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Adaptive kernel method for evaluating structural system reliability

G.S. Wang†, A. H-S. Ang‡ and J-C. Lee‡†

Department of Civil & Environmental Engineering
University of California, Irvine, California 92697, U.S.A.

Abstract. Importance sampling methods have been developed with the aim of reducing the computational costs inherent in Monte Carlo methods. This study proposes a new algorithm called the adaptive kernel method which combines and modifies some of the concepts from adaptive sampling and the simple kernel method to evaluate the structural reliability of time variant problems. The essence of the resulting algorithm is to select an appropriate starting point from which the importance sampling density can be generated efficiently. Numerical results show that the method is unbiased and substantially increases the efficiency over other methods.

Key words: importance sampling; adaptive kernel method; structural system reliability; time variant problems; random dynamic loadings; uncertain nonlinear hysteretic structures.

1. Introduction

The basic step of simulation generally consists of simulating the behavior of a system using specific realization of element random variables and determining whether a system failure occurs in each simulation. The system failure probability is estimated as the ratio of the number of failures to the total number of simulations. Because a low failure probability is usually expected in structural systems, the total number of simulations required to obtain a failure probability with low variance can be extremely large. To reduce the number of simulations and the statistical error of the failure probability in the Monte Carlo method for time-invariant systems, various important sampling schemes have been proposed. Recently, a benchmark study (Engelund and Rackwitz 1993) was performed to evaluate various importance sampling schemes. A general conclusion of the study is that for practical applications the selection among the various schemes should be based on the characteristics of the particular problem. The benchmark study identified the following 4 major schemes:

(1) direct methods (Ibrahim 1991, Melchers 1989, Schueller and Stix 1987, Maes, *et al.* 1992)
(2) updating methods (Hohenbichler and Rackwitz 1988),
(3) adaptive sampling (Bucher 1988, Melchers 1990, Karamchandani, *et al.* 1989), and
(4) spherical sampling (Bjerager 1988, Ditlevsen, *et al.* 1990).

For time-variant reliability analysis, various importance sampling methods (Melchers 1994, Mori and Ellingwood 1993) have been proposed. However, in these methods it is necessary

† Ph.D.
‡ Professor
‡† Ph.D. Student

that the conditional failure probability for a given set of time-invariant variables be derived in closed form.

As a different importance sampling method, a simple kernel formulation (Ang, *et al.* 1992) was proposed to construct the importance sampling density as a weighted sum of kernel functions. The main drawback of the simple kernel method is the need to perform Monte Carlo simulation in order to construct the kernel density function. If the probability of failure is very small, the required Monte Carlo simulations can still be costly. Importance sampling with the kernel sampling density function, however, insures unbiasedness in the results. The objective of this paper is to employ the technique of adaptive sampling to improve the simple kernel method so that an effective simulation method, called *adaptive kernel method*, can be developed to calculate the system reliability.

In the evaluation of structural reliability under stochastic dynamic loadings, structural parameters such as stiffness, damping and strength, are seldom perfectly known. The effect of these and other uncertainties may be important in the overall reliability of the structural system. In order to investigate this effect and demonstrate the ability of the proposed adaptive kernel method in solving time variant problems, a single degree-of-freedom and a multiple degree-of-freedom uncertain dynamic systems are investigated. Nonlinear hysteretic models are considered for the restoring forces of these dynamic systems.

2. Mathematical background

The basic equation for the probability of failure is

$$p_f = \int_{D_f} f_X(\boldsymbol{x}) d\boldsymbol{x} \tag{1}$$

where D_f is the failure domain, $\boldsymbol{X}$ the vector of basic variables with the joint probability density function (PDF) $f_X(\boldsymbol{x})$. Eq. (1) can also be written as

$$p_f = \int_X I[g(\boldsymbol{x})] f_X(\boldsymbol{x}) d\boldsymbol{x} \tag{2}$$

where

$$I[g(\boldsymbol{x})] = \begin{cases} 1 & \text{if } g(\boldsymbol{x}) \leq 0 \\ 0 & \text{if } g(\boldsymbol{x}) > 0 \end{cases}$$

in which $g(\boldsymbol{x})$ is the performance function. Then, by Monte Carlo simulation, the probability of failure is estimated as

$$\hat{p}_f = \frac{1}{N} \sum_{i=1}^{N} I[g(\boldsymbol{x}_i)] \tag{3}$$

where N is the number of Monte Carlo samples.

If p_f is small, however, the number of Monte Carlo sampling, N, has to be very large to obtain sufficient number of failure samples satisfying $g(\boldsymbol{x}) \leq 0$. To increase the number of failure samples for given N and, thus, reduce the variance of the probability estimator, the importance sampling technique can be used where the sampling is biased toward the failure domain as

follows:

$$p_f = \int_X I[g(\boldsymbol{x})] \frac{f_X(\boldsymbol{x})}{h_X(\boldsymbol{x})} h_X(\boldsymbol{x}) d\boldsymbol{x} \tag{4}$$

where $h_X(\boldsymbol{x})$ is the PDF of importance sampling. Accordingly, the estimate of the failure probability becomes

$$\hat{p}_f = \frac{1}{N} \sum_{i=1}^{N} I[g(\boldsymbol{x}_i)] \frac{f_X(\boldsymbol{x}_i)}{h_X(\boldsymbol{x}_i)} \tag{5}$$

and the variance of the estimator can be also estimated during the sampling process as follows:

$$Var(\hat{p}_f) = \frac{1}{N-1} \left\{ \frac{1}{N} \sum_{i=1}^{N} I[g(\boldsymbol{x}_i)] \frac{f_X^2(\boldsymbol{x}_i)}{h_X^2(\boldsymbol{x}_i)} - \hat{p}_f^2 \right\} \tag{6}$$

Clearly, the appropriate choice of the importance sampling PDF, $h_X(\boldsymbol{x})$, is crucial for the proper estimate of p_f with Eq. (5). Observe that the statistical error of Eq. (5) reduces to zero if $h_X(\boldsymbol{x})$ is the original PDF, $f_X(\boldsymbol{x})$, conditional on the failure domain D_f; i.e.,

$$h_X(\boldsymbol{x}) = f_X(\boldsymbol{x} | \boldsymbol{x} \in D_f) = \frac{I[g(\boldsymbol{x})] f_X(\boldsymbol{x})}{p_f} \tag{7}$$

It is, however, difficult to find this function. With the simple kernel method, Ang, *et al.* (1992) suggested the following for constructing the importance sampling PDF that will insure unbiasedness in the result of Eq. (5).

$$\hat{h}_X(\boldsymbol{x}) = \frac{1}{M} \sum_{i=1}^{M} \frac{1}{(\lambda_i w)^n} K\left(\frac{\boldsymbol{x} - \boldsymbol{y}_i}{\lambda_i w} \right) \tag{8}$$

where; $\boldsymbol{y}_i$ = failure sample generated from the original joint PDF, $f_X(\boldsymbol{x})$,
M = number of failure samples
$K(\cdot)$ = kernel function satisfying $\int_Y K(\boldsymbol{y}) d\boldsymbol{y} = 1$,
w = window width,
λ_i = scale parameter, and
n = number of random variables.

Criteria for selecting w and λ_i are suggested by Ang, *et al.* (1992) including a Gaussian PDF for $K(\cdot)$. The above simple kernel method suffers from having to generate the M failure samples from $f_X(\boldsymbol{x})$ through basic Monte Carlo simulation. This is the main drawback particularly when the failure probability is small.

To overcome this weakness, the simple kernel method can be modified by introducing an adaptive sampling scheme (Bucher 1988) to generate the M failure samples. These failure samples can be generated by using the original densities of the basic variables shifted to a starting point, $\overline{\boldsymbol{X}}_h$, and then $\hat{h}_X(x)$ is constructed on the basis of the kernel function in Eq. (8). The starting point, $\overline{\boldsymbol{X}}_h$, is obtained by the weighted average (Bucher 1988):

$$\overline{X}_{h_i} = \mu_{X_i} + (\overline{X}_{c_i} - \mu_{X_i}) \cdot \frac{p_{f_i}}{\sum_{k=1}^{n} p_{f_k}} \tag{9}$$

where $\bar{X}_{c_i}$ is the conditional mean value of X_i on the failure region with all other variables kept at the mean value, and p_{f_i} is the conditional failure probability. Eq. (9) may be interpreted as the mass center for masses p_{f_i} located on the axes X_i at a distance $\bar{X}_{c_i}-\mu_{X_i}$ from the mean.

For uncertain nonlinear structures under random dynamic loadings, the basic random variables include:

(1) random variables for structural properties, capacities, and uncertain loading variables (i.e., the variables defining the power spectral density and duration), $\boldsymbol{X}$,

(2) the random variables to generate the random load process, $\boldsymbol{V}$.

If the probability of failure is more sensitive to the random variables $\boldsymbol{X}$ than to the random variables $\boldsymbol{V}$, importance sampling is needed only for the variables $\boldsymbol{X}$ while the variables $\boldsymbol{V}$ can be sampled from the original probability density function. Since the variables $\boldsymbol{X}$ and $\boldsymbol{V}$ are statistically independent the integral in Eq. (4) is replaced by

$$p_f=\int_V\int_X I[g(\boldsymbol{x},\ \boldsymbol{v})]\frac{f_X(\boldsymbol{x})}{h_X(\boldsymbol{x})}h_X(\boldsymbol{x})f_V(\boldsymbol{v})\,d\boldsymbol{x}\,d\boldsymbol{v} \tag{10}$$

which is computed using the adaptive kernel method of importance sampling. In this case, the major difficulty is the computation of the conditional means $\bar{X}_{c_i}$ and conditional probabilities p_{f_i}, which are needed to compute the starting point as given by Eq. (9). One possibility is to consider a sample of $\boldsymbol{V}$ that results in a particularly severe load process and compute $\bar{X}_{c_i}$ and p_{f_i} using this sample.

3. Adaptive kernel method

The essence of the proposed adaptive kernel method is the use of the starting point of Eq. (9) from which the kernel importance sampling density can be constructed efficiently. The proposed method is different from traditional importance sampling methods. Available importance sampling methods require the determination of the "design point" or "maximum likelihood point"; in the proposed method this requirement is not necessary.

For the sake of simplicity the following algorithm for the adaptive kernel sampling is described for uncorrelated random variables. The anslysis for correlated random variables proceeds in the same way after a transformation to uncorrelated space is performed. The algorithm for the adaptive kernel sampling can be described as follows:

(1) For each random variable, find a critical value, a_{X_i}, for X_i, from the condition $g(\mu_{X_1}, \mu_{X_2}, \cdots, \mu_{X_{i-1}}, a_{X_i}, \mu_{X_{i+1}}, \cdots, \mu_{X_n}, \boldsymbol{v}_0)=0$, where $\boldsymbol{v}_0$ is the chosen set of $\boldsymbol{V}$ that gives a strong load process. An approximate value of a_{X_i} is sufficient for this method and can be obtained through a small number of iterations. The conditional failure probability $p_{f_i}=P(X_i<a_{X_i})$ or $P(X_i>a_{X_i})$ can be obtained accordingly and the corresponding conditional mean by $\bar{X}_{c_i}=E[X_i|X_i<a_{X_i}]$ or $E[X_i|X_i>a_{X_i}]$. If X_i is a normal variate, i.e., $N(\mu_{X_i}, \sigma_{X_i})$, one of the following equations is used to obtain the conditional mean:

$$\bar{X}_{c_i}=E[X_i|X_i<a_{X_i}]=\mu_{X_i}-\sigma_{X_i}\cdot\frac{\exp\left(-\frac{1}{2}z_i^2\right)}{\sqrt{2\pi}\,\Phi(z_i)} \tag{11}$$

$$\bar{X}_{c_i}=E[X_i|X_i>a_{X_i}]=\mu_{X_i}+\sigma_{X_i}\cdot\frac{\exp\left(-\frac{1}{2}z_i^2\right)}{\sqrt{2\pi}\,\Phi(-z_i)} \tag{12}$$

where

$$z_i=\frac{a_{X_i}-\mu_{X_i}}{\sigma_{X_i}}$$

The above process is repeated for $i=1, 2, \cdots, n$, to obtain the conditional means for all the random variables $\bar{X}_{c_i}$.

(2) Unimportant random variables have been known to have a significant negative effect on importance sampling. When there are a number of random variables and only a few are important, the efficiency can be improved if the importance sampling is restricted to the important variables (Karamchandani, *et al.* 1989). Prior information on which of the random variables are important is not known, but it may be assumed that the important random variables are those whose $p_{f_i}/\max_{k=1}^{n}(p_{f_k})$ are greater than 10^{-10}. Thus, the random variables are divided into a group of n_I important random variables and a group of n_U unimportant ones, where $n=n_I+n_U$.

(3) For the n_I important random variables, the starting point $\bar{X}_h$ for the importance sampling is obtained by Eq. (9). This equation yields $\bar{X}_{h_i}\cong\mu_{X_i}$ when p_{f_i} is negligibly small which is the case for unimportant variables.

(4) Using the adaptive sampling densities for the important variables that have the same forms and variances as their respective original probability densities but are centered at the starting point $\bar{X}_{h_i}$, M failure samples $\boldsymbol{y}_i$, $i=1, 2, \cdots, M$, are generated while keeping the unimportant variables at the mean values. Then, the kernel importance sampling density function $\hat{h}_{X_I}(\boldsymbol{x}_I)$ for the n_I important random variables is constructed.

(5) Since the important and unimportant random variables are statistically independent, the importance sampling density function for the entire set of random variables $\boldsymbol{V}$ is $\hat{h}_{X_I}(\boldsymbol{x}_I)\cdot f_{X_U}(\boldsymbol{x}_U)$. That is, the important random variables are sampled from $\hat{h}_{X_I}(\boldsymbol{x}_I)$ whereas the unimportant random variables are sampled from the original probability density function $f_{X_U}(\boldsymbol{x}_U)$. The sample functions of the random process are also generated from the original probability density function $f_V(\boldsymbol{v})$. From N samples for each of the entire set of random variables and process the estimate of the entire set of random variables and processs the estimate of the failure probability and its variance are calculated through Eqs. (5) and (6), respectively.

4. Numerical examples

4.1. Example 1

The first example is a single degree-of-freedom structure whose restoring force is governed by a bilinear hysteresis model. The equation of motion is

$$\frac{d^2u}{dt^2}+2\zeta\omega_n\frac{du}{dt}+q(u(t), r_1, r_2, r_3, t)=a(t) \tag{13}$$

where ω_n is the natural frequency, ζ the damping ratio, u the relative displacement of the structure with respect to the ground, $a(t)$ a Gaussian white noise with spectral density S_0 and duration t_d, and q is the nonlinear restoring force that is a bilinear hysteresis model as shown in Fig. 1. If the yield displacement is assumed to be $u_y=0.594$ in., the parameters in the bilinear hysteresis model may be defined as follows:

$$\begin{aligned} r_1&=u_y r_2 \\ r_2&=\omega_n^2 \\ r_3&=0.1\ r_2 \end{aligned} \tag{14}$$

where r_1 is the yield strength, r_2 the initial stiffiness, and r_3 the post-yield stiffness. Eq. (13) is first transformed into a set of first-order nonlinear differential equations:

$$\begin{aligned} \frac{dy_1}{dt}&=y_2 \\ \frac{dy_2}{dt}&=-2\zeta\omega_n y_2-q(u,\ r_1,\ r_2,\ r_3,\ t)+a(t) \end{aligned} \tag{15}$$

in which

$$\begin{aligned} y_1&=u \\ y_2&=\frac{du}{dt} \end{aligned}$$

The set of first-order differential in Eq. (15) is then solved by using the fourth-order Runge-Kutta method with zero initial conditions. Failure is assumed to occur when the structure experiences excessive relative displacement, e.g., $|u|>6$ in. The parameters, ω_n, ζ, S_0 and t_d, are assumed to be independent random variables with the distributions and parameters described in Table 1.

The construction of the kernel sampling density function and estimation of p_f follow the algorithm in the previous section. The important random variables are found to be ω_n and S_0. The

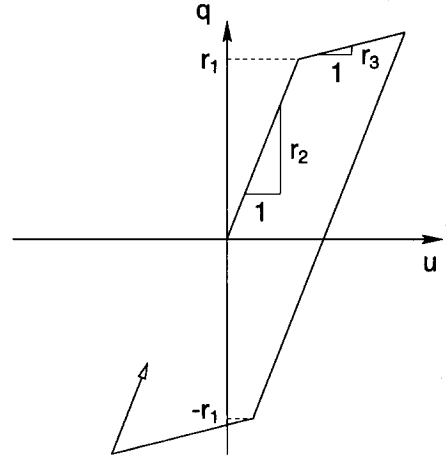

Fig. 1 Bilinear hysteresis model for Example 1.

Table 1 Statistics of random variables for Example 1

Variable	Distribution	Mean	C.O.V
ω_n[rad/sec]	Normal	4π	0.1
ζ	Lognormal	0.02	0.4
S_0[in^2/sec^3]	Type II-largest	36	0.6
t_d[sec]	Lognormal	10	0.3

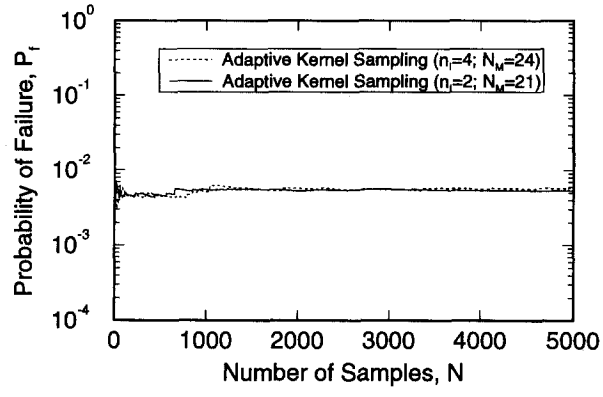

Fig. 2 P_f for Example 1.

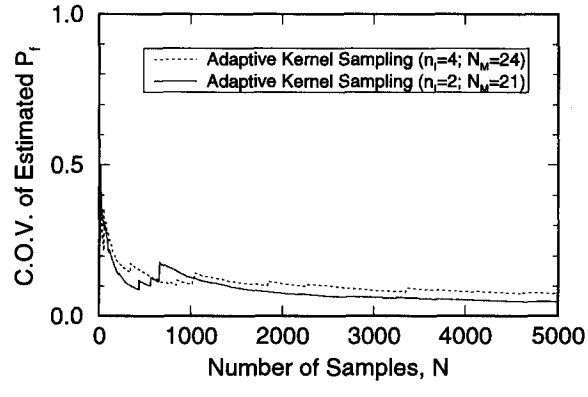

Fig. 3 Coefficient of variation of P_f for Example 1.

number of failure samples to construct the kernel sampling density function, M, is 10. Figs. 2 and 3 show the estimates of the failure probability and their coefficients of variation, respectively; these results are obtained by the proposed method by

(1) limiting the importance sampling to the important variables only and

(2) importance sampling is applied to all the variables in Table 1.

The proposed method is more efficient than the adaptive kernel method involving all the variables and also gives the same estimated failure probability of 5.5×10^{-3}.

The estimates of the failure probability and their coefficient of variation computed by other available methods are also shown in Figs. 4 and 5, respectively. A direct importance sampling has been conducted with the original density centered at the starting point. The proposed adaptive

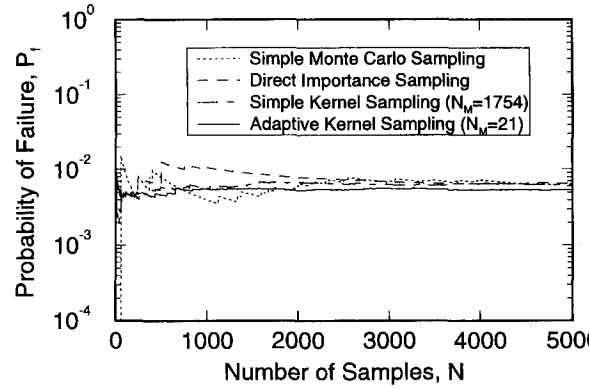

Fig. 4 P_f obtained by various methods for Example 1.

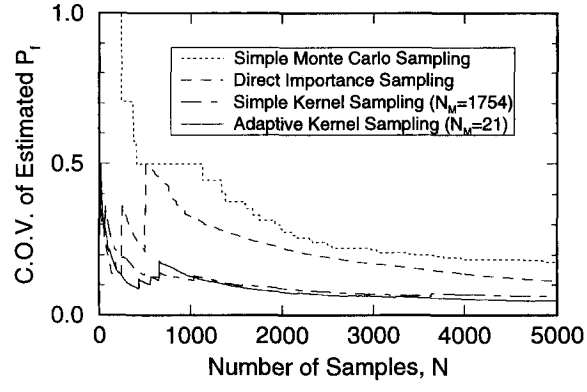

Fig. 5 Coefficient of variation of P_f obtained by various methods for Example 1.

kernel method is superior to the simple Monte Carlo method and the direct method of importance sampling. The number of simulations, N_M, to generate the M failure samples in the adaptive kernel method is 21. This is far smaller than the number required in the simple kernel method (which required N_M=1754). The efficiency of the adaptive kernel method is even more promising when the failure probability of a system is very small.

4.2. Example 2

The adaptive kernel method is used to compute the probability of exceeding a given level of global damage for the shear-beam type structure shown in Fig. 6, for various intensities of the earthquake ground motion. The equations of motion of the system may be written as

$$\ddot{u}_i-(1-\delta_{i1})\frac{q_{i-1}}{m_{i-1}}+\frac{q_i}{m_i}\left[1+(1-\delta_{i1})\frac{m_i}{m_{i-1}}\right]-(1-\delta_{i7})\frac{q_{i+1}}{m_{i+1}}\frac{m_{i+1}}{m_i}=-\ddot{u}_g,\quad i=1,\ 2,\ \cdots,\ 7 \tag{16}$$

where u_i is the interstory deformation of the ith floor, $\ddot{u}_g$ the ground acceleration, m_i the mass of the ith floor, q_i the restoring force, δ_{i1} and δ_{i7} are Kronecker deltas. It is assumed that the restoring force follows a hysteretic model (Baber and Wen 1981) as follows:

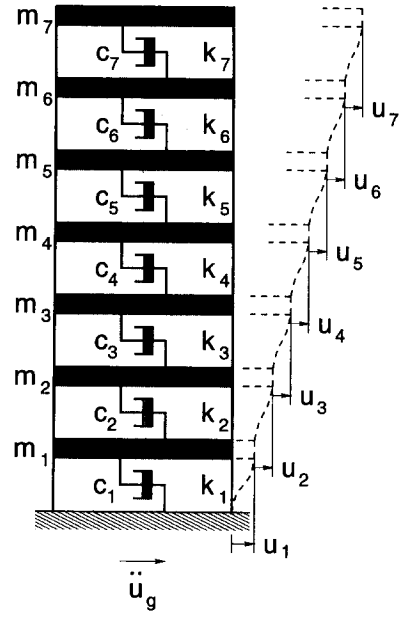

Fig. 6 A 7-story shear-beam structure (Example 2).

Table 2 Structural properties of the 7-story shear-beam structure (Example 2)

Story i	1	2	3	4	5	6	7
m_i[kip·sec^2/in]	5.0	4.9	4.9	4.9	4.9	4.9	4.8
c_i[kip·sec/in]	12.03	19.09	19.09	19.09	16.39	13.40	13.26
k_i[kip/in]	2896	7436	7436	7436	5481	3664	3664
α_i	0.02	0.02	0.02	0.02	0.02	0.02	0.02
β_i[in^{-2}]	0.507	6.848	6.848	6.848	4.420	2.530	2.530
γ_i[in^{-2}]	−0.169	−2.283	−2.283	−2.283	−1.474	−0.834	−0.834
δ_u[in]	22.5	9.0	7.0	6.0	5.0	5.5	6.3
β_{0i}	2.88	1.66	1.66	1.66	1.76	2.03	2.03
Q_{yi}[kip]	3985	3480	3480	3480	3192	2821	2821

$$q_i=c_i\dot{u}_i+a_ik_iu_i+(1-\alpha_i)k_iz_i,\ i=1,\ 2,\ \cdots,\ 7 \tag{17}$$

in which c_i is the viscous damping of the ith floor, k_i the initial stiffness of the ith floor, and z_i is the hysteretic component of the deformation u_i, which is given as

$$\dot{z}_i=\dot{u}_i-(\beta_i|\dot{u}_iz_i|z_i+\gamma_i\dot{u}_i|z_i|^2),\quad i=1,\ 2,\ \cdots,\ 7 \tag{18}$$

Values of pertinent parameters are summarized in Table 2. The ground acceleration is modeled as a zero-mean filtered Gaussian white noise and is simulated using the fast Fourier transform algorithm. The power spectral density (PSD) of the ground motion is represented by the Kanai-Tajimi PSD:

$$S_{\ddot{g}\ddot{g}}(\omega)=S_0\frac{1+4\zeta_g^2(\omega/\omega_g)^2}{[1-(\omega/\omega_g)^2]^2+4\zeta_g^2(\omega/\omega_g)^2} \tag{19}$$

where S_0 is the power spectrum ordinate of the stationary unfiltered shot noise. In this case,

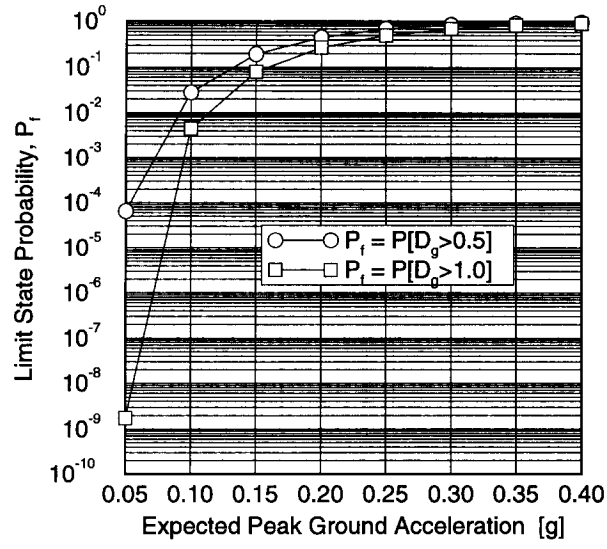

Fig. 7 Fragility curves of the 7-story shear-beam structure (Example 2).

ω_g and ζ_g are given as 3.2 rad/sec and 0.3, respectively, representing the site soil condition in Mexico City. To obtain an even more representative process for strong ground motion, the non-stationary characteristics of actual accelerograms can be considered. This appearance suggests using a nonstationary process, namely a process $\ddot{u}_g$ given by

$$\ddot{u}_g = \psi(t)\,\xi(t) \tag{20}$$

where $\psi(t)$ is the envelope function having an appropriate form based on statistical analyses of real accelerograms and is given by

$$\psi(t)=\begin{cases} (t/t_1)^2, & t\le t_1 \\ 1.0, & t_1<t\le t_2 \\ \exp[-2(t-t_2)/t_d], & t>t_2 \end{cases} \tag{21}$$

where $t_1=1.5$ sec, $t_2=11.5$ sec, and t_d=the strong motion duration$=10$ sec. The structure described above suffered substantial damage during the Mexico City earthquake of 1985. According to the Park-Ang model (Park and Ang 1985), the damage index for a structural component can be expressed in terms of the maximum interstory displacement δ_{max} and the dissipated hyteretic energy E as follows:

$$D=\frac{\delta_{max}}{\delta_u}+\beta_0\frac{\int dE}{Q_y\,\delta_u} \tag{22}$$

where D is the damage index, δ_u the ultimate deformation, Q_y the yield force capacity, and β_0 is a constant. The above parameters for each component of the building are also given in Table 2.

The hysteretic energy associated with each degree-of-freedom is described by the following differential equation:

$$\dot{E}_i=(1-\alpha_i)\,k_i\dot{u}_i z_i,\quad i=1,\ 2,\ \cdots,\ 7 \tag{23}$$

The global damage index for a structure is clearly a function of the damage indices of the component, and may be defined as follows:

$$P(D_g \geq d) = P[\bigcup_i (D_i \geq d)] \tag{24}$$

where D_g is the global damage index and D_i is the damage index of the ith story, The random variables considered in this system are m_i, c_i, k_i, α_i, δ_{u_i}, β_{0_i} and Q_{y_i}. Each one of the structural parameters is assumed to be perfectly correlated between stories and to be lognormally distributed. The coefficients of variation of these variables are 0.1, 0.5, 0.3, 0.1, 0.4, 0.6 and 0.2, respectively. The interstory deformation and hysteretic energy of each floor are obtained by solving Eqs. (16) and (23) simultaneously using Newmark's constant-average-acceleration method. After obtaining the maximum deformation and hysteretic energy, the damage index for each floor is obtained through Eq. (22) and the probability that the global damage index D_g will exceed a specified value is evaluated through Eq. (24) by the adaptive kernel method. The fragility curves for the global damage index exceeding 0.5 (irreparable damage) and 1.0 (collapse), respectively, are obtained and presented in Fig. 7, where each point is obtained for a given constant S_0. The number of failure samples used to construct the kernel sampling density function, M, is 20 and the number of samples generated from the kernel sampling density function, N, is 2000 for all the points on this fragility curve. Observe that the collapse probability drops to 1.7×10^{-9} under a mean peak ground acceleration of 0.05 g. This shows that the adaptive kernel method is equally effective even for problems involving very low failure probability. It would be difficult or expensive to apply most other methods, including the simple kernel method for problems involving such small probabilities.

5. Conclusions

An adaptive kernel method of importance sampling for a class of time-variant problems, i.e., uncertain nonlinear hysteretic structures under random dynamic loadings, is presented. The examples presented indicate that the method is effective to evaluate structural reliability for systems with very small probabilities of failure (e.g., $\leq10^{-6}$) which are often required in structural problems. Moreover, with the kernel sampling density function, unbiased failure probability is assured.

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