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3-D fracture analysis of cracked aluminum plates repaired with single and double composite patches using XFEM

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Abstract. Bonded composite-patch repair has been widely used to restore or extend the service life of damaged structures due to its effectiveness as a mechanical repair technique. In this paper using extended finite element method (XFEM), three-dimensional crack models are developed to examine the fracture behavior of centrally cracked aluminum plates repaired with single and double sided composite patches. Stress intensity factor (SIF) at the crack tip is used as the fracture criterion. In this regard, the effects of the crack lengths, patch materials, orientation of plies, adhesive and patch thickness are examined to estimate the SIF of the repaired plate and the repair performance. The obtained results show that composite patches have significant effect on reduction of the SIF at the crack tip. It is also proved that using double symmetric repair, in comparison to single one, reduces considerably SIF at the crack tip. Hence, the residual strength can be improved significantly as well as fatigue life of the structure. Investigation of ply orientation effects shows SIF increase as the ply orientation is changed from 0° (perpendicular to the advancing crack) to 90° (parallel to the crack line). However, the effectiveness of the ply orientation depends on the loading direction and the crack direction.

Keywords: 3-D crack modeling; stress intensity factor; single and double sided composite patches; adhesive; XFEM

1. Introduction

Aircrafts are subjected to severe structural and aerodynamic loads during their service life, which may result from repeated landing and take-off, maneuvering, ground handling, bird strike and environmental degradation such as stress corrosion. These loads can cause damage or weakening of the structure especially for aging military and civilian aircrafts and thereby affect the structure load carrying capabilities. Hence, repair or reinforcement of damaged or weakened part of structures has become an important issue for restoring the structural efficiency and assuring the aircraft airworthiness in recent years (Okafor *et al.* 2005). Application of an adhesively bonded composite patch for repairing cracked metallic structures is a desirable procedure since it provides a high structural efficiency and extends the flawed component life at an economical cost. A bonded composite patch reduces the stress intensity factor by bridging the stresses between itself and the cracked plate. It reduces the stress field in the crack growth and improves the fatigue life. In

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addition, bonded composite patches have the inherent advantage of a high stiffness/strength to weight ratio which makes them suitable for aerospace applications (Jones *et al.* 1982, Jones 1984, Baker *et al.* 1984, Baker 1987, Baker and Jones 1988). Several studies have been conducted to investigate the mechanics of bonded composite patches to repair cracked metallic structures, especially to analyze the stress redistribution in the repaired structure and to compute the stress intensity factor after repair. Baker pioneered in these researches at the aeronautical and maritime research laboratory for the Royal Australian Air force (Baker *et al.* 1984, Baker 1984, 1987, Baker and Jones 1988, Baker and Chester 1993, Baker 1996, 1997a ,b). However, most of these studies were based on simple analysis, experiment or experiences and were limited to the case of mode I opening of the crack. Ratwani (1977) conducted a finite element analysis where plane stress two-dimensional elements were used to model the cracked plate and the composite patch and shear spring elements were used to model the adhesive. Also, elastic singular elements were employed at the crack tip to calculate the stress intensity factor directly.

Sun and Klug (1996) presented a simple analytical method to analyze cracked aluminum plates repaired with composite patch based on Mindlin panel theory. The adhesive layer was modeled with effective springs connecting the patch to the aluminum plate. A 3-D finite element analysis was performed using a commercial code ABAQUS and a comparison was made between stress intensity factor obtained from both cases. Rose (1987), Callinan *et al.* (1997) used the finite element method to analyze performance of the bonded composite repair. They showed that the presence of the patch highly reduces the stress intensity factor at the crack tip and therefore, improves the fatigue life of the cracked structure. Jones and Chiu (1999) performed experimental and numerical studies on the repair of cracks in thick structural components. Because it is difficult to obtain an analytical solution for the composite bonded patch repairs, numerical methods such as 2-D FEA, full 3-D FEA, boundary element method, etc. were employed to analyze the stress fields at the crack tips and to assess the efficiency of bonded composite patch repair on the fracture toughness of the cracked metallic structures.

Bouiadjra *et al.* (2002) used a finite element method to analyze the behavior of repaired cracks with bonded composite patches in mode I and mixed mode by computing the stress intensity factors at the crack tip. Also, they investigated the effects of patch size and adhesive properties on the stress intensity factor variation. Megueni *et al.* (2003) carried out the finite element method to analyze the performance of the bonded composite patch for repairing cracks in pure mode II by computing the stress intensity factor at the crack tip. The comparison between the double and single patch was analyzed by Belhouari *et al.* (2004). They showed that using double symmetric patches improves the fatigue life of the repaired structures. This improvement is because of transferring double stress in the double patch configuration. In addition, the double symmetric patch annuls the bending effect due to eccentricity of the composite patch in case of single sided patch.

Ayatollahi and Hashemi (2007) carried out the finite element method to analyze the effects of bonded composite patches for repairing cracks in pure mode I and also mixed mode I/II conditions; they computed the stress intensity factor and T-stress as functions of the crack length, the crack inclination angle and the type of composite material. Madani *et al.* (2008) studied the behavior of centrally cracked aluminum panels repaired with single and double-sided Graphite/Epoxy composite patches subjected to uni-axial loading. Stress intensity factors were computed for panels repaired with 12-ply and 14-ply repairing patches with different adhesive thicknesses. Bouiadjra *et al.* (2008) investigated the effect of adhesive on repaired panels and fracture parameters. The problem was handled in plane stress and mode I conditions. Kaddouri *et* *al.* (2008), Ouinas *et al.* (2007, 2009) analyzed numerically the performance of the octagonal, circular and elliptical shape of patches. They showed that the patch shape has a significant effect on the value of the stress intensity factor at the crack tip. In addition, using appropriate patch shapes can reduce the level of the thermal residual stresses due to the adhesive curing. Megueni and Lousdad (2008) studied performance of a double sided patch and a stepped patch. The investigation was accomplished via a two-dimensional finite element method of a centre cracked metallic panel repaired with an externally bonded composite patch. Yala and Megueni (2009) studied cracks in aluminum plate repaired with bonded patches using the finite element method. Different values of patch thickness, plate thickness and adhesive thickness were considered. Ouinas *et al.* (2010) used the finite element method to analyze the behavior of bonded lap joints of dissimilar materials. The effects of the mechanical properties of the joints on the shear stress variation with and without presence of a circular notch were investigated. Albedah *et al.* (2011) applied a new approach to compare the performance of single and double-sided symmetric composite patches with circular shape for repairing cracked aircraft structures. This approach evaluated the final mass gain using the double symmetric composite patch. The 3-D finite elements method is used to compute the stress intensity factor. The obtained results showed that use of double patch technique leads to a significant reduction of the stress intensity at the crack tip. The mass gain depends on the patch shape and the adhesive properties. Gu *et al.* (2011) studied the mechanical behavior of a single edge V-notch aluminum plate repaired with 1-ply and 4-ply composite patches through the finite element method. Contour integral method was used to define and evaluate the stress intensity factors at the crack tip. The effect of the adhesive epoxy film, patch material, thickness and ply orientations on the evolution of stress intensity factor of the repaired structure was examined. Bouiadjra *et al.* (2012) investigated the effects of the adhesive disband on the efficiency of bonded composite patches in aircraft structures repair. The obtained results showed that the repair efficiency is negatively influenced if the adhesive disband grow perpendicularly to the crack. In the case of a double symmetric patch, the presence of double adhesive disband decreases the repair efficiency considerably and increases the risk of adhesion failure between the composite patch and the cracked aluminum structure.

Another approach which considers the presence of crack through the kinematic relations is XFEM. In this method presence of the crack is modelled by adding degrees of freedom to the nodes around the crack. This method was formed based on the partition of unity method (PUM) and was developed rapidly. Belytschko and Black (1999) originally used this technique in fracture mechanics problems. Using this model, crack growth can take place without re-meshing (Dolbow 1999).

In several works, the stress intensity factors are computed at the tip of a crack in 2-D bodies using domain forms of the interaction integrals (Belytschko and Black 1999, Moes *et al.* 1999, Daux *et al.* 2000, Yau 1980). Sukumar *et al.* (2000) used domain integral methods to evaluate stress intensity factors along the 3-D crack front (Moran and Shih 1987). Duarte *et al.* (2001) extracted the stress intensity factors by a least squares fit method and minimized the numerical errors of calculated stresses and their asymptotic values. Nagashima *et al.* (2003) exposed the stress intensity factor analysis of the bi-material interface crack problem. They used asymptotic solution of a homogeneous (not interface) crack to enrich the crack tip nodes, and adopted a fourth order Gauss integration for a 4-node isoparametric element with enriched nodes. Xiao and Karihaloo (2003) enhanced the accuracy of the local fields and determined the stress intensity factor directly and without extra post-processing. They enriched the nodes of the elements surrounding the crack tip with the leading as well as higher order terms of the asymptotic crack tip fields using the PUM. Liu *et al.* (2004) extend this technique for direct evaluation of mixed mode stress intensity factors in homogeneous and bi-materials. Menouillard *et al.* (2006) presented a general method for the calculation of mixed mode intensity factors for graded materials.

The aim of this paper is to investigate the behavior of repaired cracks in 2024-T3 aluminum plate with single and double sided composite patches using XFEM. The 3-D analysis is done by computing the SIF in pure mode I as a function of crack length varies between $0.1<a/w<0.5$. Also the beneficial effects of the double patches compared to single one. Furthermore, the influence of patch materials, orientation of plies, adhesive and patch thickness in maintaining the repair efficacy is investigated.

2. XFEM: a review

In general, the current numerical crack modeling methods can be classified into two broad categories of geometrical and non-geometrical presentations. In the first category, the presence of a crack in the model is explicit and the geometry and mesh are changed during the crack growth. This category includes various numerical methodologies, such as the Finite Difference Method (FDM), the Finite Element Method (FEM), the Boundary Element Method (BEM) and the meshless (meshfree) method. In the second category, the crack does not appear in the model as a physical object, but its presence affects the governing equations.

In recent years, the XFEM has emerged as a powerful numerical procedure for the analysis of crack problems. In this method, presence of crack has effects on kinematic equations. The finite element framework is not well suited for modeling crack growth because the domain of interest is defined by the mesh. At each increment of crack growth, at least the domain surrounding the crack tip must be remeshed such that the updated crack geometry is accurately represented. The XFE method can be used to alleviate many of the inconveniences of using the FE method to model the evolution of a crack and it is the main advantage of this method.

In the traditional formulation of the FEM, the existence of a crack is modelled by requiring the crack to follow the element edges. In contrast, the crack geometry in the XFEM does not need to be aligned with the element edges, so provides flexibility and versatility in modelling. The method is based on enrichment of the FE model with additional degrees of freedom (DOFs) that are tied to the nodes of the elements intersected by the crack (Moes *et al.* 1999). In this manner, the discontinuity is included in the numerical model without modifying the discretization, as the mesh is generated without taking into account the presence of the crack. One of the key steps in the implementation of XFEM is the selection of a proper region, where the field is required to be enriched. In other words, correct nodes must be selected where additional degrees of freedoms are added to the system. To do this, three sets of nodes I, J and K are defined. Where set I contains all the nodes of the body, set K contains the nodes whose support closure contains crack tips and set J, regardless of the nodes contained in set K, contains the nodes whose support is intersected by crack. Fig. 1 shows a portion of the mesh with four-node bilinear elements.

The circled nodes are the nodes enriched with two additional DOFs (total of four DOFs per node), whereas the nodes marked with a square are enriched by eight more DOFs (total of ten DOFs per node). Elements that contain at least one enriched node are known as enriched elements. Nodes with two additional DOFs (one for each coordinate direction) have shape functions that multiply the Heaviside function, $H(x)$, (function of unit magnitude whose sign changes across the crack, $H(x)=\pm 1$). Physically, this function introduces the discontinuity across the crack faces.

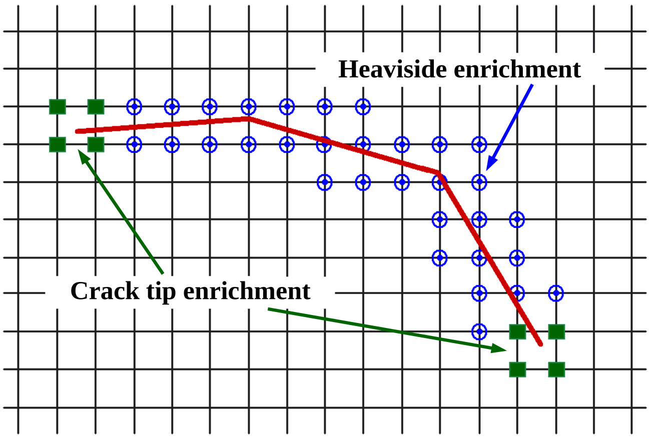

Fig. 1 Enriched nodes in the XFEM: Circles- nodes with 2 additional DOFs, Squares- nodes with 8 additional DOFs (Giner *et al.* 2009)

Nodes with eight additional DOFs are enriched in two Cartesian directions with four crack tip functions $F_\alpha(X)$ (Belytschko and Black 1999)

$$\{F_l(r,\theta)\}_{l=1}^{4} = \left\{ \sqrt{r}\cos\left(\frac{\theta}{2}\right), \sqrt{r}\sin\left(\frac{\theta}{2}\right), \sqrt{r}\sin\left(\frac{\theta}{2}\right)\sin\theta, \sqrt{r}\cos\left(\frac{\theta}{2}\right)\sin\theta \right\} \quad (1)$$

where r, θ are local polar coordinates defined at the crack tip.

The displacement approximation for crack modelling in the XFEM takes the form (Moes *et al.* 1999)

$$u_{XFEM}(x) = \sum_{i \in I} N_i(x)\, u_i + \sum_{i \in J} N_i(x)\, H(x)\, a_i + \sum_{i \in K} [N_i(x) \sum_{\alpha=1}^{4} F_\alpha(x)\, b_{i\alpha}] \quad (2)$$

where $N_i(x)$ is the nodal shape function and u_i is the standard DOF of node i (u_i represents the physical nodal displacement for non-enriched nodes only). The subsets J and K contain the nodes enriched with Heaviside function, $H(x)$, or crack-tip functions, $F_\alpha(X)$, respectively, and a_i, $b_{i\alpha}$ are the corresponding DOFs.

Thus enrichment provides modelling of the crack discontinuity within the crack-tip element and substantially increases the accuracy in the computation of the SIFs (Giner *et al.* 2009).

3. Geometry and materials models

The basic geometry of the cracked plate with single and double patches considered in this study is shown in Fig. 2. Consider an elastic aluminum 2024-T_3 plate with the following dimensions: height h=120 mm, width w=120 mm, thickness t=3 mm. The plate is subjected to a uniform linear ramping pressure of 70 MPa. A central crack with length of 2a perpendicular to the loading axis with a pre-existent crack with length of a=12 mm is supposed to exist in the plate. This crack is repaired with polymer composite patches. The dimensions of the patch and adhesive are: h_p=h_a=90 mm, w_p=w_a=180 mm, t_p=1.15 mm and t_a=0.2 mm. The optimum thickness for a patch is adapted

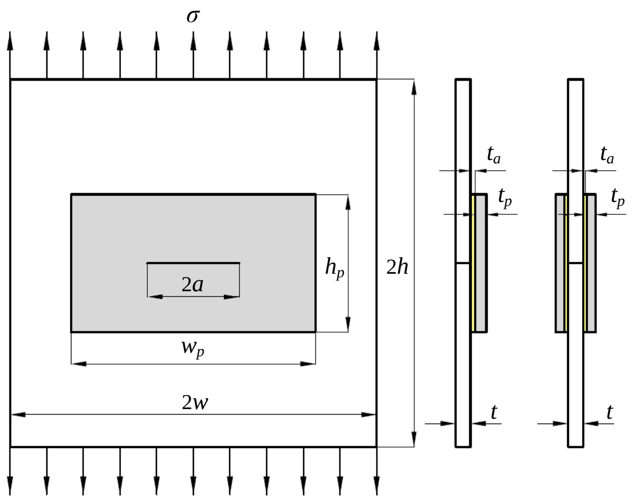

Fig. 2 Geometrical model of the centre cracked plate with single and double patch under pure mode I

Table 1 Material property of the cracked plate, patches and adhesive

Material	E_1	E_2, E_3	υ_{12}, υ_{13}	υ_{23}	G_{12}, G_{13}	G_{23}
Al 2024-T3	72	---	0.33	---	28	---
Boron/Epoxy	208	25.4	0.17	0.04	7.24	4.94
Carbon/Epoxy	172.4	10.34	0.3	0.18	4.82	3.1
FM73 adhesive	0.97	---	0.33	---	0.365	---

E and *G* are in GPa. Directions: 1- normal to crack (fiber), 2- along crack, 3- thickness.

from (Gu *et al.* 2011, Marioli-Riga *et al.* 2004) as

$$E_p t_p = Et \tag{3}$$

where E is the modulus of elasticity of the aluminum plate, E_p is modulus of elasticity of the composite patch, t is the thickness of the plate, and t_p is the thickness of the patch.

In this research, focus is on the design of repair process using Boron/Epoxy and Carbon/Epoxy composite patches due to their outstanding properties. Therefore, based on Eq. (3) the suitable thickness is selected by using the average thickness of these two materials. It should be noted that the increase in patch thickness beyond the suitable level would eventually increase the cost of repair.

Eq. (3) is valid only for pure tension loads. In order to determine the SIF as a function of crack length for pure mode I, the analysis is performed for various crack lengths. The material properties for the aluminum plate, FM 73 adhesive, composite patches are summarized in Table 1.

4. XFEM model

4.1 Numerical model and validation

The crack in XFE method is represented independent of the mesh by the enrichment functions

Table 2 Comparison of the SIF for un-patched centre cracked plate

a/w	K_I (MPa.m$^{1/2}$) Theory (Sih 1973)	K_I (MPa.m$^{1/2}$) FEM (Ayatollahi and Hashemi 2007)	Deviation %D	K_I (MPa.m$^{1/2}$) Presented XFEM	Deviation %D
0.1	13.67	13.22	3.3	13.68	0.07
0.2	19.70	19.13	2.9	19.73	0.15
0.3	24.93	24.33	2.4	25.09	0.6
0.4	30.21	29.45	2.5	30.25	0.1
0.5	36/13	35.36	2.1	35.97	0.4

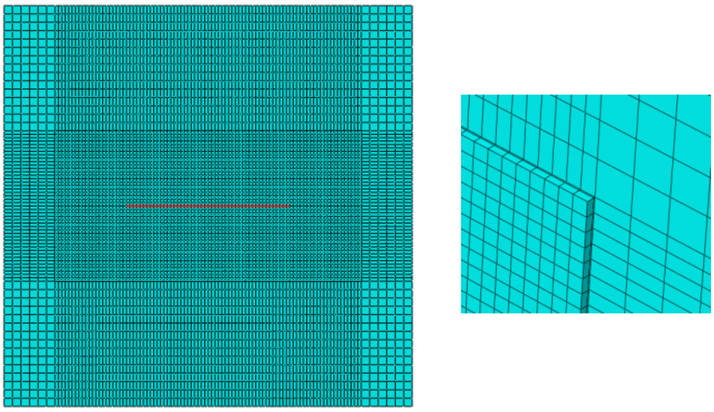

Fig. 3 The mesh of repaired cracked plate using XFEM

which allows for the crack geometry to be updated without a need to create/update a new mesh on the domain (Pais and Kim 2009). Hence modeling and analysis of complicated geometry by use of this method is done quickly.

The analytical and classical finite element (FE) solutions for mode I on un-patched center cracked plate with different crack length ratio (a/w) (Sih 1973, Ayatollahi and Hashemi 2007) are utilized to validate the 3-D numerical results by using XFEM. In this regard, a 3-D un-patched plate with a central crack, as shown in Fig. 2, is modeled and analyzed using the commercially available finite element code ABAQUS (2010). The aluminum cracked plate is meshed using 8-node linear brick elements (C3D8) without meshing the crack. The plate is subjected to a uniform linear ramping load of 70 MPa on one end while the other end is fixed.

The comparison between the XFEM results for K_I and analytical and classical FEM results are given in Table 2. It is seen that the results of the XFEM, in comparison with classical FEM, have much better agreement with analytical results for various crack lengths. After proving the proper performance of numerical modelling, the strength behavior of cracked plates before and after repair with composite patches is studied.

4.2 Numerical evaluation of crack plates with composite repairs

The analysis is carried out to investigate the effects of the repairing composite patches on the crack under pure mode I with evaluation of the SIF as a function of crack length. In this regard, a 3-D finite element model consists of three subsections including the cracked plate, the adhesive and the composite patch is created (Fig. 3).

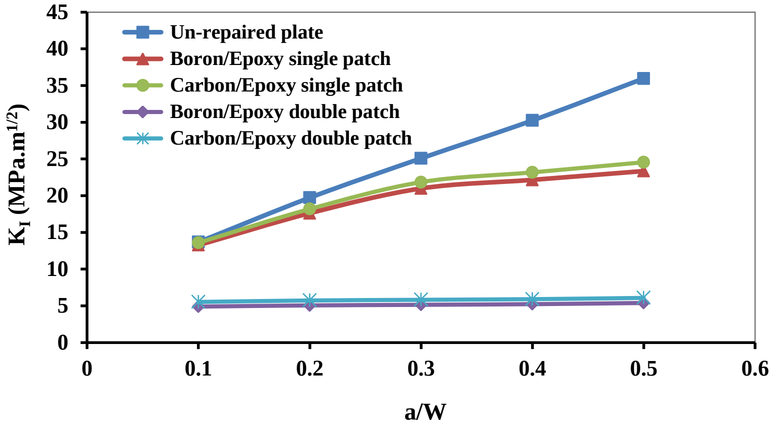

Fig. 4 Variation of K_I versus crack length for single and double patched and un-patched centre cracked plate

Table 3 Comparison of the SIF before and after repair with composite patches

Name	Patch material	Averaged K_I along crack tip (MPa.mm$^{1/2}$)
Cracked plate	---	24.944
Repaired single patch	Boron/Epoxy	19.480
Repaired single patch	Carbon/Epoxy	20.274
Repaired double patch	Boron/Epoxy	5.144
Repaired double patch	Carbon/Epoxy	5.814

The 8-node brick element (C3D8) is used in the commercial code ABAQUS for simulation of cracked plate and composite patches. Also, the 8-node three-dimensional cohesive element (COH3D8) is used for simulation of adhesive film. All of the cracked plate, repairing patch and the adhesive film are modeled with one layer of element through thickness.

As mentioned before in the XFEM, the mesh is generated without taking into account the presence of the crack. Boundary conditions and loads are consistent with the section 4.1. The analysis is performed for various crack lengths of 24, 48, 72, 96 and 120 mm.

4.2.1 Effect of single and double sided repairs

The variation of the SIF K_I with the crack length for single and symmetric double Boron/Epoxy and Carbon/Epoxy composite patches is shown in Fig. 4. It is known that the single sided repair leads to an out of plane bending due to a shift in the neutral axis of the plate under tension loading. Therefore, the crack parameters are calculated at the free side of single patched plates.

The results demonstrate a significant reduction in the SIF by using repairing composite patches, especially for large cracks. As the crack length increases, asymptotic behavior is noted for all cracked configurations; in other words, the more the crack length grows the more absorption of the stresses by the patch increases. The reduction in the K_I is about 1.3-4.9 and 1.2-4.3 times that of an un-patched plate for single and double sided Boron/Epoxy and Carbon/Epoxy patch repairs, respectively (Table 3). Also, the results show that the Boron/Epoxy patch decreases K_I more than the Carbon/Epoxy patch.

Double patched cracked plate shows more reduction in SIF, in comparison to single one. This indicates that the symmetrical double patch have significant effect in reducing the SIF and

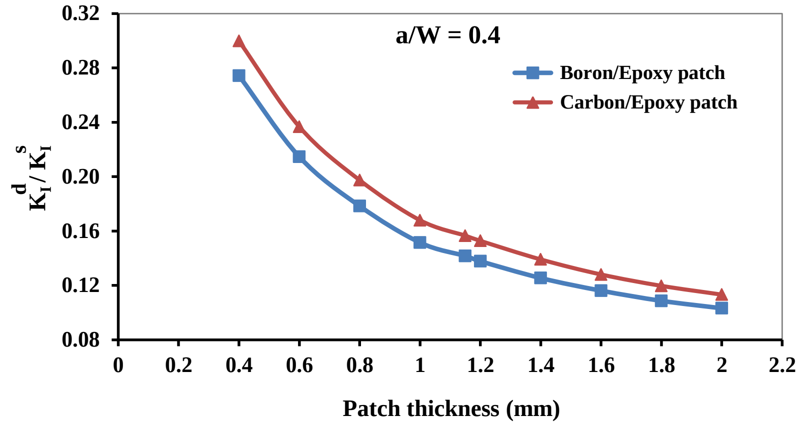

Fig. 5 Effect of patch thickness on the SIF (single and double sided repair)

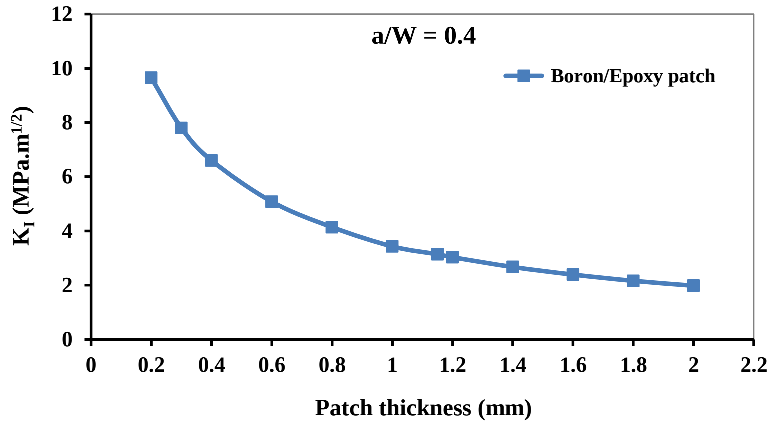

Fig. 6 Effect of double sided Boron/Epoxy composite patch thickness on K_I

decelerating the crack growth. However, there is a varying efficacy between the Boron/Epoxy and Carbon/Epoxy composite patches.

4.2.2 Effect of the patch thickness

The bonding between patch and cracked substrates affects mechanical properties of the joint (Kasavajhala and Gu 2011). The variation of the ratio of the SIF K^d/K^s versus the patch thickness for a/w=0.4 and adhesive thickness of 0.2 mm is shown in the Fig. 5. K^d and K^s are the stress intensity factor at the crack tip for double symmetric and single bonded composite repairs, respectively. The results illustrate the reduction in the K^d/K^s with increasing patch thickness. Also, it is shown that the effect of double symmetrical patch on the reduction of the SIF, compared to single patch, is remarkable.

The effect of double symmetrical Boron/Epoxy patch thickness on the SIF at the crack tip for a/w=0.4 is shown in Fig. 6. It can be seen that beyond thickness about 1.2 mm the variation of K_I versus the patch thickness reaches a plateau. Additionally, for patch thickness of 1.15 mm there is 6 times reduction in the SIF with respect to the un-patched plate. Thus, this patch thickness is desirable for improving the fatigue life of damaged structures.

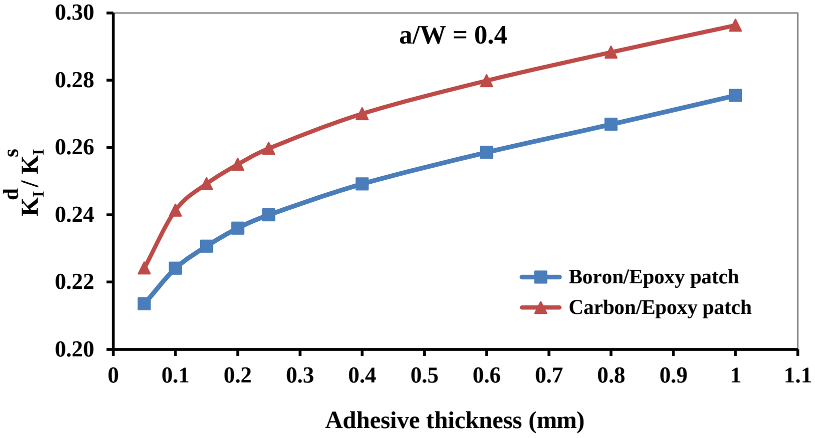

Fig. 7 Effect of adhesive thickness on the SIF (single and double sided repair)

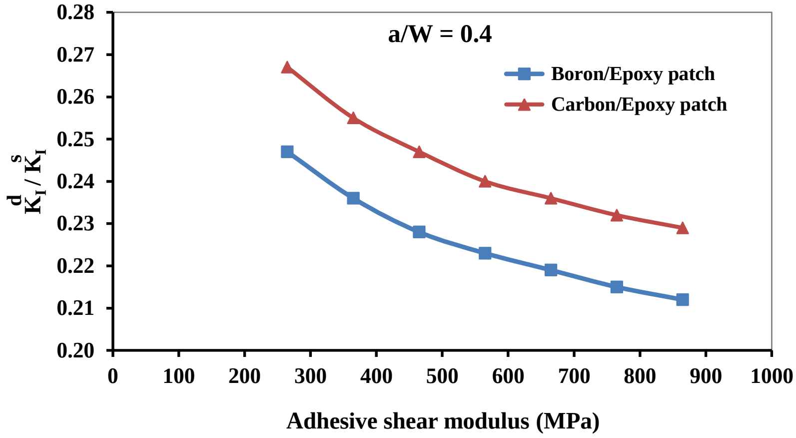

Fig. 8 Effect of adhesive shear modulus on the SIF (single and double sided repair)

4.2.3 Effect of the adhesive thickness

In the repair process, load is transferred through the adhesive from the cracked plate to the composite patch. Hence, the properties and thickness of adhesive play an essential role in restore the fracture strength of damaged structures. In this section, the effect of adhesive thickness is examined as a design variable in the repair process. Fig. 7 shows the variation of the ratio of the stress intensity factor K^d/K^s versus the adhesive thickness for a/w=0.4. There is an increase in the K^d/K^s as the thickness of the adhesive increases, which reduces the beneficial effect of the double patch. The results show a sharp increase in the SIF at the thickness around 0.1 mm and then an opposite behavior can be observed as the adhesive become thicker. Also, the results indicate that an adhesive with low thickness has a better load sharing capacity and reduced SIF. In this regard, it should be noted that the low thickness adhesive bond can involve the rupture of the adhesive layer easily. Also, when the adhesive thickness increases, the adhesive tries to absorb further amount of load that is not desirable in the repair process. By increasing the adhesive layer applied load share, due to higher shear stress and relative weaker bonding, adhesive delamination might be occurred which is not desirable (Kasavajhala and Gu 2011, Arenas *et al.* 2010).

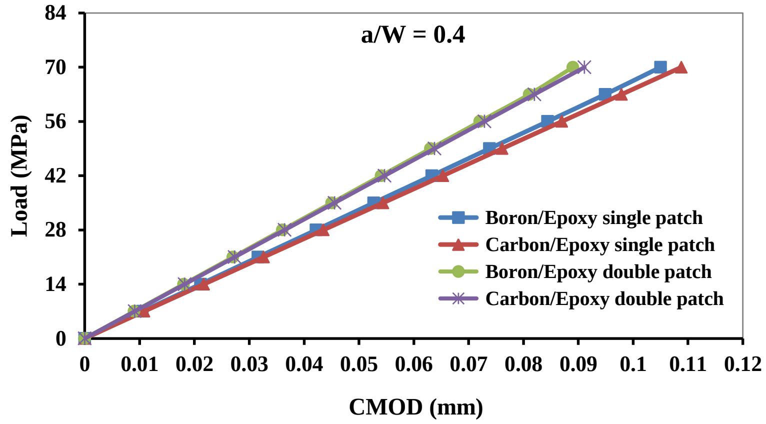

Fig. 9 Effect of single and double symmetric repairing patches on the CMOD

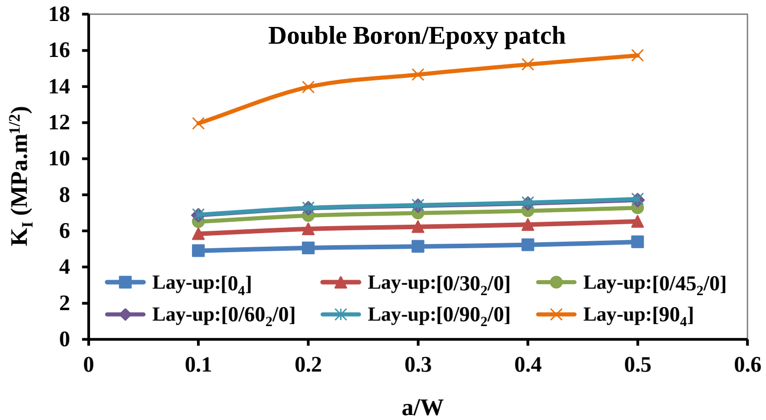

Fig. 10 Effect of double symmetric Boron/Epoxy repair ply orientations on the SIF

4.2.4 Effect of adhesive shear modulus

As mentioned above, the objective of bonding is to transfer the loads to the adhesive layer and consequently to the composite patch. Thus in addition to adhesive thickness, its shear modulus is another design parameter that should be considered in the repair process of damaged structures. The variation of the ratio of the stress intensity factor K^d/K^s versus the adhesive shear modulus for a/w=0.4 is shown in Fig. 8. The result indicates as the adhesive shear modulus increases, there is a drop in the SIF. However, this increase in the adhesive shear modulus leads to a bad adherence between the plate and the patch. Consequently, the adhesive selection for repairing cracks must be optimized to allow the transferring of the stresses towards the patch while avoiding the adhesive failure (delamination or breakage) resulted from the increase of the stresses in the adhesive layer. Also, the application of the double patch reduces the kinetics of the crack considerably and thus allows the deceleration or even stop of its propagation (Ouinas *et al.* 2009).

4.2.5 Effect of patch material and orientation of plies

Composite patch material plays an essential role in increasing the strength and thereby the service life of the repaired structure. Material type and geometry of the structure being repaired,

Table 4 Comparison of the SIF for different ply orientations

Name	Patch material	Ply orientation	Averaged K_I along crack tip (MPa.mm$^{1/2}$)
Repaired double patch	Boron/Epoxy	0/0/0/0	5.144
		0/30/30/0	6.212
		0/45/45/0	6.948
		0/60/60/0	7.348
		0/90/90/0	7.390
		90/90/90/90	14.306

loading and amount of damage are factors that determine the material of the composite patch. The effectiveness of Boron/Epoxy and Carbon/Epoxy patches for repaired aluminum plate is evaluated in terms of crack mouth opening displacement (CMOD) and orientation of plies.

The comparison of the Boron/Epoxy and Carbon/Epoxy composite material stiffness as a function of CMOD is shown in Fig. 9. It is evident from the figure that having high stiffness in the loading direction, the Boron fibers restrict the opening of crack mouth more effectively than the Carbon/Epoxy patch. It means that stiffer patch materials are desirable and plays an important role in the performance of the repair.

The effects of orientation of plies when the ply orientation changes away from 0° (perpendicular to the advancing crack) to 90° (parallel to the emanating crack) in the double sided repair process are shown in Fig. 10. The results clearly show the increase of the SIF as the ply orientation changes away from 0° in 1-direction (Table 4). This is because the effective elastic modulus of the composite patch decreases in the direction perpendicular to the crack, and thereby, the SIF at the crack tip increases. It indicates that the effectiveness of the ply orientation depends on the loading direction and the crack direction.

5. Conclusions

In this paper a 3-D finite element study is carried out to characterize the effects of single and double sided composite patches on strength behavior of a center cracked plate under mode I loading condition using XFEM. Based on the results obtained, the conclusions can be summarized as follows:

- The 3-D modeling and analysis process of center cracked plate using XFEM is validated by available analytical and classical FEM data, and the results indicate much better agreement with analytical results for various crack lengths.
- Repair by single and symmetric double composite patches show a significant reduction in KI at the crack tip, especially for the large cracks, which leads to the plate stiffness and the resistance increase.
- For the same crack length, use of the symmetric double patches shows higher reduction in the SIF as compared to single one. The reason is that in the single repair patch, the out of plane bending occurs due to nonsymmetry and the shift of the neutral axis of the plate.
- The increase in the patch thickness has a direct effect on the reduction of the SIF. However, increase of the patch thickness beyond the suitable level, increases the cost of repair eventually. Also, a thick patch in the single sided repair process may lead to excessive out of plane bending,

and thereby, significant increase in the SIF.

• The increase in the adhesive thickness shows an increase in the SIF resulted from transferring less loads to the repair patch. This indicates that a thinner adhesive thickness is desirable for repairing cracks; however, it will be susceptible to failure (delamination and breakage of the joint). Also, the results indicate that adhesive with high shear modulus, due to a more increase in transferring the load to the repair patch, reduces the SIF further.

• Boron/Epoxy composite patches compared to the Carbon/Epoxy composite patches absorb more dissipated energy which leads to a more reduction in the stress concentration and CMOD.

• In comparison to being parallel with the crack propagation direction, the SIF decreases about three times when the ply orientation is normal to the crack emanating direction.

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