Structural Engineering and Mechanics, *Vol. 48, No. 1 (2013) 93-102*
DOI: http://dx.doi.org/10.12989/sem.2013.48.1.093

Optimization of a sandwich beam design: analytical and numerical solutions

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(Received April 3, 2013, Revised August 20, 2013, Accepted October 2, 2013)

Abstract. An optimization work was developed in this work to provide design information for sandwich beam in civil engineering applications. This research is motivated by the wide-range applications of sandwich structures such as; slab, beam, girder, and railway sleeper. The design of a sandwich beam was conducted by using analytical and numerical optimization. Both analytical and numerical procedures consider the optimum design with structure mass objective minimization. Allowable deflection was considered as design constraints. It was found that the optimized core to the skins mass ratio is affected by the skin to core density and elastic modulus ratios. Finally, the optimum core to skin mass ratio cannot be constant for different skin and core materials.

Keywords: sandwich beam; mass; analytical; numerical; optimization

1. Introduction

The use of a sandwich panel has become a popular choice in the construction of structural elements due to it robust characteristics and strength. Some core materials such as the balsa wood and lightweight foam core are soft, and it may be crushed under compression load. Others such as honeycomb and trussed-core structure have a high compressive strength but low capacity to hold mechanical connections. Skin material has high density, high elastic modulus, and high strength than core material. In most applications, skin materials are either metal (steel or aluminum) or FRP composite (carbon or glass fiber) (Awad and Yusaf 2012). Recently, a fiber composite sandwich panel was developed for civil engineering applications with high core density (850 kg/m^3) (Awad *et al*. 2012). In this fiber composite panel the ratio of skin to core density is around 2. The flexural strength of the sandwich panel depends on the composite action between the skin and the core. The separation desistance between them is one of the most significant parameters (Hollaway and Head 1999, Awad *et al*. 2012).

Economic and lightweight structure design is an important goal for the designer. Many methods have been used to find the optimum design of fiber composite structures in different applications. Most of the optimization methods are service with continuous design variables. Civil engineering structural design involves selection of design variables that satisfy requirements of the practical codes. In general, these variables are discrete for most practical civil engineering problems. All

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http://www.techno-press.org/?journal=sem&subpage=8 ISSN: 1225-4568 (Print), 1598-6217 (Online)

optimization techniques try to find the global optimum design and avoid local optimum solution. The selected design optimization method might be the right choice to find an economic, lightweight and serviceable structure. The designer has to adopt design variables such as dimensions, external applied load, and serviceability requirements. In addition, design constrains are an important part in the design. Constraints could be in the form of internal stresses and deformation (Awad *et al*. 2012).

Froud (1980), Li *et al*. (2011) found that the optimum bending stiffness design is located in the point where the core mass is equal to four times the skin mass. Both assumed that the core elastic modulus is very low, and it can be neglected. In addition, they assumed that the skin thickness is very small compared to core thickness, and it can be neglected as well.

Optimization techniques have been applied to the design of the sandwich panel by the number of researchers (Ashby 2000). Theulen and Peijs (1991) for example, presented an optimization of strength and stiffness of a sandwich panel. Their research concluded that the maximum bending stiffness occurred at a core to skin mass ratio equal to 2. Walker and Smith (2003) presented a multi-objective design optimization of fiber composite structure coupling using FE and genetic algorithms (GA). They found that the mass and deflection as a multi-objective could be optimized by the GA to suit the design engineer's requirements. Hudson *et al.* (2010) presented an optimization of sandwich panels using an Ant Colony Optimization method. The procedure was applied in the design of rail vehicle floor panel. The variables considered included: facing thickness, core thickness, core material, facing material and span (Awad *et al.* 2010).

It can be noted from the literature that the previous analytical works assumed that the density and elastic modulus of skin part are very high compared to core part (Froud 1980, Li *et al.* 2011). In present work, the analytical solution was found and compared with numerical solution. Both solutions considered the effect of skin and core material properties on the optimization results. Current research was motivated by the new material's application in sandwich structures. The skin and core materials used in these panels have a density lower than metals and higher than the wood and foam cores. Previous works avoid the effect of skin to the core elastic modulus ratio and the skin to the core density ratio. In addition, this work focuses on the allowable mid-span deflection limit as an important constraint in the design optimization of civil engineering structures.

In the following, the three point-bending problem and design methodology are explained. Analytical solution is presented with the finding and discussion. Then, it is followed by the GA numerical optimization for the same problem. Finally, the major finding is summarized in the conclusion section.

2. Problem description and design methodology

This work considers a single objective optimization to design sandwich beam. The beam structure is assumed to be simply supported in three points bending as shown in Fig. 1. The sandwich element is made of top skin, bottom skin, and core material. The top and bottom skins usually made from stronger and heavier material than core part. The beam has a span (L) and single point load at the center. The total applied service load (P) on the beam as shown in Fig. 1. The optimization problem is formulated as follows:

Objective

$$min\, f_1 = \,(mass) \tag{1}$$

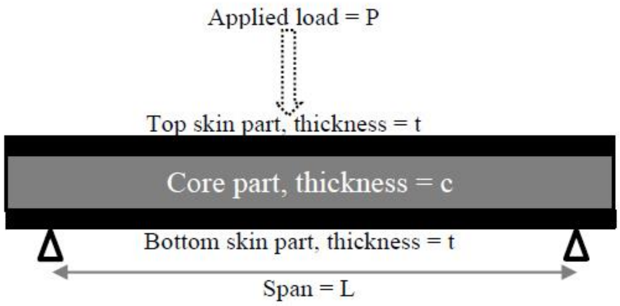


Fig. 1 Sandwich beam

Variables

$$core\ thickness(x_1)\ ,\ skin\ (x_2), and\ span\ (x_3) \tag{2}$$

Subject to the constraints

$$g_1, g_2, \dots \dots \dots .., g_n \tag{3}$$

The analytical design methodology was developed based on simple flexural and rigidity equations. Numerical design used a Genetic Algorithm (GA) method coupled with the Finite Element (FE) method. Through the design optimization, material's density and elastic modulus were changed to cover a wide range of sandwich panel applications.

3. Analytical mass design optimization

The previous studies used the sandwich beam rigidity equation to find the best core and skin thickness with minimum mass. The solution of the bending stiffness equation, and the mass equation of the sandwich beam gives values for core and skin thickness (Allen 1969, Araújo *et al*. 2009). Murthy *et al*. (2006) verified Froud's (1980) research findings by using experimental tests on a sandwich beam. Murthy *et al*. found that Froud's findings are valid for honeycomb core sandwich panels. However, Murthy *et al*. showed their finding for one type of skin material combined with one type of core material.

Previous studies assumed that the core to skin elastic modulus ratio is very high, and the skin to core thickness ratio is very small (Froud 1980, Murthy *et al.* 2006). The elastic modulus ratio of 43.75 was used in Li *et al*. (2011) work. The present procedure adds the deflection equation for the simply supported beam and considers the effect of skin to the core elastic modulus to find the optimum mass designs of the sandwich. In addition, the maximum allowable deflection is considered in the beam design for civil engineering applications. The bending stiffness (*EI*) is calculated by assuming there is full interaction between the skin and core (Froud 1980). The procedure for the single objective minimization is described by Eqs. (6) - (15) below

$$\delta = \frac{PL^3}{48\,EI} \tag{6}$$

where, δ is the mid-span deflection, P is the load, L is the span, and EI is the bending stiffness.

Allowable mid-span deflection is equal to

$$\delta = \frac{L}{B} \tag{7}$$

where, B is a factor between (250-800) depends on the structure serviceability requirement. From Eqs. (6) - (7), the following equation can be found

$$EI = PL^2\alpha \tag{8}$$

$$\alpha = \frac{B}{48} \tag{9}$$

Bending stiffness is calculated from the following equation

$$EI = 2\left(\frac{b.t^3}{12} + bt\left(\frac{c+t}{2}\right)^2\right)_{Skin} \times E_s + \left(\frac{b.c^3}{12}\right)_{core} \times E_c \tag{10}$$

where, b is the width, E_s and E_c is the skin and core elastic modulus respectively.

The term $(b.t^3/12)$ is very small and can be ignored from Eq. (10). Substituting EI from Eq. (8) into Eq. (10) leads to the equation of skin thickness

$$t = \left[\frac{2PL^2\alpha}{E_s bc^2} - \frac{E_c c}{6E_s}\right] \tag{11}$$

2.1 Mass objective:

The mass (M) equation is shown below

$$M = Mass\ of\ core + Mass\ of\ skins \tag{12}$$

$$M = \rho_c\ c\ b\ L + 2\ \rho_s\ t\ b\ L \tag{13}$$

Substituting Eq. (11) into Eq. (13), and minimizing the mass with respect to c we obtain

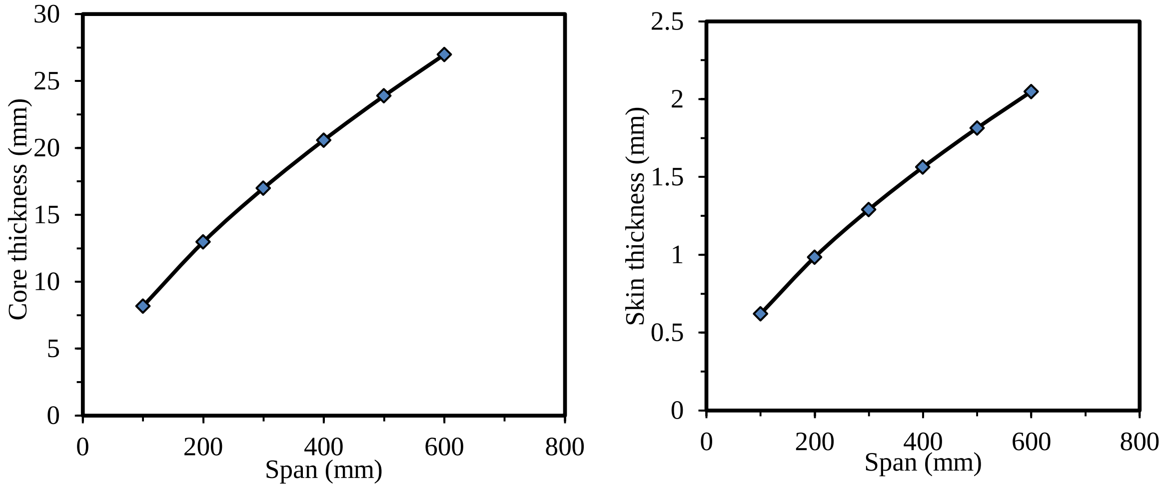
$$\frac{dM}{dc} = 0 = \rho_c bL + 2\rho_s bL\left[-\frac{6PL^2}{E_s bc^3} - \frac{E_c}{6E_s}\right] \tag{14}$$

Simplifying Eq. (14) leads to the optimum value of c

$$c = \left[\frac{12PL^2\alpha}{b\left(E_s\frac{\rho_c}{\rho_s} - \frac{E_c}{3}\right)}\right]^{1/3} \tag{15}$$

From Eqs. (11) and (15), designer can find the optimum solution for the design of the sandwich beam under mass minimization. The using of the above equations required material's properties of the skin and core components of the sandwich beam. In addition, allowable mid-span deflection factor (b) must be known.

The effect of the beam span was investigated using the present optimum solution Eqs. (9) and (15). Beam span was varied between (100-600 mm). The allowable deflection factor (b) was assumed equal to 400, and the external applied load is 400 N. The results of the optimum design are shown in Fig. 2 for core to skin density ratio equal to 2.0 and skin to core elastic modulus ratio equal to 30. It can be seen from the results that the relationship of span, skin and core thicknesses are approximately linear. In addition, this relation was control by the allowable deflection-span relation Eq. (7). Furthermore, the skin and core thickness results are influenced by the change in (b) value.



(a) Core thickness (b) Beam thickness

Fig. 2 design results of different beam spans (density ratio = 2.0, elastic modulus ratio = 30)

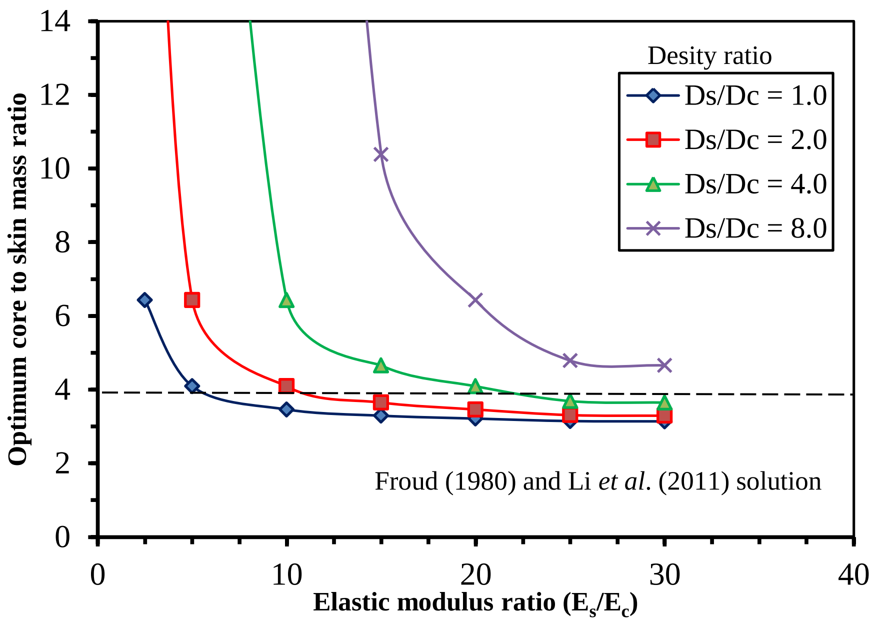


Fig. 3 Effect of mechanical properties on core to skin mass ratio

Sandwich beam usually made from a combination of different materials for the skin and core parts. These materials have different properties such as density and elastic modulus. The effects of material's mechanical properties of skin and core on the design results were studied in this part using above Eqs. (11) and (15). Both equations depend on the material's mechanical properties to calculate the optimum skin and core thicknesses for mass minimization. Effect of skin elastic modulus (E_s) to core elastic modulus (E_c) ratio on the core to skin thickness ratio was investigated.

Wide range of (E_s/E_c) ratio was used, and it was between 2.5 to 30. In addition, the effect of skin density (D_s) to core density (D_c) was investigated, and the ratio range was between 1.0 to 8.0.

The results of mechanical properties affect are shown in Fig. 3. It can be seen that the core to skin ratio is presented with the elastic modulus ratio and density ratio. The core to skin ratio shows a non-linear behavior. The core to skin ratio increases when the skin to core elastic modulus ratio decreases. The graph shows that the behavior of core to skin ratio becomes approximately linear for the high values of the skin to core elastic modulus ratio. Froud (1980) and Li *et al*. (2011) solutions are presented on Fig. 3 as well, and these solutions consider that the skin to the core elastic modulus ratio is very high. Therefore, their findings were indicated that the core to skin thickness ratio is constant. On the other hand, it can be seen that the density ratio has a big effect on the optimum core to skin results. The optimum core to skin mass ratio increases with the increasing of the skin to the core density ratio. The present solution proves that the optimum core mass to skin mass becomes constant in different stages based on the elastic modulus ratio (E_s/E_c). It can be seen in Fig. 3, the core mass to skin mass is constant for the range of (E_s/E_c) is higher than 15 and density ratio equal to 1.0. In comparison, the core mass to skin mass is constant for the range of (E_s/E_c) is higher than 25 and density ratio equal to 8.0.

4. Numerical mass design optimization

Optimum design of a GFRP sandwich beam is important to avoid any material waste and to obtain an economic product (Simoes and Negrão 2005). There are a number of studies discuss the optimization of an individual sandwich panel to optimize the cost or mass, and strength (Swanson and Kim 2002, Murthy *et al*. 2006, Meidell 2009). The optimization of the bending stiffness has been studied either with the minimum mass or minimum cost to find the best values of the core and skin thicknesses for a certain bending stiffness (Froud 1980, Gibson 1984).

Genetic algorithm (GA) is an efficient method for the optimization, which is based on a stochastic approach and relies on a survival of the fittest in the natural process. In the last few decades, GA has been widely used for structural design optimization due to its capability to deal with complicated and large variable problems. The principle of GA depends on the concept of natural selection and natural genetics. The basic idea of the GA is to generate a group of design variables randomly within the allowable values of each variable. Two features can be noticed in the GA; the first is the stochastic algorithm. This means that the random procedure is essential in both selection and reproduction (Falzon and Faggiani 2012). The second is the GA always remains all the population of solution in its memory. This allows it to recombine between different solutions to find the best one. Robustness makes the GA a great optimization tool and is essential for the algorithm success. It gives the method the ability to deal with different type of problems without particular requirements for use of the GA.

This work considers the optimization design of a sandwich simply supported beam with the mass minimization objective. The main objective is to minimize the mass while satisfying the mid-span allowable deflection. The design methodology will explore the effect of; the thicknesses of the sandwich beam components at service load, the optimum core to skin mass ratio for the sandwich beam, elastic modulus and density ratios of the core to skin. The Genetic Algorithm (GA) (Awad *et al*. 2012) was coupled with ABAQUS, and it was used in the optimization.

A two dimensional (2D) finite element (FE) model was developed for the simulation of the sandwich beam. A 2D plane stress element (CPS8R) was used in the skin and core parts. Total

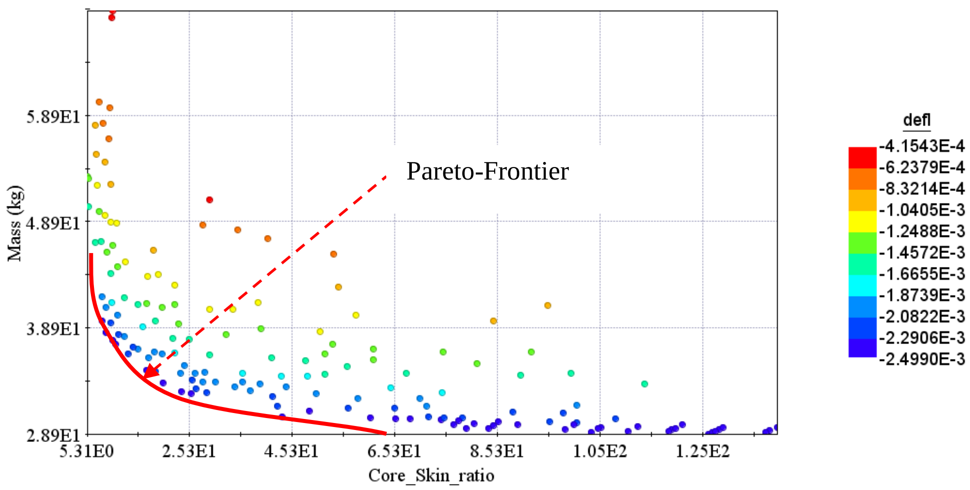


Fig. 4 Scatter chart of mass and cost of the sandwich beams

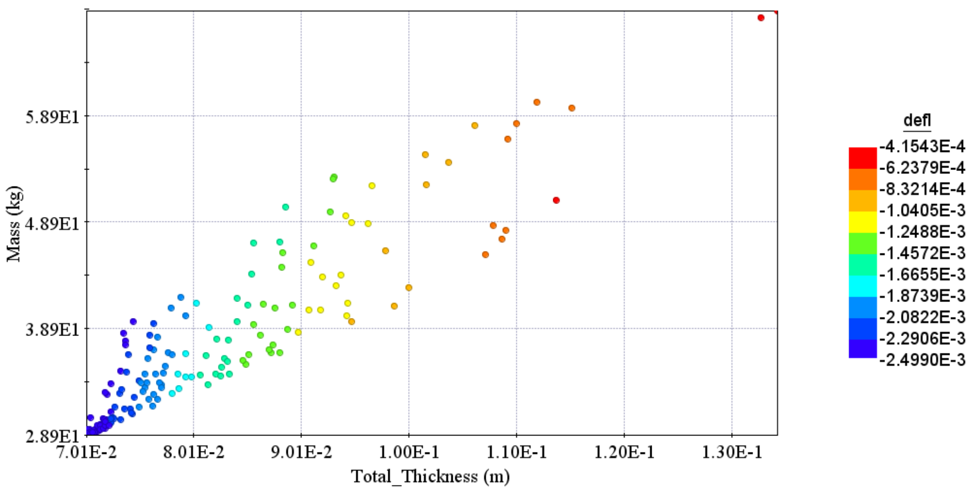


Fig. 5 Scatter chart of mass and total thickness of sandwich beams

elements are 384 for core part, and 194 elements for skin part. The behavior of the core and skin materials is assumed elastic. In addition, full bond was assumed between core and skin, and no slip was allowed.

The allowable deflection at service load is equal to span/400, and same boundary conditions were used as in analytical design. The scatter chart of total mass and core to skin ratio is shown in Fig. 4. It can be seen that the mass of the beam decreases as the core to skin ratio increases. In addition, the scatter chart of the total beam thickness and the beam mass is presented in Fig. 5. It shows how the beam thickness effect on the mass and the mid span deflection of the beam. GA optimization solution shows the distribution of design iterations with the Pareto-Frontier as shown in Fig. 4. The Pareto-Frontier explains the trade-off between the beam mass and core to skin

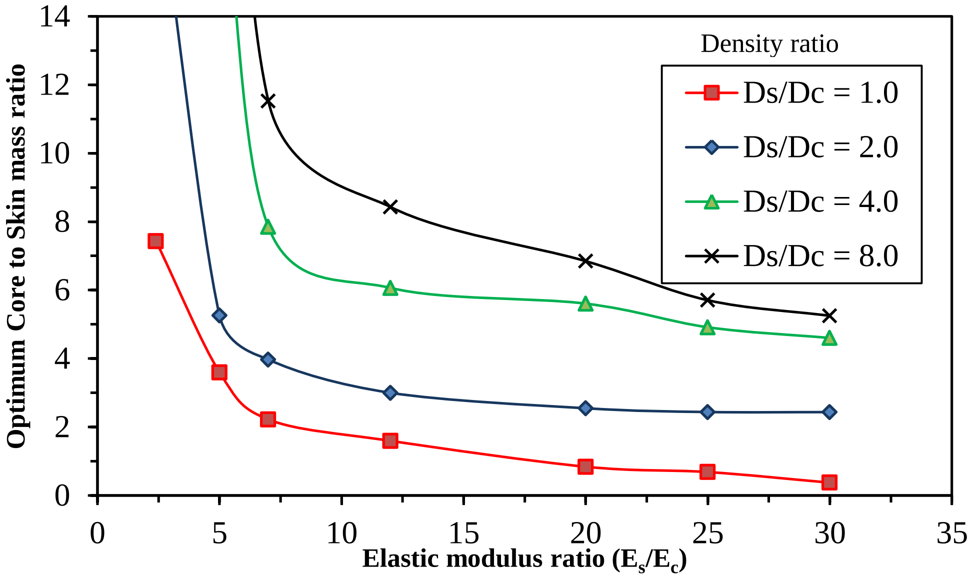
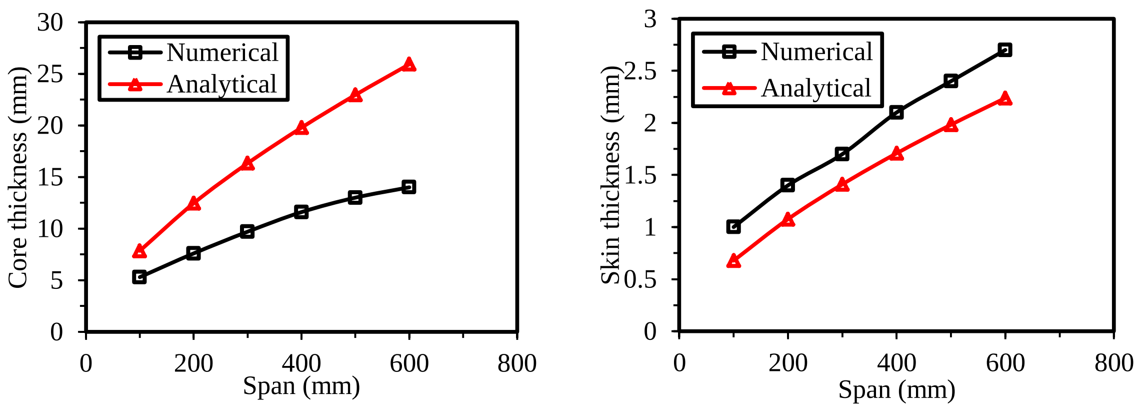


Fig. 6 Numerical optimization results



(a) Core to span relation (b) Skin to span relation

Fig. 7 Comparison between analytical and numerical design results of different beam spans (density ratio = 2.0, elastic modulus ratio = 30)

thickness ratio. The design solutions with maximum allowable deflections are located on the Pareto-Frontier.

Material mechanical property effects were investigated on the optimum design of the sandwich beam. Core to skin elastic modulus and skin to the core density ratio were studied. Both ratios showed a significant effect on the optimum core to skin mass design. The results are shown in Fig. 6. The numerical optimization showed that the optimum core to skin mass is not constant. Comparing the results of Figs. 3 and 6 shows that the numerical results are different than the analytical results. The analytical results showed that the core to skin mass ratio is approximately constant for the elastic modulus ratio more than 25, and the values of optimum core to skin mass ratio are between 3 to 4 for core to skin density ratio less than 4. Whereas, the numerical design showed a large difference between different Core to skin density ratios. The difference is attributed to one reason; the analytical solution uses the bending equation while the numerical optimization uses the FE method in the analysis of finding the mid span deflection. Finally, both analytical and numerical solutions proofs that the designer cannot ignore the contribution of the core density and core elastic modulus to the design results.

From analytical and numerical designs, it can be seen clearly that the analytical solution gives higher core to skin mass ratio compare to the numerical solution. The numerical optimization shows a core to skin mass ratio around 3.5 for high elastic modulus ratio and density ratio equal to 1.0. In comparison, the numerical optimization shows that the core to skin mass ratio is below 1.0 for a high elastic modulus ratio. On the other hand, the difference between the analytical solutions for becomes low compare to the numerical solutions in the high elastic modulus ratio.

The span effect was studied numerically as well, and the results are shown in Fig. 7. This figure confirms that the relation between skin thickness, core thickness and span of the beam is approximately linear. The numerical solution shows a lower core thickness than the analytical solution as shown in Fig. 7(a). In addition, it shows higher skin thickness than the analytical solution as shown in Fig. 7(b). Furthermore, the core to skin mass ratio of analytical solution is higher than the numerical solution. The reason behind this is the FE method was used in the numerical optimization while the flexural equation was used in the analytical solution. In addition, there is no approximation in the numerical design optimization calculations. Furthermore, finding design variables (core and skin thicknesses) in analytical optimization depends on simplified equations and in the numerical optimization depends on GA method. It is expected that using FE method allows considering the shear deformation effect in the numerical solution.

5. Conclusions

The paper discussed the optimum design of beam using analytical and numerical optimization methods. The analytical optimization shows a core to the skin mass ratio is non-linear, and its behavior is influenced by the material's mechanical properties. Both skin to the core elastic modulus and density have a big effect on the optimum core to skin mass ratio. The optimum analytical and numerical designs confirm that the core to skin mass ratio is not constant as it was mentioned throughout the literature. The analytical optimization shows a higher core to the skins mass ratio when compared with the one obtained by numerical optimization. Finally, this work showed that core elastic modulus, and density should be considered in the design optimization.

References

Allen, H.G. (1969), *Analysis and design of structural sandwich panels*, Pergamon Press Oxeford, England.

Araújo, A., Martins, P., Mota Soares, C., Mota Soares, C. and Herskovits, J. (2009), "Damping optimization of viscoelastic laminated sandwich composite structures", *Structural and Multidisciplinary Optimization*, **39**(6), 569-579.

Ashby, M. (2000), "Multi-objective optimization in material design and selection", *Acta Materialia*, **48**(1), 359-370.

Awad, Z.K., Aravinthan, T. and Manalo, A. (2012), "Geometry effect on the behaviour of single and glue-laminated glass fibre reinforced polymer composite sandwich beams loaded in four-point bending",

Materials & Design, **39**(0), 93-103.
Awad, Z.K., Aravinthan, T., Zhuge, Y. and Gonzalez, F. (2012), “A review of optimization techniques used in the design of fibre composite structures for civil engineering applications”, *Materials & Design*, **33**(1), 534-544.
Awad, Z.K., Gonzalez, F. and Aravinthan, T. (2010), “Advanced robust design optimization of FRP sandwich floor panels”, *IOP Conference Series: Materials Science and Engineering*, **10**(1), 1-9.
Awad, Z.K. and Yusaf, T. (2012), “Fibre composite railway sleeper design by using FE approach and optimization techniques”, *Structural Engineering and Mechanics*, **41**(2), 231-242.
Falzon, B.G. and Faggiani, A. (2012), “The use of a genetic algorithm to improve the postbuckling strength of stiffened composite panels susceptible to secondary instabilities”, *Composite Structures*, **94**(3), 883-895.
Froud, G. (1980), “Your sandwich order, sir(mechanical properties in bending of sandwich structures) ”, *Composites*, **11**, 133-138.
Gibson, L. (1984), “Optimization of stiffness in sandwich beams with rigid foam cores”, *Materials Science and Engineering*, **67**(2), 125-135.
Hollaway, L.C. and Head, P.R. (1999), *Composite materials and structures in civil engineering*, Elsevier, Oxford.
Hudson, C.W., Carruthers, J.J. and Robinson, A.M. (2010), “Multiple objective optimisation of composite sandwich structures for rail vehicle floor panels”, *Composite Structures*, **92**(9), 2077-2082.
Li, X., Li, G., Wang, C.H. and You, M. (2011), “Optimum Design of Composite Sandwich Structures Subjected to Combined Torsion and Bending Loads”, *Applied Composite Materials*, 1-17.
Meidell, A. (2009), “Minimum weight design of sandwich beams with honeycomb core of arbitrary density”, *Composites Part B: Engineering*, **40**(4), 284-291.
Murthy, O., Munirudrappa, N., Srikanth, L. and Rao, R. (2006), “Strength and stiffness optimization studies on honeycomb core sandwich panels”, *Journal of Reinforced Plastics and Composites*, **25**(6), 663.
Simoes, L.M.C. and Negrão, J.H.O. (2005), “Reliability-based optimum design of glulam cable-stayed footbridges”, *Journal of Bridge Engineering*, **10**(1), 39-44.
Swanson, S.R. and Kim, J. (2002), “Optimization of sandwich beams for concentrated loads”, *Journal of Sandwich Structures and Materials*, **4**(3), 273-293.
Theulen, J. and Peijs, A. (1991), “Optimization of the bending stiffness and strength of composite sandwich panels”, *Composite Structures*, **17**(1), 87-92.
Walker, M. and Smith, R.E. (2003), “A technique for the multiobjective optimisation of laminated composite structures using genetic algorithms and finite element analysis”, *Composite Structures*, **62**(1), 123-128.