Structural Engineering and Mechanics, *Vol. 50, No. 3 (2014) 365-382*
DOI: http://dx.doi.org/10.12989/sem.2014.50.3.365

Sizing, geometry and topology optimization of trusses using force method and supervised charged system search

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(Received August 10, 2012, Revised March 6, 2014, Accepted March 9, 2014)

Abstract. In this article, the force method and Charged System Search (CSS) algorithm are used for the analysis and optimal design of truss structures. The CSS algorithm is employed as the optimization tool and the force method is utilized for analysis. In this paper in addition to member's cross sections, redundant forces, geometry and topology variables are considered as the optimization variables. Minimum complementary energy principle is used directly to analyze the structure. In the presented method, redundant forces are calculated by the CSS in order to minimize the energy function. Combination of the CSS and force method leads to an efficient algorithm in comparison to some of the optimization algorithms.

Keywords: optimization; charged system search; force method; minimum complementary energy; truss structures

1. Introduction

Developing methods with higher computational efficiency is a crucial subject in advanced engineering problems of multi-physics nature. For instance, analyzing structures with larger number of members requires larger memory size and longer computation time. In addition, this costly computation has to be repeated many times, typically over 10,000 times, because the cross section size of the members is not determined in the early stages of designing such structures. Therefore, reducing the size of structural matrices and eliminating the unduly repetitions in the design and analysis procedures can lead to a considerable reduction in the computation efficiency. In this paper, this goal is achieved utilizing meta-heuristics algorithms which minimize the energy function indirectly. Besides, design procedure and minimizing the weight of the structure and improving the geometry and topology of the structures are added to the analysis procedure. One of the most reliable meta-heuristic methods recently developed is Charged System Search (CSS) (Kaveh and Talatahari 2010, 2011) that is used in here. The SCSS is an improved version of the CSS that uses a kind of agents that are called supervisor agent to increase the exploration ability of the CSS (Kaveh and Ahmadi 2013).

Analysis of structures by the force method is well established by Argyris and Kelsey (1960). Further developments are due to Henderson (1960), Henderson and Maunder (1969), Cassell *et al*.

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http://www.techno-press.org/?journal=sem&subpage=8 ISSN: 1225-4568 (Print), 1598-6217 (Online)

(1974), Felippa (1975), Kaveh (1974) among many others. A comprehensive list of references can be found in the review paper of Kaveh (1992).

Topology and geometry of structures can play an important role in reducing the weight of the structures. Optimum nodal coordinates lead to a better behavior of structure. Besides an optimum topology leads to the use of the just necessary members and has a great effect on the weight and behavior of the structure. Methods are available for topology optimization of trusses using the displacement method as the analyzer (Ohsaki and Katoh 2005, Lee *et al.* 2012, Martinez *et al.* 2007, Xie *et al*. 2009, Tohlu *et al*. 2013).

Designing structures with minimum weight and optimum topology need to analyze the structure and this can be achieved by simultaneous minimization of the energy function and weight function of the structure. Minimizing the energy function by a meta-heuristic algorithm instead of the direct solution of classic equations leads to avoid not only the repetitive computations in the design and analysis but also avoiding the computation of the inverse of the large matrices. Naturally, one needs to formulate the equations based on the minimum energy principle, and employ these in an efficient optimization algorithm. Combining the SCSS algorithm and the force method that is preferred to the displacement method due to the less number of the unknowns provides a suitable means for this purpose, since the former provides the optimization algorithm and the latter can be used to derive the energy equations.

A brief introduction to SCSS is presented in the first part of this article and energy principle is presented in the next part. Energy formulation based on the force method is derived in the fourth part. In the last part, using the SCSS, structures are analyzed and designed, topology and geometry of structures are considered as optimization variables in this part. In recent years, the CSS has been successfully applied to many engineering optimization problems. For these problems, CSS has performed very well and improved most of the resulted design parameters, nodal coordinate and topology of structures leading to smaller weight. In the simultaneous analysis and design of structures using energy function, force method and SCSS algorithm, nodal coordinate is considered variable to improve the geometry of the structure and presence or absence of a member is considered as a variable to improve the topology of the structure.

2. Supervised CSS algorithm

In the CSS algorithm, each vector of variables is an agent that moves through the search space and finds the minimal solutions (Kaveh and Talatahari 2010, 2011). Throughout the search process, an agent might go to a coordinate in the search space that already has been searched by the same agent or another. If this coordinates, have a good fitness, it will be saved in the Charged Memory (Kaveh and Talatahari 2010) but if this coordinates, does not have a good fitness, it will be saved nowhere. Therefore, this step of the search process becomes redundant. This unnecessary step adversely affects the exploration ability of the algorithm. In this paper, the supervisor agents are introduced to improve the exploration ability of the CSS algorithm. The supervisor agent is an independent agent of constant values that repels the agent if its coordinate has a bad fitness or attracts the agents if its coordinate has a good fitness. This procedure is repeated in all of the iterations and gives an overall view of the search space. The number of supervisor agents is selected at the beginning of the algorithm, and then their constant coordinates in the search space are determined as follows, Eq. (1)

$$xs_{j,i} = \frac{(i-1)[x_{max,j} - x_{min,j}]}{NOSA-1} + x_{min,j} \tag{1}$$

Where *NOSA* is the number of supervisor agents, and $xs_{j,i}$ is the j^{th} variable of the i^{th} supervisor agent; $x_{min,j}$ and $x_{max,j}$ are the minimum and the maximum limits of the j^{th} variable, respectively. The kind of the force for these agents is determined from Eq. (2)

$$p = log(\frac{\overline{fit}}{fit_i}) \tag{2}$$

where p is the same as the parameter in the original version of the CSS (Kaveh and Talatahari 2010), fit_i is equal to the fitness value of the i^{th} supervisor agent and $\overline{fit}$ is the average value of the fitness of the normal agents. Calculating other properties of the supervisor agents such as force and radius are similar to the standard CSS algorithm (Kaveh and Talatahari 2010). Supervisor agents do not move from their coordinate determined from Eq. (1), yet they apply additional forces on the normal agents. By doing so, they determine the fitness values of their fixed coordinate and its neighborhood, resulting in a better exploration ability of the CSS algorithm (Kaveh and Ahmadi 2013).

3. Minimum energy principle

As mentioned in the previous section, minimum energy principle is used directly in order to analyze the structure. In the following a brief introduction is provided to the energy method. Three main concepts of energy are strain energy, complementary strain energy, and the total potential energy that can be expressed as

$$U = \iint g(\varepsilon) d\varepsilon dV \tag{3}$$

$$U^c = \iint f(\sigma) d\sigma dV \tag{4}$$

$$V = U - \mathbf{P}^t \mathbf{u} \tag{5}$$

Where **P** is the vector of the external loads and **u** is the vector of joint displacements. $g(\varepsilon)$ and $f(\sigma)$ are the stress-strain relationship functions.

According to Castigliano's first theorem, for an elastic (linear and nonlinear) system, the potential energy in stable equilibrium is minimum. Similarly according to the second theorem, the complementary potential energy is minimum for a system of internal forces which satisfies the compatibility. In general, U corresponds to the stiffness method and U^c corresponds to the flexibility approach. In the first case one looks for the displacements and in the latter case we look for redundant forces. Since in a statically indeterminate structure, after calculating the redundant loads, the remaining member forces can easily be obtained, hence using U^c, i.e., the flexibility method, corresponds to smaller number of unknown. In the following the basic steps of the flexibility method based on the principle of complementary strain energy is described.

4. Energy function formulation using force method

In the presented approach, force method is applied to analyze the structures. Since this method leads to less number of unknowns, it is preferred to displacement method. In the force method, the redundant forces are unknowns, whereas in the displacement method, the nodal displacements are unknowns. In this method, energy function is derived using the force method and by applying the minimum energy principle as illustrated in the second section member internal forces and nodal displacements can be calculated (Kaveh 2004, 2006, Kaveh and Rahami 2006). Energy function is considered as a constraint in the objective function in the CSS algorithm and simultaneous with reducing the weight of the structure energy function is minimized. Suppose $\mathbf{\{p\}=\{p_1,p_2,\ldots,p_n\}^t}$ is the vector of nodal forces, $\mathbf{\{q\}=\{q_1,q_2,\ldots,q_r\}^t}$ is the vector of redundant forces, and $\mathbf{\{r\}=\{s_1,s_2,\ldots,s_m\}^t}$ comprising of the internal forces of the members. Equilibrium condition results in the following Eqs. (16), (17)

$$\mathbf{r} = \mathbf{B_0 p} + \mathbf{B_1 q} = \begin{bmatrix}\mathbf{B_0} & \mathbf{B_1}\end{bmatrix}\begin{bmatrix}\mathbf{p}\\ \mathbf{q}\end{bmatrix} \tag{6}$$

In addition, the complementary energy function is

$$U^c = \frac{1}{2}\mathbf{r^t F_m r} \tag{7}$$

where $\mathbf{[F_m]}$ is the unassembled flexibility matrix of the structure. According to the Castigliano's principle, a group of the redundant forces that minimize the complementary energy function is the exact solution that satisfies compatibility condition. By substituting $\mathbf{\{r\}}$ from Eq. (6) into Eq. (7), the following equation obtained

$$U^c = \frac{1}{2}\begin{bmatrix}\mathbf{p^t} & \mathbf{q^t}\end{bmatrix}\mathbf{H}\begin{bmatrix}\mathbf{p}\\ \mathbf{q}\end{bmatrix} \tag{8}$$

where $[\mathbf{H}] = \begin{bmatrix}\mathbf{B_0} & \mathbf{B_0}\end{bmatrix}^{\mathbf{t}}\mathbf{F_m}\begin{bmatrix}\mathbf{B_0} & \mathbf{B_1}\end{bmatrix}$. Decomposing matrix [**H**] into four submatrices leads to

$$U^c = \frac{1}{2}\left(\{\mathbf{p}\}^{\mathbf{t}}[\mathbf{H_{pp}}]\{\mathbf{p}\} + \{\mathbf{p}\}^{\mathbf{t}}[\mathbf{H_{pq}}]\{\mathbf{q}\} + \{\mathbf{q}\}^{\mathbf{t}}[\mathbf{H_{qp}}]\{\mathbf{p}\} + \{\mathbf{q}\}^{\mathbf{t}}[\mathbf{H_{qq}}]\{\mathbf{p}\}\right) \tag{9}$$

In the classical method, the derivative of U^c in terms of $\mathbf{\{q\}}$ is calculated and is equated to zero leading to

$$\{\mathbf{q}\} = -[\mathbf{H_{qq}}]^{-\mathbf{1}}[\mathbf{H_{qp}}]\{\mathbf{p}\} \tag{10}$$

Since $\mathbf{[H]}$ is symmetric, $\mathbf{[H_{qp}]^t = [H_{pq}]}$, Ref. (Kaveh and Rahami 2006).

Accordingly, in the classical method the inverse of $\mathbf{[H_{qq}]}$ needs to be calculated. This is a difficult task and requires extensive computer memory, especially in the case of large-scale structures. Therefore, finding $\mathbf{\{q\}}$ that minimizes the complementary energy without calculating the inverse of $\mathbf{[H_{qp}]}$ reduces the computation time and computer memory. The first term of Eq. (9) is constant and the second and third terms are equal. It can be shown that the third and fourth terms of U^c are symmetric. Therefore

$$F_u = \{\mathbf{q}\}^{\mathbf{t}}[\mathbf{H_{qp}}]\{\mathbf{p}\} \tag{11}$$

is the equation that should be minimized (Kaveh and Rahami 2006).

5. Simultaneous analysis, design and optimization formulation

In the case of simultaneous design and analysis of structures, the objective function is the weight of the structure, and the equilibrium, compatibility, and force/displacement conditions are the constraints. In summary, all these three conditions are called analysis criteria for simplicity. Other constraints such as stresses, displacements, dynamical properties, etc. can also be imposed to the fitness function. Penalty function is the most common approach to satisfying the constraints. The penalty function imposes a penalty to the fitness value of the solution, if the constraint is not satisfied

$$f = A + \alpha B \tag{12}$$

In Eq. (12), f is the fitness value, A is the objective function and B is the penalty function and α is often selected as a big number. According to this equation, when B approaches to zero and A goes to its minimum value, f approaches to the minimum value of the fitness. However, since the minimum complementary energy is not zero, this form of penalty function cannot be used. In this case, W is minimum while the corresponding U^c is not minimum, i.e. the structure is not analyzed yet. Also a small value of α does not guarantee the minimum value of the B. On the other hand, in a structure that is in equilibrium and compatible state, sum of the complementary energy U^c and the strain energy U is zero. Therefore, instead of the complementary energy, the sum of the complementary energy and the strain energy is used as the analysis criteria and is imposed to the SCSS as a constraint. The strain energy is a function of nodal displacements as follows (Kaveh and Rahami 2006)

$$\{\mathbf{d}\} = [\mathbf{B_0}]^{\mathbf{t}}[\mathbf{F_m}]([\mathbf{B_0}]\{\mathbf{p}\} + [\mathbf{B_1}]\{\mathbf{q}\}) \tag{13}$$

and

$$U = \frac{1}{2}\{\mathbf{d}\}^{\mathbf{t}}[\mathbf{K}]\{\mathbf{d}\} - \{\mathbf{d}\}^{\mathbf{t}}\{\mathbf{F}\} \tag{14}$$

Where **[K]** is the stiffness matrix and **{F}** is the nodal force vector. For equilibrium, U is negative and $U+U^c$ is equal to zero. Kaveh and Rahami used a different formulation to impose the analysis criteria as a constraint (Kaveh and Rahami 2006). In this method, using the derivative of U^c in Eq. (9) with respect to **{q}** leads to

$$\frac{\partial U^c}{\partial q} = [\mathbf{H_{qp}}]\{\mathbf{p}\} + [\mathbf{H_{qq}}]\{\mathbf{q}\} = \mathbf{0} \tag{15}$$

Eq. (15) indicates that the complementary energy of the structure is equal to its minimum value in the compatible condition. Thus, **{q}** should be selected such that Eq. (15) holds. The left hand of this equation is a zero vector and it should be changed to a scalar. The best way is calculation of the norm, because the norm of a vector is equal to zero when all the entries are equal to zero. Here, we use the equilibrium itself. For this purpose we can write

$$F(\mathbf{q},\mathbf{A},\mathbf{D}) = W(\mathbf{A})(1+\alpha norm(\lfloor \mathbf{H_{qp}} \rfloor\{\mathbf{p}\} + \lfloor \mathbf{H_{qq}} \rfloor\{\mathbf{q}\})) \tag{16}$$

Where **{q}** is the force variables vector, **{A}** is the member cross sections vector and **{D}** is the topology and geometry variables vector**.** Having these variables the magnitude of F can be calculated from Eq. (16) and its minimum for a large value of α corresponds to minimum W. Other constraints such as stress constraints, displacement constraints or dynamical properties constraints can be applied to Eq. (16) after normalizing and selecting a penalty coefficient. Therefore, the final formulation will be as follow

$$Find \longrightarrow q, A; A \in \{S_d \quad or \quad S_c\}$$

$$MinF(\mathbf{q},\mathbf{A},\mathbf{D}) = \sum_{i=1}^{ne} A_i l_i \rho_i (1+\alpha norm(\lfloor \mathbf{H_{qp}} \rfloor\{\mathbf{p}\} + \lfloor \mathbf{H_{qq}} \rfloor\{\mathbf{q}\})) + \sum_{m=1}^{nc} \max(0, g_m(A)) \tag{17}$$

Where S_d and S_c are the discrete and continuous sections, respectively. $g_m(A)$ corresponds to violation of the constraints. According to Kaveh and Ahmadi (2013), the following formulation is more efficient than Eq. (17).

$$F(\mathbf{q},\mathbf{A},\mathbf{D}) = \sum_{i=1}^{ne} A_i l_i \rho_i (1+\alpha norm(\lfloor \mathbf{H_{qp}} \rfloor\{\mathbf{p}\} + \lfloor \mathbf{H_{qq}} \rfloor\{\mathbf{q}\})) + \sum_{m=1}^{nc} \max(0, g_m(A))^{R(norm)} \tag{18}$$

Where $R(norm)$ is a function of $norm(\lfloor \mathbf{H_{qp}} \rfloor\{\mathbf{p}\} + \lfloor \mathbf{H_{qq}} \rfloor\{\mathbf{q}\})$. This function can be considered as follows:

$$R(norm) = \log(10 + NORM) \tag{19}$$

Where NORM is equal to $norm(\lfloor \mathbf{H_{qp}} \rfloor\{\mathbf{p}\} + \lfloor \mathbf{H_{qq}} \rfloor\{\mathbf{q}\})$. In all of the examples studied in the following, Eq. (18) is used in the SCSS algorithm. Geometry variables are considered as the nodal coordinates and topology variables are considered as a vector with length equal to the number of members of the structures. This vector has entries of 1 and 0 for presence or absence of a member, respectively. By altering the number of members in a structure during the algorithm, redundant members and the number of redundant forces (DSI) alter. In order to improve the performance of the algorithm, a vector of redundant force equal to the number of members of the structure is considered and each selected redundant force will be affected by the corresponding redundant forces in the other agents. Besides because of altering the number of members of the structure, the DSI of the structure will change, considering the mentioned redundant force vector leads to having constant value for the number of variables in the optimization algorithm.

6. Numerical examples

In this section, different planar and space trusses are optimized by the presented method. Results show that the combination of SCSS and force method has a good performance in comparison to some other optimization algorithms. Besides the weight of the considered examples are also less than the existing results.

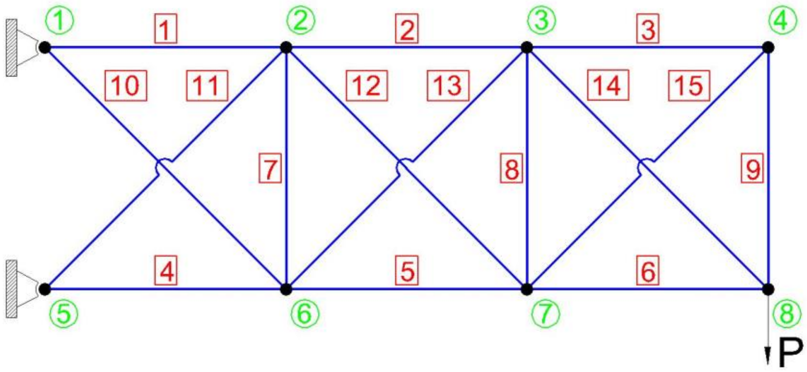


Fig. 1 The topology of a fifteen-bar truss

Table 1 Data for design of fifteen-bar planar truss

Design variables
Size variables A_i; i=1, 2, …, 15 Geometry variables x_2=x_6; x_3=x_7; y_2;y_3;y_4;y_6;y_7;y_8
Constraint data
Stress constraints $(\sigma_t)_i \leq 172.4$ MPa (25 ksi); i=1,…,15 $\lvert(\sigma_t)_i\rvert \leq 172.4$ MPa (25 ksi); i=1,…,15 Side constraint for geometry variables 254 cm (100 in) $\leq x_2 \leq$ 355.6 cm (140 in); 558 cm (220 in) $\leq x_3 \leq$ 660.4 cm (260 in) 254 cm (100 in) $\leq y_2 \leq$ 355.6 cm (140 in); 254 cm (100 in) $\leq y_3 \leq$ 355.6 cm (140 in) 127 cm (50 in) $\leq y_4 \leq$ 228.6 cm (90 in); -50.8 cm (-20 in) $\leq y_6 \leq$ 50.8 cm (20 in) -50.8 cm (-20 in) $\leq y_7 \leq$ 50.8 cm (20 in); 50.8 cm (20 in) $\leq y_8 \leq$ 152.4 cm (60 in) p=0.0254 cm (0.01 in.)
List of the available profiles
$A_i \in S$ = {0.716, 0.910, 1.123, 1.419, 1.742, 1.852, 2.239, 2.839, 3.477, 6.155, 6.974, 7.574, 8.600, 9.600, 11.381, 13.819, 17.400, 18.064, 20.200, 23.00, 24.6, 31.0, 38.4, 42.4, 46.4, 55.0, 60.0, 70.0, 86.0, 92.193, 110.774, 123.742} (cm^2) $A_i \in S$ ={0.111, 0.141, 0.174, 0.22, 0.27, 0.287, 0.347, 0.44, 0.539, 0.954, 1.081, 1.174, 1.333, 1.488, 1.764, 2.142, 2.697, 2.8, 3.131, 3.565, 3.813, 4.805, 5.952, 6.572, 7.192, 8.525, 9.3, 10.85, 13.33, 14.29, 17.17, 19.18} (in^2); i=1,…,15
Loading data

Load case	node	F_x	F_y
1	8	0.0	-44.537kN (-10.0 kips)

Material properties
Modulus of elasticity E=6.895 × 10^4 MPa (1.0 × 10^4 ksi) Density of the material ρ =0.0272 N/cm^3 (0.1 lb/in^3)

Case study 1. The firs structure is a fifteen-bar truss as shown in Fig. 1. The input data for this truss is given in Table 1. The optimization variables in this example consist of redundant force **{q}**,

Table 2 The comparison of the results of optimum weight for fifteen-bar truss with those of the other references

Design variables ($in.^2$)	Wu and Chow (1995)	Hwang and He (2006)	Tang *et al.* (1995)	Rahami *et al.* (2008)	Present work
$A_1(in.^2)$	1.174	0.954	1.081	1.081	1.081
$A_2(in.^2)$	0.954	1.081	0.539	0.539	0.539
$A_3(in.^2)$	0.440	0.440	0.287	0.287	0.287
$A_4(in.^2)$	1.333	1.174	0.954	0.954	0.954
$A_5(in.^2)$	0.954	1.488	0.954	0.539	0.539
$A_6(in.^2)$	0.174	0.270	0.220	0.141	0.141
$A_7(in.^2)$	0.440	0.270	0.111	0.111	0.111
$A_8(in.^2)$	0.440	0.347	0.111	0.111	0.111
$A_9(in.^2)$	1.081	0.220	0.287	0.539	0.440
$A_{10}(in.^2)$	1.333	0.440	0.220	0.440	0.440
$A_{11}(in.^2)$	0.174	0.220	0.440	0.539	0.539
$A_{12}(in.^2)$	0.174	0.440	0.440	0.270	0.270
$A_{13}(in.^2)$	0.347	0.347	0.111	0.220	0.220
$A_{14}(in.^2)$	0.347	0.270	0.220	0.141	0.141
$A_{15}(in.^2)$	0.440	0.220	0.347	0.287	0.287
$X_2(in)$	123.189	118.346	133.612	101.5775	102.4881
$X_3(in)$	231.595	225.209	234.752	227.9112	227.8206
$Y_2(in)$	107.189	119.046	100.449	134.7986	134.6850
$Y_3(in)$	119.175	105.086	104.738	128.2206	128.3577
$Y_4(in)$	60.462	63.375	73.762	54.8630	54.5826
$Y_6(in)$	-16.728	-20.0	-10.067	-16.4484	-16.7742
$Y_7(in)$	15.565	-20.0	-1.339	-13.3007	-13.3367
$Y_8(in)$	36.645	57.722	50.402	54.8572	54.3873
Weight (lb)	120.528	104.573	79.820	76.6854	76.8214
Weight (N)	536.1	465.1	355.0389	341.0962	341.7011

At the present work, Max stress ratio=0.9992

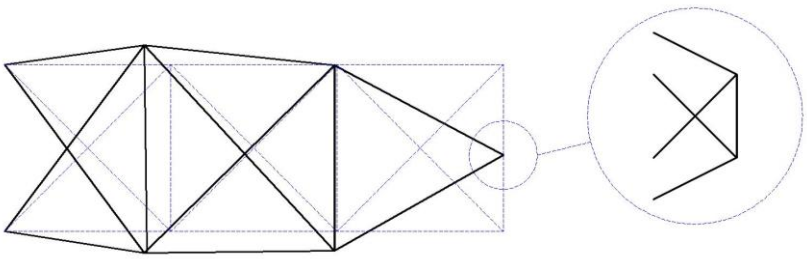


Fig. 2 Optimum geometry of fifteen-bar truss

member cross sections **{A}**, and nodal coordinates **{D}**.

One set of redundant forces can be the internal forces of the elements 11, 13 and 15. In this case, the topology of the structure is constant. Achieved results and comparison with other references is provided in Table 2. Resulted geometry of the example is shown in Fig. 2. Elapsed time for this case is 5.264266 seconds when a computer with core (TM) i5-2400 CPU @ 3.10GHz and 4.00 GB RAM is being used.

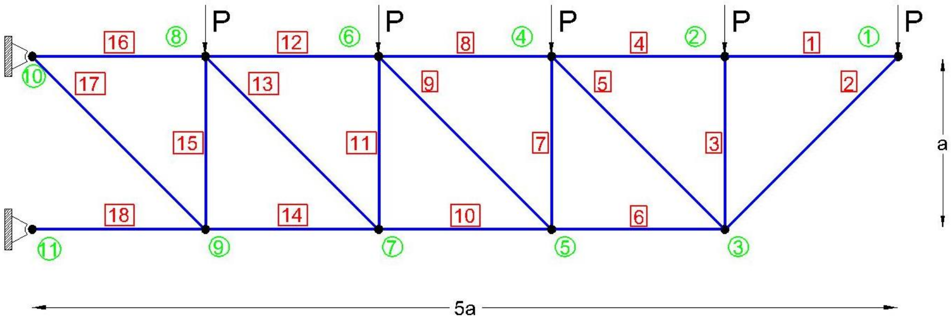


Fig. 3 The topology of an eighteen-bar planar truss (a=250 in)

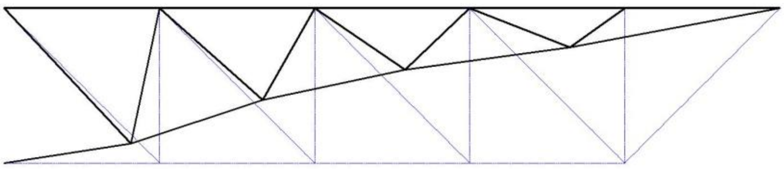


Fig. 4 The optimum geometry of eighteen-bar planar truss

Force method finite element formulation has just three unknowns (redundant forces) to solve this problem while displacement method has 12 unknowns (degrees of freedom), then using displacement method increases the optimization variables and leads to higher computational cost.

Case study 2. This example is an eighteen-bar planar truss which is statically determinate, and therefore the optimization variables do not contain any redundant force variable. The cross sections of the members in 4 set, and the coordinates of four bottom nodes, form the optimization variables. The topology of structure is shown in Fig. 3. The optimum geometry of structure is shown Fig. 4. The input data and comparison of the achieved results with other methods are provided in Table 4 and Table 5 for discrete and continuous cross sections, respectively. Elapsed time for this case is 4.050538 seconds when a computer with core (TM) i5-2400 CPU @ 3.10GHz and 4.00 GB RAM is used.

Case study 3. In this example, the presented method is applied to a twenty five-bar space truss. The topology of the truss is provided in Fig. 5. Optimization variables consist of member cross sections in 8 set, 7 redundant forces (such as elements with end nodes 1-2, 1-4, 2-6, 6-7, 4-9, 3-8, 5-10), and 8 nodal coordinates. The input data are given in Table 6. The comparison of the results with those of the other references is provided in Table 7. The SCCS is the optimization algorithm and Eq. (18) is considered as the objective function. Elapsed time for this case is 3.544595 seconds when a computer with core (TM) i5-2400 CPU @ 3.10GHz and 4.00 GB RAM is being utilized. As can be seen, force method analysis variables are seven redundant forces while displacement analysis variables are 18 free nodal displacements. Then displacement method will increase optimization problem variables and analysis system size in comparison to the force method, and this will increase computational time.

Case study 4. In this example, a ten-bar truss as shown in Fig. 6 is considered. The presence or absence of members is considered as variable. Then topology of this truss is considered as variable similar to the geometry. For this purpose, a vector of 1 and 0 is considered to determine the

Table 3 Data for design of eighteen-bar planar truss

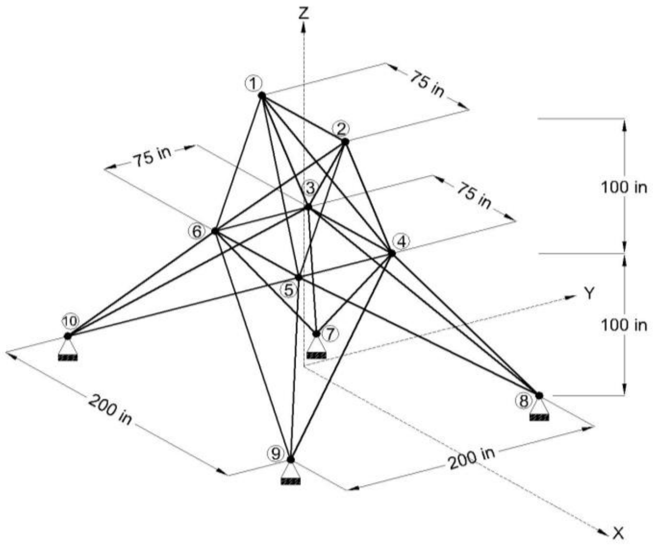
Design variables			
Size variables A_1=A_4=A_8=A_{12}=A_{16}; A_2=A_6=A_{10}=A_{14}=A_{18}; A_3=A_7=A_{11}=A_{15}; A_5=A_9=A_{13}=A_{17}			
Geometry variables x_3;y_3;x_5;y_5;x_7;y_7;x_9;y_9			
Constraint data			
Stress constraints			
$(\sigma_t)_i \le 137.9$ MPa (20 ksi); i=1,…,18			
$\mid(\sigma_c)_i\mid \le 137.9$ MPa (20 ksi); i=1,…,18			
Euler buckling stress constraints			
$\mid(\sigma_c)_i\mid \le \alpha E A_i / L_i^2$ i=1,…,18			
Side constraint for geometry variables			
-571.5 cm (-225 in) $\le y_3, y_5, y_7, y_9 \le$ 622.3 cm (245 in);			
1968.5 cm (775 in) $\le x_3 \le$ 3111.5 cm (1225 in)			
1333.5 cm (525 in) $\le x_5 \le$ 2476.5 cm (975 in)			
698.5 cm (275 in) $\le x_7 \le$ 1841.5 cm (725 in)			
63.5 cm (25 in) $\le x_9 \le$ 1206.5 cm (475 in)			
p=2.54 cm (1.0 in)			
List of the available profiles			
Discrete cross section			
$A_i \in$ S ={12.903, 14.516, …, 138.709, 140.322} (cm^2)			
$A_i \in$ S ={2.00, 2.25, …, 21.50, 21.75} (in^2); i=1,…,15			
Continuous cross section			
$3.5 \le A_i \le 18$ (in^2); i=1, …, 25			
Loading data			
Load case	node	F_x	F_y
1	1, 2, 4, 6, 8	0.0	-89.075kN (-20.0 kips)
Material properties			
Modulus of elasticity E=6.895 × 10^4 MPa (1.0 × 10^4 ksi)			
Buckling coefficient α =4			
Density of the material ρ =0.0272 N/cm^3 (0.1 lb/in.^{3})			

presence or absence of the members. Geometry variables are considered as the coordinates of the 3 top nodes in y direction. Connectivity of the example is provided in Fig. 6. Modulus of elasticity for this example is 10^7 psi, weight density is considered as 0.1 lb/in^3, allowable stress in tension or compression is 25000 psi and the maximum displacement in y direction is limited to 2 in. Loading is as shown in Fig. 6. In some references for size and topology optimization and fixed geometry, the maximum displacement in y direction is limited to 2.05 in, while the allowable displacement is 2.0. In the present article, both cases are considered and results and comparison with those in the other references are provided in Tables 8 and 9. Optimum topology and geometry of this example is provided in Fig. 7. Elapsed time for this case is 10.817588 seconds when a computer with core

Table 4 Results and comparison with other references for eighteen-bar truss by discrete cross sections

Design variables	Hasancebi and Erbatuer (2001)	Kaveh and Kalatjari (2004)	Rahami *et al*. (2008)	Present work
A_1(in.^{2})	12.50	12.25	12.75	12.50
A_2(in.^{2})	18.25	18.0	18.50	18.00
A_3(in.^{2})	5.5	5.25	4.75	5.25
A_4(in.^{2})	3.75	4.25	3.25	3.75
X_3(in)	933.0	913.0	917.4475	911.7926
Y_3(in)	188.0	186.8	193.7899	186.8277
X_5(in)	658.0	650.0	654.3243	645.6011
Y_5(in)	148.0	150.0	159.9436	150.2578
X_7(in)	422.0	418.8	424.4821	416.4751
Y_7(in)	100.0	97.4	108.5779	101.5338
X_9(in)	205.0	204.8	208.4691	204.1336
Y_9(in)	32.0	26.7	37.6349	31.6508
Weight (lb)	4574.28	4547.9	4530.7	4527.6952
Weight (N)	2034.6	2022.9	2015.3	2013.9

Maximum stress ratio for the present work=0.999988



Group Number	Members
1	1-2
2	1-4,2-3,1-5,2-6
3	2-5,2-4,1-3,2-6
4	3-6,4-5
5	3-4,5-6
6	3-10,6-7,4-9,5-8
7	3-8,4-7,6-9,5-10
8	3-7,4-8,5-9,6-10

Fig. 5 Topology of a twenty five-bar space truss (Case study 3) and grouping of the members

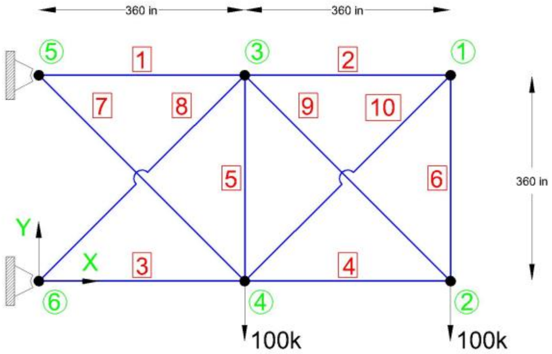


Fig. 6 The primary geometry and topology of a ten-bar truss

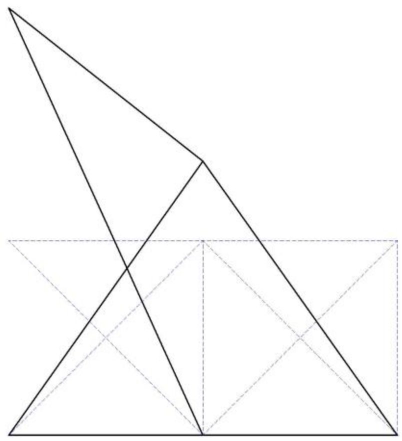


Fig. 7 Optimum topology and geometry of a ten bar planar truss

(TM) i5-2400 CPU @ 3.10GHz and 4.00 GB RAM is used. This problem redundancy depends on the number of nodes and members are being considered, but in the case that all nodes and members are presence, the force method finite element has only two unknowns while displacement method has eight unknowns.

As shown in the Fig. 7, the node number 1 has been eliminated in the optimum topology. This happens because of eliminating all of the members of a node. Because of existing of loads on the nodes number 2 and 4, these nodes cannot be eliminated. Obviously, support nodes also cannot be eliminated. Because of presence and absence of members, redundant forces cannot be considered fixed. In this manner, algebraic force method that finds redundant forces using considering independent columns of equilibrium matrix is used here in the examples with member presence and absence variables.

Case study 5. In this example, a fifteen-bar truss shown in Fig. 1 is considered to optimize its topology and geometry. A vector of 1 and 0 is considered to determine the presence or absence of the members. Input data for this example is the same as those of Table 1. The achieved results and comparison with those of other references are provided in Table 10. Fig. 8 shows the achieved optimum topology and geometry of the example. Elapsed time for this case was 8.083642 seconds when a computer with core (TM) i5-2400 CPU @ 3.10GHz and 4.00 GB RAM is used.

Table 5 Results and comparison with other references for eighteen-bar truss by continuous cross sections

Design variables (in.^{2})	Imai and Schmit (1981)	Felix (1981)	Yang (1996)	Soh and Yang (1996)	Rajeev and Krishnamoorthy (1997)	Yang and Soh (1997)	Kang and Zong (2005)	Rahami *et al.* (2008)	Present work
A_1(in.^{2})	11.24	11.34	12.61	12.59		12.33	12.65	12.55	12.4106
A_2(in.^{2})	15.68	19.28	18.10	17.91	12.50	17.97	7.22	18.02	17.8115
A_3(in.^{2})	7.93	10.97	5.470	5.50	16.25	5.60	6.17	5.11	5.3029
A_4(in.^{2})	6.49	5.30	3.540	3.55	8.00	3.66	3.55	3.57	3.8306
X_3(in)	891.10	994.60	914.5	909.8	4.00 891.90	907.20	903.10	912.96	911.4261
Y_3(in)	143.60	162.30	183.0	184.5	145.30	184.20	174.30	188.06	185.4620
X_5(in)	608.20	747.40	647.0	640.3	610.60	643.30	630.30	646.45	643.4574
Y_5(in)	105.40	102.90	147.0	147.8	118.20	149.20	163.30	150.61	147.2265
X_7(in)	381.70	482.90	414.2	410.0	385.40	413.90	402.10	416.61	413.7991
Y_7(in)	57.10	33.00	100.4	97.00	72.50 184.40	102.00	90.50	102.52	98.3685
X_9(in)	181.00	221.70	200.0	200.9	23.40	202.10	195.30	204.28	202.4140
Y_9(in)	-3.20	17.10	31.90	32.00	4616.8	30.90	30.60	32.65	29.3862
Weight(lb)	4667.9	5713.0	4552.8	4531.9	2053.5	4520.0	4515.6	4511.4	4509.1083
Weight (N)	2076.3	2541.1	2025.1	2015.8		2010.5	2008.5	2006.7	2005.6

At the present work, Max stress ratio=0.99999075

Table 6 Data for design of twenty five-bar space truss

Design variables

Size variables A_1; A_2; A_3; A_4; A_5; A_6; A_7; A_8

Geometry variables $x_4=x_5=-x_3=-x_6$; $x_8=x_9=-x_7=-x_{10}$;
$y_3=y_4=-y_5=-y_6$; $y_7=y_8=-y_9=-y_{10}$; $z_3=z_4=z_5=z_6$

Constraint data

Stress constraints

$(\sigma_t)_i \le 275.8$ MPa (40 ksi); i=1,…,25

$|(\sigma_c)|_i \le 275.8$ MPa (40 ksi); i=1,…,25

Displacement constraint in all direction of the coordinate system

$|\Delta_i| \le 0.89$ cm (0.35 in); i=1,…,18

Side constraint for geometry variables

50.8 cm (20 in) $\le x_4 \le$ 152 cm (60 in); 101.6 cm (40 in) $\le x_4 \le$ 203.2 cm (80 in);

101.6 cm (40 in) $\le y_4 \le$ 203.2 cm (80 in); 254 cm (100 in) $\le y_8 \le$ 355.6 cm (140 in);

228.6 cm (90 in) $\le z_4 \le$ 330.2 cm (130 in);

P=0.0254 cm (0.01 in.)

List of the available profiles

$A_i \in$ S ={0.645I (I=1, …, 26), 18.064, 19.355, 20.645, 21.935} (cm^2)

$A_i \in$ S ={0.1I(I=1, …, 26), 2.8, 3.0, 3.2, 3.4} (in^2); i=1,…,25

Loading data

Load case	Node	F_x kN (kips)	F_y kN (kips)	F_z kN (kips)
1	1	4.454 (1.0)	-44.537 (-10.0)	-44.537 (-10.0)
	2	0.0	-44.537 (-10.0)	-44.537 (-10.0)
	3	2.227 (0.5)	0.0	0.0
	6	2.672 (0.6)	0.0	0.0

Material properties

Modulus of elasticity $E=6.895 \times 10^4$ MPa (1.0×10^4 ksi)

Density of the material $\rho=0.0272$ N/cm^3 (0.1 lb/in.^{3})

Table 7 Results and comparison with other references for twenty five-bar space truss

Design variables (in.^{2})	Wu and Chow (1995)	Tang *et al*. (1995)	Kaveh and Kalatjari (2004)	Rahami *et al*. (2008)	Present work
A_1(in.^{2})	0.1	0.1	0.1	0.1	0.1
A_2(in.^{2})	0.2	0.1	0.1	0.1	0.1
A_3(in.^{2})	1.1	1.1	1.1	1.1	0.9
A_4(in.^{2})	0.2	0.1	0.1	0.1	0.1
A_5(in.^{2})	0.3	0.1	0.1	0.1	0.1
A_6(in.^{2})	0.1	0.2	0.1	0.1	0.1
A_7(in.^{2})	0.2	0.2	0.1	0.2	0.2
A_8(in.^{2})	0.9	0.7	1.0	0.8	0.9
X_4(in)	41.07	35.47	36.23	33.0487	32.9609
Y_4(in)	53.47	60.37	58.56	53.5663	53.6141
Z_4(in)	124.6	129.07	115.59	129.9022	129.8648
X_8(in)	50.8	45.06	46.46	43.7826	43.6204
Y_8(in)	131.48	137.04	127.95	136.8381	137.2674
Weight (lb)	136.2	124.94	124.0	120.1149	119.3354
Weight (N)	605.8168	555.7323	551.5512	534.2703	530.8031

At the present work, maximum displacement=0.34978946 in

Table 8 Results and comparison with other references for ten-bar truss (With fixed geometry and maximum displacement=2.05 in)

Design variables (in.^{2})	Rajan (1995)	Tang *et al*. (1995)	Rahami *et al*. (2008)	Present work
A_1	30.0	30.0	30.0	30.0652
A_2	0.0	0.0	0.0	0.0
A_3	19.9	26.5	19.9	21.4747
A_4	15.5	14.2	15.5	14.2382
A_5	0.0	0.0	0.0	0.0
A_6	0.0	0.0	0.0	0.0
A_7	7.22	7.97	7.22	5.9382
A_8	22.0	19.9	19.9	20.4454
A_9	22.0	18.8	22.0	20.9991
A_{10}	0.0	0.0	0.0	0.0
Weight (lb)	4962.1	4921.25	4855.2	4780.3
Weight (N)	2207.1	2189.0	2159.6	2126.3
Max displacement (in)	2.07	2.05	2.0486	2.0500

Case study 6. The twenty five-bar space truss shown in Fig. 5 is optimized for topology and geometry variables in addition to the redundant forces and cross section variables. Input data for this case is considered the same as Table 6. The achieved results and comparison with those of other references are provided in Table 11. Elapsed time for this case is 5.326514 seconds when a computer with core (TM) i5-2400 CPU @ 3.10GHz and 4.00 GB RAM is being used. Fig. 9 shows the optimum topology and geometry of this case.

Table 12 shows the connectivity of members that are eliminated in the process of topology optimization of the twenty five-bar space truss.

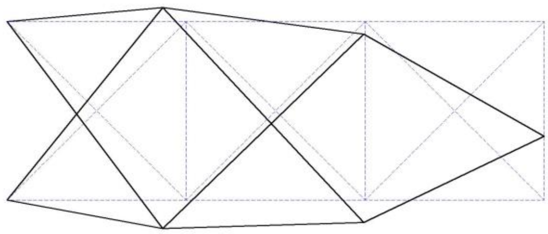


Fig. 8 Optimum topology and geometry of the fifteen-bar truss

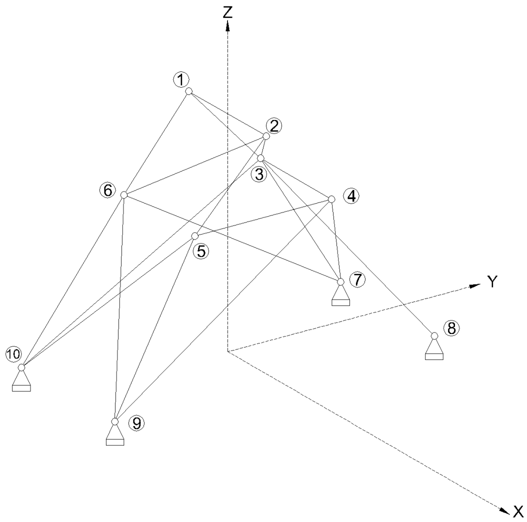


Fig. 9 Optimum topology and geometry of the twenty five-bar space truss

Table 9 Results and comparison with other references for ten-bar truss (Geometry and topology optimization with maximum displacement=2.05 in)

Design variables (in^2)	Rajan (1995)	Tang *et al*. (1995)	Rahami *et al*. (2008)	Present work
A_1(in.^{2})	9.9	13.5	11.5	11.9651
A_2(in.^{2})	9.4	0.0	0.0	0
A_3(in.^{2})	11.5	7.97	11.5	10.0722
A_4(in.^{2})	1.5	7.22	5.74	6.9273
A_5(in.^{2})	0.0	1.62	0.0	0
A_6(in.^{2})	12.0	0.0	0.0	0
A_7(in.^{2})	11.5	4.49	5.74	5.7919
A_8(in.^{2})	3.6	3.13	3.84	3.7217
A_9(in.^{2})	0.0	13.5	13.5	12.9009
A_{10}(in.^{2})	10.4	0.0	0.0	0
Y_1(in)	186.5	-	-	-
Y_2(in)	554.5	527.9	506.4203	506.4203
Y_3(in)	786.9	888.8	789.7306	789.7306
Weight (lb)	3254.0	2813.8	2723.05	2695.6
Weight (N)	1447.4	1251.6	1211.2	1199.0
Max stress ratio	0.6240	0.7400	0.7659	0.7590
Max displacement (in)	1.99	1.9998	1.999996	1.9999935

Table 10 Results and comparison with other references for fifteen-bar truss topology and geometry optimization

Design variables(in.^{2})	Rajan (1995)	Tang *et al*. (1995)	Rahami *et al*. (2008)	Present work
A_1(in.^{2})	1.174	1.081	0.954	0.9540
A_2(in.^{2})	0.954	0.539	0.954	0.5390
A_3(in.^{2})	0.440	0.000	0.000	0.00
A_4(in.^{2})	1.333	1.081	1.081	0.9540
A_5(in.^{2})	0.954	0.954	0.539	0.5390
A_6(in.^{2})	0.174	0.440	0.539	0.4400
A_7(in.^{2})	0.440	0.000	0.000	0.00
A_8(in.^{2})	0.440	0.141	0.000	0.00
A_9(in.^{2})	1.081	0.000	0.000	0.00
A_{10}(in.^{2})	1.333	0.270	0.440	0.4400
A_{11}(in.^{2})	0.174	0.270	0.220	0.4400
A_{12}(in.^{2})	0.174	0.539	0.111	0.2700
A_{13}(in.^{2})	0.347	0.141	0.347	0.2200
A_{14}(in.^{2})	0.347	0.440	0.539	0.4400
A_{15}(in.^{2})	0.440	0.000	0.000	0.00
X_2(in)	123.189	111.85	107.3896	104.3184
X_3(in)	231.595	242.45	244.4534	238.9108
Y_2(in)	107.189	104.02	125.4198	129.4551
Y_3(in)	119.175	109.22	117.2854	111.4651
Y_4(in)	60.462	-	-	-
Y_6(in)	-16.728	-10.82	-1.6249	-19.1032
Y_7(in)	15.565	-11.12	18.0828	-15.1112
Y_8(in)	36.645	48.84	50.2040	42.8964
Weight (lb)	120.528	77.84	75.0966	71.1417
Weight (N)	536.1078	346.2318	334.0292	316.5016

Table 11 Results and comparison with other references for twenty five-bar truss Topology and geometry optimization

Design variables(in.^{2})	Wu and Chow (1995)	Tang *et al*. (1995)	Rahami *et al*. (2008)	Present work
A_1(in.^{2})	0.1	0.0	0.0	0.10
A_2(in.^{2})	0.2	0.1	0.1	0.20
A_3(in.^{2})	1.1	0.9	0.9	0.90
A_4(in.^{2})	0.2	0.0	0.0	0.30
A_5(in.^{2})	0.3	0.0	0.0	0.10
A_6(in.^{2})	0.1	0.1	0.1	0.20
A_7(in.^{2})	0.2	0.1	0.1	0.30
A_8(in.^{2})	0.9	1.0	1.0	0.80
X_4(in)	41.07	39.91	38.7913	34.4947
Y_4(in)	53.47	61.99	66.1110	59.4592
Z_4(in)	124.6	118.23	112.9787	129.9686
X_8(in)	50.8	53.13	48.7924	45.3730
Y_8(in)	131.48	138.49	138.8910	139.0046
Wight (lb)	136.2	114.74	114.3701	106.0518
Weight (N)	605.82	510.36	508.72	471.7178
Maximum stress ratio	0.3897	0.4338	0.4438	0.4016
Maximum displacement (in.)	0.347	0.3500	0.34999896	0.35000

Table 12 The eliminated members and end nodes for the case study 6

Member number	End nodes
2	(1,4)
4	(1,5)
7	(2,4)
10	(3,6)
13	(5,6)
17	(5,8)
23	(4,8)

7. Conclusions

The presented algorithm could achieve better results in all of the cases studied in comparison to those of other references. This method increases the exploitation and exploration ability of the algorithm in comparison to the classical methods. Geometry and topology optimization needs more iterations and agents if inversion processes of the matrices are not eliminated. Force method is used as an approach for driving energy functions of structures. This method helps to reduce the number of variables in comparison to the displacement method. Besides the SCSS algorithm shows a good performance in reducing the weight of the structure and minimizing its energy function. In order to keep the length of the variable vector constant in the topology optimization, a vector of 0 and 1 with length of member numbers is considered. Also a vector with the length equal to the memebr numbers is considered as the redundant force variables for keeping the redundant variables constant by changing the DSI of the structure during the process of topology optimization. Finally combination of the force method as analyser and the SCSS algorithm as an optimizer leads to an algorithm that improves all of the results achieved by other considered methods.

Acknowledgements

The first author is grateful to the Iran National Science Foundation for the support.

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