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Nonlinear dynamic analysis of laterally loaded pile

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Abstract. In the present study a parametric analysis is conducted to study the effect of pile dimension and soil properties on the nonlinear dynamic response of pile subjected to lateral sinusoidal load at the pile head. The study is conducted on soil-pile model of different pile diameter, pile length and soil modulus, and results are compared to get the effect. The soil-pile system is modelled using Finite element method. The programming is done in MATLAB. Time history analysis of model is done for varying non-dimensional frequency of load and the results are compared to get the non-dimensional frequency at which pile head displacement is maximum in each case. Maximum possible bending moment and soil-pile interacting forces for the dynamic excitation of the pile is also compared. When results are compared with the linear response, it is observed that non-dimensional frequency is reduced in nonlinear response on account of reduction in the soil stiffness due to yielding. Nonlinear response curve shows high amplitude as compared to linear response curve.

Keywords: soil-pile interaction; consistent mass matrix; Rayleigh damping; dynamic lateral loading; Numerical modeling

1. Introduction

Pile foundation may be subjected to variety of loading conditions, such as static, cyclic or dynamic loads. The design of laterally loaded pile involves consideration of usually large ratio of lateral to vertical loads, particularly in areas subjected to severe storms. The design become complex when the load is dynamic in nature as the dynamic response is found to be more than the static response for same equivalent force. In the past many studies have been devoted to lateral response of single piles. Various approaches have been developed for the static and dynamic lateral response of piles such as boundary element analysis, Winkler approach and finite element analysis. Boundary element approach was effectively applied for analysis of laterally loaded pile in the linear-elastic domain (Banerjee and Davies 1978, Kaynia and Kausel 1982). Basu *et al*. (2009) presented a continuum based model for the analysis of laterally loaded pile in layered soils. However, the inclusion of soil nonlinear behavior in this approach is difficult. Three dimensional finite element analysis can be developed for analysis of pile to include various conditions like material nonlinearity, pile soil separation (Karthigeyan *et al*. 2007, Dewaikar *et al*. 2007). Chore *et*

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al. (2010) discussed the effects of pile spacing, pile diameter and configuration on response of pile group. Emani and Maheshwari (2009) presented the dynamic impedances for the pile groups with pile cap embedded in soil. Sawant and Ladhane (2012) presented dynamic response of pile groups for different configurations assuming soil as continuous media. Nogami and Konagai (1986, 1988) analyzed the dynamic response of pile foundations in the time domain using a Winkler approach. El Naggar and Novak (1995, 1996) presented a nonlinear analysis for pile groups in the time domain within the framework of the Winkler hypothesis. However, proper representation of damping and inertia effects of continuous soil media is difficult with such discrete systems. In the current study, nonlinear dynamic analyses were performed to investigate the effect of changing soil parameters and pile dimension on the dynamic lateral behavior of the pile.

2. Modeling soil-pile system

The modelling of soil-pile system includes discretization of the soil pile system and formation of stiffness, mass and force matrix. In this section, the 1D mesh, boundary conditions and properties of soil and pile will be discussed. In the pile-soil model for single piles, the pile-soil system is divided into horizontal slices containing the pile segment and homogeneous soil layers. The pile is modelled using a series of linear or nonlinear frame elements, and the soil is modelled using a series of linear or nonlinear springs and dashpots attached to each node along the length of the pile as shown in Fig. 1. Details of modelling the pile and its surrounding soil are discussed in subsequent sections. The near-field soil reaction which is modelled by a linear or nonlinear spring is placed in series with the far-field soil reaction which is modelled by a set of a linear spring and dashpot. Viscous damper that is used to account for radiation damping effects is placed in series with the hysteretic soil model as shown in Fig.1. Such a method has been adopted by Nogami and Konagai (1987, 1988), El- Naggar and Novak (1996). The cyclic non-linear models that follow the actual stress-strain path during cyclic load can be used to represent the nonlinear stress-strain behaviour of soil.

The backbone function, $F_{bb}(\gamma)$, can be described by a hyperbola (see Fig. 2)

$$Fbb = \frac{G_{\max}\gamma}{1 + (G_{\max} / \tau_{\max})|\gamma|} \tag{1}$$

The response of the soil to the cyclic loading is governed by the Masing rules: 1) For initial loading, the stress-strain curve follows the backbone curve; 2) If a stress reversal occurs at a point defined by (γ_r, τ_r), the stress-strain curve follows a path given by

$$\frac{\tau - \tau_r}{2} = F_{bb}\left(\frac{\gamma - \gamma_r}{2}\right) \tag{2}$$

The soil in the immediate vicinity of the pile shows nonlinear behaviour and is represented by near field elements. It is modelled by nonlinear spring and consistent mass matrix, m_n, as proposed by Nogami *et al.* (1992). Assuming that variation of the soil displacement with the radial distance from the pile is linear, the consistent mass matrix is defined by

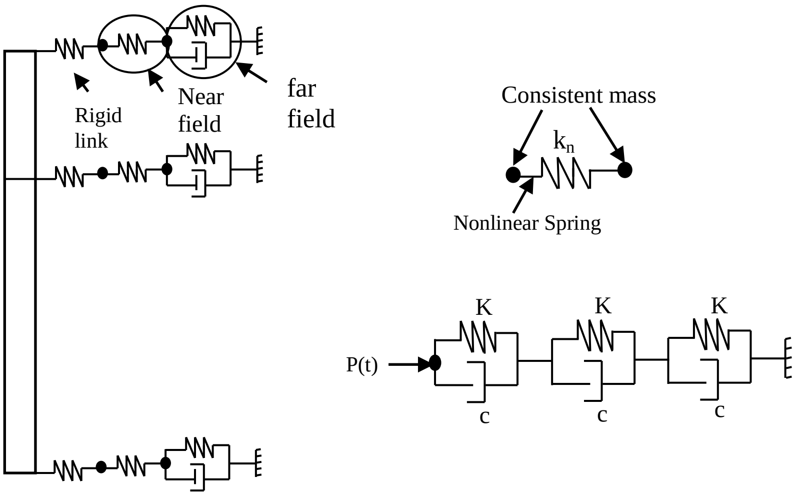


Fig. 1 Various elements of the model for nonlinear dynamic analysis

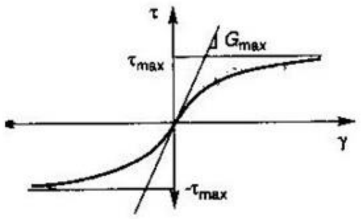


Fig. 2 Backbone curve

$$m_n = \frac{\pi \rho r_0^2}{6}\left(\frac{r_1}{r_0}-1\right)\begin{bmatrix} \frac{r_1}{r_0}+1 & -\left(3\frac{r_1}{r_0}+1\right) \\ -\left(3\frac{r_1}{r_0}+1\right) & \frac{r_1}{r_0}+1 \end{bmatrix} \tag{3}$$

and

$$k_n = G_{\max}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \tag{4}$$

In which, ρ is the density of soil, $\mathbf{r}_0$ is the radius of pile, r_1 is the artificial distance from the centre of the loaded pile shaft beyond which soil behaviour is assumed to be more or less elastic and $G_{\max}$ is the maximum shear modulus, respectively.

The far-field medium can be modelled by springs and dashpots, as proposed by Nogami and Konagai (1988). The model parameters are as follows

$$m_s = \xi_m(\nu_s)\rho_s \pi r_0^2 \tag{5}$$

$$k_n = \xi_k(\nu_s)G_s \begin{cases} 3.518, & n=1 \\ 3.581, & n=2 \\ 5.529, & n=3 \end{cases} \tag{6}$$

$$c_n = \xi_k(\nu_s)\frac{G_s r_0}{V_s} \begin{cases} 113.097, & n=1 \\ 25.133, & n=2 \\ 9.362, & n=3 \end{cases} \tag{7}$$

In which, ρ_s is the mass per unit volume of the medium, $\xi_k(\nu_s)$ and $\xi_m(\nu_s)$ are the functions of Poisson's ratio of soil and given in Nogami and Konagai (1988).

The stiffness, mass and damping matrix of far field model are given by

$$k_f = (1/k_1 + 1/k_2 + 1/k_3)^{-1} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \tag{8}$$

$$m_f = \xi_m(\nu_s)\rho_s \pi r_0^2 \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \tag{9}$$

$$c_f = (1/c_1 + 1/c_2 + 1/c_3)^{-1} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \tag{10}$$

The pile was modelled using two noded beam column elements with six degrees of freedom in the local system and was discretized into L elements. The local consistent mass matrix $\overline{M}^e$ Stiffness matrix $\overline{K}^e$ and Damping matrix $\overline{C}^e$ is obtained as

$$\overline{M}^e = \frac{\rho AL}{420} \begin{bmatrix} 140 & 0 & 0 & 70 & 0 & 0 \\ 0 & 156 & 22L & 0 & 54 & -13L \\ 0 & 22L & 4L^2 & 0 & 13L & -3L^2 \\ 70 & 0 & 0 & 140 & 0 & 0 \\ 0 & 54 & 13L & 0 & 156 & -22L \\ 0 & -13L & -3L^2 & 0 & -22L & 4L^2 \end{bmatrix} \tag{11}$$

$$\overline{K}^e = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{4EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{2EI}{L} & 0 & \frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} \tag{12}$$

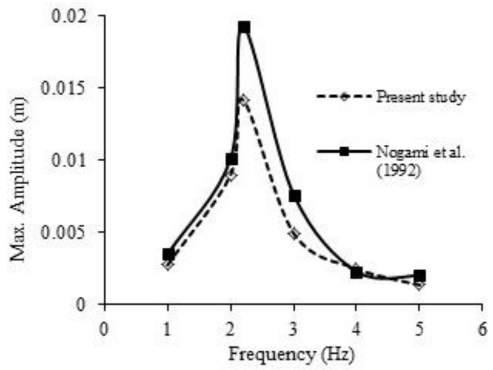


Fig. 3 Comparison of frequency amplitude response of present study with Nogami *et al*. (1992)

$$\overline{C}^e = a\overline{M}^e + b\overline{K}^e \tag{13}$$

In which, a and b are Rayleigh damping constant.

Element mass matrix, stiffness matrix and damping matrix is calculated by using transformation matrix G which contain direction cosines as

$$M^e = G^T \overline{M}^e G; \quad K^e = G^T \overline{K}^e G; \quad C^e = G^T \overline{C}^e G$$

Where

$$G = \begin{bmatrix} n(1) & n(2) & 0 & 0 & 0 & 0 \\ -n(2) & n(1) & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & n(1) & n(2) & 0 \\ 0 & 0 & 0 & -n(2) & n(1) & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Equivalent nodal load vector is computed according to

$$\overline{f_l}^e = \int_0^L q_s N^T dx \tag{14}$$

In which, q_s is the intensity of uniformly distributed load over the cross sectional area, N is the shape functions. The internal forces in a two dimensional nonlinear beam element are calculated by using Olsson (1996) method. In the present study Newmark-β method with average acceleration and Newton Raphson iteration technique has been used and convergence criterion was displacement dependent.

3. Validation

To validate the developed algorithm, the results obtained from the present formulation are compared with the result presented by Nogami *et al*. (1992) on steel pile with wall thickness 0.93 cm and outside diameter 0.273 m. The pile was driven to a depth of 15 m. Ratio of modulus of elasticity of pile and soil was taken as 10000. Pile was subjected to dynamic force at pile head as $P_0 \sin\omega t$ with value of P_0=3 kN and ω is the forcing frequency. Comparison of frequency amplitude response variation from present study with Nogami *et al.* (1992) is depicted in Fig. 3. The variation in amplitude is observed to be about 35 % and corresponding frequencies varies by 2%. It can be seen that a fairy good agreement is seen the results.

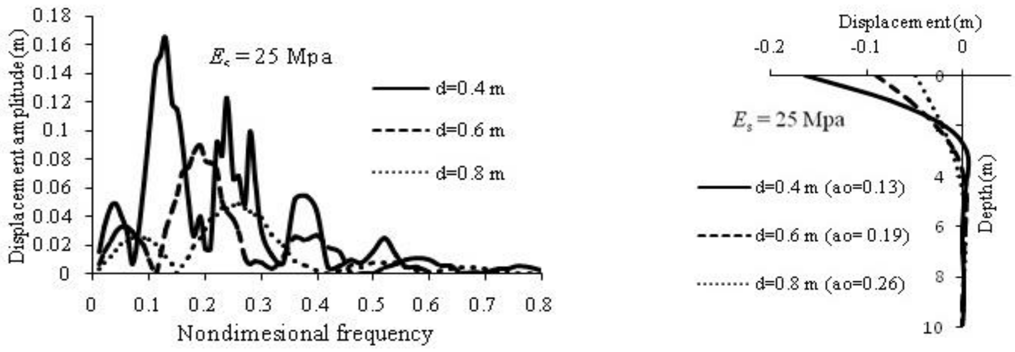
4. Parametric study

A parametric study is conducted to examine the effect of various key parameters including pile diameter, pile length, Soil modulus, Poisson's ratio, soil density and material nonlinearity. The dynamic force applied in the present case is given by $P_0 \sin\omega t$ (for the present study P_0=100 kN) is applied.

Pile displacements and bending moments in the pile are mainly considered as state variables. Effect on these two state variables is discussed in detail. It is observed from Fig. 4 that pile displacement at the pile head is reducing with an increase in pile diameter. This is due to the fact that the passive resistance zone increases with an increase of pile diameter and pile length under lateral dynamic shaking. Also with the increase in pile diameter, stiffness of the system increases and hence the maximum deflection is supposed to be decrease. Peak amplitudes and corresponding non-dimensional frequencies are summarized in Table 1. As the diameter of pile increases the non-dimensional frequency increases.

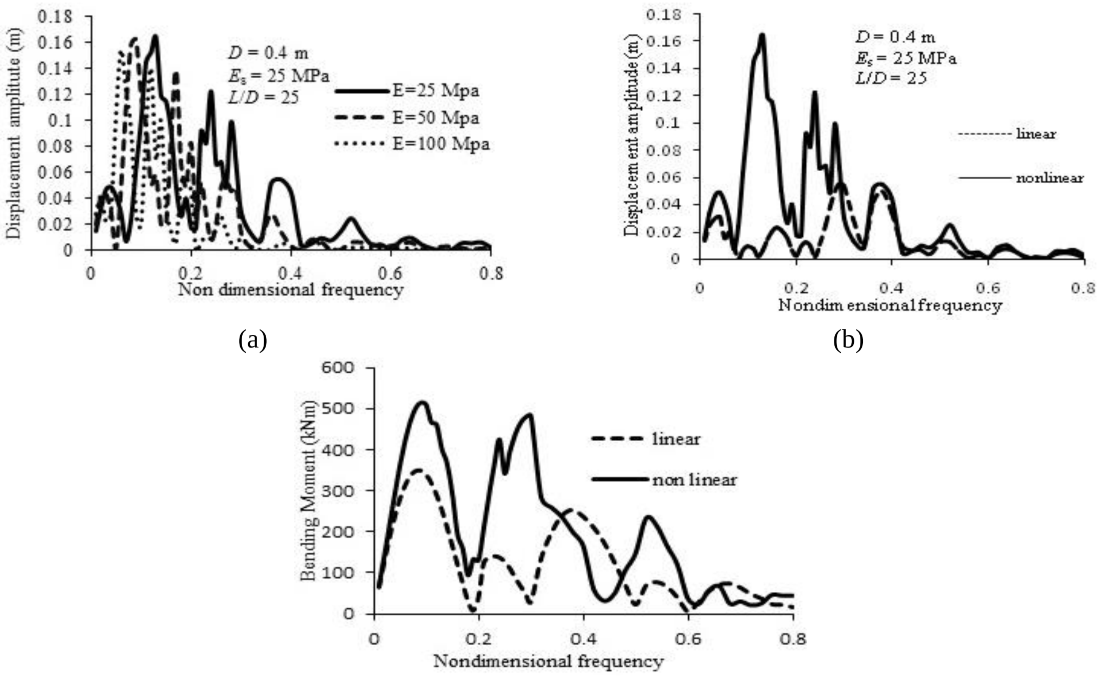
Table 1 Peak amplitude and corresponding non-dimensional frequency

Diameter of Pile (m)	E_s=25 MPa		E_s=50 MPa		E_s=100 MPa	
	Disp (mm)	a_o	Disp (mm)	a_o	Disp (mm)	a_o
0.4	163.99	0.13	162.44	0.09	154.37	0.06
0.6	89.90	0.19	88.06	0.13	85.90	0.09
0.8	49.28	0.26	49.36	0.18	50.22	0.13



(a) (b)

Fig. 4 (a) Maximum displacement vs nondimensional frequency, (b) displacement along length of pile



(c)

Fig. 5 (a) Maximum deflection vs nondimensional frequency, (b) Comparison of linear and nonlinear response for displacement amplitude, (c) Comparison of linear and nonlinear response for Bending Moment

Fig. 5 (a) shows the response of single pile for different soil modulus. Maximum amplitudes are observed to be decreased with increase in the soil modulus but this reduction is very small. This very much matches the expected result that with increase in elastic modulus of soil maximum deflection should decrease due to increase in the stiffness of the system. However, it is noted that the rate of reduction of maximum deflection is relatively less when compare to one in very low soil modulus (E_s= 25 MPa). For soil modulus of 25 MPa the displacement amplitude decrease by 25.25% from peak 1 to peak 2 whereas 14.42 % and 9.06 % are observed for soil modulus 50 MPa and 100 MPa respectively. This is probably because of hysteretic damping in addition to radiation damping that prevails in soil of low elastic modulus (25 MPa) thus leading to high rate of reduction of maximum deflection.

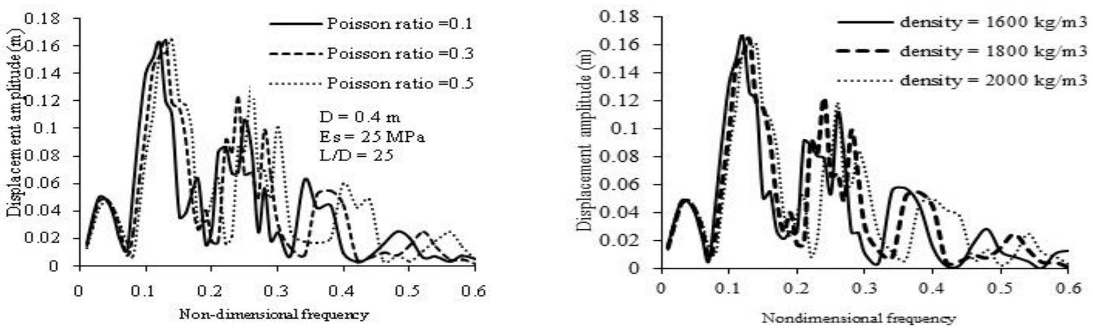
Fig. 5(b) and 5(c) shows the comparison of linear and nonlinear response of single pile for L/D =25, E_s= 25000 kPa and D= 0.4 m. It is observed from the figure that response curve shows high amplitude as compare to linear response curve. When compared with linear response it is also observed that non-dimensional frequencies corresponding to peak are smaller on account of reduction in soil stiffness due to yielding. The bending moments are significantly higher. Maximum moment in case of linear analysis was 349 kNm at non-dimensional frequency 0.09. Same is increased to 514 kNm in case of nonlinear analysis with increase of 47 %.

The pile is analysed for varying length with all other condition remaining constant. The other properties used for the pile are as follows:

Diameter of pile = 0.4 m; E_p= 25000 MPa; E_s= 25 MPa

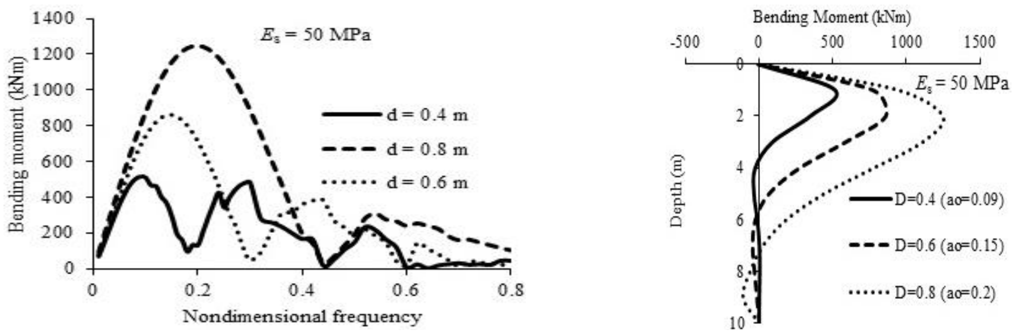
Table 2 Maximum deflection for different length of pile

Length (m)	8	10	12	14
Max. deflection (mm)	165.49	164.99	164.08	164.08



(a) (b)

Fig. 6 (a) of variation of sand Poisson's ratio, (b) Effect of variation of density of soil



(a) (b)

Fig. 7 (a) Maximum Bending moment vs. non-dimensional frequency for different diameters of pile (E_s= 50 MPa), (b) Bending Moment along the length of pile for different diameter of pile (E_s=50 MPa)

Table 2 shows that as the length of pile increases, maximum pile head deflection first decreases and then become constant at a particular length known as length of fixity, so if the pile length is longer than this length it is called as long pile, otherwise called short pile, even though it still depends on other factors. In short pile the sensitivity of maximum displacement of pile is greater than the pile length, however with increasing the length, this sensitivity decreased. Another point is that short pile behaves linearly but long pile behaves non-linearly.

Fig. 6(a) shows the response of single pile for different Poisson's ratio. It may be observed from the figure that as the Poisson's ratio increases, the maximum pile amplitude slightly increases at the first peak but significant difference may occur at second and third peak. This is attributed by the fact that higher soil Poisson's ratio indicate a higher tendency for the soil to bulge upward at the unconstrained soil surface at the ground level. Thus the higher the Poisson's ratio the higher the soil moves upward at the ground surface. This freedom to move upward reduces the bearing strength of the soil to lateral pile movements thus resulting in higher pile displacement. Overall, a

Table 3 Maximum bending moment, corresponding non-dimensional frequency and depth of occurrence for different diameter and elastic modulus of soil

E_s (MPa)	D= 0.4 m			D= 0.6 m			D= 0.8 m		
	B.M (kNm)	a_0	Depth (m)	B.M (kNm)	a_0	Depth (m)	B.M (kNm)	Depth (m)	a_0
25	514.967	0.09	1.3	861.058	0.15	1.9	1247.97	2.2	0.20
50	506.461	0.07	1.2	850.083	0.10	1.8	1240.17	2.2	0.14
100	504.404	0.05	1.1	841.730	0.07	1.6	1234.32	2.2	0.10

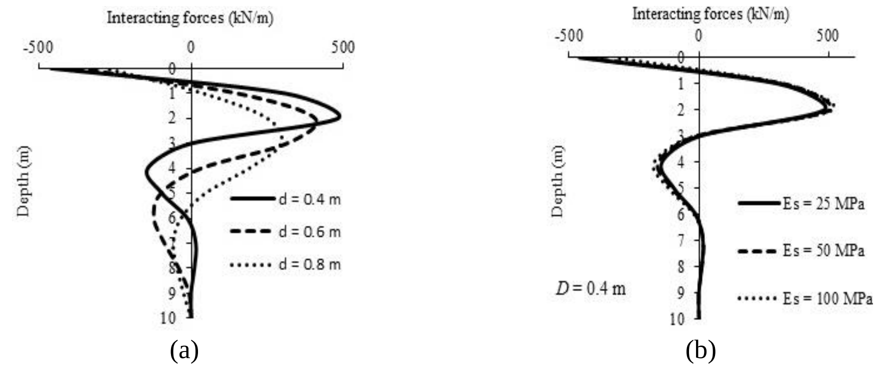


Fig. 8 (a) Variation of interacting force along the length of pile (a) for different diameter of pile (E_s=25MPa), (b) for different Elastic modulus of soil (D= 0.4 m)

higher Poisson's ratio lowers the stiffness of the soil, increases the relative stiffness between the pile and the soil, thus increasing the burden on the pile to resist lateral load.

Fig. 6(b) shows the response of single pile for different density of soil. It is observed that as the density of soil increase maximum displacement decreases. This very much matches the expected result because increase in stiffness and density of soil, increase the lateral load resistance and hence maximum displacement decrease.

It is observed from Fig. 7 and Table 3 that Bending moment profile are almost following the same trend, but it is only magnitude of bending moment that increases with increase pile diameters. This may be due to the increase of flexural rigidity with pile diameter which indirectly causes the increase in bending moment. Also the relative stiffness between the pile and the soil increases, the maximum bending moment occurs at deeper location along the pile length. The variation of the bending moment near the bottom of the pile is dependent on the stiffness of soil-pile system although its value is small. As the stiffness of the pile increases and relative stiffness of pile with respect to soil increases, the bending moment near the bottom of the pile are developed, indicating that the pile starts to play a larger role in resisting lateral loads.

It may be observed from Fig. 8 that as the diameter of pile and modulus of elasticity of soil increases, interacting force decreases at the pile top but the effect of variation of modulus of elasticity of soil is not significant in case of interacting forces. The pile displacement for larger diameter pile is lower, which results in lower soil resistance. Therefore the interacting forces decreases with increase in the pile diameter.

To examine the effect of ultimate yield stress τ_{max}, the ratio of τ_{max}/G_{max} was varied from 0.01 to 0.5. Effect of τ_{max}/G_{max} on variation in maximum amplitude with non-dimensional frequency is highlighted in Fig. 9 and Table 4. It is observed that for τ_{max}/G_{max}=0.01, the maximum amplitude

Table 4 Effcet of τ_{max}/G_{max} on maximum amplitudes

τ_{max}/G_{max}	0.01	0.03	0.05	0.1	0.2	0.5
Displacement amplitude (m)	0.163986	0.078715	0.046097	0.05871	0.064858	0.059572
Non-dimensional frequency a_0	0.13	0.24	0.29	0.3	0.3	0.3

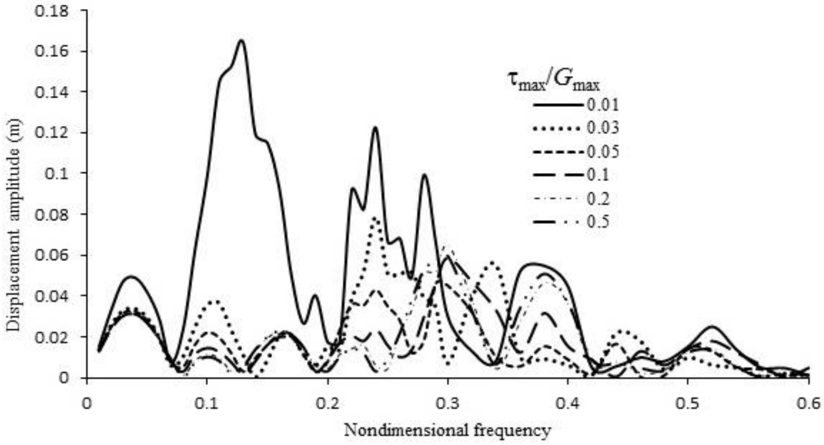


Fig. 9 Effect of τ_{max}/G_{max} on amplitude frequency response

was 164 mm at non-dimensional frequency 0.13. With increase in value of τ_{max}/G_{max}, the maximum amplitude was decreasing to 58 mm approaching towards linear response vindicating clear effect of τ_{max} on the response.

5. Conclusions

1. Higher response is observed at all non-dimensional frequencies with decrease in the diameter. Maximum amplitude is observed to be decreasing with increase in diameter.
2. Lower value of non-dimensional frequency corresponds to higher soil modulus and is increasing with decreases in the modulus.
3. Nonlinear response curve shows high amplitudes as compared to linear response curve.
4. As the length of pile increases, the maximum deflection first decreases and then becomes constant.
5. Sand Poisson's ratio and density does not have any considerable effect on Pile head displacement
6. Maximum bending moment decreases as the elastic modulus of soil increases and increases as the diameter of pile increases.
7. Depth of maximum bending moment increases with the increase in diameter of pile.
8. Interacting forces between pile and soil increases as the diameter of pile increase but effect of variation of modulus of elasticity of soil is not significant in case of interacting forces.
9. With increase in value of τ_{max}/G_{max}, the maximum amplitude decreases and approaches towards linear response vindicating clear effect of τ_{max} on the response.

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