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Progressive failure of symmetrically laminated plates under uni-axial compression

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Abstract. The objective of this work is to predict the failure loads, associated maximum transverse displacements, locations and the modes of failure, including the onset of delamination, of thin, flat, square symmetric laminates under the action of uni-axial compression. Two progressive failure analyses, one using Hashin criterion and the other using Tensor polynomial criteria, are used in conjunction with the finite element method. First order shear deformation theory and geometric nonlinearity in the von Karman sense have been employed. Five different types of lay-up sequence are considered for laminates with all edges simply supported. In addition, two boundary conditions, one with all edges fixed and other with mixed boundary conditions for $(+45/-45/0/90)_{2s}$ quasi-isotropic laminate have also been considered to study the effect of boundary restraints on the failure loads and the corresponding modes of failure. A comparison of linear and nonlinear results is also made for $(\pm 45/0/90)_{2s}$ quasi-isotropic laminate. It is observed that the maximum difference between the failure loads predicted by various criteria depend strongly on the laminate lay-ups and the flexural boundary restraints. Laminates with clamped edges are found to be more susceptible to failure due to the transverse shear and delamination, while those with the simply supported edges undergo total collapse at a load slightly higher than the fiber failure load.

Key words: progressive failure; laminated plate; failure criteria; uni-axial compression.

1. Introduction

Laminated composite materials are being widely used in the construction of mechanical, aerospace, marine and automotive structures. These require high reliability as well as safety. It is well known that the total failure of a laminated composite panel does not always occur at the load corresponding to the first-ply failure. The panel failure in a broad sense could be considered to have occurred when a structural element ceased to function satisfactorily; thus the definition of failure varies from one case to another. The failure charactistics of heterogeneous and anisotropic composite laminates is completely different from that of the isotropic ones. The appearance of detectable cracks in metals is generally considered to be unsafe since a slight amount of damage could rapidly progress into a catastrophic fracture. However, this is not true in the case of composite materials i.e., although internal damage might appear very early, its propagation is arrested by the internal configuration of the structures. Therefore, compo-

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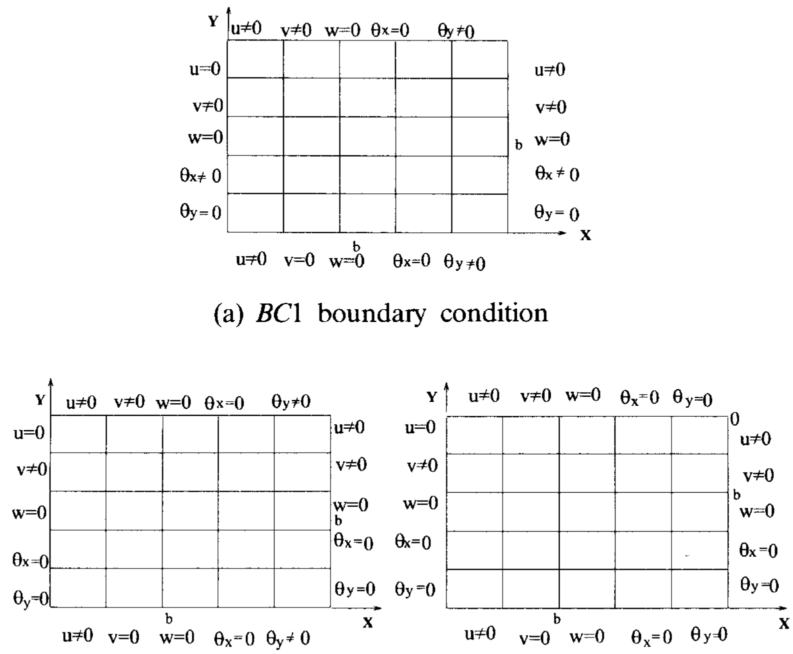
site laminates can still sustain a much higher load after the occurrence of localized damage such as matrix cracking, fiber breaks or delamination (Agrawal and Broutman, 1980 and Jones 1975). Hence the knowledge of the first-ply failure load and the ultimate load of such structures is essential so that these panels can be designed efficiently and economically by fully utilizing its postbuckling strength with appropriate reliability and safety. Thus, to accurately predict the failure loads of such structures, the progressive failure analysis has become an important subject of research. Of the early investigations related to the failure of laminated plates are the work by Turvey (1980a, b, c, 1981, 1982, 1987) in which analytical solutions for the first-ply failure load are presented for symmetric and antisymmetric laminates with simply supported boundary conditions under transverse loads. The finite element procedure for the prediction of linear first-ply failure loads of composite laminates subjected to transverse and in-plane (tensile) loading was presented by Reddy and Pandey (1987). Another study by Reddy and Reddy (1992) used the first order shear deformation theory in the finite element modeling to present the linear and nonlinear failure analysis. Engelstad, *et al.* (1992) investigated the postbuckling response and failure characteristics of graphite-epoxy panels with and without circular hole in axial compression using a progressive damage failure mechanism in conjunction with a 3-D degenerated shell element. Lee and Hyer (1993) studied postbuckling failure characteristics of square, symmetrically laminated plate with a circular hole under uni-axial compression using the maximum stress failure criterion. Very recently, Kam and Sher (1995) studied the nonlinear behaviour and the first ply failure strength of centrally loaded laminated composite plates with semi-clamped edges using a method developed from the von Karman-Mindlin plate theory in conjunction with the Ritz method.

There are many investigations in literature which deal with the failure of laminated composite plates under in-plane (tensile) and transverse loadings. However, not much effort has been made towards the understanding of the nonlinear failure behaviour of such structures especially under in-plane loads. The present study deals with the investigation of the first-ply failure and the subsequent progressive failure (till the ultimate failure) of thin, square and symmetrically laminated composite plates with various lay-ups (Table 1) and boundary conditions (Fig. 1) under the actions of uni-axial compressive load. Two progressive failure procedures are used, one with the Hashin criterion and the other with the Tensor polynomial forms of the maximum stress, maximum strain, Tsai-Hill, Hoffman and Tsai-Wu criteria, with the primary objective to evaluate all these failure criteria . Different material property degradation models for the failed lamina have been considered; the model for the tensor polynomial criteria is based on Engelstad, *et al.* (1992), whereas that for the Hashin criterion is based on Tsai (1986). In addition, two levels of the application of the material property degradation model for the failed lamina are considered in this work. One level of the application is based on the stiffness reduction for the lamina as a whole while the other is based on the stiffness reduction over the element only.

Table 1 Lamination scheme of symmetric laminates

Lamination scheme	$(\pm 45/0/90)_{2s}$	$(\pm 45/0_2)_{2s}$	$(\pm 45)_{4s}$	$(\pm 45/0_6)_s$	$(0/90)_{4s}$
Type	A	B	C	D	E



(a) *BC*1 boundary condition

(b) *BC*2 boundary condition (c) *BC*3 boundary condition

Fig. 1 Details of the various boundary conditions for full plate.

2. Failure criteria

A review of the Tensor polynomial failure criterion, with its various degenerate cases, and other independent failure criteria (excluding the Hashin criterion) has been presented by Reddy and Pandey (1987) and Reddy and Reddy (1992). However, for the sake of completeness, various failure criteria used in the present study are given below and the details of these criteria are presented in appendix.

1. Hashin criterion
2. Tensor polynomial failure criterion: Degenerate cases of this criterion are as follows:
(a) Maximum stress criterion
(b) Maximum strain criterion
(c) Tsai-Hill criterion
(d) Hoffman criterion
(e) Tsai-Wu criterion

3. Methodology

The study is based on the finite element formulation using the first order shear deformation theory with nine noded Lagrangian element having five degrees of freedom per node. Geometric nonlinearity based on von Karman's assumptions has been incorporated. The nonlinear algebraic equations are solved by Newton-Raphson technique.The calculation of stresses is done on the nodal points as well as on the gauss points. Due to connectivity of a particular node to various elements, nodal point stresses are calculated taking the average value of stresses at that node from various elements associated with that node. All the six stress components are calculated at each node point and at the gauss point. However, to predict the failure of a lamina only five stress components (three in-plane stress and two transverse shear stress) are used in the selected failure criterion. To predict the onset of delamination two transverse shear stress components and one transverse normal stress component are used in the maximum stress failure criterion. Delamination at any interface is said to have occurred when any of the transverse stress components in any of the two layers adjacent to interface becomes equal to or greater than its corresponding strength. The ply failure is said to have occurred when state of stress at any point within the lamina satisfies the selected failure criterion. The first-ply failure refers to the first instant at which one or more than one plies fail at the same load. After the first-ply failure, the progressive failure analysis is carried out using progressive failure procedure appropriate to the selected failure criterion. The two progressive failure procedures employed are described below:

3.1. Tensor polynomial progressive failure procedure

At each load step, nodal point stresses are used in the selected failure criterion. If failure occurs at a point in a layer a reduction in the lamina stiffness is applied which causes the changes in the overall laminate stiffness. Following terms are used to determine the failure modes.

$$H_1 = F_1\sigma_1 + F_{11}\sigma_1^2; \quad H_2 = F_2\sigma_2 + F_{22}\sigma_2^2$$

$$H_4 = F_{44}\sigma_4^2; \quad H_5 = F_{55}\sigma_5^2; \quad H_6 = F_{66}\delta_6^2$$

Notations in above expressions are defined in the appendix.

The largest H_i term is selected to be the dominant failure mode and the corresponding modulus is reduced to zero. H_1 corresponds to the modulus E_1; H_2 to E_2; H_4 to G_{23}; H_5 to G_{13} and H_6 to G_{12}, An outline of the steps required is as follows:

(1) After nonlinear iterative convergence is achieved, calculate the stresses at the middle of the each layer and at its interfaces with the adjacent layers at each of the nodal and the gauss point.
(2) Transform the stresses to principal stresses.
(3) Compute failure indices, H_1, H_2, $\cdots$
(4) If failure occurs reduce the appropriate lamina moduli and recompute laminate stiffness and restart nonlinear analysis at the same load step.
(5) If no failure occurs, proceed to the next load step.
(6) Final failure is said to have occurred when delamination occurs or when the plate is no longer able to carry any further load because of very large deflection.

3.2. Hashin progressive failure procedure

As per the Hashin criterion, failure of the lamina occurs if any of the four failure criteria

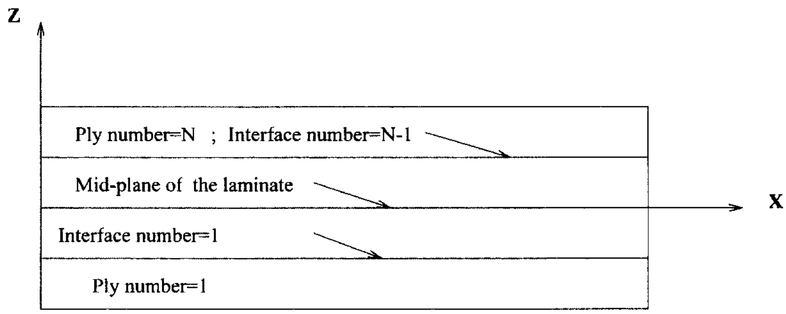


Fig. 2 Ply and interface numbering within the laminate.

Table 2 Material properties of T300/5208 (pre-peg) graphite-epoxy

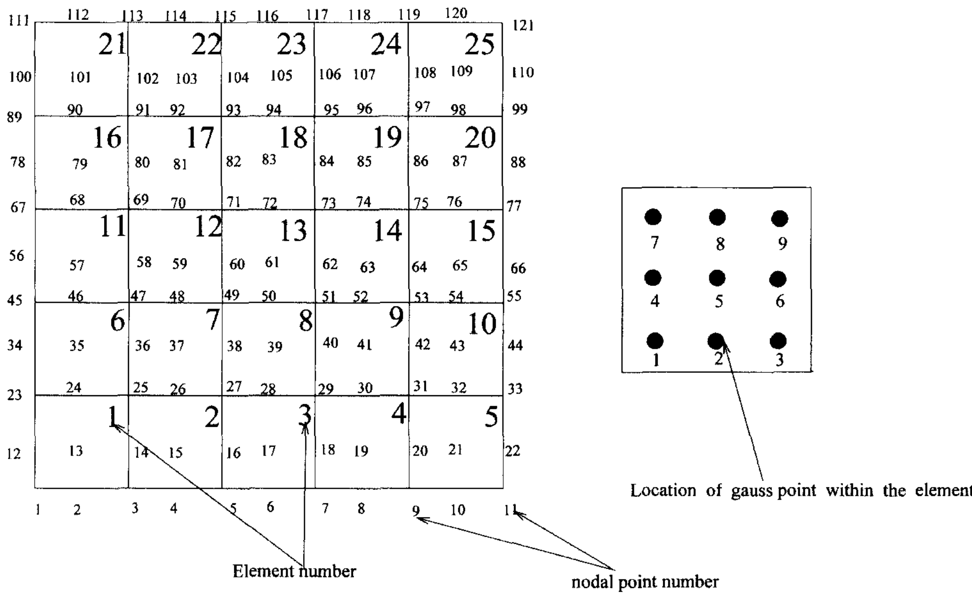
Mechanical properties	Values	Strength properties	Values
E_1	132.58 Gpa	X_t	1.515 Gpa
E_2	10.8 Gpa	X_c	1.697 Gpa
E_3	10.8 Gpa	$Y_t=Z_t$	43.8 Mpa
$G_{12}=G_{13}$	5.7 Gpa	$Y_c=Z_c$	43.8 Mpa
$\nu_{12}=\nu_{13}$	0.24	R	67.6 Mpa
ν_{23}	0.49	$S=T$	86.9 Mpa

is satisfied at any point in a lamina of the laminate and the corresponding mode of failure is also determined with the possibility of occurrence of two modes (fiber and matrix mode simultaneosly). An outline of the steps required in this procedure is as follows:

- Steps (1) and (2) are the same as with Tensor polynomial criterion.
- If matrix failure occurs, reduce the lamina modulli as per recommendations in (Tsai 1986), which are
  1. Reduce E_2 to 45% of its original value.
  2. Reduce shear modulus to 35% of its original value.
  3. Reduce major Poisson's ratio to 30% of its original value.
- If fiber failure occurs then reduce E_1 to zero.
- Recompute the laminate stiffness and restart nonlinear analysis at the same load step.
- Steps (5) and (6) are the same as with the Tensor polynomial criteria.

A total of five symmetric lamination schemes are employed to understand the progressive failure. The individual laminates are designated from *A* to *E* for identification. The details of the lamination schemes are shown in Table 1. The ply and the interface numbering scheme within the laminate is shown in Fig. 2. Properties of the material of the laminate (Reddy and Reddy 1992) are presented in Table 2.

In the above table E_1, E_2, E_3 are the principal Young's moduli while G_{12}, G_{13}, G_{23} are the shear moduli corresponding to the planes 1-2; 1-3; and 2-3 respectively and ν_{12}, ν_{13}, ν_{23} are the corresponding Poisson's ratios. In this study a full square plate of width b is used with 5×5 (25 element mesh). Details of the finite element mesh and the location of gauss points within the element are shown in Fig. 3. Three types of flexural boundary conditions, namely *BC*1,



a) Finite element mesh for full plate (5×5 of nine noded quadratic elements).

Fig. 3 Finite element mesh for full plate (5×5 of nine noded quadratic elements).

*BC*2, *BC*3, have been considered; *BC*1- refers to a plate with all edges simply supported, *BC*2- refers to a plate with two logitudinal edges ($Y=0$ and $Y=b$) simply supported and the other two edges clamped and *BC*3- refers to a plate with all edges clamped. In all three cases, the in-plane boundary conditions are identical and the compression load is applied on the edge $X=b$. The details are shown in Fig. 1.

The accuracy of the programme has been checked by comparing the first-ply failure loads with those obtained by Engelstad, *et al.* (1992). A very small difference in the two results is attributed to the initial imperfection considered by Engelstad, *et al.* (1992). While presenting results in tabular forms, failure loads and corresponding displacements are presented in the following nondimensionalized forms:

Uni-axial compression$=N_x b^2/E_2 h^3$

Maximum transverse displacement$=w_{max}/h$

where h is the total thickness of the laminate and N_x is the applied uni-axial compression per unit length. The quantities in parentheses indicate the percentage difference in failure loads as compared with those by the Tsai-Wu criterion.

4. Results and discussion

4.1. Progressive failure of laminates with BC1 boundary condition

Progressive failure results of various laminates with *BC*1 boundary condition are presented in Table 3. First ply failure loads predicted by various failure criteria are found to differ from one another by a maximum of 7.5% for laminate A, 8% for laminate B, 8.4% for laminate C,

Table 3a Progressive failure results for $(\pm 45/0/90)_{2s}$ laminate with *BC*1 boundary condition

Failure criteria	First-ply failure load	Ultimate failure load	$\left(\frac{w_{max}}{h}\right)^{\oplus}$	$FL^{\dagger}$	$FP^{\ddagger}$	Mode of first-ply failure
Maximum stress	58.09 (1.5)*	77.45 (−4.3)	3.7	1	109	Transverse
Maximum strain	54.22 (−5.3)	67.98 (−16.0)	3.4	1	109	Transverse
Tsai-Hill	58.09 (1.5)	79.60 (−1.6)	3.7	1	109	Transverse
Tsai-Wu	57.24 (0.0)	80.89 (0.0)	3.65	1	109	Transverse
Hoffman	58.09 (1.5)	79.60 (−1.6)	3.70	1	109	Transverse
Hashin	53.79 (−6.0)	79.17 (−2.2)	3.41	1	109	Compressive matrix

$^{\oplus}$Non-dimensionalized maximum transverse displacement in the plate at the first-ply failure.
†First failed layer number
‡First failed point number
*Percentage difference based on Tsai-Wu criterion

Table 3b Progressive failure results for $(\pm 45/02)_{2s}$ laminate with *BC*1 boundary condition

Failure criteria	First-ply failure load	Ultimate failure load	$\left(\frac{w_{max}}{h}\right)^{\oplus}$	$FL^{\dagger}$	$FP^{\ddagger}$	Mode of first-ply failure
Maximum stress	55.94 (4.0)*	72.29 (1.8)	4.01	3	77	Transverse
Maximum strain	51.63 (−4.0)	62.83 (−11.5)	3.61	16	121	Transverse
Tsai-Hill	55.94 (4.0)	68.84 (−3.0)	4.01	3	77	Transverse
Tsai-Wu	53.79 (0.0)	71.0 (0.0)	3.83	16	121	Transverse
Hoffman	55.94 (4.0)	69.28 (−2.4)	4.01	3	77	Transverse
Hashin	53.36 (−0.80)	71.86 (1.2)	3.8	16	97	Tensile matrix

$^{\oplus}$Non-dimensionalized maximum transverse displacement in the plate at the first-ply failure.
†First failed layer number
‡First failed point number
*Percentage difference based on Tsai-Wu criterion

Table 3c Progressive failure results for $(\pm 45)_{2s}$ laminate with *BC*1 boundary condition

Failure criteria	First-ply failure load	Ultimate failure load	$\left(\frac{w_{max}}{h}\right)^{\oplus}$	$FL^{\dagger}$	$FP^{\ddagger}$	Mode of first-ply failure
Maximum stress	49.06 (5.6)*	65.83 (7.0)	4.33	16	121	Transverse
Maximum strain	45.18 (−2.8)	55.08 (−10.5)	3.92	16	121	Transverse
Tsai-Hill	48.6 (4.6)	61.53 (0.0)	4.32	16	121	Transverse
Tsai-Wu	46.47 (0.0)	61.53 (0.0)	4.04	16	121	Transverse
Hoffman	48.63 (4.6)	61.53 (0.0)	4.3	16	121	Transverse
Hashin	46.04 (−0.90)	47.76 (−22.4)	4.0	16	121	Tensile matrix

$^{\oplus}$Non-dimensionalized maximum transverse displacement in the plate at the first-ply failure
†First failed layer number
‡First failed point number
*Percentage difference based on Tsai-Wu criterion

Table 3d Progressive failure results for $(\pm 45/0_6)_{2s}$ laminate with *BC*1 boundary condition

Failure criteria	First-ply failure load	Ultimate failure load	$\left(\frac{w_{max}}{h}\right)^{\oplus}$	$FL^{\dagger}$	$FP^{\ddagger}$	Mode of first-ply failure
Maximum stress	50.78 (1.7)*	58.09 (0.0)	3.89	3	77	Transverse
Maximum strain	48.19 (−3.5)	52.49 (−9.6)	3.70	1	11	Transverse
Tsai-Hill	50.78 (1.7)	58.52 (0.74)	3.89	3	77	Transverse
Tsai-Wu	49.92 (0.0)	58.09 (0.0)	3.87	1	11	Transverse
Hoffman	50.78 (1.7)	58.09 (0.0)	3.89	3	77	Transverse
Hashin	48.19 (−3.5)	64.11 (−10.4)	3.69	1	11	Tensile matrix

$^{\oplus}$Non-dimensionalized maximum transverse displacement in the plate at the first-ply failure
†First failed layer number
‡First failed point number
*Percentage difference based on Tsai-Wu criterion

Table 3e Progressive failure results for $(0/90)_{4s}$ laminate with *BC*1 boundary condition

Failure criteria	First-ply failure load	Ultimate failure load	$\left(\frac{w_{max}}{h}\right)^{\oplus}$	$FL^{\dagger}$	$FP^{\ddagger}$	Mode of first-ply failure
Mximum stress	60.24 (−7.3)*	71.86 (−4.6)	4.31	16	97	Transverse
Maximum strain	58.52 (−9.9)	71.86 (−4.6)	4.21	16	97	Transverse
Tsai-Hill	60.67 (−6.6)	72.72 (−3.4)	4.38	16	97	Transverse
Tsai-Wu	64.98 (0.0)	75.30 (0.0)	4.58	16	97	Transverse
Hoffman	60.24 (−7.3)	71.86 (−4.6)	4.31	16	97	Transverse
Hashin	53.36 (−17.9)	67.55 (−10.29)	3.89	16	97	Tensile matrix

$^{\oplus}$Non-dimensionalized maximum transverse displacement in the plate at the first-ply failure
†First failed layer number
‡First failed point number
*Prcentage difference based on Tsai-Wu criterion

5.2% for laminate D and 17.9% for laminate E respectively, while ultimate loads are found to differ from one another by a maximum of about 16% for laminate A, 13.3% for laminate B, 29.4% for laminate C, 20% for laminate D and 10.3% for laminate E respectively. Nodal locations corresponding to the first-ply failure, predicted by various criteria are found to be different for different laminates. Tensor polynomial criteria in general predict the same mode of failure (transverse) while Hashin criterion predicts matrix failure. Hence the inference is that progressive failure initiates primarily due to in-plane normal stresses transverse to the fiber direction, followed by its combination with transverse shear stresses leading to the fiber failure at a load closer to the ultimate load. It is also worth mentioning that delamination does not occur before the ultimate load is reached in all cases. Hence, for the *BC*1 boundary condition, ultimate load is the one at which plate has virtually no reserve strength (which is indicated by very large deflection at a constant load). The absolute maximum value of the deflections, (w_{max}/h), predicted by various failure criteria, just before the ultimate load, occurs for the laminate B and is equal to 8.2. However, the average value of the maximum transverse displacements predicted by the various failure criteria for this laminate is 6.1. The average value of the first-ply failure loads predicted by various failure criteria are found to be about 3 times the buckling load for laminate A, 2.9 times for laminate B, 2.2 times for laminate C, 2.8 times for laminate D and 4.6 times for laminate E, while the corresponding values for ultimate loads ultimate loads are found to be 4.1 times the buckling load for laminate A, 3.7 times the buckling load for laminate A, 3.7 times for laminate B, 2.7 times for laminate C, 3.3 times for laminate D and 5.6 times for laminate E.

Progressive failure results for laminate E using gauss point stresses and with elemental lamina stiffness reduction for the failed laminae are presented in Table 4. It is seen that the first-ply

Table 4 Progressive failure results for $(0/90)_{4s}$ laminate with *BC*1 boundary condition and elemental lamina stiffness reduction

Failure criteria	First-ply failure load	Ultimate failure load	$\left(\frac{w_{max}}{h}\right)^{\oplus}$	$FL^{\dagger}$	$FE^{\ddagger}$	$FG^{\S}$	Mode of first-ply failure
Maximum stress	61.53 (−7.2)*	79.17 (−4.3)	4.39	16	25	1	Transverse
Maximum strain	60.24 (−9.0)	78.75 (−4.7)	4.31	16	25	1	Transverse
Tsai-Hill (−5.8)	62.40 (−4.7)	78.74 (−4.7)	4.43	16	25	1	Transverse
Tsai-Wu (0.0)	66.27 (0.0)	82.62 (0.0)	4.65	16	25	1	Transverse
Hoffman (−6.5)	61.96 (−3.7)	79.6 (−3.7)	4.41	16	25	1	Transverse
Hashin (−17.5)	54.65 (−0.52)	82.19 (−0.52)	3.97	16	25	1	Compressive matrix

$^{\oplus}$Non-dimensionalized maximum transverse displacement in the plate at the first-ply failure

†First failed layer number

‡First failed element number

§First failed gauss point

*Percentage difference based on Tsai-Wu criterion

failure loads predicted by various failure criteria in this case differ from one another by a maximum of 18%, while ultimate loads differ from by a maximum of 5%. Hence, it is worth noting that the maximum difference in ultimate failure loads predicted by various failure criteria, using elemental stiffness reduction procedure for the failed laminae, is reduced to half of that predicted with stiffness reduction for the failed laminae as a whole. Further, the maximum difference in failure loads predicted by Tensor polynomial criteria in general is almost the same with both the stiffness reduction levels. Further, ultimate failure loads predicted by Hashin criterion using elemental stiffness reduction procedure is very close to that predicted by Tsai-Wu criterion. The average value of the first-ply failure loads predicted with gauss point stresses is found to be 4.75 times the buckling load while that of the ultimate loads is found to be 6.2 times the buckling load. Thus, there is a slight increase in the first ply failure loads and the larger for the ultimate loads predicted with gauss point stresses and elemental lamina stiffness reduction procedure. Also the maximum value of the w_{max}/h, predicted by various failure criterion just before the ultimate load is found to be 6.62 while its average value is 5.6.

4.2. Progressive failure results for $(\pm 45/0/90)_{2s}$ laminate with different boundary conditions

Progressive failure results for a thin, square, flat, sixteen layers symmetric quasi-isotropic laminate with three different boundary conditions (*BC*1, *BC*2, *BC*3) are presented in Table 5. It is observed that the first-ply failure loads predicted by various failure criteria differ from one another by a maximum of 6% for *BC*1 boundary condition, 5.5% for *BC*2 boundary condition and about 13% for *BC*3 boundary condition, while that for ultimate loads the corresponding values are

Table 5a Progressive failure results for $(\pm 45/0/90)_{2s}$ laminate with $BC1$ boundary condition

Failure criteria	First-ply failure load	Ultimate failure load	$\left(\frac{w_{max}}{h}\right)^{\oplus}$	$FL^{\dagger}$	$FP^{\ddagger}$	Mode of first-ply failure
Maximum stress	58.09 (1.5)*	77.45 (−4.3)	3.7	1	109	Transverse
Maximum	54.22 (−5.3)	67.98 (−16.0)	3.4	1	109	Transverse
Tsai-Hill	58.09 (1.5)	79.60 (−1.6)	3.7	1	109	Transverse
Tsai-Wu	57.24 (0.0)	80.89 (0.0)	3.65	1	109	Transverse
Hoffman	58.09 (1.5)	79.60 (−1.6)	3.70	1	109	Transverse
Hashin	53.79 (−6.0)	79.17 (−2.2)	3.41	1	109	Compressive matrix

⊕Non-dimensionalized maximum transverse displacement in the plate at the first-ply failure
†First failed layer number
‡First failed point number
*Percentage difference based on Tsai-Wu criterion

Table 5b Progressive failure results for $(\pm 45/0/90)_{2s}$ laminates with $BC2$ boundary condition

Failure criteria	First-ply failure load	Ultimate failure load	$\left(\frac{w_{max}}{h}\right)^{\oplus}$	$FL^{\dagger}$	$FP^{\ddagger}$	Mode of first-ply failure
Maximum stress	91.65 (2.9)*	94.67 (0.46)	3.72	1	108	Transverse
Maximum strain	85.20 (−4.3)	91.23 (−3.2)	3.31	1	108	Transverse
Tsai-Hill	90.80 (1.9)	95.53 (−1.4)	3.50	1	108	Transverse
Tsai-Wu	89.07 (0.0)	94.24 (0.0)	3.44	1	108	Transverse
Hoffman	90.37 (1.5)	95.53 (−1.4)	3.48	1	108	Transverse
Hashin	84.34 (−5.3)	90.79 (−3.7)	3.28	1	108	Compressive matrix

⊕Non-dimensionalized maximum transverse displacement in the plate at the first-ply failure
†First failed layer number
‡First failed point number
*Percentage difference based on Tsai-Wu criterion

Table 5c Progressive failure results for $(\pm 45/0/90)_{2s}$ laminates with *BC*3 boundary condition

Failure criteria	First-ply failure load	Ultimate failure load	$\left(\frac{w_{max}}{h}\right)^{\oplus}$	$FL^{\dagger}$	$FP^{\ddagger}$	Mode of first-ply failure
Maximum stress	84.34 (1.6)*	95.09 (6.2)	1.63	13	8	Transverse
Maximum strain	81.33 (−3.1)	91.23 (1.9)	1.56	16	66	Transverse
Tsai-Hill	81.76 (−2.6)	89.50 (0.0)	1.56	16	66	Transverse
Tsai-Wu	83.91 (0.0)	89.51 (0.0)	1.66	16	77	Transverse
Hoffman	82.19 (−2.1)	89.51 (0.0)	1.58	16	66	Transverse
Hashin	73.15 (−12.8)	109.73 (22.6)	1.82	16	117	Compressive matrix

⊕Non-dimensionalized maximum transverse displacement in the plate at the first-ply failure.
†First failed layer number
‡First failed point number
*Percentage difference based on Tsai-Wu criterion

16%, 4% and 23% respectively. It is also observed that the modes of the first-ply failure predicted by various failure criteria are identical irrespective of boundary conditions (except that the maximum stress criterion predicts transverse shear mode of first-ply failure for *BC* boundary condition). However, the locations of the first-ply failure are not the same. It is also noted that the higher the flexural restraint, higher is the buckling load, and the strength for a fixed value of maximum transverse displacement.

It is also seen that the ultimate load is limited by the onset of delaminations for boundary conditions *BC*2 and *BC*3. However, for the boundary condition *BC*1, delamination occurs when the plate has virtually no reserve strength to carry any further load. Hence the efficient utilization of material strength is observed with *BC*1 boundary condition. The progressive failure response specific to this laminate and for all the boundary conditions is shown in Figs. 4 and 5 using the Tsai-Wu and the Hashin criterion, respectively. It is seen that, for *BC*3 boundary condition, a change in buckled configuration takes place before the first-ply failure. It is to be noted that the kinks in the curves represent failure points during the progressive loading. Average value of first-ply failure loads predicted by various failure criterion is found to be about 3.0 times the buckling load for *BC*1 boundary condition, 2.9 times for *BC*2 boundary condition and 1.85 times the buckling load for *BC*3 boundary condition, while the ultimate loads are found to be 4.1 times the buckling load for *BC*1 boundary conditions, 3.1 times for *BC*2 boundary condition and 2.1 times for *BC*3 boundary conditions.Absolute maximum value of the maximum transverse displacements (w_{max}/h) predicted by various failure criteria just before the ultimate load is found to be for *BC*1 and is equal to 6.0

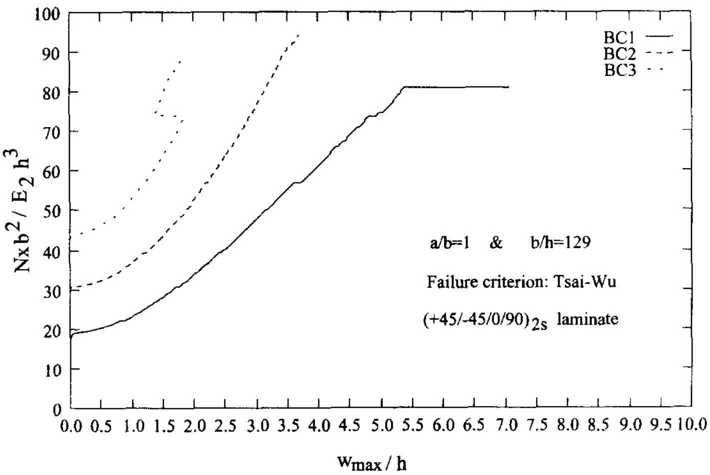


Fig. 4 Progressive failure response of $(\pm 45/0/90)_{2s}$ quasi-isotropic laminate with Tsai-Wu criterion.

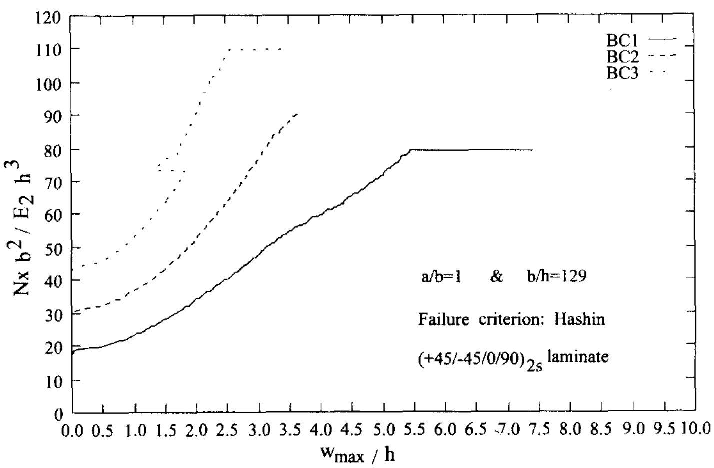


Fig. 5 Progressive failure response of $(\pm 45/0/90)_{2s}$ quasi-isotropic laminate with Hashin criterion.

4.3. Comparison of the linear and nonlinear progressive failure results of $(\pm 45/0/90)_{2s}$ laminate with BC1 boundary condition

The first-ply failure load, the ultimate failure load and the mode of first-ply failure of $(\pm 45/0/90)_{2s}$ quasi-isotropic laminate are also obtained for various failure criteria by using the linear analysis and the comparison of results with those obtained by nonlinear analysis is presented in Table 6. It may be noted that linear results are obtained by dropping the nonlinear terms in the nonlinear strain displacement relationships. It is seen that linear failure loads are too high as compared to nonlinear failure loads. Here, it is worth mentioning that results obtained by the linear analysis can not be construed as realistic because the buckling of this laminate can not be precluded before such high loads of failure. However, it is worth noting that modes of first-plyfailure are the same in both cases. It is observed that, in linear analysis, the maximum strain, the Tsai-Hill and the Tsai-Wu criteria predict the onset of delamination before the fiber failure.

Table 6 Linear and nonlinear progressive failure results for $(\pm 45/0/90)_{2s}$ laminates with *BC*1 boundary condition

Failure criteria	Linear			Nonlinear		
	First-ply failure load	Utimate load	Mode of first-ply failure	First-ply failure load	Ultimate load	Mode of first-ply failure
Maximum stress	350.70	884.71	Transverse	58.09	77.45	Transverse
Maximum strain	305.09	410.086	Transverse	54.22	67.98	Transverse
Tsai-Hill	351.99	427.30	Transverse	58.09	79.60	Transverse
Tsai-Wu	334.35	422.14	Transverse	57.24	80.89	Transverse
Hoffman	350.273	884.71	Transverse	58.09	79.60	Transverse
Hashin	402.34	930.76	Compressive matrix	53.79	79.17	Compressive matrix

However, the onset of delamination is not predicted in the case of nonlinear analysis, irrespective of the failure crieria used. It is also important to note that locations of the first-ply failure predicted by the linear and the nonlinear analysis (though not shown in the table) are different.

5. Concluding remarks

Based on the results presented in the previous section, following useful observation can be made.

- The maximum difference in the first-ply failure loads and the ultimate loads predicted by various failure criteria are strongly dependent on the type of laminate lay-ups and the flexural boundary restraints.
- Among all the tensor polynomial criteria, the maximum strain criterion is found to give large inconsistent results. Hashin criterion predicts even more inconsistent failure loads, especially for cross-ply laminates.
- Difference between the ultimate failure loads of cross-ply laminates predicted by the Hashin criterion and the various Tensor polynomial criteria is, in general, drastically reduced if the Hashin criterion is used with the stiffness reduction for the failed laminae over the failed elements only.
- Maximum difference between the failure loads of cross-ply laminates predicted by all tensor polynomial criteria under uni-axial compression with stiffness reduction for the failed laminae as a whole is found to be of the same order of magnitude as that obtained with the stiffness reduction of the failed laminae over the failed elements only.
- Average value of the ultimate failure loads for symmetric cross-ply laminates obtained by progressive failure analysis using gauss point stresses and the elemental lamina stiffness reduction model for the failed lamina is found to be larger in comparison to that obtained by progressive failure analysis using nodal point stresses and the lamina stiffness reduction model for the failed lamina as a whole.
- Failure mode of the first-ply failure is associated with the localised matrix cracking and

occurs primarily due to in-plane normal stresses transverse to the fiber directions irrespective of the laminate lay-ups and the boundary conditions.

- First-ply failure locations is found to be sharply dependendent on the boundary conditions. Most critical points of failure lie near the loaded edges of the plate.
- Laminates with two opposite edges or all the edges clamped are more susceptible to ultimate failure due to transverse shear and the delamination.
- Maximum value of the maximum transverse displacements (w_{max}/h) just before the ultimate load predicted by all the failure criteria is found to be 8.2 irrespective of boundary conditions and types of laminate. However, the average value of the maximum transverse displacements (w_{max}/h) is found to be less than 6.0. Hence the use of non-linear theory in the von Karman sense is validated for all the laminates under consideration till the failure load is reached.
- It is observed that the fiber breakage precedes very closely the ultimate loads for simply supported laminates and this mode of failure is not predicted in laminates with clamped edges.
- The first-ply failure loads and the ultimate failure loads of $(+45/-45/0/90)_{2s}$, quasi-isotropic laminates (with respect to the buckling load) are found to be largest for *BC*1 boundary condition.
- The maximum difference in the first-ply failure loads of $(+45/-45/0/90)_{2s}$, quasi-isotropic laminate, predicted by various failure criteria, is found to be minimum in the case of *BC*2 boundary conditions and maximum in the case of *BC*3 boundary condition. The same holds good for ultimate loads as well.
- Linear failure loads of simply supported $(\pm 45/0/90)_{2s}$ quasi-isotropic laminate are too high as compared to nonlinear failure loads. However, modes of the first-ply failure predicted by the linear and the nonlinear analysis are same.

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Appendix

1. Hashin criterion

In this criterion (Hashin 1980) four distinct failure modes-tensile matrix, tensile fiber, compressive matrix and compressive fiber- are modelled separately, resulting in a piece-wise smooth failure surface. Another unique feature of this failure criterion is that it avoids prediction of multi-axial tensile (compressive) modes in terms of compressive (tensile) failure stresses. The four criteria corresponding to the different failure modes are:

(1) Tensile fiber mode $\sigma_1 > 0.0$

$$\left(\frac{\sigma_1}{X_t}\right)^2 + \frac{1}{T^2}(\sigma_6^2 + \sigma_5^2) = 1 \tag{1}$$

(2) Tensile matrix mode $\sigma_2 + \sigma_3 > 0.0$

$$\frac{1}{Y_t^2}(\sigma_2 + \sigma_3)^2 + \frac{1}{R^2}(\sigma_4^2 - \sigma_2\sigma_3) + \frac{1}{T^2}(\sigma_6^2 + \sigma_5^2) = 1 \tag{2}$$

(3) Compressive fiber mode $\sigma_1 > 0.0$

$$\sigma_1 = X_c \tag{3}$$

(4) Compressive matrix mode $\sigma_2 + \sigma_3 < 0.00$

$$\frac{1}{Y_c}\left[\left(\frac{Y_c}{2R}\right)^2 - 1\right](\sigma_2 + \sigma_3) + \frac{1}{4R^2}(\sigma_2 + \sigma_3)^2 + \frac{1}{R^2}(\sigma_4^2 - \sigma_2\sigma_3) + \frac{1}{T^2}(\sigma_6^2 + \sigma_5^2) = 1 \tag{4}$$

In above expressions σ_1, σ_2, σ_3 are the normal stress components; σ_4, σ_5, σ_6 are the shear stress components in the principal material directions (the subscript 1 refering to the fiber direction); X_t, Y_t are the tensile strengths of the lamina in the fiber direction and transverse to it respectively; X_c, Y_c are the corresponding compressive strengths. R and T are the shear strengths of lamina in planes 2-3 and 1-2 respectively.The shear strength in plane 1-3 will be designated by S in the expressions to follow.

2. Tensor polynomial .failure criteria

The most general polynomial failure criterion,as proposed by Tsai (1984) is expressed as

$$\begin{aligned} &F_1\sigma_1 + F_2\sigma_2 + F_3\sigma_3 + 2F_{12}\sigma_1\sigma_2 + 2F_{13}\sigma_1\sigma_3 + 2F_{23}\sigma_2\sigma_3 + \\ &F_{11}\sigma_1^2 + F_{22}\sigma_2^2 + F_{33}\sigma_3^2 + F_{44}\sigma_4^2 + F_{55}\sigma_5^2 + F_{66}\sigma_6^2 + \cdots \geq 1 \end{aligned} \tag{5}$$

Particular cases of the above criterion differ from one another by their strength tensors F_i. Hence, various degenerate cases of the Tensor polynomial criterion can be obtained by subtituting the appropriate tensor strength factors F_i in Eq. (5). Tensor strength factors appropriate to the various polynomial criteria are given below:

(a) Maximum stress criterion:

$$
\begin{aligned}
&F_1=\frac{1}{X_t}-\frac{1}{X_c};\quad F_2=\frac{1}{Y_t}-\frac{1}{Y_c};\quad F_3=\frac{1}{Z_t}-\frac{1}{Z_c}\\
&F_{11}=\frac{1}{X_tX_c};\quad F_{22}=\frac{1}{Y_tY_c};\quad F_{33}=\frac{1}{Z_tZ_c}\\
&F_{44}=\frac{1}{R^2};\quad F_{55}=\frac{1}{S^2};\quad F_{66}=\frac{1}{T^2}\\
&F_{12}=-\frac{F_1F_2}{2};\quad F_{13}=-\frac{F_1F_3}{2};\quad F_{23}=-\frac{F_2F_3}{2}
\end{aligned}
\tag{6}
$$

The remaining strength tensor terms are zero.

In the above expressions Z_t, and Z_c are the tensile and the compressive strength, respectively, in the principal direction 3 of the lamina and the other strength terms are the same as described in the Hashin criterion.

(b) Maximum strain criterion:

$$F_1=F_1^A+\frac{S_{12}}{S_{11}}F_2^A+\frac{S_{13}}{S_{33}}F_3^A$$

$$F_2=\frac{S_{12}}{S_{11}}F_1^A+F_2^A+\frac{S_{23}}{S_{33}}F_3^A$$

$$F_3=\frac{S_{13}}{S_{11}}F_1^A+\frac{S_{23}}{S_{22}}F_2^A+F_3^A$$

$$F_{11}=\frac{1}{X_tX_c}+\left(\frac{S_{12}}{S_{22}}\right)^2\frac{1}{Y_tY_c}+\left(\frac{S_{13}}{S_{33}}\right)^2\frac{1}{Z_tZ_c}-\frac{S_{13}}{S_{33}}F_1^AF_3^A-\frac{S_{12}}{S_{22}}F_1^AF_2^A-\frac{S_{12}S_{13}}{S_{22}S_{23}}F_2^AF_3^A$$

$$F_{22}=\frac{1}{Y_tY_c}+\left(\frac{S_{12}}{S_{11}}\right)^2\frac{1}{X_tX_c}+\left(\frac{S_{23}}{S_{33}}\right)^2\frac{1}{Z_tZ_c}-\frac{S_{12}}{S_{11}}F_1^AF_2^A-\frac{S_{23}}{S_{33}}F_2^AF_3^A-\frac{S_{12}S_{23}}{S_{11}S_{33}}F_1^AF_3^A$$

$$F_{33}=\frac{1}{Z_tZ_c}+\left(\frac{S_{13}}{S_{11}}\right)^2\frac{1}{X_tX_c}+\left(\frac{S_{23}}{S_{22}}\right)^2\frac{1}{Y_tY_c}-\frac{S_{13}}{S_{11}}F_1^AF_3^A-\frac{S_{23}}{S_{22}}F_2^AF_3^A-\frac{S_{13}S_{23}}{S_{11}S_{22}}F_1^AF_2^{\mathrm{A}}$$

$$F_{44}=\frac{1}{R^2};\quad F_{55}=\frac{1}{S^2};\quad F_{66}=\frac{1}{T^2}$$

$$
\begin{aligned}
F_{12}=&\frac{S_{12}}{S_{11}}\frac{1}{X_tX_c}+\frac{S_{12}}{S_{22}}\frac{1}{Y_tY_c}+\frac{S_{13}S_{23}}{S_{33}^2}\frac{1}{Z_tZ_c}-\frac{1}{2}\left(\frac{S_{12}^2}{S_{11}S_{22}}+1\right)F_1^AF_2^A\\
&-\frac{1}{2}\left(\frac{S_{13}S_{12}}{S_{11}S_{33}}+\frac{S_{23}}{S_{33}}\right)F_1^AF_3^A-\frac{1}{2}\left(\frac{S_{12}S_{23}}{S_{22}S_{33}}+\frac{S_{13}}{S_{33}}\right)F_2^AF_3^A
\end{aligned}
$$

$$
\begin{aligned}
F_{13}=&\frac{S_{13}}{S_{11}}\frac{1}{X_tX_c}+\frac{S_{13}}{S_{33}}\frac{1}{Z_tZ_c}+\frac{S_{12}S_{23}}{S_{22}^2}\frac{1}{Y_tY_c}-\frac{1}{2}\left(\frac{S_{13}^2}{S_{11}S_{33}}+1\right)F_1^AF_2^A\\
&-\frac{1}{2}\left(\frac{S_{12}S_{13}}{S_{11}S_{22}}+\frac{S_{23}}{S_{22}}\right)F_1^AF_2^A-\frac{1}{2}\left(\frac{S_{13}S_{23}}{S_{23}S_{33}}+\frac{S_{12}}{S_{22}}\right)F_2^AF_3^A
\end{aligned}
$$

$$
\begin{aligned}
F_{23}=&\frac{S_{23}}{S_{22}}\frac{1}{Y_tY_c}+\frac{S_{23}}{S_{33}}\frac{1}{Z_tZ_c}+\frac{S_{12}S_{13}}{S_{11}^2}\frac{1}{X_tX_c}-\frac{1}{2}\left(\frac{S_{23}^2}{S_{22}S_{23}}+1\right)F_1^AF_2^A\\
&-\frac{1}{2}\left(\frac{S_{12}S_{23}}{S_{11}S_{22}}+\frac{S_{13}}{S_{11}}\right)F_1^AF_2^A-\frac{1}{2}\left(\frac{S_{23}S_{13}}{S_{11}S_{33}}+\frac{S_{12}}{S_{11}}\right)F_2^AF_3^A
\end{aligned}
$$

In the above expressions S_{11}, S_{12} etc. are the components of the compliance matrix and F_1^A, F_2^A, F_3^A are the expressions given for F_1, F_2, F_3 in the maximumstress criterion.

(c) Tsai-Hill criterion:

$$F_1=F_2=F_3=0;\quad F_{11}=\frac{1}{X^2};\quad F_{22}=\frac{1}{Y^2};\quad F_{33}=\frac{1}{Z^2}$$

$$F_{44}=\frac{1}{R^2};\quad F_{55}=\frac{1}{S^2};\quad F_{66}=\frac{1}{T^2};\quad F_{12}=-\frac{1}{2}\left(\frac{1}{X^2}+\frac{1}{Y^2}-\frac{1}{Z^2}\right)$$

$$F_{13}=-\frac{1}{2}\left(\frac{1}{Z^2}+\frac{1}{X^2}-\frac{1}{Y^2}\right);\quad F_{23}=-\frac{1}{2}\left(\frac{1}{Y^2}+\frac{1}{Z^2}-\frac{1}{X^2}\right) \tag{8}$$

The values of X, Y, Z are taken as either X_t, Y_t, Z_t or as X_c, Y_c, Z_c, depending upon the sign of σ_1, σ_2, σ_3.

(d) Hoffman criterion:

$$F_1=\frac{1}{X_t}-\frac{1}{X_c};\quad F_2=\frac{1}{Y_t}-\frac{1}{Y_c};\quad F_3=\frac{1}{Z_t}-\frac{1}{Z_c}$$

$$F_{11}=\frac{1}{X_tX_c};\quad F_{22}=\frac{1}{Y_tY_c};\quad F_{33}=\frac{1}{Z_tZ_c}$$

$$F_{44}=\frac{1}{R^2};\quad F_{55}=\frac{1}{S^2};\quad F_{66}=\frac{1}{T^2}$$

$$F_{12}=-\frac{1}{2}\left(\frac{1}{X_tX_c}+\frac{1}{Y_tY_c}-\frac{1}{Z_tZ_c}\right)$$

$$F_{13}=-\frac{1}{2}\left(\frac{1}{X_tX_c}+\frac{1}{Z_tZ_c}-\frac{1}{Y_tY_c}\right)$$

$$F_{23}=-\frac{1}{2}\left(\frac{1}{Z_tZ_c}+\frac{1}{Y_tY_c}-\frac{1}{X_tX_c}\right)$$

The other strength tensor terms are zero.

(e) Tsai-Wu criterion:

$$F_1=\frac{1}{X_t}-\frac{1}{X_c};\quad F_2=\frac{1}{Y_t}-\frac{1}{Y_c};\quad F_3=\frac{1}{Z_t}-\frac{1}{Z_c}$$

$$F_{11}=\frac{1}{X_tX_c};\quad F_{22}=\frac{1}{Y_tY_c};\quad F_{33}=\frac{1}{Z_tZ_c}$$

$$F_{44}=\frac{1}{R^2};\quad F_{55}=\frac{1}{S^2};\quad F_{66}=\frac{1}{T^2}$$

$$F_{12}=-\frac{1}{2}\left(\frac{1}{\sqrt{X_tX_cY_tY_c}}\right)$$

$$F_{13}=-\frac{1}{2}\left(\frac{1}{\sqrt{X_tX_cZ_tZ_c}}\right)$$

$$F_{23}=-\frac{1}{2}\left(\frac{1}{\sqrt{Y_tY_cZ_tZ_c}}\right)$$

All other strength tensor components are zero.