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# Inelastic lateral-torsional buckling strengths of stepped I-beams subjected to general loading condition

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**Abstract.** The cross sections of multi-span beams are sometimes suddenly increased at the interior support of continuous beams to resist high negative moment. An earlier study on elastic lateral torsional buckling of stepped beams was conducted to propose new design equations. This research aims to continue the earlier study by considering the effect of inelastic buckling of stepped beams subjected to pure bending and general loading condition. A three-dimensional finite element-program ABAQUS and a statistical program MINITAB were used in the development of new design equations. The inelastic lateral torsional buckling strengths of 36 and 27 models for singly and doubly stepped beams, respectively, were investigated. The general loading condition consists of 15 loading cases based on the number of inflection point within the unbraced length of the stepped beams. The combined effects of residual stresses and geometrical imperfection were also considered to evaluate the inelastic buckling strengths. The proposed equations in this study will definitely improve current design methods for the inelastic lateral-torsional buckling of stepped beams and will increase efficiency in building and bridge design.

**Keywords:** inelastic buckling; buckling strength; stepped beam; moment gradient factor; beam design

## 1. Introduction

Stepped beams are often used in bridges due to economy in materials since the size of the beam are reduced abruptly in areas with low moments. Stepped beams used in continuous beams can either be doubly stepped beam (DSB), with abrupt increase of cross section at both ends or singly stepped beam (SSB), with abrupt increase of cross section at one end. Stepped beams are either constructed by increasing the flange thickness of a welded beam or by welding additional flange plates to a hot-rolled I-section.

Although its popularity, only few researchers have studied on lateral-torsional buckling (LTB) of stepped beams. Trahair and Kitipornchai (1971) investigated the effect of lateral-torsional buckling strengths on simply supported beam stepped at midspan. Lellep and Kraav (2011) made a study which focuses on the elastic buckling capacity of stepped beams having piece wise dimensions with cracks. Park and Stallings (2003, 2005) wrote researches regarding lateral-

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torsional buckling strength of beams that are stepped at the ends. Design equations for these stepped beams were presented to consider general loading cases, stepped conditions, and the effect of $L_b/h$ on lateral torsional buckling. All of these researches considered elastic lateral torsional buckling of stepped beams.

This paper investigates to further explore and extend the study on lateral torsional buckling of beams with steps at one end and at both ends by considering the effect of inelastic beam. This research also presents new design equations that are similar to current lateral torsional buckling solutions which use a modifier $C_{ist}$ to consider the effect of the change in cross section and $C_{ibst}$ to account for the varying moment along the unbraced length. The design equations were compared with finite element analyses results and a calculation procedure using the new equations was provided.

## 2. Background and previous studies

Kitipornchai and Trahair (1975) studied inelastic buckling of simply supported steel I-beam and presented some limited experimental evidences. Nethercot and Trahair (1976) investigated inelastic buckling predictions for hot-rolled prismatic beams with unequal end moments, and transverse loads in the unbraced lengths. Trahair (1993) presented approximate methods that have been developed for analyzing the inelastic buckling of beams which are prevented from deflecting and twisting at their supports and brace points. This simplest method is to ignore the buckling interactions which take place between adjacent segments during buckling. A lower bound may then be obtained from the lowest of the inelastic buckling load factors calculated for the segments of the beam. Mohebkhah (2010) showed the results of his study on nonlinear inelastic lateral torsional buckling of hot rolled steel I-beams having a wide variety of overall slenderness under moment gradient and subjected to off-shear center loading. Also, Mohebkhah and Chegeni (2012) made a study involving the nominal flexural capacity of I-beam sections having compact webs and noncompact or slender flanges taking into consideration on the interaction between the flange local buckling and lateral torsional buckling. They concluded that the flange local buckling (FLB) and LTB limit states for beams in the inelastic range has no significant interaction. On the other hand, Li (2007) studied the lateral torsional buckling behavior of general prismatic and tapered steel members having doubly and singly symmetric sections. He proposed equations that can calculate the inelastic lateral-torsional capacity by using straight-line transition method and for determining the limiting unbraced member lengths. Meanwhile, Trahair (2011) made a study on the inelastic buckling of monosymmetric steel I-beams under moment gradient and compared it with design recommendations. These studies deal with inelastic buckling of beams but do not cover the buckling of stepped beams.

American Institute of Steel Construction (AISC) *Specifications* (2010) defines inelastic LTB moment capacity as below

$$\mathbf{M_n} = \mathbf{C_b}\left[\mathbf{M_p} - (\mathbf{M_p} - \mathbf{M_r})\left(\frac{\mathbf{L_b} - \mathbf{L_p}}{\mathbf{L_r} - \mathbf{L_p}}\right)\right] \leq \mathbf{M_p} \tag{1}$$

where $C_b$ is the moment gradient modifier; $M_p$ is the plastic bending strength of a beam; $M_r$ is the limiting buckling moment; $L_b$ is the laterally unbraced length; $L_p$ and $L_r$ are the limiting lengths for plastic and elastic zone, respectively. Eq. (1) with $C_b$=1 is the inelastic lateral-torsional buckling resistance for an I-shaped prismatic section under the action of constant moment over the laterally unbraced length.

The AISC *Specifications* (2010) and AASHTO *LRFD Bridge Design Specifications* (2010) which is based on the study of Kirby and Nethercot (1979) have incorporated the following expression for $C_b$, which is applicable for linear and nonlinear moment diagrams

$$\mathbf{C_b} = \frac{\mathbf{12.5M_{max}}}{\mathbf{2.5M_{max}+3M_A+4M_B+3M_C}} \tag{2}$$

where $M_{\max}$ is the maximum moment along $L_b$, $M_A$, $M_B$, and $M_C$ are the respective moments at $L_b/4$, $L_b/2$, and $3L_b/4$ and $L_b$ is the distance between the braced points. Absolute value is used for all moments.

There are also studies made by Nawaz (2009) and Serna *et al*. (2006) for the equivalent uniform moment factors for lateral torsional buckling of steel members which can be used instead of the one given by AISC. However, for this study, the AISC specifications (2005) are followed.

For studies involving stepped beams, Trahair and Kitipornchai (1971) have previously studied lateral torsional buckling of beams with step at midspan. They discussed how steps can affect the minor axis flexural rigidity, torsional rigidity, and warping rigidity of beams. A series of studies on the LTB capacity of stepped beams were published by Park and Stallings (2003, 2005). All the researches focused on two general types of stepped beams, which are doubly stepped beams (DSBs) and singly stepped beams **(**SSBs). Park and Stallings (2003) suggested the following equation for the lateral torsional buckling of the DSB and the SSB subjected to pure bending

$$M_{ost} = C_{st} M_{ocr} \tag{3a}$$

$$\text{with } C_{st} = C_0 + 6\alpha^2(\beta\gamma^{1.3}-1) \quad \text{for DSB} \tag{3b}$$

$$C_{st} = C_0 + 1.5\alpha^{1.6}(\beta\gamma^{1.2}-1) \text{ for SSB} \tag{3c}$$

where $M_{ocr}$ is the LTB strength of prismatic beam with smaller cross section and $\alpha$, $\beta$ and $\gamma$ are the ratios of stepped lengths, flange widths and flange thicknesses, respectively. Park and Stallings (2003) proposed an equation applicable to various loadings conditions. The presented equation for $C_{bst}$ depends on the number of inflection point (IP).

$$M_{st} = C_{bst} C_{st} M_{ocr} \tag{4a}$$

$$\text{with } C_{bst} = \frac{12.5\ M_{max}}{2.5M_{max}+3M_A+4M_B+3M_C} \text{ for IP=0} \tag{4b}$$

$$C_{bst} = \frac{10\ M_{max}}{4M_{max}+M_A+7M_B+M_C} \text{ for IP=1 or IP=2} \tag{4c}$$

where $C_{st}$ is given by Eqs. (3b) and (3c). $C_0$ values for load cases with IP of zero or one are 1.0 while $C_0$ value for load cases with IP of two is 0.85. All proposed equations from Park and Stallings (2003, 2005) can be used for stepped beam within elastic buckling region.

## 3. Inelastic finite element modeling

A finite element computer program ABAQUS (2011) was used to perform the numerical analyses of the lateral torsional buckling capacities of inelastic stepped I-beams. A linear, four-noded shell element, S4R, was chosen to model the beams due to its capability to provide enough degrees of freedom to clearly model the inelastic buckling deformations of the beam. Park and

Park (2013) presented the comparison study on experimental tests, proposed equations, and finite element analyses for monosymmetric stepped beams subjected to a concentrated load on the centre of the span having the unbraced length within inelastic strength zone. The study also used the program ABAQUS for numerical analyses and showed that the results from the finite element analyses were similar to the values yielded by the experimental test.

Fig. 1 shows the cross section and dimension of the smaller beam. The flange width and thickness of the smaller beam were fixed at 305 mm and 25.4 mm, respectively. The thickness of the web and height of beam, 16.5 mm and 890 mm, respectively, were kept constant. Figs. 2-3 show the two general types of stepped beams being studied here, doubly stepped beam (DSB) and singly stepped beam (SSB). Both flange thickness and flange width were varied at both ends for DSB and at one end for SSB. The material and section properties of the beam shown in Fig.1 are modulus of elasticity of 210GPa, shear modulus of 80.77GPa, second moment of inertia of $1.20\times10^{-4}\mathrm{m}^4$, torsional constant of $4.63\times10^{-6}\ \mathrm{m}^4$, and warping constant of $2.38\times10^{-5}\ \mathrm{m}^6$. Figs. 2-3 show the elevation and plan view of the DSB and the SSB, respectively. The ratio of the flange thicknesses, $\gamma$, the ratio of the flange widths, $\beta$, and the ratio of stepped lengths, $\alpha$, are also defined in these figures.

Kim *et al*. (2008) presented the inelastic buckling behaviors of stepped I-beams subjected to pure bending and provided the design equations for LTB strengths. The modeling approach and the equations presented were reviewed to extend for stepped I-beams subjected to general loading conditions. First of all, boundary conditions for three-dimensional modeling were investigated based on comparisons between finite element analyses (FEA) and theoretical equations for elastic and inelastic LTB strengths of prismatic beams. Second, a convergence test was held to choose the optimum mesh for the models. Tables 1-2 show the stepped parameters for doubly and singly stepped beams. The parameters used for stepped beams are taken from the geometry of real bridges.

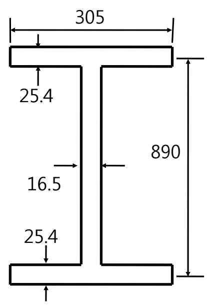
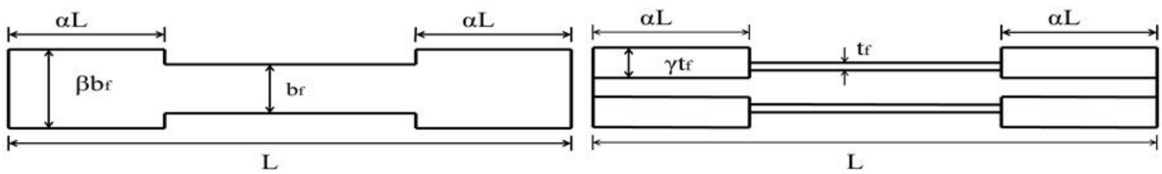
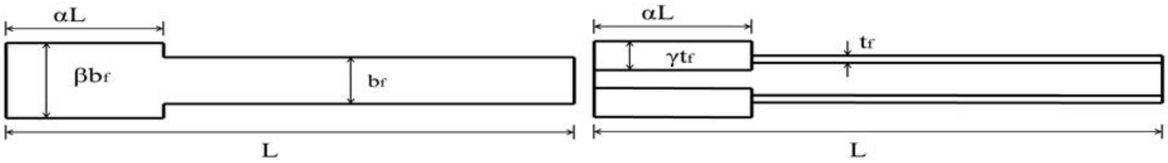



Fig. 1 Cross section for analytical model (unit:mm)




(a) Plan (b) Elevation

Fig. 2 Definition of $\alpha$, $\beta$ and $\gamma$ for doubly stepped beam




(a) Plan (b) Elevation

Fig. 3 Definition of $\alpha$, $\beta$ and $\gamma$ for singly stepped beam

Table 1 Parameters for doubly stepped beams

| $\alpha$ | $\beta$ | $\gamma$ |
|---|---|---|
| 0.167 | 1.0 | 1.2; 1.4; 1.8 |
| 0.167 | 1.2 | 1.0; 1.4; 1.8 |
| 0.167 | 1.4 | 1.0; 1.4; 1.8 |
| 0.25 | 1.0 | 1.2; 1.4; 1.8 |
| 0.25 | 1.2 | 1.0; 1.4; 1.8 |
| 0.25 | 1.4 | 1.0; 1.4; 1.8 |
| 0.333 | 1.0 | 1.2; 1.4; 1.8 |
| 0.333 | 1.2 | 1.0; 1.4; 1.8 |
| 0.333 | 1.4 | 1.0; 1.4; 1.8 |

Table 2 Parameters for singly stepped beams

| $\alpha$ | $\beta$ | $\gamma$ |
|---|---|---|
| 0.167 | 1.0 | 1.2; 1.4; 1.8 |
| 0.167 | 1.2 | 1.0; 1.4; 1.8 |
| 0.167 | 1.4 | 1.0; 1.4; 1.8 |
| 0.25 | 1.0 | 1.2; 1.4; 1.8 |
| 0.25 | 1.2 | 1.0; 1.4; 1.8 |
| 0.25 | 1.4 | 1.0; 1.4; 1.8 |
| 0.333 | 1.0 | 1.2; 1.4; 1.8 |
| 0.333 | 1.2 | 1.0; 1.4; 1.8 |
| 0.333 | 1.4 | 1.0; 1.4; 1.8 |
| 0.5 | 1.0 | 1.2; 1.4; 1.8 |
| 0.5 | 1.2 | 1.0; 1.4; 1.8 |
| 0.5 | 1.4 | 1.0; 1.4; 1.8 |

AISC *Specifications* (2010) categorize buckling failure based on the unbraced length of the beam. The basic relationship between the flexural strength and unbraced length is shown in Fig. 4. Beams with unbraced length falling between the limiting lengths $L_p$ and $L_r$ usually fails by inelastic buckling. Five length models were considered for inelastic buckling strengths of stepped beams subjected to pure bending and three length models were investigated for stepped beams subjected to general loading conditions, each picked in sections near plastic, at the middle of inelastic zone, and near elastic zone. The three lengths are $L_b$ =3.56m ($L_b/h$=4.0), $L_b$ =5m ($L_b/h$=5.6) and $L_b$ =8.5m ($L_b/h$=9.6), respectively, as shown in Fig. 4.

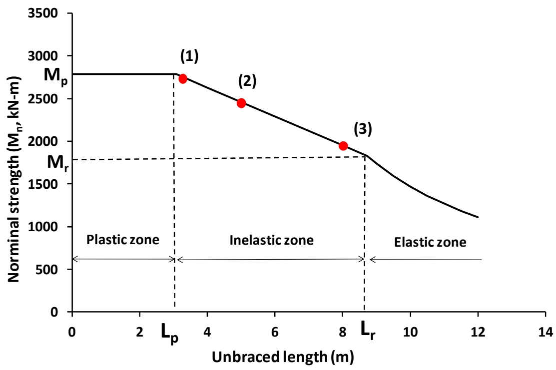



Fig. 4 Nominal flexural strength as a function of unbraced length

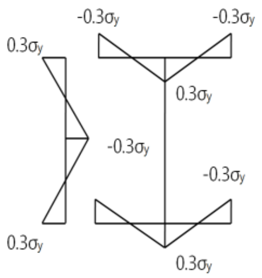



Fig. 5 Distribution of residual stress

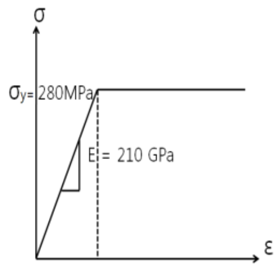



Fig. 6 Stress-strain relationship of analytical model

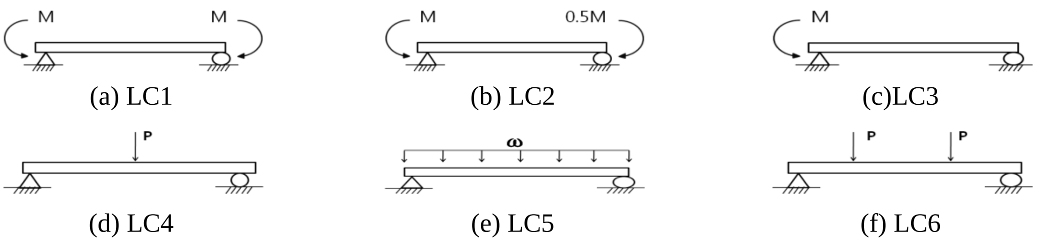
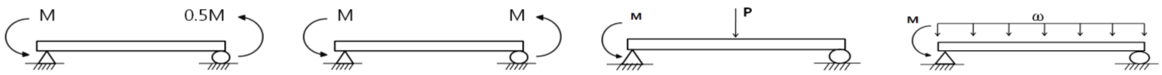



Fig. 7 Load cases for inflection point of zero (IP=0)

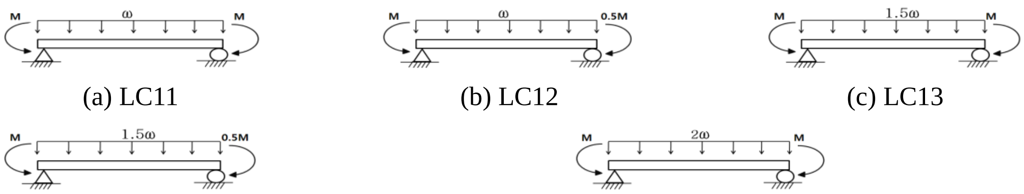
To have an accurate inelastic model, the effect of residual stresses and geometric imperfections must be realistically applied to the model. The linearly distributed residual stress for hot rolled section shown in Fig. 5 was adopted from Pi and Trahair (1995). This was modeled in ABAQUS using initial conditions option with type=stress. The initial geometric imperfection of the beam is set by central displacement and is equal to 0.1% of the unbraced length of the beam suggested by Hyundai Steel (2006). This is introduced using imperfection option where the buckling mode shape came from the elastic analysis. Fig. 6 shows the stress-strain relationship used, which has a yield stress of 280 MPa and a modulus of elasticity of 210GPa.

Stepped beams were subjected to load cases (LC) of 15 and were divided into groups based on the number of inflection points. Fig. 7 shows the load cases with no inflection point (IP=0). There are six load cases under this group. Fig. 8 shows the four load cases considered with one inflection point (IP=1). Fig. 9 shows the five load cases considered with two inflection point (IP=2). The end




(a) LC7 (b) LC8 (c) LC9 (d) LC10

Fig. 8 Load cases for inflection point of one (IP=1)




(a) LC11 (b) LC12 (c) LC13

(d) LC14 (e) LC15

Fig. 9 Load cases for inflection point of two (IP=2)

moments M at LC9 and LC10 are given as 3PL/16 and $\omega L^2/12$, respectively. For the load cases of the Fig. 9, the end moment $M$ is $\omega L^2/12$.

## 4. Finite element results

### *4.1 Beams subjected to pure bending*

Parametric analyses were conducted to determine the relationship between all the parameters discussed previously and the LTB strengths of beams. Doubly stepped beams having 135 models and singly stepped beams having 180 models with $L_b$ of 4m, 5m, 6m, 7m, and 8m were investigated. The buckling moment results from ABAQUS (2011) were used to compute the stepped beam correction factor, $C_{ist}$, which is defined as the ratio between the moment capacity of the stepped beam and the buckling capacity of the prismatic beam having the smaller section subjected to pure bending. A regression program, MINITAB (2007), was used for developing new equations.

The proposed inelastic lateral torsional buckling equation for stepped beam subjected to pure bending along the unbraced span is

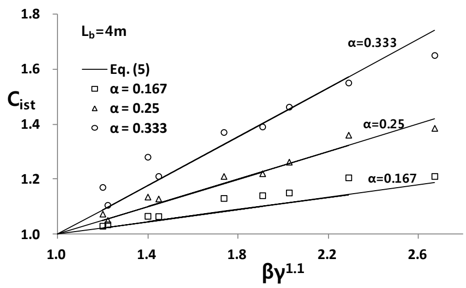
$$\mathbf{M_{ist} = C_{ist} M_{icr}} \tag{5a}$$

$$\text{with} \quad C_{ist} = 1 + 4\alpha^2(\beta\gamma^{1.1}-1) \qquad \text{for DSB} \tag{5b}$$

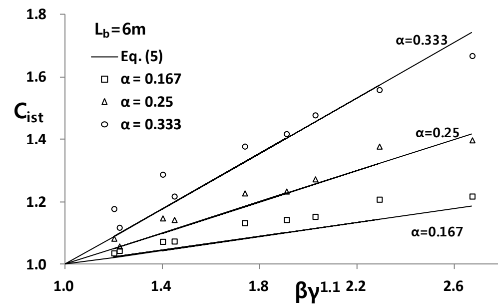
$$C_{ist} = 1 + 0.7\alpha^2(\beta\gamma^{1.05}-1) \text{ for SSB} \tag{5c}$$

where $M_{icr}$ is the inelastic LTB strength of beam using Eq. (1) and $\alpha$, $\beta$ and $\gamma$ are the stepped length ratio, flange width ratio and flange thickness ratio as defined in Figs. 2 and 3 (Kim *et al*. 2008).

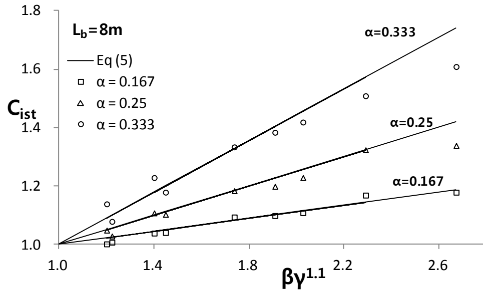
Fig. 10 presents comparisons between the results of FEA and the proposed equation for doubly stepped beams with $L_b$ of 4m, 6m, 8m and total results. The graph shows that the dots represent the FEA results and the solid line represents the proposed equation. The maximum difference of the conservative estimate is 9.5% with $\alpha$=0.333, $\beta$=1.4, $\gamma$=1.0 and $L_b$ =6m. The maximum difference of the unconservative estimate is -7.6% with $\alpha$=0.333, $\beta$=1.4, $\gamma$=1.8, and $L_b$ =8m. Fig. 11 also shows



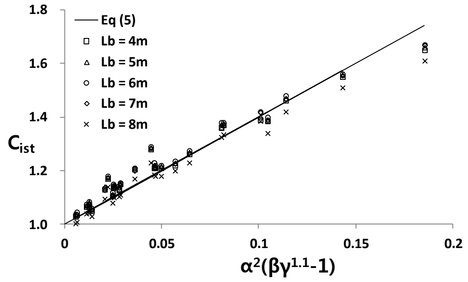

(a) $L_b/h$=4.5




(b) $L_b/h$=6.7

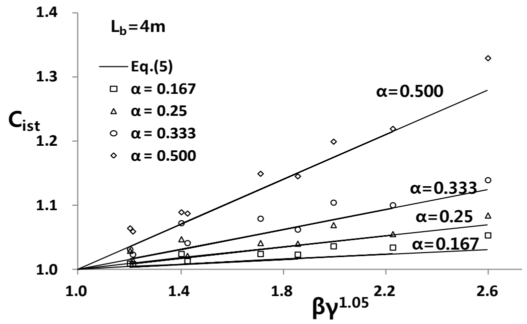



(c) $L_b/h$=9.0

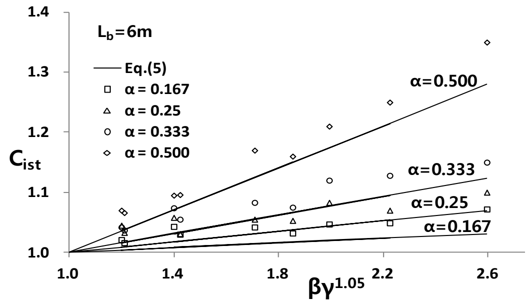



(d) All

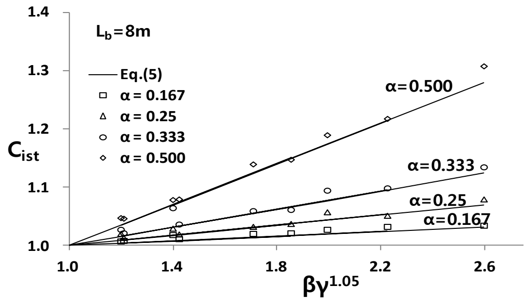
Fig. 10 Comparisons between FEA results and proposed equation for doubly stepped beams



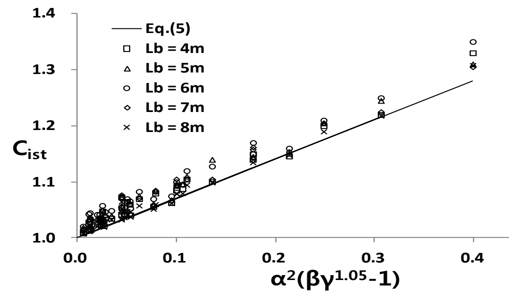

(a) $L_b/h$=4.5




(b) $L_b/h$=6.7




(c) $L_b/h$=9.0




(d) All

Fig. 11 Comparisons between FEA results and proposed equation for singly stepped beams inelastic LTB strengths for singly stepped beams having $L_b$ of 4m, 6m, 8m and total results. The dots and solid line denotes the FEA results and the proposed equation, respectively. The results of the analyses show that Eq. (5c) gives conservative results for most cases, with a difference ranging from -0.4% to 5.5%. The solution is the most conservative for beams with $\alpha$=0.5, $\beta$=1.4, $\gamma$=1.8, and $L_b$ =6m and the most unconservative with $\alpha$=0.333, $\beta$=1.0, $\gamma$=1.8, and $L_b$=8m. Figs. 10(d) and 11(d) present all results of the models with $L_b$ of 4m, 5m, 6m, 7m, and 8m. The new equations presented have results that are either conservative or unconservative as mentioned and shown in Figs. 10 and 11. However, it is a simple equation that produces reasonable estimates with $C_{ist}$ from 1.03 to 1.74 for DSBs and from 1.01 to 1.28 for SSBs.

### *4.2 Beams subjected to general loading*

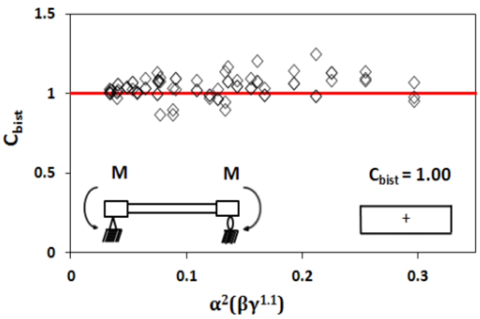
The general equation was developed based on 15 different load cases which were applied to 81 doubly stepped beams and 108 singly stepped beams with $L_b/h$ of 4, 5.6, and 9.6. The load cases were divided by the number of inflection points within the unbraced length. The load cases investigated are shown in Figs. 7, 8 and 9. The proposed design equation for inelastic LTB resistances of stepped beams is

$$\mathrm{M_{ist}} = \mathrm{C_{bist}} \mathrm{C_{ist}} \mathrm{M_{icr}} \tag{6}$$

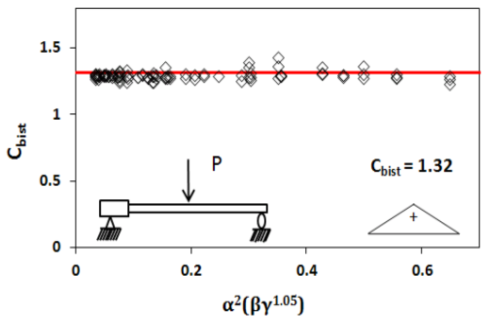
where $M_{icr}$ is the inelastic buckling strength of the beam with the smaller section at mid span which is given by Eq. (1), $C_{ist}$ is the correction factor to account for the steps at the ends of the beam, defined by Eq. (5b) for DSB and Eq. (5c) for SSB, and $C_{bist}$ is the correction factor for varying moment along the unbraced beam length with respect to the number of inflection points.

Load cases from LC1 to LC6 shown in Fig. 7 were investigated for stepped beams with IP=0. The results of the finite element analyses showed that the Eq. (2) from the AISC *Specifications* (2010) can be used for inelastic LTB strengths of stepped beams with IP=0. Comparisons of FEA results and Eq. (2) for doubly and singly stepped beams are presented in Fig. 12. The solid line represents Eq. (2) and the dots are the values obtained from the finite element analyses. Fig 12(a) shows the results for DSB under pure bending and the Eq. (2) produces a result of $C_{bist}$ = 1, with the difference between the proposed equation and FEM results of -5% to 8%. In Fig. 12(b), the results for the SSB subjected to a concentrated load at the midspan are considered with $C_{bist}$=1.32 and the difference between the proposed equation and FEM results is of -3% to 3.5%. Fig. 12(c) also presents comparisons of results for DSBs under a distributed load, having a difference ranging from -9% to 12%. The Eq. (2) gives the $C_{bist}$ value of 1.14. Fig. 12(d) shows the results of SSBs for LC6 which is subjected to two concentrated loads located at $L_b$/3 and 2$L_b$/3. Since most of the results for stepped beams with IP=0 show a difference within 10%, use of the Eq. (2) from the AISC *Specifications* (2010) was therefore concluded to be suitable.

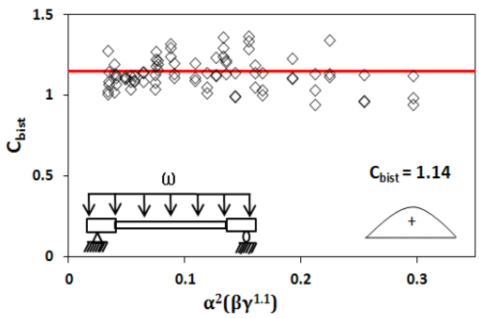
Fig. 13 shows comparisons of the results from the proposed Eq. (7) and FEA results for doubly and singly stepped beams with IP=1. The solid lines and dots represent the results of the proposed equation and FEM results, respectively. Fig. 13(a) is the result for SSBs subjected to LC7 with $C_{bist}$=1.66 and a difference between proposed equation and FEM results of -2.5% to 24%. Fig. 13(b) shows the results for DSBs subjected to LC8 with $C_{bist}$=1.69 having a difference of -2.9% to 15%. The $C_{bist}$ of the SSB subjected to LC9 is of 1.29 from Eq. (7) and the conservative and unconservative differences are of -6% to 12% as shown in Fig. 13(c). Fig. 13(d) shows the comparisons for DSBs subjected to one end moment with an uniformly distributed load and the difference of each result is of -19% to 18%. For beams under a linear moment diagram such as



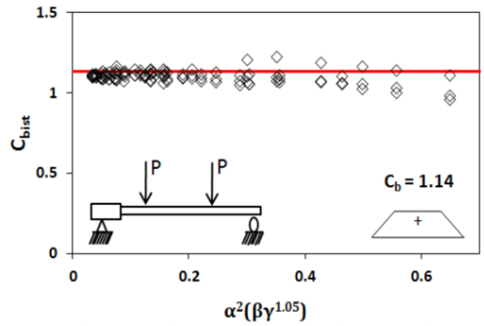

(a) LC1, DSB




(b) LC4, SSB

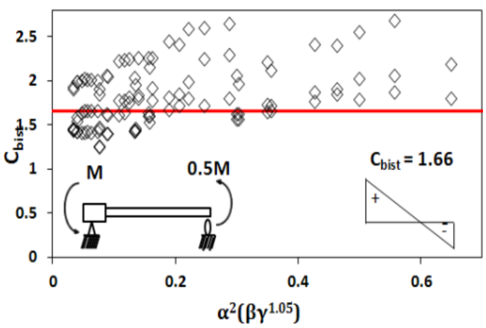



(c) LC5, DSB

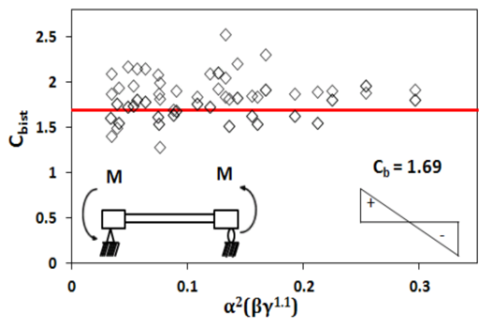



(d) LC6, SSB

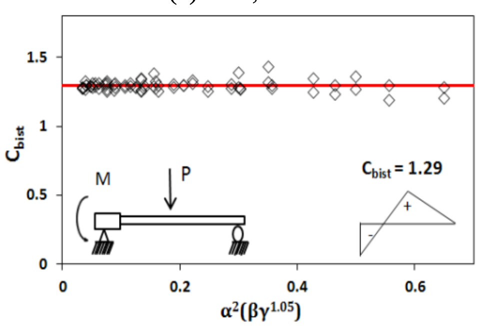
Fig. 12 Comparisons between FEA results and proposed Solution with inflection point of zero



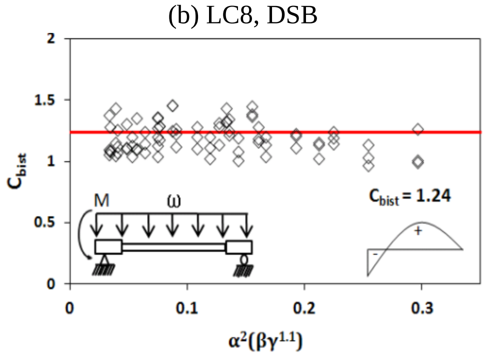

(a) LC7, SSB




(b) LC8, DSB

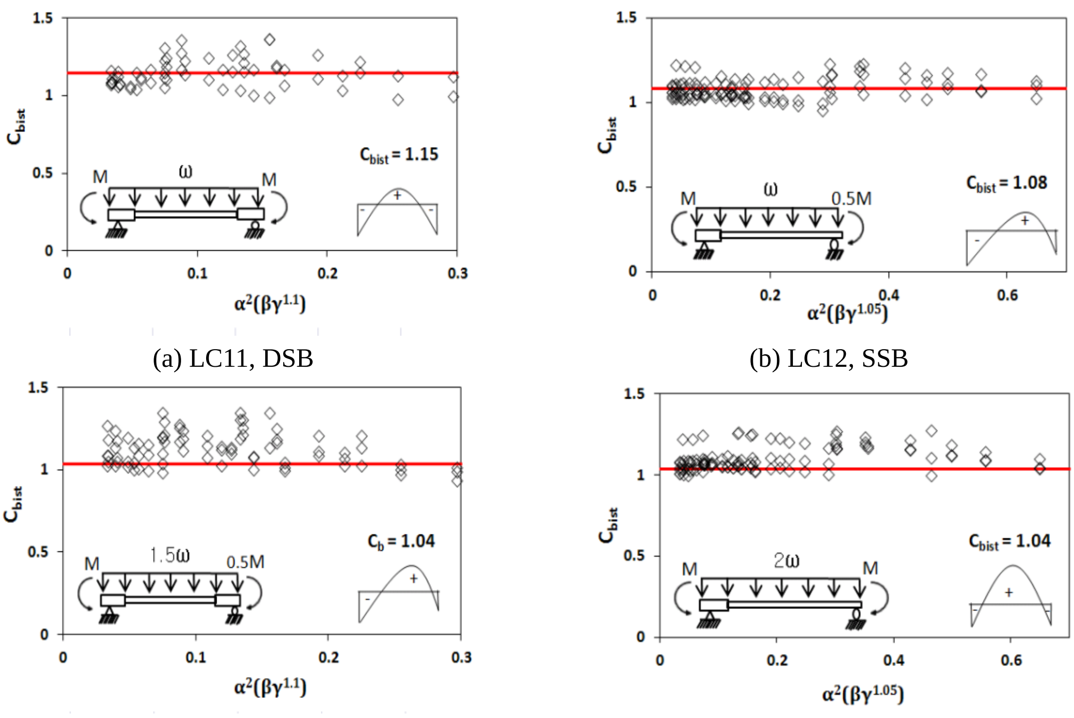



(c) LC9, SSB




(d) LC10, DSB

Fig. 13 Comparisons between FEA results and proposed Solution with inflection point of one




(a) LC11, DSB

(b) LC12, SSB

(c) LC14, DSB

(d) LC15, SSB

Fig. 14 Comparisons between FEA results and proposed Solution with inflection point of two

$$C_{bist} = \frac{11M_{max}}{4M_{max}+2.5M_A+3M_B+2.5M_C} \tag{7}$$

LC7 and LC8, FEA results are scattered mostly above the solid line and has a high difference between the proposed equation and FEA results. The Eq. (7) can be simply used for stepped beam design. Another simple and easy method can be used for stepped beam with IP=1. The moment gradient factors can be conservatively obtained using Eq. (2) multiplied by 0.8 for the load cases of LC7, LC8, and LC10 and by 0.9 for the load case of LC9.

$$C_{bist} = \frac{10M_{max}}{8M_{max}+0.5M_A+M_B+0.5M_C} \tag{8}$$

Fig. 14 shows comparisons of the results of proposed Eq. (8) and the FEA results for load cases with IP=2. The results of four load cases were shown in Figs. 14(a) to (d). Fig. 14(a) presents the results for stepped beams with two end moments and uniformly distributed load. The difference between the results is of -9% to 11%. Fig. 14(b) shows results for LC12 with $C_{bist}$=1.08 and the distributed difference of -6% to 8%. The results for DSBs subjected to LC14 are shown in Fig. 12(c) having a difference of -5% to 11%. For SSBs subjected to LC15, the analytical results and proposed equation are shown in Fig. 14(d). The difference between the proposed solution and the FEM results is of -3% to 10%.

## 5. Conclusions

This research presents the inelastic lateral torsional buckling strengths of stepped beam subjected to pure bending and general loading condition. A finite element program ABAQUS (2011) was used for parametric analyses and a statistical program MINITAB (2007) was used to develop the new design equations. First of all, stepped beams subjected to pure bending were considered to evaluate inelastic lateral torsional buckling strengths. Two design equations of $C_{ist}$ considering stepped ratios were developed and extended for stepped beams subjected to general loading conditions. The general loading conditions were categorized according to the number of inflection points. The $C_b$ formula suggested by AISC *Specifications* (2010) can be used for stepped beams with inflection point of zero. For load cases with inflection point of one and two, some modifications were applied to develop new equations for more accuracy. The proposed equations produce results that are either conservative or unconservative depending on the load cases and variables involved. The proposed equations will definitely improve current design methods for the inelastic lateral-torsional buckling of stepped beams and will increase efficiency in building and bridge design.

## Acknowledgments

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## Appendix. example problem

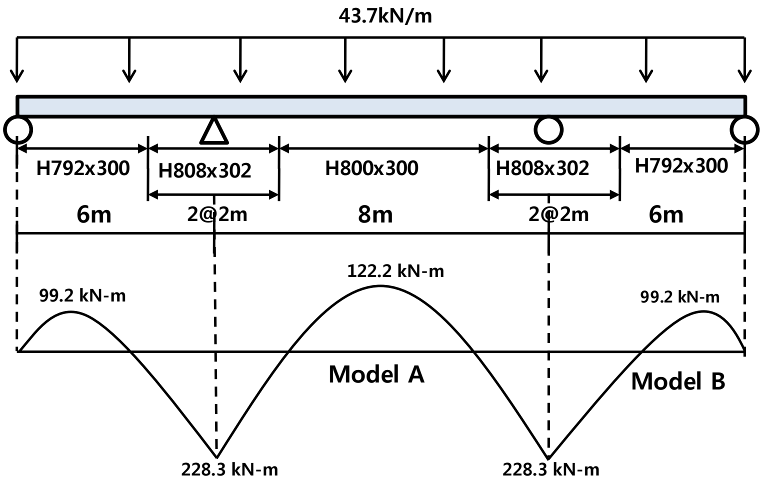

Fig. 15 Three-span continuous beam

Two example applications of the proposed equation, Eq. (6), are illustrated in the Appendix. Eq. (6) provides an estimate of the resistance for the limit state of lateral-torsional buckling. Current design codes generally provide separate methods that should be used for evaluating the resistance to other limit states such as local buckling and yielding. The example uses a three-span beam with uniform loading where bracing is initially assumed to be provided only at the supports. Hence, the center span is a doubly-stepped beam with inflection points of two (Model A) and end span is a singly stepped beam with inflection points of one (Model B).

Determine the LTB capacity of the center span of the continuous beam shown in Fig.15 during the construction of the concrete slab. Steps in the cross section are shown in Fig. 15.

(1) Doubly-stepped beam (model A)

$L_b$ = 8 m, $L_b/h$ = 10.39, $r_y$ = 67mm, $S_x$ = 7.06mm, $Z_x$ = 7.9mm, J = 4.2×$10^{-3}$mm$^4$, $C_w$ = 1.75×$10^{-2}$ mm$^6$, $X_1$ = 1.36×$10^{13}$, $X_2$ = 2.59×$10^{-13}$, E = 210 GPa, v = 0.3, $F_y$ = 280 MPa, $F_r$ = 68.9 MPa, $\mathbf{M_r = (F_y - F_r)S_x}$ = 1,491.2 kN-m, $\mathbf{L_p = 1.7 r_y \sqrt{\frac{E}{F_y}}}$ = 3.23m, $\mathbf{L_r = \frac{r_y X_1}{F_L}\sqrt{1+\sqrt{1+X_2 F_L{}^2}}}$ = 9.17m, $\mathbf{M_p = F_y Z_x}$=2,210kN-m,

α= 2/8= 0.25, β= 302/300= 1.01, γ=30/26= 1.15,

$\mathbf{M_n = M_p - (M_p - M_r)\left(\frac{L_b - L_p}{L_r - L_p}\right)}$ =1,634kN-m

$\mathbf{C_{ist} = 1 + 4\alpha^2(\beta\gamma^{1.1} - 1)}$ = 1.04, $\mathbf{C_{bist} = \frac{10M_{max}}{8M_{max}+0.5M_A+M_B+0.5M_C}}$=1.15

$\mathbf{M_{ist} = C_{bist}C_{ist}M_{icr}}$=1,954kN-m

The difference between the proposed equation and finite element result of 1940kN-m is of -1.07%.

(2) Singly-stepped beam (model B)

$L_b$ = 6 m, $L_b/h$ = 7.79, $r_y$ = 64.73 mm, $S_x$ = 6.18 mm, $Z_x$ = 7 mm, J = 2.81×10$^{-3}$mm$^4$ ,
$C_w$ = 1.47×10$^{-2}$ mm$^6$ , $X_1$ = 1.21×10$^{13}$ , $X_2$ = 4.37×10$^{-13}$ , E = 210 GPa, v = 0.3, $F_y$ = 280 MPa,
$F_r$ = 68.9MPa, $\mathbf{M_r = (F_y - F_y)S_x}$= 1,303.6kN-m, $\mathbf{L_p = 1.7 r_y \sqrt{\frac{E}{F_y}}}$=3.12m,

$\mathbf{L_r = \frac{r_y X_1}{F_L}\sqrt{1 + \sqrt{1 + X_2 F_L{}^2}}}$=8.72m, $\mathbf{M_p = F_y Z_x}$=1,970kN-m,

α = 2/6 = 0.4, β = 302/300 = 1.01, γ = 30/20 = 1.5

$\mathbf{M_n = M_p - (M_p - M_r)\left(\frac{L_b - L_p}{L_r - L_p}\right)}$ = 1,626kN-m,

$\mathbf{C_{ist} = 1 + 0.7\alpha^2(\beta\gamma^{1.05} - 1) = 1.02}$, $\mathbf{C_{bist} = \frac{11M_{max}}{4M_{max} + 2.5M_A + 3M_B + 2.5M_C}}$ = 1.86,

$\mathbf{M_{ist} = C_{bist} C_{ist} M_{icr}}$ **= 3,085kN-m**

The difference between the proposed equation and finite element result of 2900kN-m is of -6.0%.