Structural Engineering and Mechanics, *Vol. 48, No. 3 (2013) 383-394*
DOI: http://dx.doi.org/10.12989/sem.2013.48.3.383

Cold-formed steel channel columns optimization with simulated annealing method

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(Received April 23, 2012, Revised September 12, 2013, Accepted October 26, 2013)

Abstract. Cold-formed profiles have been largely used in the building industry because they can be easily produced and because they allow for a wide range of sections and thus can be utilized to meet different project requirements. Attainment of maximum performance by structural elements with low use of material is a challenge for engineering projects. This paper presents a numerical study aimed at minimizing the weight of lipped and unlipped cold-formed channel columns, following the AISI 2007 specification. Flexural, torsional and torsional–flexural buckling of columns was considered as constraints. The simulated annealing method was used for optimization. Several numerical simulations are presented and discussed to validate the proposal, in addition to an experimental example that qualifies its implementation. The ratios between lips, web width, and flange width are analyzed. Finally, it may be concluded that the optimization process yields excellent results in terms of cross-sectional area reduction.

Keywords: cold-formed structures; steel; optimization; simulated annealing; columns; compression

1. Introduction

Cold-formed steel structures are an alternative to small and medium-sized steel frames. The great advantage of these structures is the possibility of adjusting the form to the needs of the member and of the overall structure, which is obtained with the minimum possible weight. In practice, this advantage is not fully explored because the simple search of a section with a maximum strength-to-weight ratio is not trivial in engineers' daily practice. Therefore, the present paper aims to investigate the optimization of the components of cold-formed structures. First, we introduce the procedure used to calculate lipped and unlipped channel sections to determine the load capacity.

The simulated annealing (SA) technique is applied to formulate the optimization problem. The web width to flange width ratio is evaluated for lipped and unlipped channel sections. The effective width method is used to determine the load capacity. All the calculations follow the American Standard AISI (2007) specifications. According to this procedure, each of the possible elastic buckling loads (global flexural, torsional, local, and their combinations) is obtained, and the smallest of them determines the maximum load. Distortional buckling mode of the section is not

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http://www.techno-press.org/?journal=sem&subpage=8 ISSN: 1225-4568 (Print), 1598-6217 (Online)

addressed in the present study.

The optimization of cold-formed steel members is still incipient in the literature. Seaburg and Salmon (1971) assessed the minimum weight of top hat sections using direct methods and the gradient for minimization of the weight function, but they could provide only a single example due to the numerical complexity of the problem.

Dinovitzer (1992) optimized the lip length of a channel section based on the Canadian cold-formed steel design standard. The procedure used for optimization was not based on the loading resistance of the member, but rather on the capacity to increase the resistant moment of the section and, at the same time, reduce the cross-sectional area.

Castelluci *et al*. (1997) presented a lipped channel section optimized for bending, including two intermediate web stiffeners. Those authors conclude that it was possible to obtain 15% more capacity of the section with only a 5% increase in the area. Their work, however, does not mention the optimization technique used.

Adeli and Karim (1997) developed a neural network model for nonlinear problems and applied it to simply supported beams with *I* and *Z* sections. For assessment of the sections, they used the 1996 AISI allowable stress design (ASD) specification.

Al-Mosawi and Saka (2000) included the bending stress of cold-formed sections in their optimization procedure and obtained optimal symmetric and asymmetric channel sections and *Z* sections subjected to uniform loads. However, they used only normal stress and displacement as constraints.

Tian (2003) conducted a theoretical and experimental study for obtaining the minimum weight of channel sections subjected to compression and recommended a fixed length and a resistant axial load. The BS 5959 British standard and sequential quadratic programming (SQP), a nonlinear method, were used to calculate the load design. A simplified procedure, in which buckling stresses are equivalent was also used. The author concluded that there existed an optimal web width to flange width ratio. The analyzed sections increased loading resistance to axial compression by 50% without increasing the area.

Kolcu *et al.* (2010) sought to maximize the critical buckling load of steel columns by imposing constraints on the volume of material used. The post-buckling performance of optimized cold-formed steel columns was also investigated using nonlinear variable thickness finite strip analysis. The optimization was performed by sequential quadratic mathematical programming. In a similar way, Leng, Guest and Schafer (2011) aimed to maximize the capacity of a steel column subjected to uniform compression, cold formed from a fixed width coil of sheet steel, discretized into narrow strips in order to obtain nonconventional shapes. Three optimization algorithms are explored: the gradient-based steepest descent method, genetic algorithms and simulated annealing. Compared with a standard cold-formed steel lipped channel section, the final optimized capacities are found to be more than twice larger than the original design.

The aim of the present study is to optimize the cross section of cold-formed steel members according to the AISI specifications, providing metal frame designers with optimized solutions. All types of buckling without simplifications are taken into account. Experimental results are also shown with the purpose to validate the numerical results obtained.

Simulated annealing was chosen due to its success in other applications developed by the authors (2008). Also, a research study carried out by Degertekin (2007) showed that simulated annealing provided lighter frames than did genetic algorithms; similar conclusions were obtained by Grigoletti (2008).

2. Methodology

2.1 Verification of lipped and unlipped channel sections subjected to compression

The sections were assessed according to the AISI (2007) specifications, briefly described in what follows.

According to Section C4 of AISI (2007), the nominal axial resistance P_n should be defined as follows

$$P_n = A_e F_n \tag{1}$$

Where A_e is the effective area at stress F_n, calculated as follows

If $\lambda c \leq 1.5$, then

$$F_n = \left(0{,}658^{\lambda_c^2}\right)F_y \tag{2}$$

For $\lambda c > 1.5$, then

$$F_n = \left[\frac{0{,}877}{\lambda_c^2}\right]F_y \tag{3}$$

Where

$$\lambda = \sqrt{\frac{F_y}{F_e}} \tag{4}$$

In the previous expression, F_e is the minimum elastic compressive flexural (σ_{ex}, σ_{ey}), torsional (σ_t) or flexural-torsional (F_{elt}) buckling, based on

$$\sigma_{ex} = \frac{\pi^2 E}{\left(\frac{KL_x}{r_x}\right)^2} \tag{5}$$

$$\sigma_{ey} = \frac{\pi^2 E}{\left(\frac{KL_y}{r_y}\right)^2} \tag{6}$$

$$\sigma_t = \frac{1}{A_g . r_0^2}\left[E_r . J + \frac{\pi^2 . E . C_w}{(K . L_t)}\right] \tag{7}$$

$$F_{elt} = \frac{1}{2.\beta}\left[(\sigma_{ex} + \sigma_t) - \sqrt{(\sigma_{ex} + \sigma_t)^2 - 4.\beta.\sigma_{ex}.\sigma_t}\right] \tag{8}$$

The calculation of the area is in line with the effective width method, according to Section B2 of AISI (2007)

$$b = w \quad \text{if} \quad \lambda \leq 0{,}673 \tag{9}$$

$$b = \rho.w \quad \text{if} \quad \lambda \leq 0{,}673 \tag{10}$$

$$\rho = (1 - 0{,}22/\lambda)/\lambda \tag{11}$$

$$\lambda = \frac{1{,}052}{\sqrt{k}}\left(\frac{w}{t}\right)\sqrt{\frac{f}{E}}\,,\ f = F_n \tag{12}$$

where w is the flat width of the element analyzed in the section. The formulation above is valid for elements fixed at both ends. For simply supported and lipped elements, we use the formulations described in Sections B3.1 and B4.2 of the AISI specification.

2.2 Optimization formulation for cold-formed steel columns

Optimum design formulation of cold-formed steel columns is based on AISI specification (2007) as outlined in Section 2.1. The objective function is the area of the cross section (see Fig. 1) with four design variables: web length (A), flange length (B), lip length (C) and thickness (t). Eq. (13) represents the objective function, and Eqs. (14) through (19) express the calculations for the gross sectional area properties.

$$f(A, B, C, t) = A_g \tag{13}$$

Where

$$A_g = t[a + 2b + 2u + \alpha(2c + 2u)] \tag{14}$$

$$a = A - (2r + t) \tag{15}$$

$$b = B - \left[r + \frac{t}{2} + \alpha(r + \frac{t}{2})\right] \tag{16}$$

$$c = \alpha\left[c - \left(r + \frac{t}{2}\right)\right] \tag{17}$$

$$\mathrm{u} = \frac{\pi \mathrm{r}}{2} \tag{18}$$

$$\alpha \begin{cases} 1 \to C \neq 0 \\ 0 \to C = 0 \end{cases} \tag{19}$$

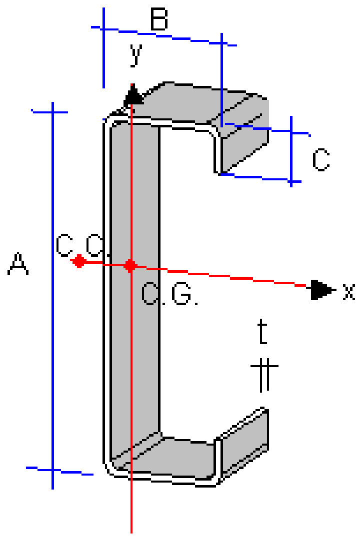

Fig. 1 Channel section

For the sake of simplification, the thickness value is used for the corner radius. In the case of channel columns with unlipped section, the number of variables in the objective function is reduced to three.

The dimensions were considered as continuous variables, with lower limits and upper limits for each dimension defined as a function of the limitations that are inherent to the production process.

In addition to having to follow size specifications, the strength of the section must be equal to or greater than the working stress. For the optimized section, the ultimate strength *Pd* will be very close to the load applied to *Ps*.

Based on the previous considerations, the problem of minimize the section have the following restrictions

$$P_s \le P_d \tag{20}$$

$$50\text{mm} \le \text{A} \le 1000\text{mm} \tag{21}$$

$$30\text{mm} \le \text{B} \le 1000\text{mm} \tag{22}$$

$$1\text{mm} \le \text{t} \le 25\text{mm} \tag{23}$$

The formulation was implemented by simulated annealing, which consists of a heuristic method for global optimization, developed by Kirkpatrick *et al.* (1997). Simulated annealing utilizes a non-descending strategy, unlike conventional optimization approaches, in an attempt to avoid convergence to a local minimum, also accepting, according to a specific criterion, solutions that increase the value of the function. The method draws an analogy with annealing in solids, when a minimum energy state is searched. The term "annealing" refers to the process of heating a solid to its point of fusion, followed by slow cooling. In this process, slow cooling is essential to maintain a thermal equilibrium in which the atoms are able to rearrange into a minimum energy structure. If a solid is cooled abruptly, its atoms will form an irregular and weak structure with high energy as a consequence of the internal stress. In computational terms, annealing can be seen as a stochastic procedure used to determine the organization of atoms with minimum energy. At high temperatures, the atoms move freely and are very likely to achieve positions that increase the energy of the system. When the temperature is reduced, the atoms can move gradually to form a regular structure, and the probability of energy increase is also reduced.

In the optimization technique, the objective function corresponds to the energy of a solid. Similarly to annealing in thermodynamics, the process initiates with a high value of T, for which a new solution is generated. This new solution will be automatically accepted if it decreases the value of the function. If the new value of the function is greater than the previous one, it will be accepted according to a probabilistic criterion, and the accept function

$$p = \exp\left(\frac{-\Delta f}{T}\right) \tag{24}$$

and the new solution will be accepted if p is larger than a randomly generated number between zero and one. While T is high, most of the solutions are accepted, and T is then reduced gradually, for each trial series, to the neighborhoods of the current solution.

3. Numerical results

Some results were obtained from the program developed by the authors, using the formulation proposed in this paper. The following data were considered in all of the numerical simulations: Buckling length coefficients $K_x = K_y = 1$ and $K_t = 0.7$, longitudinal elasticity module $E = 200$ GPa, Poisson coefficient $\upsilon = 0.3$ and yield stress of $f_y = 350$ MPa.

The first analyses were developed in a section with the following characteristics: height $A = 88.5$ mm, width $B = 37.5$ mm, lip height $C = 7.65$ mm, thickness $t = 1.5$ mm and total length $L = 2400$ mm (generating the effective lengths $K_{lx} = K_{ly} = 2400$ mm and $K_{lt} = 1680$ mm). For the described section, a resistant load of $P_n = 10.72$ kN was obtained.

Subsequently, several analyses (at least 10 runs) were made with the developed computational code, attempting to obtain the smallest cross section Ag with the originally proposed load. Table 1 presents the results obtained from two simulations, namely, unlipped and lipped sections (designated, respectively, by the initials PSE and PCE), comparing them with the original section (designated by P0). In this table, the last column indicates the percentage of area reduction (and, consequently, the weight).

The data in Table 1 show a great reduction in the section area obtained from the optimization process. As expected, the presence of lips considerably increases its efficiency, generating an additional reduction in the total amount of material (approximately 15%). Also, we observed that the resistance of the initial section P0 is limited by the flexural buckling stress in relation to the vertical axis y, while in optimized sections we have the simultaneous action of flexural and torsional-flexural buckling.

It is important to emphasize that such economy in the section area (weight reduction) does not compromise the safety conditions. The values of the load capacity for the three channel columns analyzed are exactly the same.

Table 1 Initial reference section and optimized section with and without lips

Section	A (mm)	B (mm)	C (mm)	t (mm)	$F = A_G$ (mm²)	Reduction (%)
P0	88.50	37.50	7.65	1.50	253.41	-
PSE	80.86	48.62	-	1.15	200.84	20.70
PCE	69.33	40.07	13.52	1.00	169.96	32.90

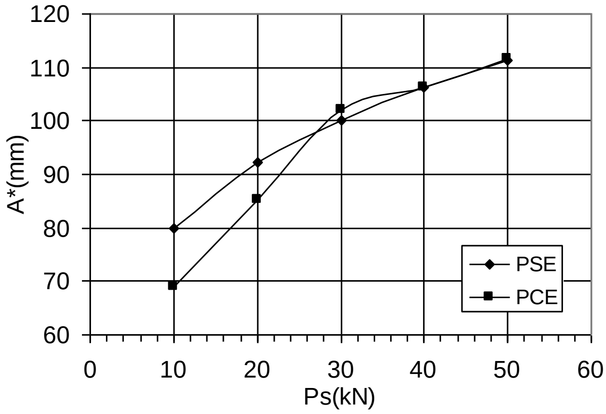

Fig. 2 Load (*Ps*) to web length (*A**) ratio

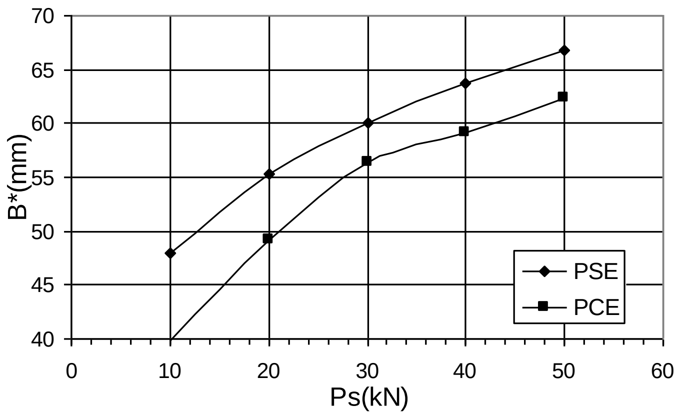

Fig. 3 Load (*Ps*) to flange length (*B**) ratio

In order to validate the procedure employed for optimization of the section, several analyses were performed for the channel column section with different initial values for each variable. In all analyses, the final result does not depend on the initial values adopted.

A second series of analyses was made for the same data used in the previous analyses, but with increasing loads. Considering *Ps* with values of 10, 20, 30, 40 and 50 kN, we sought to determine the optimal dimensions for lipped and unlipped sections. The results of these analyses are displayed in Figs. 2 through 6. As occurred in the previous analyses, we have flexural and flexural-torsional buckling at the same time.

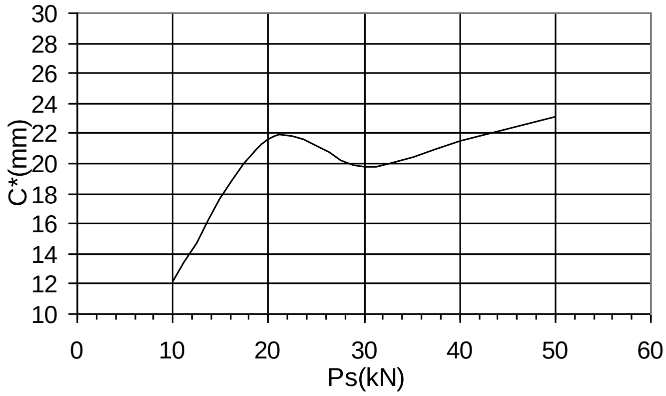

Fig. 4 Load (*Ps*) to lip length (*C**) ratio

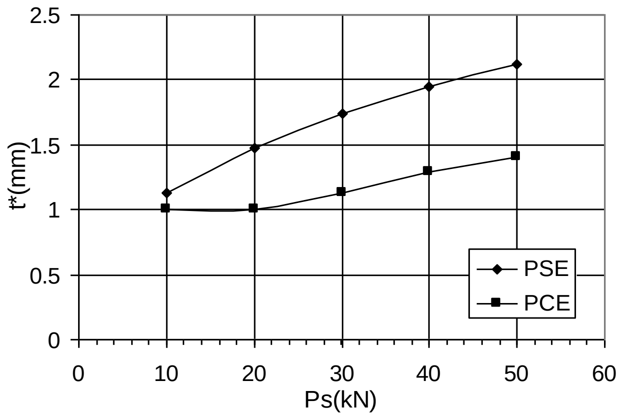

Fig. 5 Load (*Ps*) to thickness (*t**) ratio

The figures show a linear relation in section dimensions as the load is increased. This relationship, however, is not verified for lip length (Fig. 4). Nevertheless, the significant contribution of the lips to the reduction of the section area is evident. In the cases analyzed herein, this reduction ranged from 14.2 to 25.3%.

In spite of the variation of section dimensions with the increase in load, the ratio between the height and base of the unlipped profile is constant (1.66). For lipped sections, this ratio varies from 1.73 to 1.81.

We highlight that the width to thickness ratio, recommended by AISI (2007), was not imposed as constraints in the present work, but the optimized section dimensions were below those limits (200 for the web and 60 for the flanges).

4. Experimental analysis

In order to verify the optimization procedure proposed herein, we built a 110×55×1.2mm

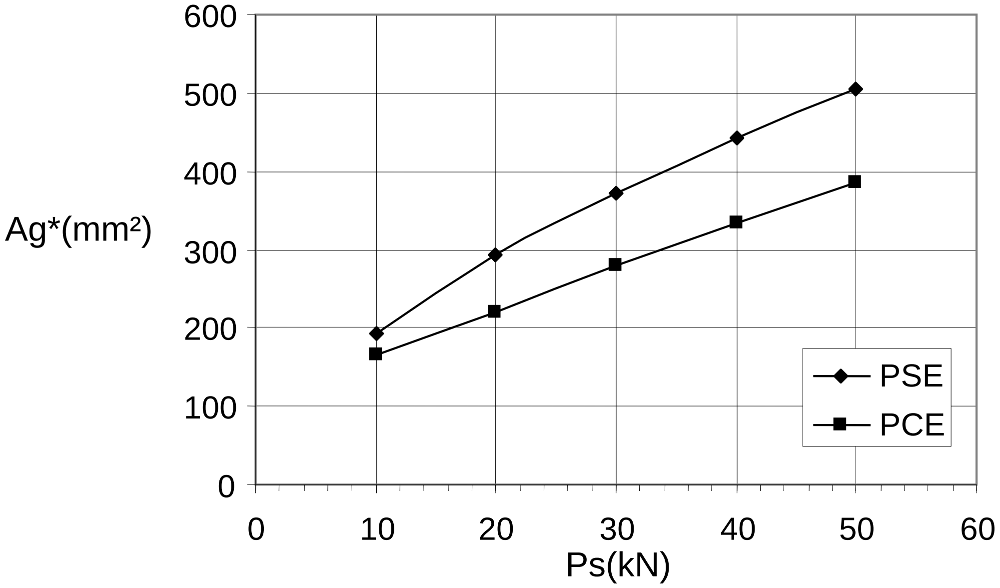

Fig. 6 Load (*Ps*) to Gross Section (*Ag**) ratio

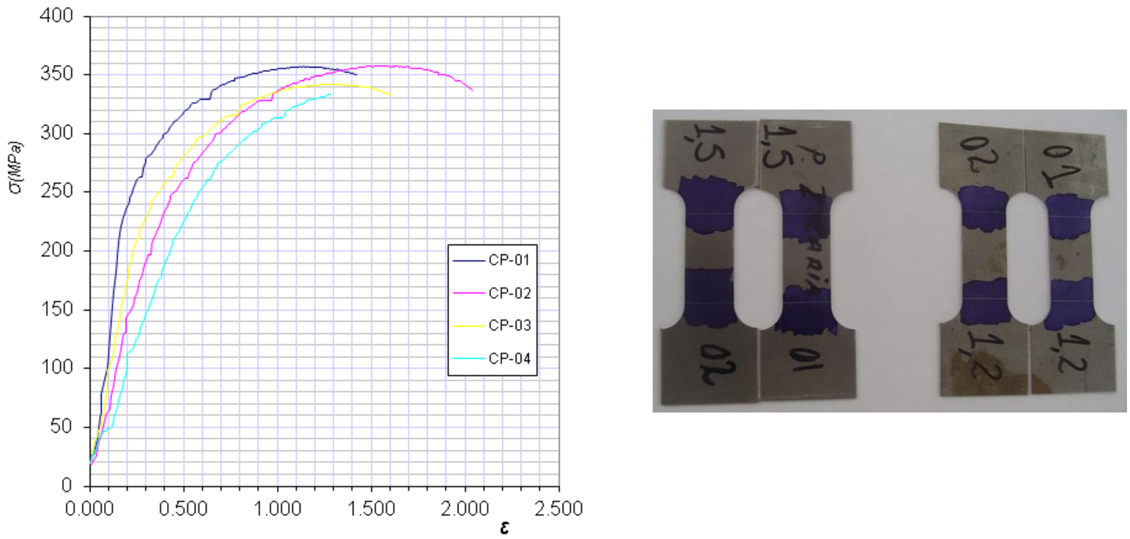

Fig. 7 Stress vs. strain curve (left) and the samples tested (right)

section, with a length of 1200mm. Using the value of the load capacity, we obtained an optimized section with 65×35×1.5mm. The thicknesses were limited to commercial values.

The yield stress was measured in four samples (see right-hand side of Fig. 7), two with 1.2mm and two with 1.5mm in thickness. The results are summarized in Table 2, and the stress vs. strain curve is shown in Fig. 7.

Table 2 Tensile stress results for thicknesses of 1.2 and 1.5mm

Sample	*fy* (MPa)	*fu* (MPa)	*t* (mm)
CP-01	221.9	352.8	1.2
CP-02	231.7	353.3	1.2
CP-03	201.4	337.8	1.5
CP-04	213.7	335.0	1.5
Mean	217.2	344.7	
St. dev.	12.8	9.7	
CV	5.9	2.8	

Fig. 8 View of the four channel columns used on the experimental tests (U110×55×1.2 and U65×35×1.5)

Fig. 9 Assembly of column tests

Fig. 10 Tested sections with local buckling deformation

Table 3 Values of the experimentally tested sections

Section	*Ag* (mm^2)	Experimental *P* (kN)	Reduction area (%)	Variation of collapse load (%)
110×55×1.2	259.27	16.70	-	-
65×35×1.5	197.00	16.08	24%	3.71

Two specimens of each section were built (U110×55×1.2 and U65×35×1.5) in order to test compression (see Fig. 8). An overview of the test assembly is presented in Fig. 9. The tested sections are shown in Fig. 10.

The comparison of the experimental results (see Table 3) shows that the weight of the optimized section was reduced by 24% in relation to the original section. A small variation was observed in the collapse load (3.7%) between both of the tested sections, which does not invalidate the results obtained herein.

We suggest that this work be extended, including more numerical and experimental tests, which could confirm the applicability of the optimization method applied here.

5. Conclusions

In this study, cold-formed steel lipped and unlipped channel columns under axial compression were optimized by the simulated annealing method. The design followed the 2007 edition of the North American Specification for the Design of Cold-Formed Steel Structural Members.

The numerical results demonstrate an important weight reduction in analyzed members. After experimentally evaluating the optimization process and considering the commercially available thickness specifications, we optimized a section with a defined compression resistance: whereas the original section measured 110x55x1.2, the optimized section dimensions were 65x35x1.5. The experimental results show a 24% decrease in weight.

Both reference sections investigated here showed lip local buckling deformation, combined with global flexural buckling. In the case of optimized sections, there was lip local buckling combined with flexural-torsional buckling.

References

Adeli, H. and Karim, A. (1997), “Neural network model for optimization of cold-formed steel sections”, *Journal of Structural Engineering*, ASCE, **123**(11), 1525-1543.

Al-Mosawi, S. and Saka, M.P. (2000), “Optimum shape design of cold-formed thin-walled steel sections”, *Advances in Engineering Software*, **31**, 851-862

American Iron and Steel Institute, AISI, (2007), *Specification for the Design of Cold-Formed Steel Structural Members*, Washington, D.C., USA

Castellucci, M.A., Pillinger, I., Hartley, P. and Deeley, G.T. (1997), “The optimization of cold rolled formed products”, *Thin-Walled Structures*, **29**, 159-174

Degertekin, S.O. (2007), “A comparison of simulated annealing and genetic algorithm for optimum design of nonlinear steel space frames”, *Struct. Multidisc Optim*, **34**, 347-359.

Dinovitzer, A.S. (1992), “Optimization of cold formed steel C-sections using Standard Can/CSA-S316-M89”, Canadian Journal of Civil Engineering, No. 19, 39-50.

Drehmer, G.A., Kripka, M. and Chamberlain Pravia, Z.M. (2008), “Pré-dimensionamento de perfis I soldados sujeitos a flexão usando técnicas de otimização”, *Construção Metálica*, **86**, 23-26.

Grigoletti, G.C. (2008), “Otimização de vigas metálicas de chapa dobrada utilizando o método da Resistência Direta”, Tese (PhD Thesis) - Universidade Federal do Rio Grande do Sul.

Kirkpatrick, S., Gelatt, C.D. and Vecchi, M.P. (1983), *Optimization by Simulated Annealing*, Science 220, 4598, 671-680.

Kolcu, F., Ekmekyapar, T. and Özakça, M. (2010), “Linear buckling optimization and post-buckling behavior of optimized cold formed steel members”, *Scientific Research and Essays*, **5**(14), 1916-1924.

Leng, J., Guest, J.K. and Schafer, B.W. (2011), “Shape optimization of cold-formed steel columns”, *Thin-Walled Structures*, **49**, 1492-1503.

Seaburg, P.A. and Salmon, C.G. (1971), “Minimum weight design of light gauge steel members”, *Journal of Structural Division*, ASCE, **97**(1), 203-222.

Tian, Y.S. (2003), “Optimal design of cold-formed steel sections and panels”, Ph.D. Thesis, Engineering Department, Cambridge University.