixed support at the bottom and HS parameters; HMS, HMCR and PAR were defined as 5, 0.5 and 0.2, respectively. The optimum results are found after between 500000-1000000 iterations depending to the problem. The robustness of all numerical examples is investigated by conducting several optimization processes. The same optimum solutions were found for all runs, so a single run is sufficient for all problems.

3.1 Example 1

Optimum cost and optimum wall thickness were investigated for tanks with different compressive strength of concrete and radius of wall. The density of liquid is taken as 9.81 kN/m^3. Constant tank volume was taken as 10000 m^3, 15000 m^3 and 20000 m^3 for different optimization cases. The optimum parameters were found for different radiuses taken between 20 m-40 m for every 0.5 m increase and different compressive strength taken 20 MPa-50 MPa for every 5 MPa increases. Elasticity modulus was calculated with expression given in ACI 318 code and the poisson ratio was defined as 0.15. Material cost was taken as 35 $/ m^3 for 20 MPa compressive strength of concrete and the cost was increased 5 $ for every 5 MPa. Reinforced concrete steel price was taken 315$/ton and form work was taken 5 $/m^2. For different compressive strength of concrete, optimum cost of the 10000 m^3 tank depends on radius graph seen in Fig. 4.

The difference between minimum (33513 $ for 20 MPa) and maximum (44323 $ for 50 MPa) cost for 20 m wall radius is 32.25%. This value is decreased with increasing wall radius and for 40 m, it is calculated as 16.83% (8922 $-10424 $). Due to constant volume of the tank, the height

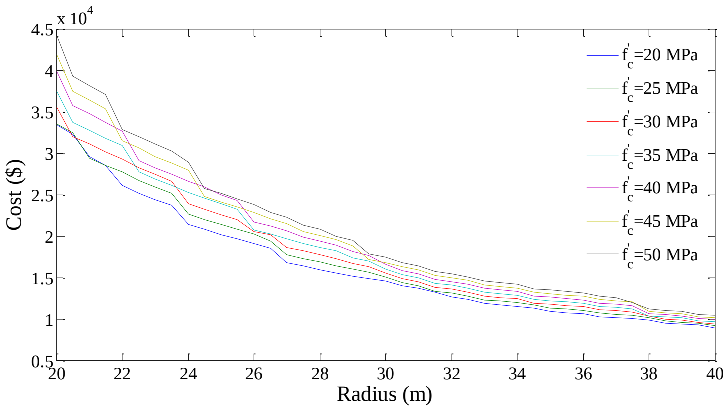

Fig. 4 Optimum cost vs. radius (10000 m^3)

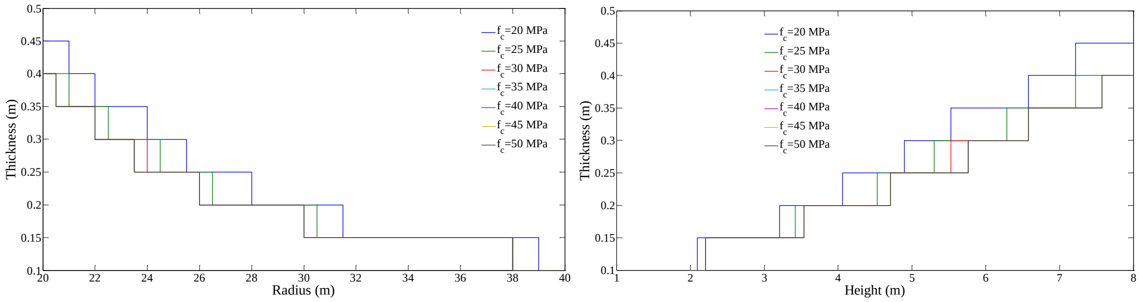

Fig. 5 Optimum thickness vs. radius (left) and thickness vs. height (right) (10000 m^3)

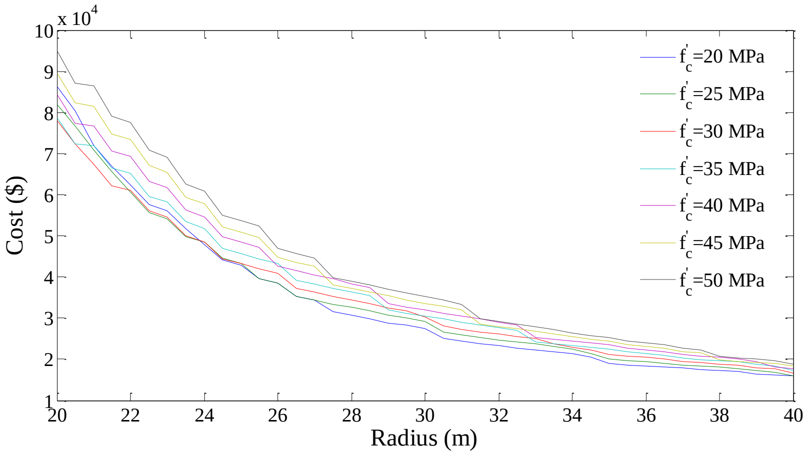

Fig. 6 Optimum cost vs. radius (15000 m^3)

of the wall (in connection with load intensity) is decreased while the radius is increasing.

According to analyses results, the minimum cost values are generally obtained for 20 MPa compressive strength. The only expectation of these values is obtained for radius range between 20.5 m-21.5 m. However, differences between these costs and 20 MPa costs are very low.

The relationship between radius-optimum wall thickness and height-optimum wall thickness are given in Fig. 5. As seen in figure, optimum thickness values vary between 0.45 m and 0.1 m. When thickness values related to compressive strength of concrete are examined, thickness is different when radius is between 20 m-28 m, while it is nearly the same between 28 m-40 m radius for all concrete values. According to results, the ratio between optimum thickness and wall height is approximately 0.057 for 20 MPa.

In Fig. 6, optimum cost depends on radius of the wall for 15000 m^3 is given. Also, the optimum thickness according to radius and height are given in Fig. 7. According to analysis results, minimum cost for radius between 20 m (height 11.94 m)-21.5 m (height 10.33 m) are obtained for 30 MPa compressive strength of concrete.

For radius 22 m-23.5 m, minimum cost are obtained at 25 MPa concrete. For the other ranges of radius, the minimum cost values are generally obtained for 20 MPa. For high radius values (32 m-40 m), when optimum thickness values are examined, it is seen that the thicknesses are

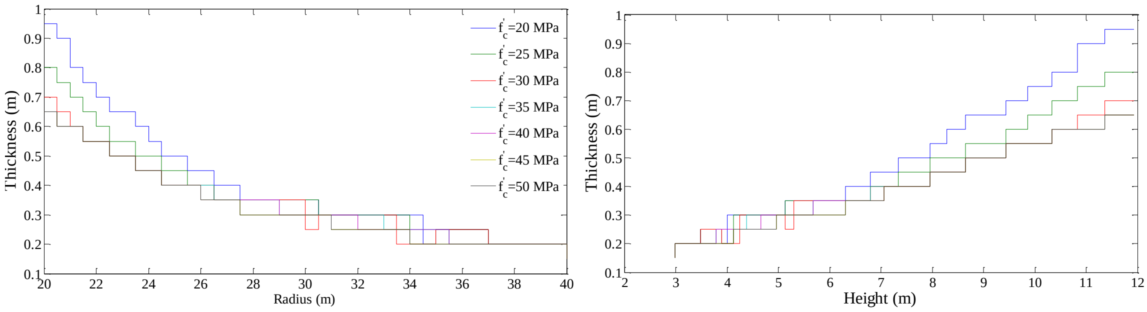

Fig. 7 Optimum thickness vs. radius (left) and thickness vs. height (right) (15000 m^3)

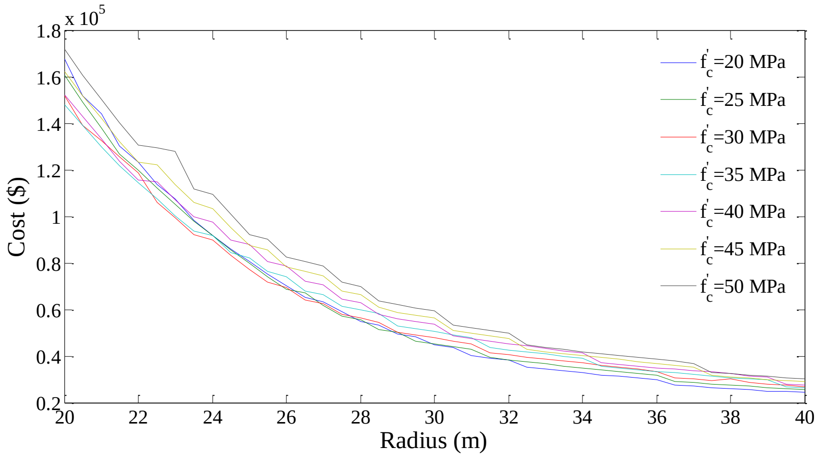

Fig. 8 Optimum cost vs. radius (20000 m^3)

Table 2 Compressive strengths of concretes with minimum costs

Radius range	Compressive strength
20 m -22 m	35 MPa
22.5 m -25.5 m	30 MPa
26 m -29 m	25 MPa
29.5 m-40 m	20 MPa

nearly same for all concretes strengths. According to results, the ratio between optimum thickness and wall height is approximately 0.078 for 20 MPa.

The relationship of optimum cost and radius of the wall for the 20000 m^3 volume tanks can be seen in Fig. 8. In Table 2, compressive strengths of concretes with minimum costs are given for different radius ranges.

As seen from the results, very high optimum thickness values for 20 MPa compressive strength are obtained in low radius ranges (Fig. 9). As an example, the optimum thickness value is obtained as 1.6 m for 20 m radius. However, this value is calculated as 1.0 m for 35 MPa. It can also be seen from Fig. 9 that, the effect of concrete strength on wall thickness decreases as radius gets bigger and tank height gets smaller.

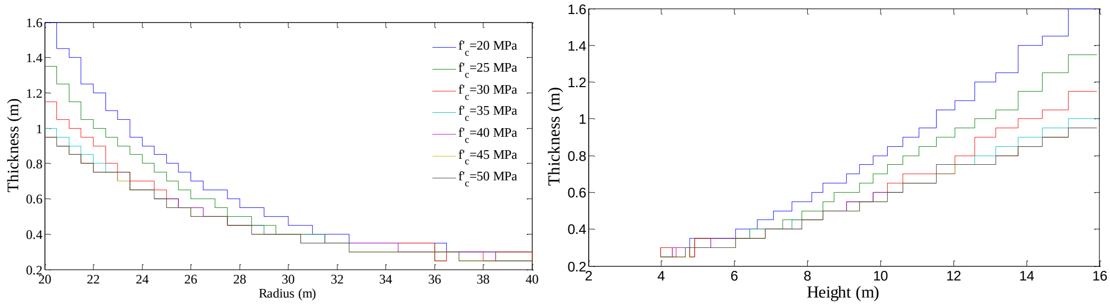

Fig. 9 Optimum thickness vs. radius (left) and thickness vs. height (right) (20000 m^3)

Table 3 The relationship between compressive strength of concrete obtained for minimum cost and height ranges

Height range	Compressive strength
<7 m	20 MPa
7 m -9.5 m	25 MPa
9.5 m -12.5 m	30 MPa
12.5 m<	35 MPa

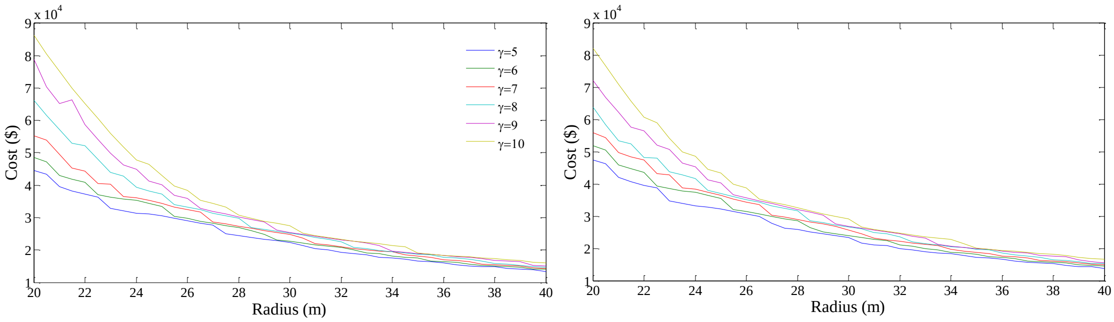

Fig. 10 The relationship between optimum cost and radius of wall thickness for 20 MPa (left) and 25 MPa (right)

The differences between thickness values are getting lower when radius is increasing and the thickness becomes nearly the same for 37 m and higher values. According to results, the ratio between optimum thickness and wall height is approximately 0.1 and 0.086 for 20 MPa and 25 MPa, respectively.

When all three analyses are examined, the relationship between compressive strength of concrete obtained for minimum cost and height ranges can be given as seen in Table 3.

3.2 Example 2

In the section, effect of loading condition to optimum cost and thickness is investigated. The analyses were performed on the same model with 15000 m^3 tank given in the first example. According to the results of the first example, generally optimum cost values were obtained for 20 MPa and 25 MPa. Thus, the optimization analyses were performed only for these concretes.

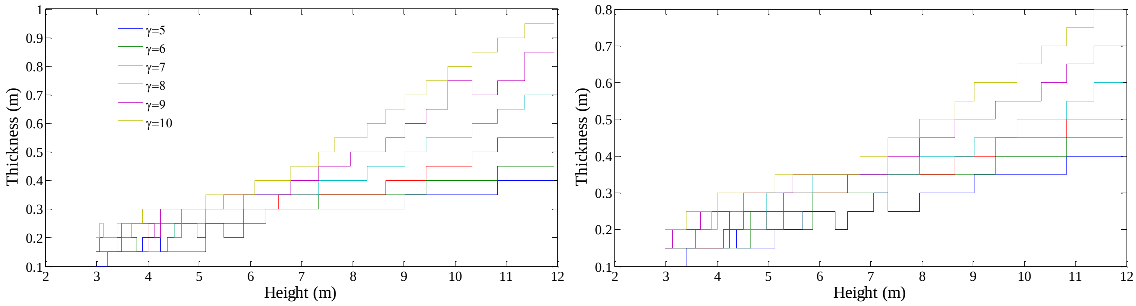

Fig. 11 The relationship between height and optimum wall thickness for densities for 20 MPa (left) and 25 MPa (right) (Example 2)

Table 4 Optimization results of Example 3

H (m)	h (m)	f_c' (MPa)	Vertical reinforcement for inner face (mm/mm)	Vertical reinforcement for exterior surface (mm/mm)	Horizontal reinforcement (mm/mm)	Cost ($)
5.0	0.25	20	Φ10/110	Φ16/400	Φ10/350	13218
7.5	0.35	25	Φ10/70	Φ20/410	Φ12/370	26204
10.0	0.50	30	Φ14/100	Φ14/160	Φ10/450	45897
12.5	0.70	30	Φ14/70	Φ26/450	Φ10/450	73855
15.0	0.85	35	Φ18/90	Φ26/380	Φ10/450	111900

Load condition is changed by taking density of liquid between 5 kN/m^3-10 kN/m^3.

For both concretes, the relationship between optimum cost and radius of wall thickness is given in Fig. 10. This figure shows that the optimum cost values obtained from 20 MPa is lower than 25 MPa for densities between 5 kN/m^3-7 kN/m^3. For these densities, the cost differences of concretes are changed between 7.03% and 0.39%. By the increase of density, the optimum costs for 25 MPa becomes lower than 20 MPa results. Radius ranges with the lowest costs are between 20 m-22 m for 8 kN/m^3, 20 m-22.5 m for 9 kN/m^3 and 20 m-24.5 m for 10 kN/m^3. As it expected, the number of lowest cost values obtained from the analyses of 25 MPa is increased, while the density is increasing.

The relationship between height and optimum wall thickness for densities is given in Fig. 11. According to results of 20 MPa concrete, the ratio between optimum thickness and wall height is approximately 0.035, 0.038, 0.047, 0.058, 0.072 and 0.0853 for 5 kN/m^3, 6 kN/m^3, 7 kN/m^3, 8 kN/m^3, 9 kN/m^3 and 10 kN/m^3 densities, respectively. For 25 MPa concrete, these values are respectively obtained as 0.036, 0.04, 0.045, 0.049, 0.057 and 0.0656.

3.3 Example 3

For the third example, optimum thickness of the wall and compressive strength of the concrete are optimized for five different heights of the walls. The density of liquid is taken as the same as the example 1. In order to show the optimization method is effective on finding the optimum characteristics of the design more than one design variables this example was handled. In Table 4, the optimum design variables and costs are given.

Table 5 Optimization results for constant compressive strength of concrete

	$f_c' = 20\,MPa$		$f_c' = 25\,MPa$		$f_c' = 30\,MPa$	
H (m)	h (m)	Cost ($)	h (m)	Cost ($)	h (m)	Cost ($)
5.0	***0.25***	***13218***	0.25	13930	0.25	14644
7.5	0.40	26442	***0.35***	***26204***	0.35	27696
10.0	0.65	49106	0.55	45966	***0.50***	***45897***
12.5	1.00	85341	0.80	75785	***0.70***	***73855***
15.0	1.35	125755	1.10	116372	1.00	117885

For the verification of the optimization method, Table 5 shows the optimum thicknesses and cost with constant compressive strength of concrete. When two variables are optimized, the results highlighted as bold and italic in Table 5 are found. Also, the optimum results in Table 5 verify that these compressive strengths of concrete are the most economical ones.

4. Conclusions

In this paper, cost optimization of axially symmetric cylindrical reinforced concrete (RC) walls were investigated. A computer program was developed by modifying harmony search (HS) algorithm to perform optimization process. Numerical analyses of the wall were conducted by using superposition method (SPM) considering ACI 318-Building code requirements for structural concrete.

In the study, thickness-height relationship was investigated under three cases related with compressive strength of concrete and density of the stored liquid.

The results obtained from the optimization process are summarized below.

-The results of the first example shows that, concrete with 20 MPa compressive strength is the most economical solution for the walls with a height lower than 7 m. For the walls with heights 7 m-9.5 m, 9.5 m-12.5 m and more than 12.5 m, the optimum compressive strengths of concrete are 25 MPa, 30 MPa and 35 MPa, respectively.

- In the second example, the optimum compressive strength of concrete was found as 20 MPa for the liquid with 5-7 kN/m^3 density. For other densities the usage of concrete with 25 MPa is sometimes more effective according to height of the wall.

-The third example was done to show that the optimization method is also effective on finding optimum compressive strength of concrete. Also, the results of the third example were verified by conducting optimization cases with constant compressive strengths. In addition to that the results of example one and three are compatible with each other.

The proposed method is effective on finding optimum design with minimum cost as verified numerical examples.

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