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Time dependent service load behaviour of prestressed composite tee beams

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Abstract. This paper is concerned with the time dependent service load behaviour of prestressed composite tee beams. The effects of creep and shrinkage of the concrete slab are modelled using the age adjusted effective modulus method and a relaxation approach. The tendon strain is determined considering compatibility of deformations and equilibrium of forces between the tendon and the composite tee beam. A parametric study is undertaken to study the influence of various aspects on the stress, strain and deformations of the concrete slab, steel beam and prestressing tendon. The effect of loading type and tendon relaxation has also been considered for various types of prestressing tendon materials. Recommendations are then made in relation to adequate span to depth ratios for varying levels of prestressing force.

Key words: beams; composite construction; creep; prestressing; shrinkage; time effects.

1. Introduction

Composite tee-beams may have their strength and stiffness increased by the application of a prestressing force subjected to the member as shown in Fig. 1. This is becoming an increasingly popular form of reparation and rehabilitation throughout the world as infrastructure such as bridges are deteriorating due to increased traffic loads and due to harsh environmental conditions. Also this application has several other advantages including elastic behaviour to higher loads, increased ultimate capacity, reduced structural steel weight and improved fatigue and fracture behaviour. Therefore, the method has applicability in both buildings and bridges.

For new construction the steel beam may be pretensioned before the concrete is placed, or it may be post-tensioned after full composite action has been achieved. For rehabilitation, post-tensioning of existing composite bridges has proven to be the more popular. This paper considers post-tensioning of composite beams only.

Currently in the USA over 50 % of highway bridges are required to be rehabilitated or replaced due to increased traffic loads and environmental degradation, (Li, *et al.* 1995). The most economical and attractive method of rehabilitation of existing steel and composite bridges is that of external prestressing, (Li, *et al.* 1995). Research into external prestressing of composite beams has been undertaken for over 35 years. The earliest work appears to be that of Szilard (1959) who developed design equations covering the problems of prestressing composite structures. Hoadley (1963) analysed the ductile behaviour of simply supported prestressed composite beams with a constant eccentric prestressing force. He calculated the stresses in the steel beam and concrete deck, and

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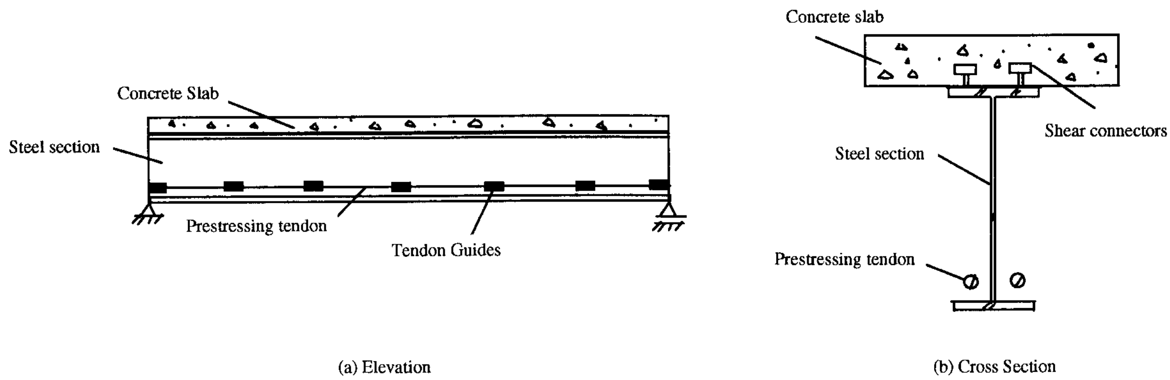


Fig. 1 Prestressed composite tee beam.

derived an expression for the increase in tendon force with applied load using the strain energy method and assuming elastic behaviour. Further work incorporating the effect of shear connectors was carried out by Reagan and Krahl (1967).

Troitsky, *et al.* (1989) researched pre-tensioned and post-tensioned composite girders both analytically and experimentally. The analysis was based on simply supported bridge girders using straight, bent up, or short straight tendons. Perfect shear connection between the steel and concrete was assumed, and the effects of losses due to creep, shrinkage, tendon relaxation, and the effects of residual stresses in the steel beam were not considered.

Saadatmanesh, *et al.* (1989a and 1989b) considered prestressed composite tee beams under positive moment with theoretical and experimental studies being carried out. Saadatmanesh, *et al.* (1992a and 1992b) then considered prestressed composite tee beams under negative moment with theoretical and experimental studies being carried out. These studies were also useful in identifying the relationship between tendon and beam strains at the level of the prestressing tendon.

Uy and Bradford (1993) developed a simple cross-sectional analysis method to study the moment-curvature response of composite steel-concrete tee-beams subjected to an eccentric prestress near the bottom flange of the steel. A parametric study was undertaken to study the influence of prestress and slip as well as strengths and geometry of the cross-section. The ramifications of these on the strength and ductility of prestressed composite beams at ultimate loads were therefore developed.

Bradford (1994) illustrated the strength gain from prestressing the steel portion of an existing simply supported composite tee beam with a straight tendon. The distortional instability of the steel beam which may occur under eccentric prestress was then analysed. Design charts for calculating the elastic critical stress in the bottom flange, and hence the prestressing force to cause elastic distortional buckling were also developed.

Most recently Li, *et al.* (1995) and Albrecht, *et al.* (1995) considered the fatigue strength of prestressed composite steel-concrete beams theoretically and experimentally. These results are of particular importance in assessing the remaining life of bridge girders when rehabilitated using the prestressing technique.

In rehabilitation of bridges, decks are often replaced or upgraded by pouring of fresh concrete. Also, if a composite steel-concrete beam is prestressed very early in its life then creep and shrinkage of the concrete slab can have an effect on the stress in the tendon and the long term deformations of prestressed composite beams. This study involves the long term behaviour of prestressed composite tee beams by considering the creep and shrinkage of the concrete slab as well as the relaxation of the steel tendon. A numerical model is presented which is initially calibrated against existing experimental results for conventional composite beams.

2. Time dependent behaviour

As serviceability is often the governing criteria in the design of structural members in bridges and buildings, the long term behaviour has therefore had close attention over the last decade. Time dependent studies on composite beams have been carried out by numerous investigators in both an experimental and theoretical capacity.

Gilbert (1989) developed an age adjusted effective modulus method (AEMM) and a relaxation procedure to study the time dependent behaviour of composite steel-concrete cross-sections. Bradford and Gilbert (1989) also developed a model to include all non-linearities of materials incorporating residual stresses in the steel section and creep and shrinkage in the concrete. Lawther and Gilbert (1990 and 1992) studied the behaviour of composite steel-concrete cross-sections using the rate of creep method and established a method for the calculation of beam deformations. Bradford (1991) then developed a theoretical model for the moment-curvature response due to time dependent deformations of creep and shrinkage.

Benchmark experiments were undertaken by Bradford and Gilbert (1991a and 1991b) on beams subjected to self weight and sustained load to determine the effects of shrinkage and creep independently. Wright *et al.* (1992) undertook an experimental study for composite beams with partial shear connection and developed a numerical model which showed good agreement with the experiments. Dezi *et al.* (1993) also developed a creep model to study the time dependent behaviour of composite beams.

The study of prestressed composite beams for the influence of creep and shrinkage has been limited to studies by Nakai *et al.* (1995) where a stiffness analysis method was developed to incorporate the external prestressing cable. This study was limited to studying the effect of tendons relaxation for various span configurations. This paper is concerned with undertaking a parametric study to evaluate the influence of various material and geometric properties on the long term behaviour of a prestressed composite beam. It is also undertaken to determine appropriate span to depth ratios for various levels of prestressing force which are of direct use and benefit to practising structural engineers.

3. Analysis method

The analysis method chosen to establish the long term behaviour of strains, stresses and deflections throughout the member is to firstly undertake the following three steps as described by Gilbert (1988):

(1) short term analysis
(2) time analysis
(3) member analysis

For the analysis, each cross section may be subjected to a combination of axial force and bending moment as shown in Fig. 2. Mechanical shear connectors are usually used to ensure that the steel-concrete interface is capable of carrying the imposed horizontal shear, and that the steel and concrete portions act compositely. In the service load analysis of such cross sections, it is therefore usual to assume that no slip occurs between the steel and the concrete. This assumption is justified, since at service loads the amount of slip is generally negligible, (Gilbert 1988). Full interaction between the steel and the concrete is therefore assumed in the ensuing analyses.

3.1. Short term analysis

The short term analysis was undertaken using the method developed by Gilbert (1989) for conventional composite beams. Consider the transformed section shown in Fig. 2. The area A and the first and second moments of area of the transformed section about the top fibre, B and I, respectively are

$$A = bD_c + (n_s - 1)A_{s1} + n_{ss}A_{ss} \tag{1}$$

$$B = \frac{1}{2}bD_c^2 + (n_{s1} - 1)A_{s1}d_{s1} + n_{ss}A_{ss}d_{ss} \tag{2}$$

$$I = \frac{1}{3}bD_c^3 + (n_{s1} - 1)A_{s1}d_{s1}^2 + n_{ss}(I_{ss} + A_{ss}d_{ss}^2) \tag{3}$$

where $n_s = \dfrac{E_s}{E_c}$ and $n_{ss} = \dfrac{E_{ss}}{E_c}$ are the modular ratios of the reinforcing steel and steel section respectively.

The strain distribution under initial loading may be represented as

$$\varepsilon_i = \varepsilon_{0i} - y\rho_i \tag{4}$$

where the top fibre strain is given by

$$\varepsilon_{0i} = \frac{BM_i + IN_i}{E_c(AI - B^2)} \tag{5}$$

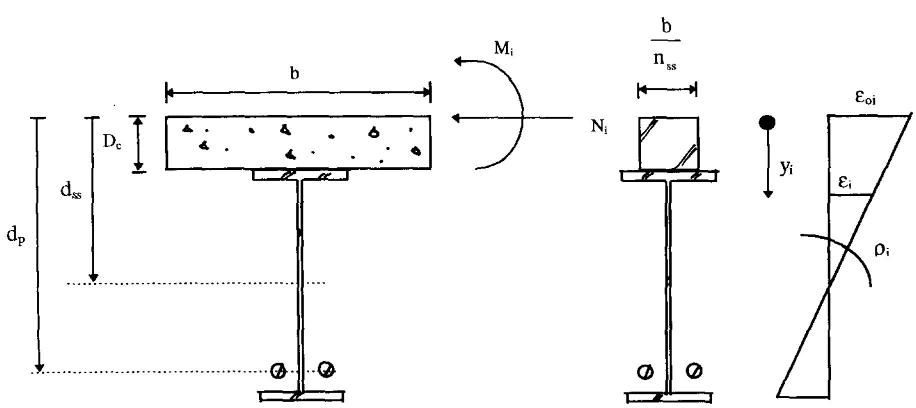


Fig. 2 Prestressed composite tee beam transformed section and strain diagram.

and the curvature is represented by

$$\rho_i = \frac{AM_i + BN_i}{E_c(AI - B^2)} \tag{6}$$

where N_i is the resultant axial force and M_i is the resultant moment about the top of the section. Now the resultant axial force on the cross-section is equal to the prestressing force P_i

$$N_i = -P_i \tag{7}$$

and the resulting moment about the top fibre is due to the positive applied service moment, M_s and the negative moment due to prestress $(P_i d_p)$

$$M_i = M_s - (P_i d_p) \tag{8}$$

Concrete and steel stresses are readily obtained from the strain diagram using

$$\sigma_i = E_c(\varepsilon_{0i} - y\rho_i) \tag{9}$$

$$\sigma_{si} = E_{ss}(\varepsilon_{0i} - y\rho_i) \tag{10}$$

$$\sigma_{tendon} = E_p\,\varepsilon_{tendon} \tag{11}$$

where the tendon strain is a function of the initial prestressing force as well as the applied loading. Saadatmanesh, *et al.* (1989b) developed a method for obtaining the strain in the tendon due to the applied loading which involves the consideration of compatibility between the steel tendon and steel beam and this is used here.

If due to the applied loading the tendon elongates by an amount δ_t, then by using the compatibility of deformations this must be equivalent to the elongation of the beam at that level, δ_s. Now the elongation of the beam will be dependent on the applied loading and this study considers both uniform loading and point loading at the center. The elongation of the beam at the level of the tendon is given by the following expression

$$\delta_s = \int_0^L \varepsilon_s\,dx \tag{12}$$

where ε_s is the strain in the steel beam at the level of the tendon. The elongation of the tendon is given by the following relationship

$$\delta_t = \int_0^L \varepsilon_t\,dx \tag{13}$$

where ε_t is the strain in the steel tendon due to the applied loading. By equating the two elongations one can solve for the tendon strain for a given load condition knowing the strain distribution throughout the beam. This procedure is valid when there are sufficient anchors along the length of the beam to ensure that the beam and tendon have the same deformation at that level. Generally anchors will be at close spacing in order to reduce the likelihood of distortional buckling of the thin web as suggested by Bradford (1994).

3.1.1. Uniform loading

For the case of uniform loading over the length of the span the expression to determine the strain in the tendon as shown in Fig. 3 was determined by solving the equations in Eqs.

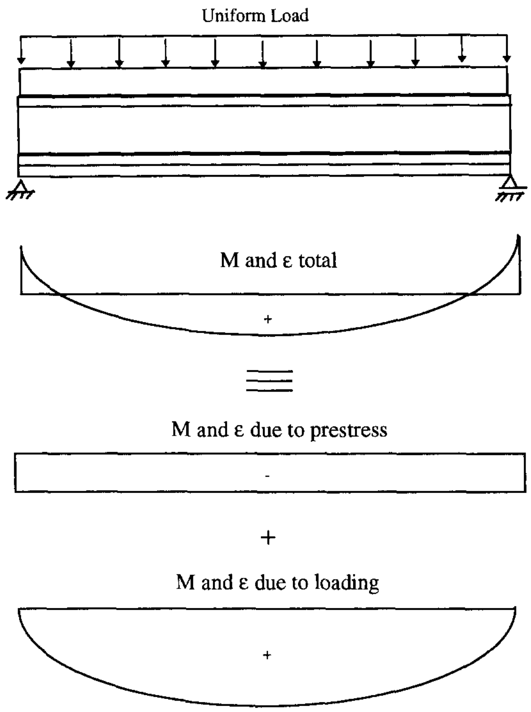


Fig. 3 Moment and strain distributions for uniform load on beam.

(12) and (13), so that the tendon strain is constant along the whole length

$$\varepsilon_t = \left[\frac{2\varepsilon_{mid} - \varepsilon_{end}}{3}\right] \tag{14}$$

3.1.2. Point loading

For the case of point loading at the centre of the beam as shown in Fig. 4 the following expression is derived for the strain in the tendon which is assumed constant along the whole length and is calculated as

$$\varepsilon_t = \left[\frac{\varepsilon_{mid} - \varepsilon_{end}}{2}\right] \tag{15}$$

where ε_{mid} and ε_{end} are the strains at the midspan and end sections of the steel beam at the level of the tendon. In addition to the strains caused through the application of load the tendon strain is initially strained by an initial strain of an amount in tension

$$\varepsilon_{ti} = \frac{P_i}{E_p A_p} \tag{16}$$

so that the total strain in the tendon for the short term analysis is given by the expression

$$\varepsilon_{tendon} = \varepsilon_{ti} + \varepsilon_t \tag{17}$$

3.2. Time analysis

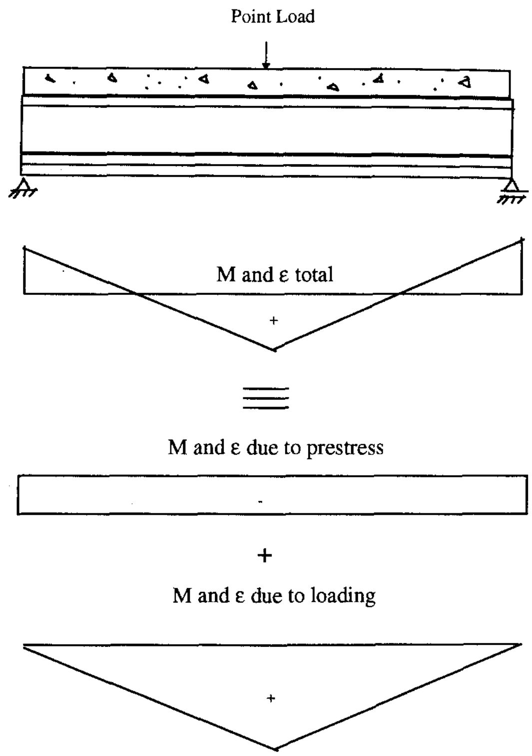


Fig. 4 Moment and strain distributions for point load at centre.

Using the age adjusted effective modulus method and a relaxation method developed by Gilbert (1989) for conventional composite beams the restraining actions $-\Delta N$ and $-\Delta M$ which are required to prevent the free development of creep and shrinkage in the concrete deck are determined using Eqs. (18) and (19)

$$-\Delta N = -\overline{E}_e[\phi(A_c\varepsilon_{0i} - B_c\rho_i) + \varepsilon_{sh}A_c] + R \tag{18}$$

$$-\Delta M = -\overline{E}_e[\phi(B_c\varepsilon_{0i} + I_c\rho_i) - \varepsilon_{sh}B_c] + Rd_p \tag{19}$$

where A_c, B_c and I_c represent the area, first moment of area and second moment of area of the concrete slab about the top fibre and R represents the relaxation of the steel tendon. The creep coefficient ϕ and the shrinkage strain ε_{sh} were based on the AS3600 (SAA 1994) model. The change of strain distribution with time is calculated by the following expressions in Eqs. (20) and (21). The top fibre concrete strain change is given by

$$\Delta\varepsilon_0 = \frac{\overline{B}_e\Delta M + \overline{B}_e\Delta N}{\overline{E}_e(\overline{A}_e\overline{I}_e - \overline{B}_e^2)} \tag{20}$$

and the change in curvature is given as

$$\Delta\rho = \frac{\overline{B}_e\Delta M + \overline{I}_e\Delta N}{\overline{E}_e(\overline{A}_e\overline{I}_e - \overline{B}_e^2)} \tag{21}$$

where the age adjusted effective modulus is calculated in Eq. (22) as

$$\overline{E}_e=\frac{E_c(\tau)}{1+\chi(t,\ \tau)\,\phi(t,\ \tau)} \tag{22}$$

and $\chi(t,\ t)$ is taken as a constant of 0.80 which is representative of a constant load history when loading takes place at about 10 days and the final creep coefficient ϕ^* is 3.0 which is representative of normal strength concretes. The age adjusted effective section properties of area, first and second moments of area are calculated using Eqs. (23), (24) and (25) respectively

$$\overline{A}_e=bD_c+(\overline{n}_s-1)A_{s1}+\overline{n}_{ss}A_{ss} \tag{23}$$

$$\overline{B}_e=\frac{1}{2}bD_c^2+(\overline{n}_{s1}-1)A_{s1}d_{s1}+\overline{n}_{ss}A_{ss}d_{ss} \tag{24}$$

$$\overline{I}_e=\frac{1}{3}bD_c^3+(\overline{n}_{s1}-1)A_{s1}d_{s1}^2+\overline{n}_{ss}(I_{ss}+A_{ss}d_{ss}^2) \tag{25}$$

where $\overline{n}_s=\frac{E_s}{\overline{E}_e}$ and $\overline{n}_{ss}=\frac{E_{ss}}{\overline{E}_e}$ are the modular ratios of the reinforcing steel and steel section respectively.

The change of stress in the concrete deck at any point y below the top fibre may be obtained by summing the stress loss due to relaxation and the stress caused by the application of ΔN and ΔM to the age adjusted transformed section.

$$\Delta\sigma=-\overline{E}_e[\phi(\varepsilon_{0i}-y\rho_i)+\varepsilon_{sh}-(\Delta\varepsilon_0-y\Delta\rho)] \tag{26}$$

As in the previous analyses, the time-dependent change of steel stress in the steel section and steel tendon is calculated using Eqs. (27) and (28) respectively

$$\Delta\sigma_{ss}=E_{ss}[\Delta\varepsilon_0-y\Delta\rho] \tag{27}$$

$$\Delta\sigma_{tendon}=E_p\,\Delta\varepsilon_{tendon} \tag{28}$$

where the change in the tendon strain due to the loading is calculated using Eqs. (29) and (30) depending on the loading type

3.2.1. Uniform loading

$$\Delta\varepsilon_t=\left[\frac{2\Delta\varepsilon_{mid}-\Delta\varepsilon_{end}}{3}\right] \tag{29}$$

3.2.2. Point loading

$$\Delta\varepsilon_t=\left[\frac{\Delta\varepsilon_{mid}-\Delta\varepsilon_{end}}{2}\right] \tag{30}$$

so that the change in the tendon strain is a function of the relaxation and the creep and shrinkage induced deformations, such that

$$\Delta\varepsilon_{tendon}=\Delta\varepsilon_{tr}+\Delta\varepsilon_{tl} \tag{31}$$

where the relaxation strain chosen for the tendon was a linear model which assumed the maximum relaxation was reached after ten years. The model used was

$$\Delta\varepsilon_{tr} = \varepsilon_{tr\,(max)} \cdot \left(\frac{t}{3650}\right) \tag{32}$$

where $\varepsilon_{tr\,(\max)}$ is the maximum relaxation of the tendon after ten years and depends on the type of material used.

3.3. Member analysis

The above procedures have been developed for analysing the cross-sectional behaviour of a prestressed composite beam. The results can then be used to look at the global system and the behaviour of the member. The beam in Fig. 5 is shown acted upon by a force $\widetilde{P}$ positioned, a from the left hand end. The corresponding virtual moment $\widetilde{m}(x)$ is given by

$$\widetilde{m}(x) = \begin{cases} \dfrac{\widetilde{P}bx}{L} & 0 \le x \le a \\ \dfrac{\widetilde{P}bx}{L} - \widetilde{P}(x-a) & a \le x \le b \end{cases} \tag{33}$$

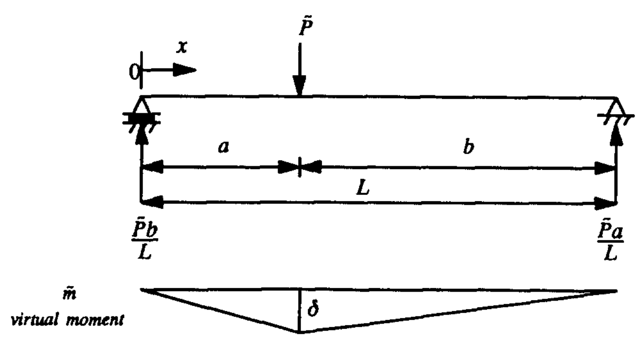
If q represents the real displacement conjugate with the virtual force $\widetilde{P}$, then the external virtual work done U_e is

$$U_e = \int_0^L \widetilde{P} q \, dx \tag{34}$$

while the internal virtual work done due to flexure U_i is

$$U_i = \int_0^L \frac{M}{EI}(x)\,\widetilde{m}(x)\,dx \tag{35}$$

where $M=$ the real moment which varies with x and $EI=$ the flexural rigidity which also varies with x due to cracking and time dependent effects. If the deflection δ at the position of the virtual force $\widetilde{P}$ is required, $\widetilde{P}$ is set equal to unity and then



$0 \le x \le a \quad \tilde{m}(x) = \frac{\tilde{P}b}{L}x$

$a \le x \le b \quad \tilde{m}(x) = \frac{\tilde{P}b}{L}x - \tilde{P}(x-a)$

Fig. 5 Virtual work formulation.

$$\delta = \int_0^L \rho(x)\,\widetilde{m}(x)\,dx \tag{36}$$

where $\rho(x)=$ the curvature due to the loading and $\widetilde{m}(x)=$ the virtual bending moment. This integral can be simplified using Simpsons rule of integration to yield the following relationship for the midspan deflection

$$\delta = \frac{L}{12}\{\rho_{s1}\widetilde{m}_{s1} + 4\rho_{q1}\widetilde{m}_{q1} + 2\rho_m\widetilde{m}_m + 4\rho_{qr}\widetilde{m}_{qr} + \rho_{sr}\widetilde{m}_{sr}\} \tag{37}$$

where the subscripts s_1 and sr refer to the left and right hand supports and ql and qr refer to the left and right hand quarter points respectively. The subscript m refers to the value at midspan. The deflection δ at any other point along the span of the beam may be defined in a similar fashion.

4. Calibration of theory

In order to verify the theoretical model the program was compared with the benchmark experiments undertaken by Bradford and Gilbert (1991b) for composite steel-concrete beams which were non prestressed. The experimental results were reported for 220 days of data and the theoretical AEMM model developed herein is shown in Fig. 6. The results of the experiments are compared with the analytical model in Table 1 for the initial and final deflections of the creep beams and creep and shrinkage beams.

The self weight loading for the creep beams $B2$ and $B4$ was 1.95 kN/m and the total load on the creep and shrinkage beams $B1$ and $B3$ was 9.45 kN/m, which represents a service loading of about 3 kPa depending on the beam spacing in a structure. Most buildings or bridges with permanent formwork for slabs spanning between beams have a beam spacing of about 2.5 m and thus this would be equivalent to 3 kPa which is an acceptable service live load for most international design standards.

The results of Table 1 show that the AEMM model slightly underestimates the experimental deflections. This slight discrepancy may be due to the inadequate nature of the use of the AS 3600 (SAA 1994) model to represent the creep and shrinkage deformations of the concrete slab.

5. Parametric study

A parametric study is undertaken here to identify the influence of various parameters on the time dependent behaviour of prestressed composite beams. The cross section is shown in Fig. 7. The properties of the cross-section are

$b=1000$ mm
$D_c=70$ mm
$d_{ss}=171.5$ mm
$A_{ss}=3{,}230$ mm^2
$I_{ss}=23.6\times10^6$ mm^4
$d_p=0.9\ D_{tot}=243$ mm

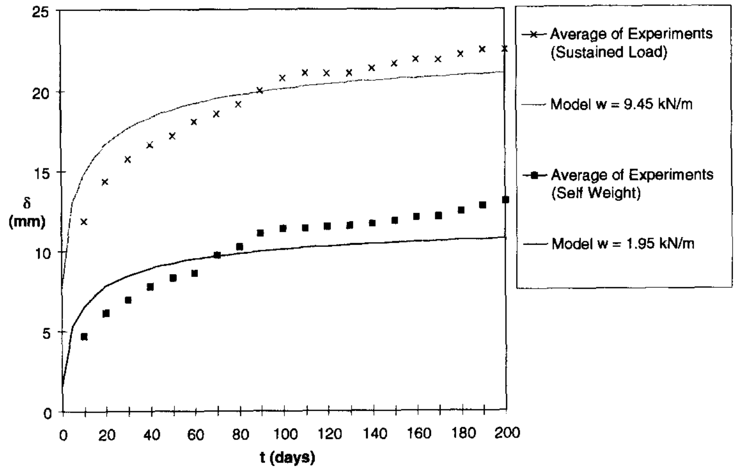


Fig. 6 Calibration of AEMM model with composite beams (Bradford and Gilbert 1991b).

Table 1 Comparison of experiments and AEMM model developed

Beam Number	Init. Defln Expts (mm)	Init. Definn (AEMM) (mm)	Final Defln Expts. (mm)	Final Defln (AEMM) (mm)
B2, B4	2.7	2.5	12	10
B1, B3	9	8	22	20

$D=270$ mm for the entire parameter study
$L/D=20$ unless noted otherwise

Unless noted otherwise the total service load applied to the beam was 9.5 kN/m which represents a 3 kPa service live load in addition to the self weight of the beam. The values of creep and shrinkage strain were determined using the (SAA 1994) model for creep and shrinkage and depends on the cross-section and the strength of concrete used. Unless noted otherwise the concrete strength used was 25 MPa which has a modulus of elasticity at 28 days of 25,000 MPa.

5.1. *Initial prestressing force, P_i*

The effect of the initial prestression force was considered by using the same calibration beam and load as that used by Bradford and Gilbert (1991b). The initial prestressing force was varied from zero to the full force to cause a balanced load condition at midspan. The effect of the initial prestressing force on the long term deflection is shown in Fig. 8. The initial deflection can be reduced to zero which may be beneficial for some instances, and span to depth ratios may be increased as a result of the initial prestressing force.

The effect of the initial prestressing force on the top fibre concrete stress is shown in Fig. 9 where it is seen that the top fibre compression is increased as a result of the prestressing force. The maximum stress is approximately 6 MPa which is less than $0.45\,f_c'$ which is about 10 MPa and represents the point at which the concrete becomes non-linear in nature. The bottom fibre concrete stress shows that the effect of increasing the initial prestressing force is to increase the compression in the slab so that tensile stresses are eliminated as illustrated in

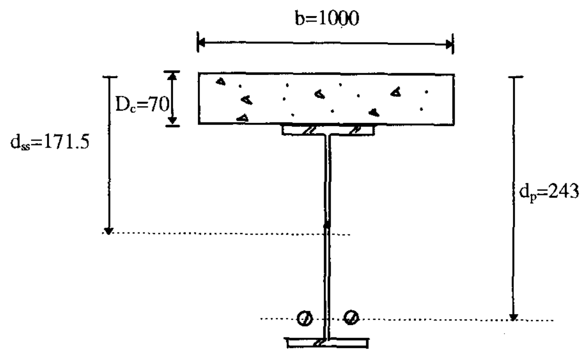


Fig. 7 Prestressed composite tee beam for parametric study.

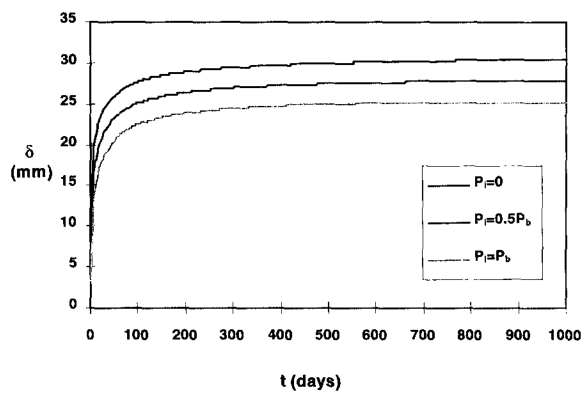


Fig. 8 Effect of initial prestressing force on long term deflection.

Fig. 10.

The effect of the initial prestressing force on the bottom fibre steel stress was plotted in Fig. 11 where it is seen that by increasing the prestressing force the tensile stress can be decreased considerably. The long term stress in the beam without any prestress was 170 MPa compared with 80 MPa for the beam which was fully balanced at initial loading. The top fibre steel stresses are increased in compression as the level of prestress is increased as shown in Fig. 12 and this may cause problems with local or lateral buckling. The design of the top flange plate and the spacing of the shear connectors can therefore have a considerable influence on the local and lateral buckling of prestressed composite beams, and this needs to be investigated further.

5.2. Span to depth ratio, L/D

For routine design situations structural engineers often use span to depth ratios as a means of sizing cross-sections. These span to depth ratios should be based on long term deflection requirements which incorporate creep and shrinkage deformations. For conventional composite beams, Johnson (1994) suggests a span to depth ratio of 20 in order to satisfy serviceability limit states without considering creep and shrinkage.

The effect of prestressing force was considered in Figs 13, 14 and 15 for the span to depth

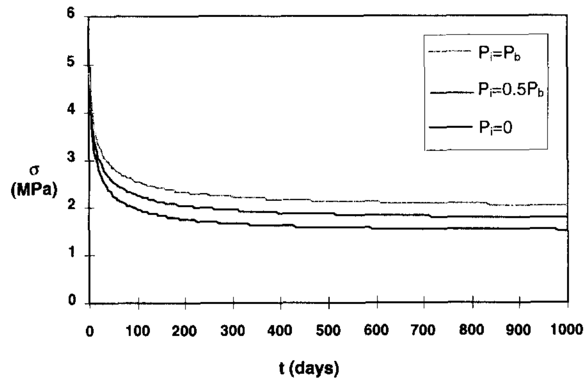


Fig. 9 Effect of initial prestressing force on top fibre concrete stress.

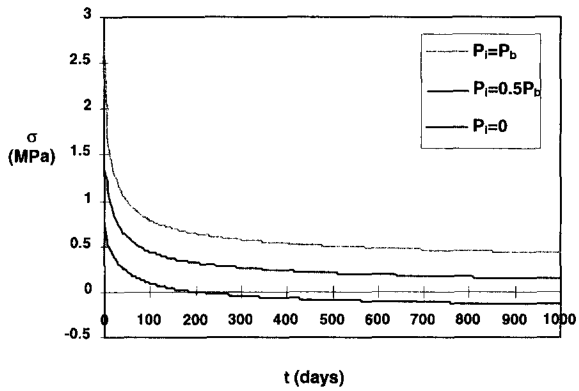


Fig. 10 Effect of initial prestressing force on bottom fibre concrete stress.

ratio of 20, 25 and 30. Table 2 summarises the results of the long term deflections and the allowable span to depth ratios can be established from these. If one takes $L/180$ as the allowable maximum deflection of a beam, then the maximum span to depth ratios which are able to be achieved are 22.5, 23.5 and 25 for each of the values of initial prestressing force of 0, 0.5 P_b and P_b respectively. This is illustrated in Fig. 16 which suggests a linear relationship for deflection versus span to depth ratios.

5.3. *Tendon modulus, E_p*

The effect of tendon modulus E_p on the long term deflection was considered. The materials considered were low relaxation strand, low relaxation bar, carbon fibre bar and arapree strand. The properties of each of these materials was obtained from the study by Gilbert and Gowripalan (1994) and these are summarised in Table 3. Fig. 17 shows that the effect of varying the material type on the long term deflection is minimal, however there is a considerable effect on the stress in the tendon since each material has a different breaking stress and therefore is stressed to a different level and the results of Fig. 18 illustrate this. Each tendon was stressed to 60% of the breaking stress, however it is shown that the change in stress in the tendon over time is virtually zero for all beams. The strains in the tendon at prestressing are very high, however

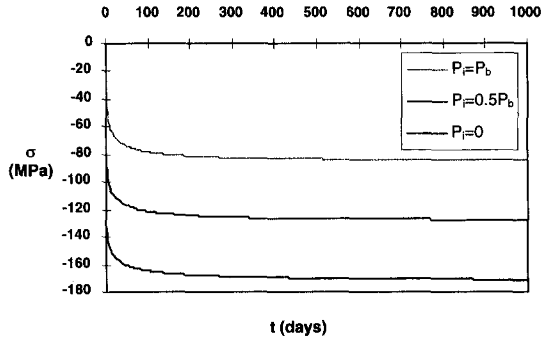


Fig. 11 Effect of initial prestressing force on bottom fibre steel stress.

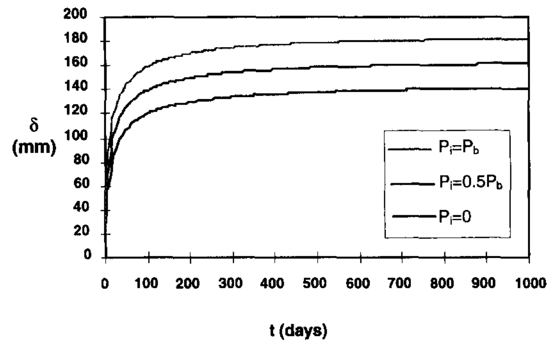


Fig. 12 Effect of initial prestressing force on top fibre steel stress.

the relaxation in the tendon due to losses and creep and shrinkage are minimal up until 1000 days. The results of these losses reflect the results of Nakai *et al.* (1995) who showed that for simply supported beams with eccentric tendons the losses are zero for a very large range of prestressing forces.

5.4. Concrete strength f'_c, E_c

The effect of the concrete strength on the long term deflection was also considered. As the concrete strength was increased, so to was the elastic modulus and thus the long term stiffness. These properties are highlighted in Table 4 which were used in the analysis. However, there was a minimal change in long term deflection by increasing the concrete strength from 25 to 32 and 40 MPa respectively as shown in Fig. 19.

5.5. Eccentricity, d_p

Another parameter to be varied was the eccentricity of the prestressing tendon. It is shown in Fig. 20 that by increasing the eccentricity by 10% of the section depth the deflection may be reduced by 10 % for both initial and long term deflection. This is very important for construction reasons, as a slight error in the location of the stressing tendon can have a dramatic effect on the behaviour under large prestressing forces.

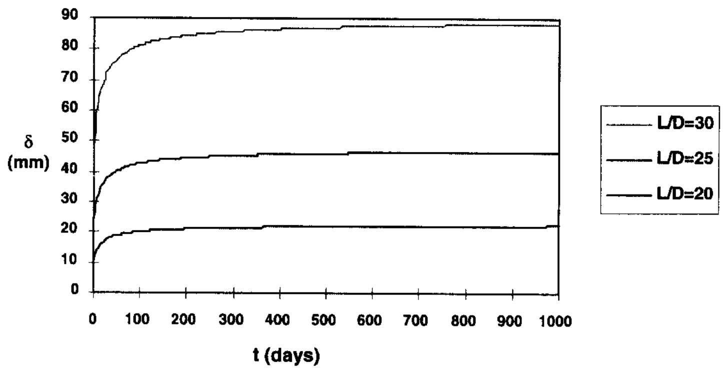


Fig. 13 Effect of span to depth ratio on long term deflection ($P_i=0$).

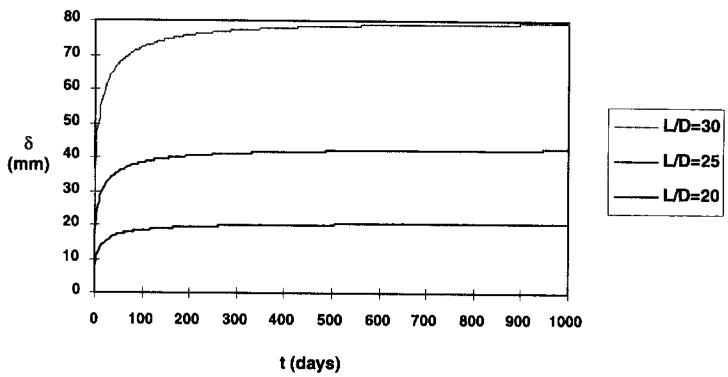


Fig. 14 Effect of span to depth ratio on oong term deflection ($P_i=0.5P_b$).

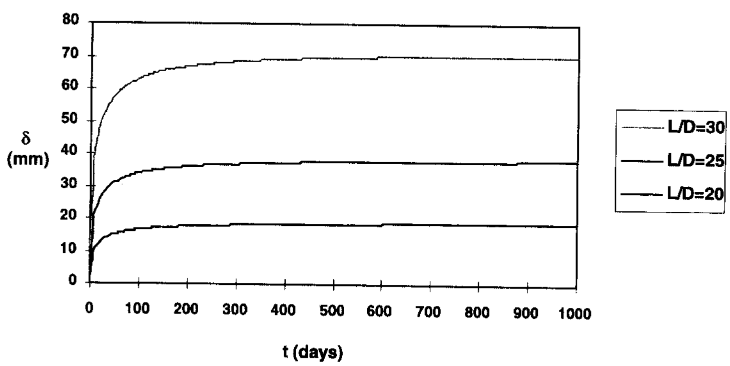


Fig. 15 Effect of span to depth ratio on long term deflection ($P_i=P_b$).

5.6. Loading type

The effect of the loading type on the long term behaviour of the prestressed composite beam was considered. The two types of loading cases chosen were the uniformly distributed load and the point load. The uniformly distributed load was 9.5 kN/m to be consistent with the previous studies. The point load chosen was 26 kN to produce the same bending moment at

Table 2 Span to deflection ratios for various levels of prestress

Prestressing force (P_i/P_b)	Span to depth ratio (L/D)	Span to deflection ratio (L/δ)
0	20	240
0	25	144
0	30	92
0.5	20	262
0.5	25	158
0.5	30	103
1	20	287
1	25	177
1	30	115

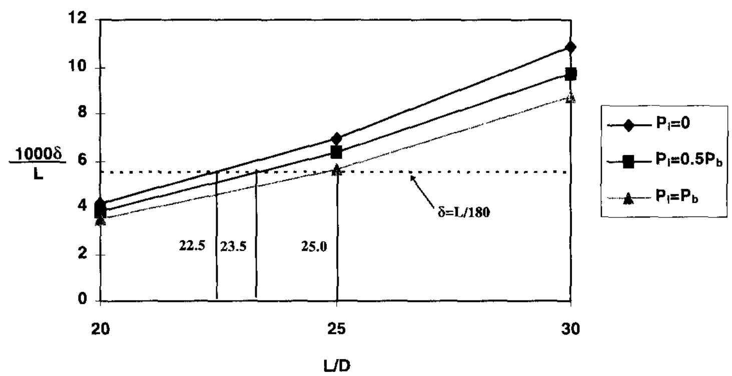


Fig. 16 Allowable span to depth ratios for various levels of prestress.

Table 3 Tendon material properties

Tendon material type	Elastic modulus, E_p $\times 10^3$ (MPa)	Breaking stress. f_{up}(MPa)	Maximum tendon relaxation (%)
Steel strand	200	1930	3
Steel bar	170	1120	3
Carbon fibre bar	140	1800	5
Arapree stand	130	1930	15

the centre as the uniform load case. Each of these beams was prestressed with a tendon force to balance the applied bending moment at the centre. It was found that the long term deflections were similar in proportion to the initial deflections. The uniform beam initially deflected 25% more than the point loaded beam whilst the long term deflection of the uniformly loaded beam was approximately 17% greater than the point loaded beam. The long term deflections values for both types of loading are illustrated in Fig. 21.

6. Conclusions

This paper has introduced and reviewed the concept of prestressing composite beams in buildi-

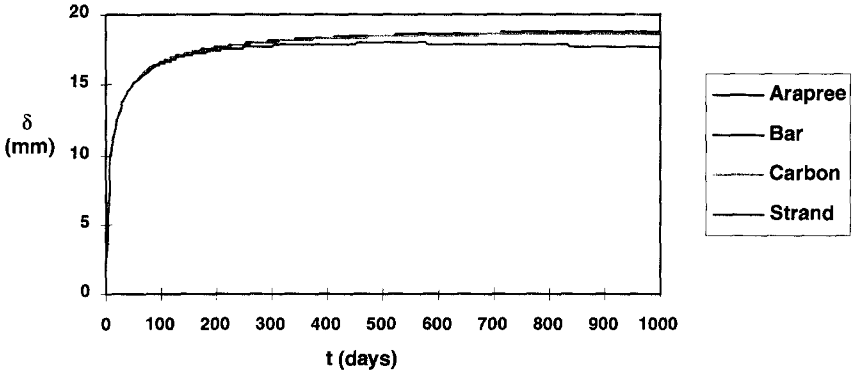


Fig. 17 Effect of tendon type on long term deflection ($P_i = P_b$).

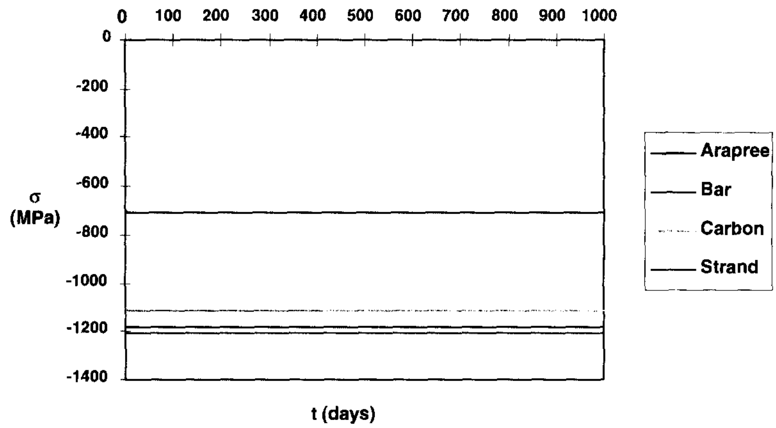


Fig. 18 Effect of tendon type of long term tendon stress ($P_i = P_b$).

Table 4 Concrete material properties

Concrete strength f_c' (MPa)	Elastic modulus, E_c (MPa)
25	25,000
32	28,250
40	31,500

ngs and bridges. The advantages of this concept have been outlined and discussed. The long term behaviour of composite beams has been reviewed and it was shown that the long term behaviour of prestressed composite beams was important when considering the application of prestressing in new construction and when fresh concrete is involved during reparation and retrofitting.

An analytical model based on the age adjusted effective modulus method coupled with a relaxation method and the principles of equilibrium of forces and compatibility of deformations was developed to consider the behaviour of prestressed composite beams. This model was calibrated successfully against existing benchmark experiments for non prestressed composite steel-concrete beams by Bradford and Gilbert (1991b).

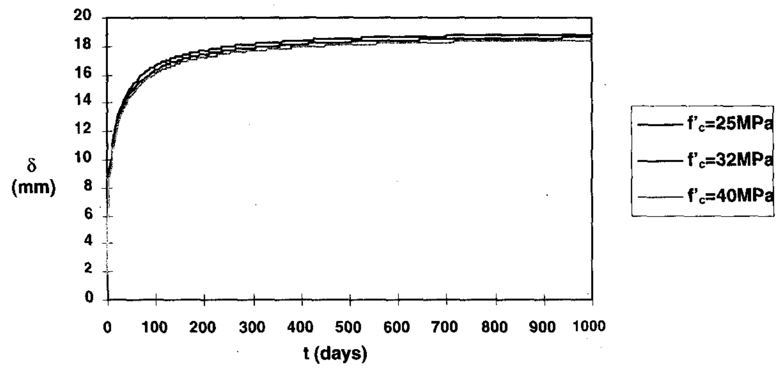


Fig. 19 Effect of concrete strength on long term deflection ($P_i = P_b$).

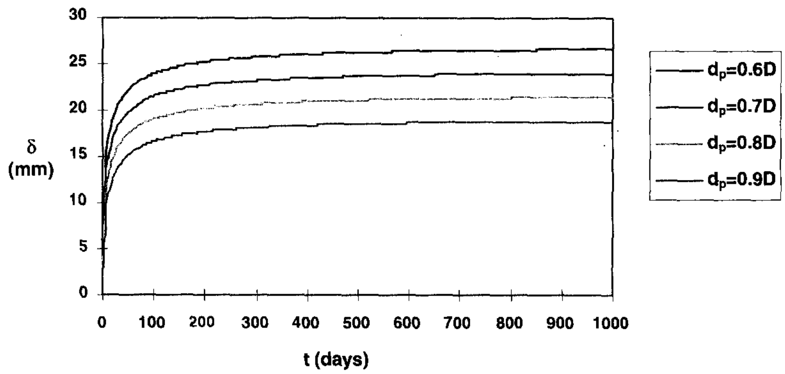


Fig. 20 Effect of eccentricity of long term deflection ($P_i = P_b$).

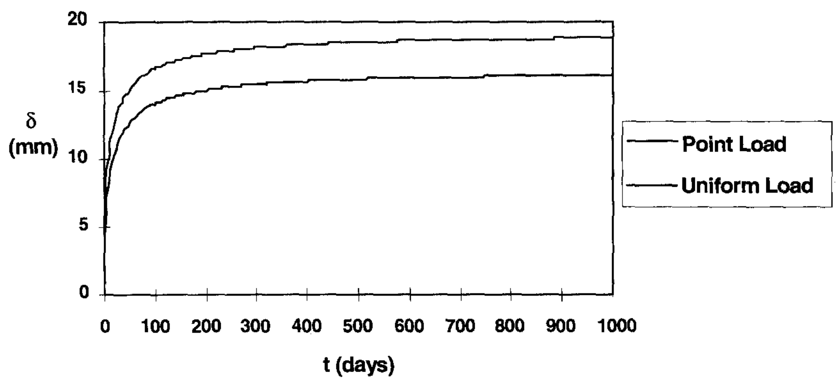


Fig. 21 Variation of deflection on load type.

A parametric study was established to look at various material and geometric factors which influence the long term behaviour. It was found that the concept of prestressing reduced the overall long term deflections of the composite beams. Furthermore a reduction in the long term stresses of the steel beam in tension was a result, with an increase in the compressive stresses in the slab and the top of the steel beam. These however were within acceptable limits for the range of prestressing forces applied.

The concept of span to depth ratios was considered as these are often used by structural engineers for initial design concepts. It was found that if a span to deflection criteria of $L/180$ was considered then a span to depth ratio of 25 could be achieved by balancing the deflection at initial loading. If the beam was conventional a span to depth ratio of 22.5 could be achieved. Thus by balancing the initial deflection an increase of about 10% can be achieved in the span to depth ratio by prestressing.

It can be concluded that all the deformations and stresses due to creep and shrinkage in this study converged after 1000 days, and thus if prestressing is used for reparation and retrofitting more than 3 years after the concrete has been poured, then creep and shrinkage deformations will be relatively small but may still require attention. In conclusion a simple model is therefore suggested where the long term deflection of the prestressed composite beam is simply the long term deflection of a conventional composite beam, minus the initial camber caused by the prestressing force.

7. Further research

The results considered in this study pertain to simply supported composite beams which are prestressed with a constant eccentric prestressing force. Further research is required in the following areas:

(1) different tendon configurations, including draped and bent up tendons;
(2) consideration of varying loading history;
(3) non-linear behaviour for higher values of service load; and
(4) continuous beams where moment redistribution takes place and the tendon configuration adds complexity to the analysis due to increased losses of prestress.

References

Albrecht, P., Li, W. and Saadatmanesh, H. (1995), "Fatigue strength of prestressed composite steel-concrete beams", *Journal of Structural Engineering, ASCE,* **121**, 1850-1856.

Bradford, M.A. (1991) "Deflections of composite steel-concrete beams subjected to creep and shrinkage", *ACI Structural Journal,* **88**(5), 610-614.

Bradford, M.A. (1994), "Buckling of post-tensioned composite beams", *Structural Engineering and Mechanics,* **2**(1), 113-123.

Bradford, M.A. and Gilbert, R.I. (1989), "Non-linear behaviour of composite beams at service loads", *The Structural Engineer,* **67**(14), 263-267.

Bradford, M.A. and Gilbert, R.I. (1991a), "Time dependent behaviour of simply-supported steel-concrete composite beams", *Magazine of Concrete Research,* **43**(157), 265-274.

Bradford, M.A. and Gilbert, R.I. (1991b), "Experiments on composite beams at service loads", *Civil Engineering Transactions, Institution of Engineers Australia,* **33**(4), 285-291.

Dezi, L., Ianni, C., and Tarantino, A.M. (1993), "Simplified creep analysis of composite beams with flexible connectors", *Journal of Structural Engineering, ASCE,* **119**(5), 1484-1497.

Gilbert, R.I. (1988), "Time effects in concrete structures", Elsevier, Amsterdam.

Gilbert, R.I. (1989), "Time-dependent analysis of composite steel-concrete sections", *Journal of Structural Engineering, ASCE,* **115**(11), 2687-2705.

Gilbert, R.I. and Gowripalan, N. (1994), "Time dependent behaviour of prestressed beams using non-metallic tendons", *Proceedings of the Australasian Structural Engineering Conference,* Sydney, 571-576.

Hoadley, P.G. (1963), "Behaviour of prestressed composite steel beams", *Journal of the Structural Division,*

ASCE, **89**(1), 21-34.
Johnson, R.P. (1995), *Composite Structures of Steel and Concrete.* Vol. **1** *Beams, Slabs, Columns and Frames for Buildings.* 2nd Edition, Blackwell Scientific.
Lawther, R. and Gilbert, R.I. (1990), "A rate of creep analysis of composite steel-concrete cross-sections", *The Structural Engineer,* **68**(11), 208-213.
Lawther, R. and Gilbert, R.I. (1992), "Deflection analysis of composite structures using the rate of creep method", *The Structural Engineer,* **70**(12), 220-223.
Li, W., Albrecht, P. and Saadatmanesh, H. (1995), "Strengthening of composite steel-concrete bridges", *Journal of Structural Engineering, ASCE,* **121**, 1842-1849.
Nakai, H., Kurita, A., Tomita, K. and Seno, Y. (1995), "Analysis for creep and shrinkage of steel-concrete composite girder bridges with external prestressing cables", *Proceedings of the Fifth East Asian Structural Engineering and Construction Conference, Building for the 21st Century,* 273-278.
Reagan, R.S. and Krahl, N.W. (1967), "Behavior of prestressed composite beams", *Journal of the Structural Division, ASCE,* **93**(6), 87-108.
Saadatmanesh, H., Albrecht, P., and Ayyub, B.M. (1989a), "Experimental study of prestressed composite beams", *Journal of Structural Engineering, ASCE,* **115**, 2348-2363.
Saadatmanesh, H., Albrecht, P., and Ayyub, B.M. (1989b), "Analytical study of prestressed composite beams", *Journal of Structural Engineering, ASCE,* **115**, 2364-2381.
Saadatmanesh, H., Albrecht, P., and Ayyub, B.M. (1992a), "Prestressed composite girders. I: Experimental study for negative moment", *Journal of Structural Engineering, ASCE,* **115**, 2743-2762.
Saadatmanesh, H., Albrecht, P., and Ayyub, B.M. (1992b), "Prestressed composite girders. I: Analytical study for negative moment", *Journal of Structural Engineering, ASCE,* **115**, 2763-2783.
Standards Association of Australia (1994), *AS3600 Concrete Structures,* Standards Association of Australia, Sydney, Australia.
Szilard, R. (1959), "Design of prestressed composite steel structures", *Journal of the Structural Division, ASCE,* **85** 97-123.
Troitsky, M.S., Zielinski, Z.A. and Nouraeyan, A. (1989), "Pre-tensioned and post-tensioned composite girders", *Journal of Structural Engineering, ASCE,* **115**, 3142-3153.
Uy, B. and Bradford, M.A. (1993), "Cross-sectional deformation of prestressed composite tee-beams", *Structural Engineering Review,* **5**(1), 63-70.
Wright, H.D., Vitek, J.L. and Rakib, S.N. (1992), "Long-term creep and shrinkage in composite beams with partial connection", *Structures and Buildings, Proceedings of the Institution of Civil Engineers,* **94**, 187-195.

Notations

A area
A_{sl} area of reinforcing steel
A_{ss} area of steel section
b width of concrete slab
B first moment of area
d_p distance to prestressing steel centroid
d_{ss} distance to steel section centroid
D depth of composite tee beam
D_c depth of concrete slab
E_c elastic modulus of concrete
E_s elastic modulus of reinforcing steel
E_{ss} elastic modulus of steel section
I second moment of area
M_i resultant bending moment
M_s applied service load bending moment
n modular ratio
N_i resultant axial force

P_b balanced prestressing force
P_i initial prestressing force
R relaxation force in prestressing tendon
t time in days
y distance from top fibre
χ aging coefficient
δ deflection
ε strain
ϕ creep coefficient
ρ curvature
σ stress
τ age at loading