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Rate-sensitive analysis of framed structures Part I: model formulation and verification

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Abstract. This paper presents a new uniaxial material model for rate-sensitive analysis addressing both the transient and steady-state responses. The new model adopts visco-plastic theory for the rate-sensitive response, and employs a three-parameter representation of the overstress as a function of the strain-rate. The third parameter is introduced in the new model to control its transient response characteristics, and to provide flexibility in fitting test data on the variation of overstress with strain-rate. Since the governing visco-plastic differential equation cannot be integrated analytically due to its inherent nonlinearity, a new single-step numerical integration procedure is proposed, which leads to high levels of accuracy almost independent of the size of the integration time-step. The new model is implemented within the nonlinear analysis program ADAPTIC, which is used to provide several verification examples and comparison with other experimental and numerical results. The companion paper extends the three-parameter model to trilinear static stress-strain relationships for steel and concrete, and presents application examples of the proposed models.

Key words: strain-rate effect; material model; nonlinear structural response.

1. Introduction

Recent advances in computational methods, coupled with the emergence of powerful computers, have provided a tremendous opportunity for the realistic assessment of the response of structures subject to severe dynamic loads, such as earthquakes and explosions. Although it is well established that the rate of straining can affect significantly the material response, it is still common practice to use material models which do not account for the strain-rate effect when assessing the dynamic response of structures.

The most notable outcome of experiments undertaken on structural steel is that the yield stress increases with the rate of straining, and hence a distinction can be made between *static* and *dynamic* yielding. Several material models were proposed to reflect the effect of the strain-rate on the elasto-plastic material response (Malvern 1951, Bodner and Symonds 1960, Perzyna 1966, Chang, *et al.* 1989, Manzocchi 1991). Amongst these models, the elastic/visco-plastic model incorporating the so-called *overstress* concept, which was developed by Malvern (1951) and Perzyna (1966), received much attention mainly due to its simplicity. Since overstress models are established within the framework of the classic theory of plasticity, different hardening rules can be readily incorporated to represent the material response more accurately under general loading.

In this paper, a new elastic/visco-plastic model is proposed, which is based on a bilinear

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static stress-strain relationship with kinematic strain hardening. The new model employs a three-parameter rate-function, which is an extension of the two-parameter function used by Manzocchi (1991). Apart from the additional flexibility that this provides in fitting experimental data, the third parameter is shown to control the transient response of the model, particularly with regard to sudden transitions between high and low strain-rates.

The new three-parameter model is implemented within the nonlinear analysis program ADAPTIC (Izzuddin 1991), where it is utilised by an elasto-plastic beam-column formulation (Izzuddin & Elnashai 1993a) accounting for the spread of plasticity within the cross-section and along the element. The response of structural steel to monotonic and cyclic loading, including variable strain-rates and stress relaxation, is investigated using the proposed model, and comparisons are made against available experimental and numerical results. The companion paper (Fang & Izzuddin 1997) extends the proposed model to trilinear static stress-strain relationships for steel and concrete, and presents several application examples of these models.

2. Elastic/visco-plastic theory

The proposed rate-sensitive model is based on the general elastic/visco-plastic equations, proposed by Malvern (1951) and Perzyna (1966). According to Perzyna (1966), the response of an elastic/visco-plastic material consists of an elastic part, which develops instantaneously, and a time-dependent visco-plastic part, which is related to the overstress. For a uniaxial stress state, the visco-plastic rate-sensitive response is hence described by the following equations:

$$\dot{\varepsilon}=\dot{\varepsilon}_e+\dot{\varepsilon}_p \tag{1}$$

$$\dot{\varepsilon}_e=\frac{\dot{\sigma}}{E} \tag{2}$$

$$\dot{\varepsilon}_p=\frac{f\langle X\rangle}{E} \tag{3}$$

$$X=\sigma-g(\varepsilon) \tag{4}$$

where (ε) is the total strain, (ε_e) is the elastic strain, (ε_p) is the plastic strain, (σ) is the stress, (E) is Young's elastic modulus, (X) is the overstress, the dot ($\cdot$) denotes the rate of variation with respect to time, and $g(\varepsilon)$ represents the static stress-strain relationship in the plastic range. The function $f\langle X\rangle$ is the *rate-function*, which reflects the rate-sensitivity of the material. In this function, the bracket $\langle\ \rangle$ implies that the expression is activated only when ($X>0$), that is when the stress point is outside the plastic limit curve.

For analytical purposes, it is more convenient to combine Eqs. (1)-(3) into the following expression:

$$E\dot{\varepsilon}-\dot{\sigma}=f\langle X\rangle \tag{5}$$

In the case of a bilinear static stress-strain relationship, shown in Fig. 1, the plastic limit curve is given by:

$$g(\varepsilon)=\sigma_y+\mu E(\varepsilon-\varepsilon_y) \tag{6}$$

where (μ) is the hardening parameter of the material.

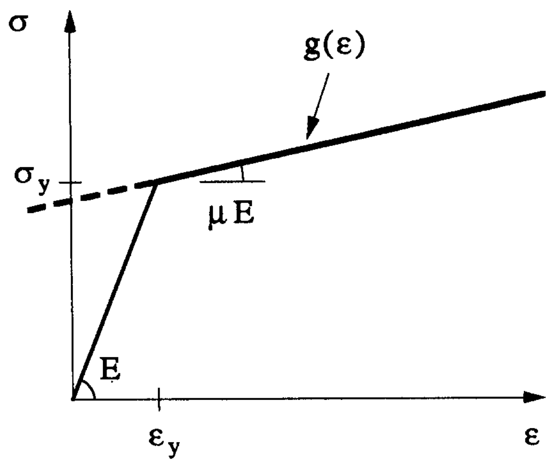

Fig. 1 Bilinear stress-strain relationship.

The combination of Eq. (4) and Eq. (6) leads to the following relationship between the stress-rate, overstress-rate and strain-rate:

$$\dot{\sigma}=\dot{X}+\dot{g}(\varepsilon)=\dot{X}+\mu E\,\dot{\varepsilon} \tag{7}$$

which can be substituted in Eq. (5) to provide a first-order differential equation for (X), given a prescribed strain-rate ($\dot{\varepsilon}$):

$$\dot{X}+f\langle X\rangle=E(1-\mu)\dot{\varepsilon} \tag{8}$$

For a constant strain-rate ($\dot{\varepsilon}$), the overstress (X) can be shown to achieve a *steady-state* value (i.e., $\dot{X}=0$) which is obtained from the inverse of the rate function:

$$X=f^{-1}\langle E(1-\mu)\dot{\varepsilon}\rangle=F\langle\dot{\varepsilon}\rangle \tag{9}$$

The rate-function $f\langle X\rangle$ also affects the *transient* variation of overstress (X), which can be determined through the integration of the differntial equation in Eq. (8). Depending on the choice of $f\langle X\rangle$, the integration may be performed analytically, as with Manzocchi's two-parameter function (1991), but for the majority of rate-functions the integration can only be evaluated numerically.

3. Rate-functions for steel

Experiments conducted on structural steel demonstrate that the yield stress increases with the strain-rate, and that the plastic plateau region is more rate-sensitive than the strain-hardening region. This behaviour is often idealised by assuming a bilinear static stress-strain relationship, with the rate-sensitive response determined from a series of experiments at constant strain-rates. Essentially, these experiments are concerned with the determination of values for the steady-state overstress (X) at prescribed constant strain-rates ($\dot{\varepsilon}$), which can be used to establish the rate-function $f\langle X\rangle$, or its inverse $F\langle\dot{\varepsilon}\rangle$, which best fit the experimental results.

Several forms of $F\langle\dot{\varepsilon}\rangle$, have been proposed for structural steel, such as the empirical expression given by Soroushian and Choi (1987), which is valid for strain-rates ranging from (10^{-5} sec^{-1}) to (10 sec^{-1}):

$$X=\sigma_y[(0.46-0.451\times10^{-6}\sigma_y)+(0.0927-9.2\times10^{-7}\sigma_y)\log_{10}(\dot{\varepsilon})] \tag{10}$$

where (σ_y) is the static yield strength of steel in (psi).

Another expression recommended by the CEB (1988) for steel is given by:

$$X=c\ ln\left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_s}\right) \tag{11}$$

where (c) and ($\dot{\varepsilon}_s$) are material constants, with suggested values of (c=6 MPa) and ($\dot{\varepsilon}_s=5\times10^{-5}$ sec^{-1}).

Alternative descriptions of the steady-state rate-sensitive response of steel have been made in terms of relationships between the dynamic yield stress and the total or plastic strain rate. Bodner and Symonds (1960) adopted the former description in their proposed expression:

$$\frac{\sigma}{\sigma_y}=1+\left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_0}\right)^{\alpha} \tag{12}$$

where (σ_y) is the static yield stress, (σ) is the steady-state dynamic yield stress, and (α) and ($\dot{\varepsilon}_0$) are material constants with suggested values for mild steel of (α=0.2) and ($\dot{\varepsilon}_0$=40 sec^{-1}).

Malvern (1951), on the other hand, related the plastic flow stress (σ) to the plastic strain-rate ($\dot{\varepsilon}_p$) using the following expression:

$$\sigma=g(\varepsilon)+s\ ln(1+b\ \dot{\varepsilon}_p) \tag{13}$$

in which (s) and (b) are material constants, and $g(\varepsilon)$ is the static stress-strain curve in the plastic range.

It is worth noting that Malvern's relationship in Eq. (13) is valid for both the steady-state and transient responses, since it expresses the overstress in terms of the plastic strain-rate, which is the inverse from of Eq. (3). For a bilinear stress-strain curve, the plastic strain-rate at steady-state (i.e., $\dot{X}=0$) is proportional to the total strain-rate, as can be established from Eqs. (3) and (8):

$$\dot{\varepsilon}_p=(1-\mu)\dot{\varepsilon} \tag{14}$$

Therefore, Malvern's expression Eq. (13) is equivalent to the two-parameter steady-state function employed by Manzocchi (1991):

$$X=S\ ln\left(1+\frac{\dot{\varepsilon}}{\dot{\varepsilon}*}\right) \tag{15}$$

where (S) and ($\dot{\varepsilon}*$) are material constants.

Comparison of the previous relationships at steady-state shows that the expressions can vary considerably, as shown in Fig. 2, which may be an indication of the scatter of available experimental data. Whilst the relationship in Eq. (12) utilises a power function, the other proposed relationships are logarithmic functions or they converge to logarithmic funcitions at high strain-rates. The use of such expressions within rate-sensitive models does not only lead to different steady-state responses, but it also results in different transient behavior.

The use of two-parameter logarithmic rate-functions, such as the functions proposed by Malvern

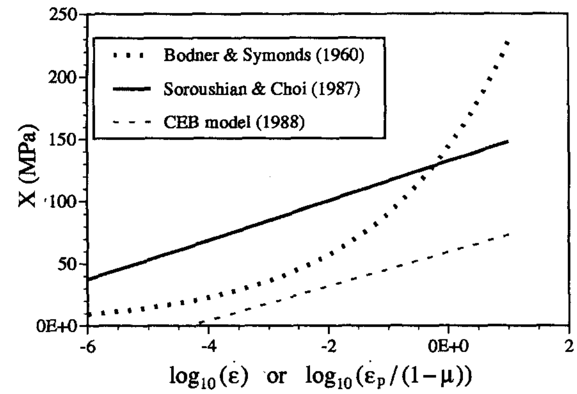

Fig. 2 Comparison of rate-functions for steel (σ_y=300 MPa).

(1951) and Manzocchi (1991), leads to excessive rates of relaxation, as shown in a later comparison against experimental results by Chang, *et al.* (1989). In addition, the application of such expressions to other materials, such as concrete, could lead to considerable inaccuracies, since the variation of overstress with strain-rate may not be logarithmic. As demonstrated later, the overall structural response can be considerably influenced by the choice of the particular rate-function, and hence it is important to choose one which describes accurately the variation of overstress over a wide range of strain-rates.

In the following section, a new three-parameter rate-function is proposed which addresses the aforementioned issues, and which is used as a basis for formulating a new rate-sensitive model for uniaxial stress states.

4. Three-parameter rate-sensitive model

A new rate-sensitive uniaxial material model is proposed, which is based on a three-parameter relationship between the steady-state overstress (X) and strain-rate ($\dot{\varepsilon}$):

$$X = F\langle\dot{\varepsilon}\rangle = SN\,ln\left(1+\left\{\frac{\dot{\varepsilon}}{\dot{\varepsilon}*}\right\}^{1/N}\right) \tag{16}$$

in which (S), ($\dot{\varepsilon}*$) and (N) are material constants.

The corresponding rate-function $f\langle X\rangle$, which relates the plastic strain-rate ($\dot{\varepsilon}_p$) to the overstress (X) as given by Eq. (3), can be obtained from the inverse of $F\langle\dot{\varepsilon}\rangle$ according to Eq. (9):

$$f\langle X\rangle = E(1-\mu)F^{-1}\langle X\rangle = E(1-\mu)\dot{\varepsilon}*\,(e^{X/SN}-1)^N \tag{17}$$

It is noted that the proposed three-parameter relationship reduces for (N=1) to the two-parameter expression employed by Manzocchi (1991). The introduction of the third parameter (N) is shown to control mainly the rate-sensitive response at low strain-rates, and hence enables more accurate prediction of the transient response, including stress-relaxation upon sudden reduction in the strain-rate. In addition, the three-parameter relationship provides more flexibility in fitting experimental data over a wider range of strain-rates, and is consequently applicable not only to steel but also to concrete and other materials.

4.1. Numerical integration

The proposed rate-sensitive model is implemented within an incremental procedure, where the strain-rate ($\dot{\varepsilon}$) is assumed constant for each time-step (Δt). To enable accurate representation of the transient response, the steady-state value of the overstress (X) corresponding to the strain-rate ($\dot{\varepsilon}$), as given by Eq. (16), is not instantly achieved. Instead, the solution to the viscoplastic differential equation in Eq. (8) is sought as an initial-value problem, where the overstress at the start of the time-step (X_0) is known, and its value at the end of time-step (X_1) is required.

The governing first-order differential equation can be obtained by substituting in Eq. (8) the rate-function proposed in Eq. (17):

$$\dot{X}+E(1-\mu)\,\dot{\varepsilon}_*(e^{X/SN}-1)^N=E(1-\mu)\,\dot{\varepsilon} \tag{18}$$

The variation of overstress (X) over the time-step (Δt) is illustrated in Fig. 3, where the overstress-rate ($\dot{X}$) is plotted aginst (X). Two forms of the transient response are established for which the initial overstress (X_0) is different from the steady-state value denoted henceforth by (X_s). The first, which corresponds to ($X_0<X_s$), is an *overstressing* response, since the overstress-rate is positive ($\dot{X}>0$). The second, which corresponds to ($X_0>X_s$), is a *relaxation* response, since the overstress-rate is negative ($\dot{X}<0$).

Except for the case ($N=1$), the solution of Eq. (18) can only be obtained numerically due to its inherent nonlinerity in the time-dependent variable (X). Two numerical integration techniques, based on *tangent* and *secant* approximation, are considered hereafter, and a new single-step numerical integration method combining the advantages of these two techniques is proposed.

4.1.1. Tangent approximation

The tangent approach is concerned with the linear expansion of the differential equation in Eq. (18) with respect to (X) about (X_0), as depicted graphically in Fig. 4. The resulting approximation leads to a linear first-order differential equation, given by:

$$\dot{X}+a_t X=a_t X_t \tag{19a}$$

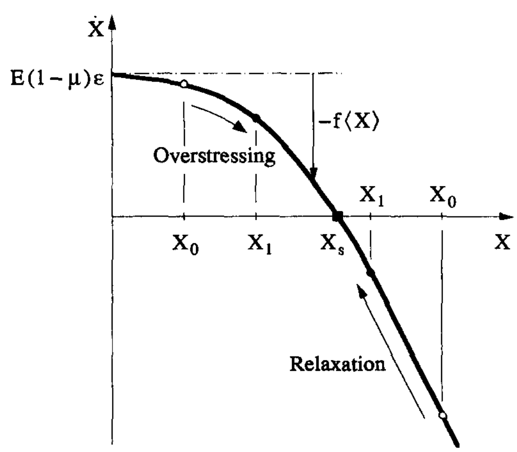

Fig. 3 Relaxation and overstressing transient states.

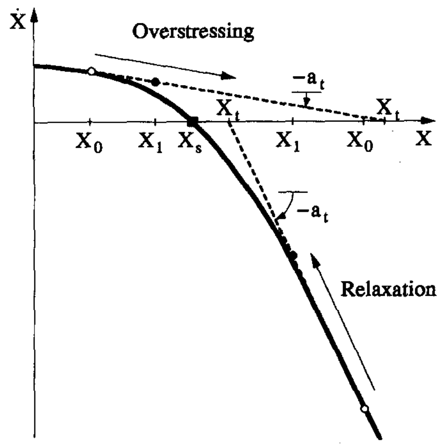

Fig. 4 Overstress prediction using tangent approach.

where,

$$a_t = \frac{E(1-\mu)\dot{\varepsilon}*}{S} e^{X_0/SN}(e^{X_0/SN}-1)^{N-1} \tag{19b}$$

$$X_t = X_0 + \frac{E(1-\mu)}{a_t}[\dot{\varepsilon} - \dot{\varepsilon}*(e^{X_0/SN}-1)^N] \tag{19c}$$

Therefore, a tangent approximation for the overstress (X_1) at the end of the time-step (Δt) can be obtained from the exponential solution of Eq. (19)

$$X_1 = (X_0 - X_t)e^{-a_t \Delta t} + X_t \tag{20}$$

This approachs is relatively accurate for a small time-step (Δt); however, if (Δt) is not sufficiently small, significant inaccuracies may result in the transient response prediction. As can be concluded from Fig. 4, the overstress (X_1) could overshoot considerably the steady-state value (X_s) in the overstressing transient phase, since (X_s) is much smaller than its tangent approximation (X_t). The same reason may also lead to the underestimation of overstress (X_1) in the case of relaxation.

4.1.2. Secant approximation

The secant approach is based on the linearisation of the differential equation in Eq. (18) with respect to (X) using its values at (X_0) and (X_s), as depicted graphically in Fig. 5. The resulting approximation leads to a linear first-order differential equation, given by:

$$\dot{X} + a_s X = a_s X_s \tag{21a}$$

where,

$$X_s = SN\, ln\left(1 + \left\{\frac{\dot{\varepsilon}}{\dot{\varepsilon}*}\right\}^{1/N}\right) \tag{21b}$$

$$a_s = \frac{E(1-\mu)}{(X_s - X_0)}[\dot{\varepsilon} - \dot{\varepsilon}*(e^{X_0/SN}-1)^N] \tag{21c}$$

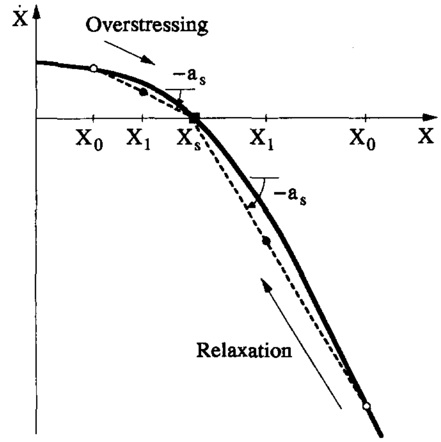

Fig. 5 Overstress prediction using secant approach.

A secant approximation for the overstress (X_1) at the end of the time-step (Δt) can be obtained from the exponential solution of Eq. (21):

$$X_1=(X_0-X_s)\,e^{-a_s\Delta t}+X_s \tag{22}$$

Contrary to the tangent approach, the secant approach is accurate for a large time-step (Δt), for which the overstress (X_1) approaches (X_s). However, this approach is inaccurate for small to moderate (Δt), since it leads to slower rates of overstressing and higher rates of relaxation in comparison with the tangent approach.

4.1.3. Proposed method

It has been shown in the previous sections that the tangent approach is accurate for a very small time-step ($\Delta t\to 0$), and that the secant approach is accurate for a very large time-step ($\Delta t\to\infty$). In view of this, a new integration method is proposed, which is also based on the linearisation of the governing differential equation; however, a modified slope (a_p) is chosen, as illustrated in Fig. 6, which depends on (Δt):

$$\dot{X}+a_pX=a_pX_p \tag{23a}$$

where,

$$X_p=X_0+\frac{E(1-\mu)}{a_p}[\dot{\varepsilon}-\dot{\varepsilon}*(e^{X_0/SN}-1)^N] \tag{23b}$$

Evidently, accuracy demands that (a_p) converges to the tangent slope (a_t) when ($\Delta t\to 0$), and that (a_p) converges to the secant slope (a_s) when ($\Delta t\to\infty$). In addition, for the intermediate range of ($\Delta t\in[0,\ \infty]$), the value of (a_p) should be between (a_t) and (a_s). In the proposed method, the proximity of (a_p) to (a_t) and (a_s) is determined by the relative value of (Δt) with respect to a reference value (Δt_r), which is obtained differently for the overstressing and relaxation transient responses.

For the overstressing response, (Δt_r) is chosen as the value of the time-step for which the

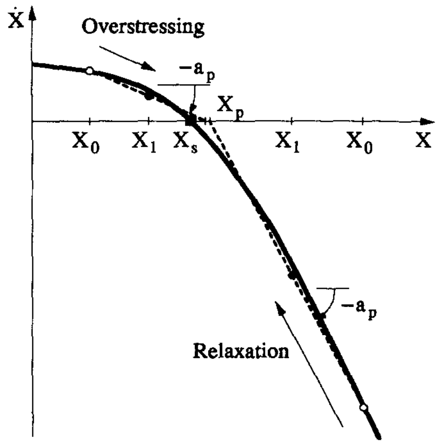

Fig. 6 Overstress prediction using proposed approach.

tangent approximation of the overstress in Eq. (20) is identical to the steady-state value (X_s), as illustrated graphically in Fig. 7a:

$$\Delta t_r = -\frac{1}{a_t} ln\left(\frac{X_s - X_t}{X_0 - X_t}\right) \quad \text{for } (X_0 < X_s) \tag{24a}$$

The dependence of (a_p) on (Δt) for overstressing is expressed through a negative exponential function, as depicted in Fig. 8:

$$a_p = a_s - (a_s - a_t) e^{-\Delta t/\Delta t_r} \quad \text{for } (X_0 < X_s) \tag{24b}$$

The above function satisfies the accuracy requirements for the limits ($\Delta t \to 0$) and ($\Delta t \to \infty$), and is shown to provide very good accuracy for the overstressing response in the intermediate range of (Δt).

For the relaxation response, (Δt_r) is chosen as the value of the time-step for which the secant approximation of the overstress-rate, using Eq. (21a) and Eq. (22), is identical to the exact overstress-rate at the tangent steady-state overstress (X_t), as illustrated graphically in Fig. 7b:

$$\Delta t_r = -\frac{1}{a_s} ln\left(\frac{\dot{X}_t}{a_s (X_s - X_0)}\right) \quad \text{for } (X_0 > X_s) \tag{25a}$$

where,

$$\dot{X}_t = E(1-\mu)\dot{\varepsilon} - f\langle X_t \rangle = E(1-\mu)[\dot{\varepsilon} - \dot{\varepsilon}*(e^{X_t/SN} - 1)^N] \tag{25b}$$

The dependence of (a_p) on (Δt) for relaxation is expressed through a negative exponential function operating on the reciprocal of (Δt), which is depicted in Fig. 8:

$$a_p = a_t + (a_s - a_t) e^{-\Delta t_r/\Delta t} \quad \text{for } (X_0 > X_s) \tag{25c}$$

As for the overstressing case, the above function satisfies the accuracy requirements for the limits ($\Delta t \to 0$) and ($\Delta t \to \infty$). In addition, the particular choice of (Δt_r) and (a_p) according to Eq. (25) is shown to provide very good accuracy for the relaxation response in the intermediate range of (Δt).

Once the value of (a_p) is established using Eq. (24) or Eq. (25), the overstress at the end

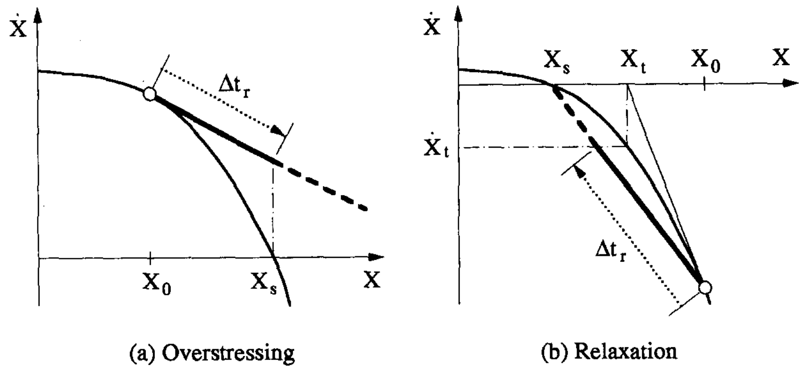

Fig. 7 Graphical representation of (Δt_r) for overstressing and relaxation.

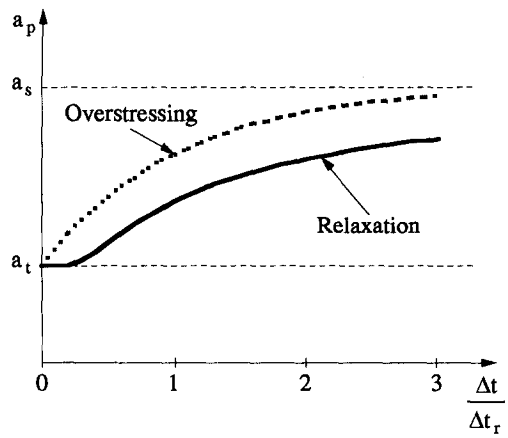

Fig. 8 Variation of (a_p) with the time-step (Δt).

of the timestep (X_1) is obtained from the solution of the linear differential equation in Eq. (23):

$$X_1 = (X_0 - X_p)\, e^{-a_p \Delta t} + X_p \tag{26}$$

In the special case when the previous overstress is identical to the current steady-state value ($X_0 = X_s$), the overstess at the end of the time-step is also identical to the steady-state value ($X_1 = X_s$).

The proposed method is a single-step integration technique, which avoids sub-incrementation in the material stress calculations when the time-step (Δt) is not sufficiently small. This leads to considerable computational savings in determining the material rate-sensitive response, which are reflected by commensurate efficiency in the prediction of the overall structural response.

4.2. Stress and tangent modulus calculation

As pointed out earlier, the proposed three-parameter model is implemented within an incremental procedure, where the stress and strain at the beginning of the increment, (σ_0) and (ε_0), are known. In addition, the strain at the end of the increment (ε_1) is established from the iterative element displacements, as described in the following section, for which the corresponding stress at the end of the increment (σ_1) is sought.

Within the current increment or time-step (Δt), the strain-rate ($\dot{\varepsilon}$) is assumed to be constant, as given by:

$$\dot{\varepsilon}=\frac{\varepsilon_1-\varepsilon_0}{\Delta t} \tag{27}$$

For the case of a plastic stress-state at the beginning of the increment (i.e., $X_0 \geq 0$), the overstress at the end of the increment (X_1) can be determined using the numerical integration approach proposed in the previous section. Therefore, the stress at the end of the increment (σ_1) for a bilinear static stress-strain relationship can be obtained from (X_1) using Eqs. (4) and (6):

$$\sigma_1=X_1+\sigma_y+\mu E(\varepsilon_1-\varepsilon_y) \tag{28}$$

In order to guide the iterative solution procedure during the current increment, a tangent modulus (E_t) is required, which expresses the infinitesimal variation of (σ_1) with respect to (ε_1):

$$E_t=\frac{d\sigma_1}{d\varepsilon_1} \tag{29}$$

Combining Eqs. (27)-(29), the tangent modulus (E_t) can be related to the first derivative of (X_1) with respect to ($\dot{\varepsilon}$), which can be determined from Eq. (26):

$$E_t=\frac{1}{\Delta t}\frac{dX_1}{d\dot{\varepsilon}}+\mu E \tag{30a}$$

$$\frac{dX_1}{d\dot{\varepsilon}}=\{1-e^{-a_p\Delta t}\}\frac{dX_p}{d\dot{\varepsilon}}-\{\Delta t(X_0-X_p)\,e^{-a_p\Delta t}\}\frac{da_p}{d\dot{\varepsilon}} \tag{30b}$$

The first derivatives of (a_p) and (X_p) are established from the proposed equations for the overstressing and relaxation transient responses, noting that in these equations (X_t), (a_s), (X_s) and (Δt_r) are dependent on the strain-rate ($\dot{\varepsilon}$).

The companion paper (Fang & Izzuddin 1997) provides full implementation details for the case of an elastic stress-state at the beginning of the increment as well as the case of plastic-unloading. In addition, the companion paper discusses the extension of the bilinear static stress-strain relationship to trilinear relationships for steel and concrete.

5. Beam-column formulation

The proposed three-parameter model can be employed by any formulation requiring the uni-axial rate-sensitive material response. In the present work, the new model is utilised by a one-dimensional elasto-plastic cubic formulation (Izzuddin & Elnashai 1993a) derived in a local Eulerian system (Izzuddin & Elnashai 1993b), where the effects of large displacements and finite rotations are accounted for.

The cubic formulation is capable of modelling the spread of plasticity over the cross-section and along the member, through the use of monitoring areas over cross-sections at two Gauss integration points. In addition, the cubic formulation is implemented within the nonlinear analysis program ADAPTIC (Izzuddin 1991), where it is utilised within an elasto-plastic adaptive procedure (Izzuddin & Elnashai 1993a). Through the application of selective mesh refinement *when* and *where* necessary, during analysis and within the structure respectively, adaptive elasto-plastic analy-

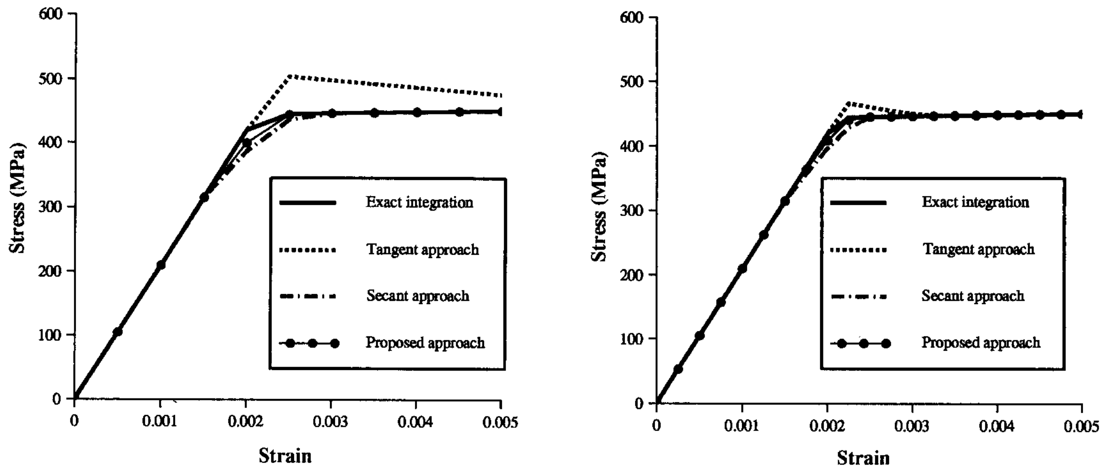

Fig. 9 (a) Overstressing transient response (Δt=0.05 msec),
(b) Overstressing transient response (Δt=0.025 msec).

sis has been shown to achieve considerable modelling and computational savings, often in excess of 80% (Izzuddin & Elnashai 1993a).

6. Verification

Several examples are presented hereafter to demonstrate the response characteristics of the proposed three-parameter rate-sensitive model, as well as to illustrate the effectiveness of the proposed single-step numerical integration method. For this purpose, ADAPTIC *v*2.5.1. (Izzuddin 1991) is used to facilitate comparison of the results from the proposed model against those from other analytical and experimental research work, on both the material and overall structural levels.

6.1. Numerical integration

In order to demonstrate the accuracy of the proposed numerical integration method, a rates ensitive material with a bilinear static stress-strain relationship ($E=210\times10^3$ MPa, $\sigma_y=300$ MPa, $\mu=0.01$) and rate-function ($S=6.7$ MPa, $\dot{\varepsilon}*=5.3\times10^{-9}$ sec^{-1}, $N=1$) is subjected to a constant strain-rate ($\dot{\varepsilon}=10$ sec^{-1}). The material constant ($N=1$) for rate sensitivity is specified to enable comparison of the proposed integration method against exact integration (Manzocchi 1991), since analytical integration is only possible for this special case. The results in Fig. 9a, for a time-step ($\Delta t=0.05$ msec), show that the tangent approach leads to considerable overshooting of the steady-state overstress, whereas the secant approach underestimates the transient response to a greater extent than the proposed approach. The use of a smaller time-step, ($\Delta t=0.025$ m sec), improves marginally the predictions of the tangent and secant methods, but leads to superior accuracy with the proposed integration method, as shown in Fig. 9b.

The accuracy of the various integration methods is investigated for stress-relaxation, where the strain-rate is reduced at a strain ($\varepsilon=0.01$) from ($\dot{\varepsilon}=10$ sec^{-1}) to ($\dot{\varepsilon}=10^{-4}$ sec^{-1}). The results in Fig. 10 show that the tangent approach predicts a slow rate of relaxation, although the predic-

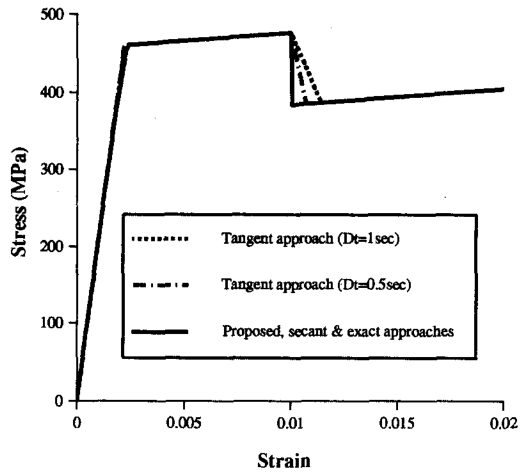

Fig. 10 Relaxation transient response.

tion is marginally improved when the time-step used in the relaxation phase is reduced from ($\Delta t = 1$ sec) to ($\Delta t = 0.5$ sec). The results also demonstrate the accuracy of the proposed integration method, which provides an exact prediction of the stress-relaxation transient response.

6.2. Stress-relaxation

Chang, *et al.* (1989) presented the results of experiments on mild steel, where the rate-sensitive material response was investigated at two constant strain-rates of (10^{-5} sec^{-1}) and (10^{-2} sec^{-1}). The two corresponding steady-state overstress values in the plastic plateau range, ($X = 5.5$ ksi) and ($X = 13.5$ ksi), are used in this example to determine the material constants for the three-parameter rate-function, ($S = 2.87$ ksi) and ($\dot{\varepsilon}* = 3.54 \times 10^{-3}$ sec^{-1}), assuming that ($N = 6$). The same two steady-state points can be fitted by the two-parameter function employed by Manzocchi (1991), for which the resulting material constants would be ($S = 1.159$ ksi) and ($\dot{\varepsilon}* = 8.793 \times 10^{-8}$ sec^{-1}).

The two-parameter function and the three-parameter function with ($N = 6$) are plotted in Fig. 11, where they are shown to coincide at the two constant strain-rates, (10^{-5} sec^{-1}) and (10^{-2} sec^{-1}), assumed in the experiments (Chang, *et al.* 1989). However, there are considerable differences betwen the two functions at very low strain-rates, for which there is no direct experimental data. The importance of such differences becomes apparent in the transient material response from high to low strain-rates, which leads to stress-relaxation.

To illustrate the above point, successive programmes, consisting of a constant strain-rate ($\dot{\varepsilon} = 10^{-4}$ sec^{-1}) followed by (10 min) of stress-relaxation at zero strain-rate, are applied. The resulting material response, using the proposed three-parameter model and Manzocchi's two-parameter model (1991), is depicted in Figs. 12a and 12b in the form of stress-strain and stress-time curves, respectively. Whilst experimental evidence (Chang, *et al.* 1989) shows that the overstress relaxes by approximately (60%) by the end of (10 min), Manzocchi's two-parameter model (1991) achieves a (90%) relaxation within (5 min), and almost full relaxation within (10 min). The proposed three-parameter model, on the other hand, is much closer to the experimental evidence, where a (68%) relaxation is achieved by the end of (10 min).

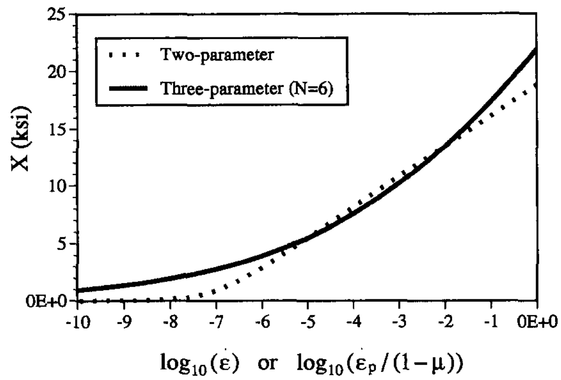

Fig. 11 Fitting two steady-state points using two- and three-parameter functions.

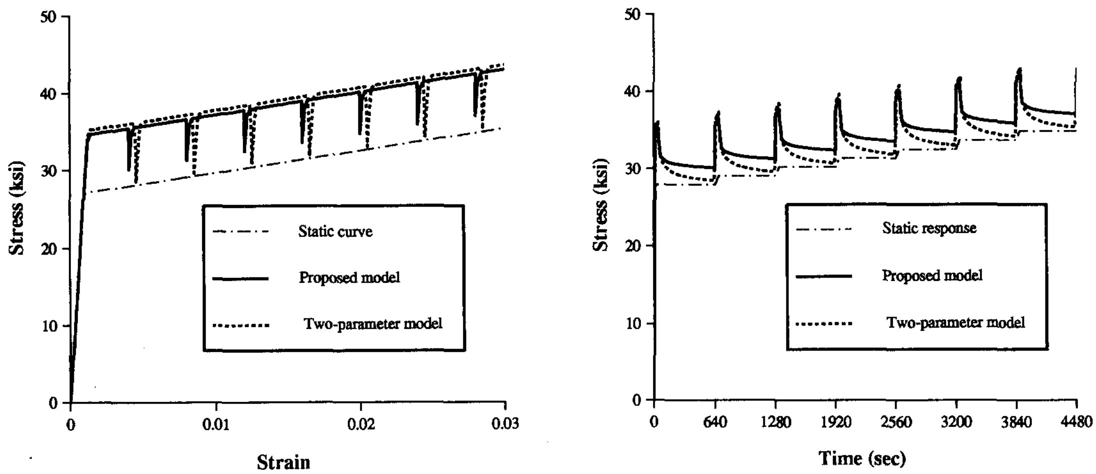

Fig. 12 (a) Stress-relaxation: Variation of stress with strain,
(b) Stress-relaxation: Variation of stress with time.

6.3. Beam subjected to blast loading

A simply supported beam is subjected to a blast loading, which is simulated by a midspan force varying according to a triangular pulse, as shwon in Fig. 13. Considering the problem symmetry, only half of the beam length is modelled, where 10 cubic elements are employed. The proposed three-parameter function and Manzocchi's (1991) two-parameter function are used to fit the expression suggested by Bodner and Symonds (1960) for the steady-state overstress, where the method of least squares is employed to determine the material constants of rate-sensitivity. As shown in Fig. 14, the proposed three-parameter rate-function, with (S=53.05 MPa), ($\dot{\varepsilon}*$=0.2445 sec^{-1}) and (N=3), enables a more accurate representation of the Bodner and Symonds expression than the two-parameter function, for which (S=31.19 MPa) and ($\dot{\varepsilon}*$=4.65$\times$10^{-3} sec^{-1}).

The effect of rate-sensitivity on the I-beam response is depicted in Fig. 15, where it is shown that the midspan displacement is overestimated by approximately (450%) if the strain-rate effect is ignored. A detailed comparison between the two-parameter model (Manzocchi 1991) and the

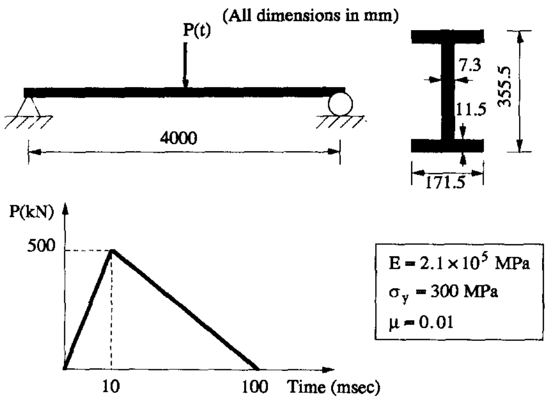

Fig. 13 Geometric configuration and loading of *I*-beam.

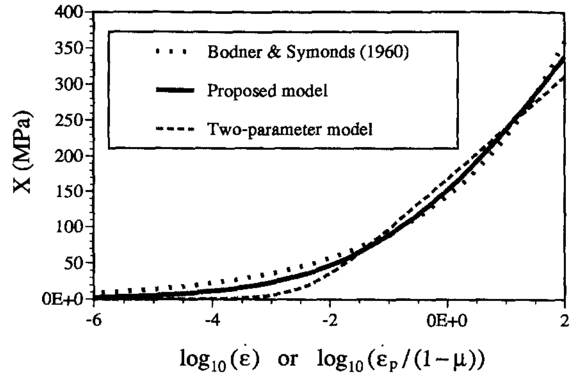

Fig. 14 Curve fitting using two-parameter and proposed three-parameter functions.

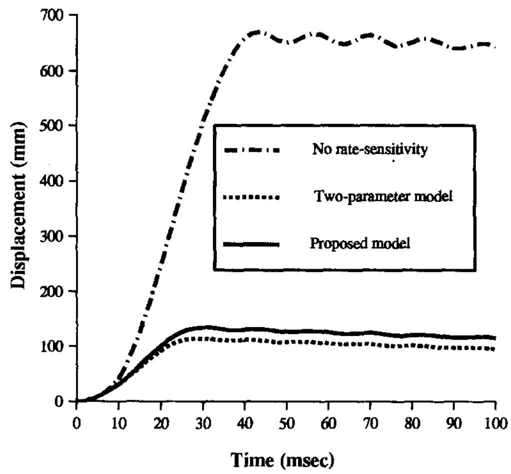

Fig. 15 Effect of rate-sensitivity on the midspan displacement of the I-beam.

proposed three-parameter model is provided in Fig. 16, where three different time-steps are used ($\Delta t = 0.1$, 1 and 2 mesc). The results indicate that the maximum midspan displacement predicted by the two-parameter model is approximately (15%) less than that predicted by the proposed model. The use of a larger time-step leads to slightly larger displacements with both models, mainly due to inaccurate integration of the governing equations of motion.

The (15%) discrepancy between the predictions of the two-parameter model and the proposed three-parameter model (with the original material constants) is attributed mainly to the inability of the two-parameter rate-function to represent the Bodner and Symonds expression accurately. As shown in Fig. 17a, the I-beam response is almost quasi-static, where the variation of the extreme fibre midspan stress with time is similar to the applied pulse, and where only minor differences are observed between the predictions of the two- and three-parameter models. However, the results of the two models for the variation of extreme fibre midspan strain with time demonstrate considerable discrepancies in Fig. 17b, with the two-parameter model underestimating the maximum strain by approximately (12%). This is attributed to the two-parameter model underesti-

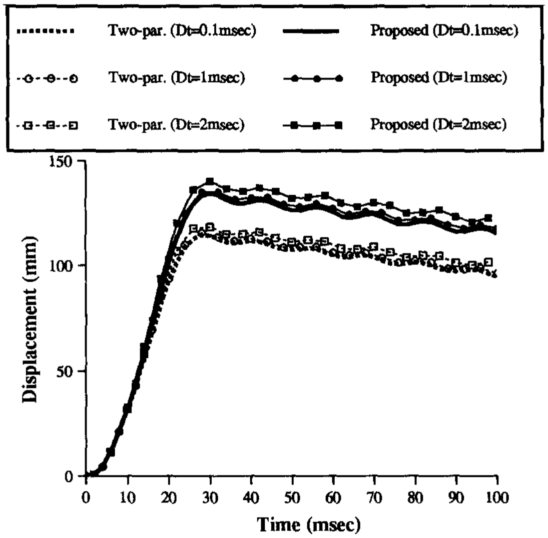

Fig. 16 Rate-sensitive response using two- and three-parameter models.

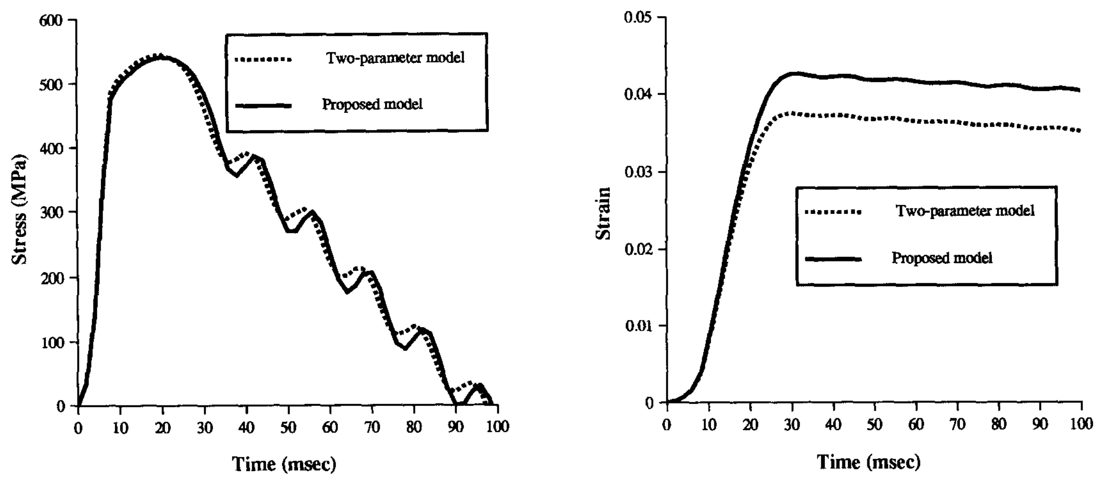

Fig. 17 (a) Variation of midspan stress with time,
(b) Variation of midspan strain with time.

mating the plastic strain-rate ($\dot{\varepsilon}_p$) for overstresses in the range (100-250 MPa) (Fig. 14), which is the range of significant overstresses for this problem, as can be inferred from Fig. 17a.

7. Conclusions

This paper presents a new uniaxial material model which accounts for rate-sensitivity according to visco-plastic theory. The proposed model is based on a three-parameter rate-function, as well as a bilinear static stress-strain relationship. The three-parameter function subsumes a previous commonly used two-parameter function, fits experimental data more closely, and enables the transient response, including stress-relaxation, to be modelled more accurately.

A new single-step method is proposed for the integration of the resulting visco-plastic differential equation, which is shown to be fairly insensitive to the size of the integration time-step. The calculation of stresses from strains at the end of the current incremental step is outlined, and the incorporation of the proposed model within an elasto-plastic cubic formulation is described.

Several examples are used to show the effectiveness of the proposed integration method, the accuracy of the proposed rate-sensitive model in comparison with experimental results, as well as the ability of the new model to predict stress-relaxation more accurately than the previous two-parameter model. The last example demonstrates that the three-parameter rate-function enables better fitting of steady-state overstress expressions than the two-parameter function, and shows that difference between the two rate-functions could lead to considerable discrepancies in the overall structural response.

The companion paper extends the proposed three-parameter model to trilinear static stress-strain relationships for steel and concrete, providing for steel a more accurate representation of the rate-sensitive response in the strain-hardening range. The paper also discusses the implementation of the rate-sensitive models, and presents several verification and application examples showing the utility of the proposed models.

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