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Flexural behaviour of reinforced low-strength concrete beams strengthened with CFRP plates

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Abstract. This paper summarises the results of an experimental study to investigate the flexural behaviour of reinforced concrete beams strengthened using carbon-fibre reinforced polymer (CFRP) laminate in four-point bending. The experimental parameters included are the reinforcing bar ratio ρ_s and preload level. Four bar ratios were selected (ρ_s = 0.13 to 0.86%), representing the section of two longitudinal tensile reinforcements, with diameters of 8, 14, 16, and 20 mm in order to reveal the effect of bar ratio on failure load and failure mode. Eight beams that could be considered "full-scale" in size, measuring 200 mm in width, 400 mm in total height and 2300 mm in length, were tested. Three beams were selected with different bar ratios (ρ_1, ρ_2, ρ_3), and considered as control specimens (without), while three other beams identical to the control beams with the same CFRP laminates ratio and a seventh beam with $\rho_{\min}$ (the lowest bar ratio) were also used. In the second part of the study, two beams with the bar ratio ρ_2 were preloaded at two levels, 50 and 100% of their ultimate loads, and then repaired. This experimental investigation was consolidated using an analytical model. The experimental and analytical results indicate that the flexional capacity and stiffness of strengthened and repaired beams using CFRP laminate were increased compared to those of control beams, and the behaviour of repaired beams was nearly similar to the undamaged and strengthened beams; unlike the ductility of strengthened beams, which was greatly reduced compared to the control.

Keywords: four-point bending; failure; CFRP-strengthening; repair; preload; reinforced concrete; beam

1. Introduction

Extensive research projects have been undertaken on flexural behaviour of reinforced concrete beams strengthened by externally bonded FRP over the last decades. Because of increased spending on infrastructure rehabilitation, the organisation and rationalisation of maintenance has become more than necessary (Ceroni 2010, Hollaway 2010). To enable planning and reduce expenses, these economic challenges have been behind the establishment of maintenance plans, ranging from diagnosis to strengthening (Akbarzadeh and Maghsoudi 2010, Toutanji *et al.* 2006).

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Many civil engineering infrastructures have suffered from various damages during their life span; the causes of these damages are due either to mechanical phenomena, physicochemical phenomena (Pelà *et al.* 2012) or errors in calculation and design. Nowadays, bonding with FRP is one of the most frequently used repair and strengthening methods in this growing market (Abdessemed *et al.* 2011, Pesic and Pilakoutas 2005, Benjeddou *et al.* 2007, Ibell 2009). This method presents greater advantages compared to conventional ones, due to the high strength, low weight and improved durability of composite materials (Hollaway2010, Esfahani 2007).

In the post-cracking stage the stiffness of control beam decrease notably due to cracking, the decrease in stiffness is smaller for retrofitted beams since the CFRP prevents cracks to develop and widen (Obaidat *et al.* 2012, Pelà *et al.* 2012).The strengthening by FRP decrease the ductility and increase the stiffness of beams compared to those unstrengthened due to the brittle failure caused by the occurrence of plate debonding (Kantar and Anil 2012). The ductility can be attained by introducing anchorage devices that avoid or delay the debonding, the best result was reached by applying distributed U-shaped FRP strips (Ceroni 2010). That it make repaired the earthquake damaged structure possible (Abdessemed *et al.* 2011).

The study of structures or structural elements made using reinforced concrete has developed extensively from an experimental, analytical, numerical and/or graphical point of view. However, whereas structural elements are strengthened, the modelling approaches differ from one author to another (Akbarzadeh and Maghsoudi 2010, Rahimi and Hutchinson 2001, Toutanji 2006, Teng *et al.* 2002, Esfahani 2007).

The main objective of this paper is to conduct an experimental program on reinforced concrete beams under four-point bending, made using a low strength concrete, in order to bring out the strengthening effect. This is truer and closer to the reality of older structures across the world.

2. Research significance

The present study highlighted the effectiveness of strengthening of the reinforced concrete structures with a low characteristic resistance by CFRP laminate. As well it showed the effect of this strengthening on the behaviour of a structural reinforced concrete element submitted to the pure bending and this for various reinforcement bars ratios. The result obtained indicate that more the bar ratio increases more the analytical approach used gives appropriate results, according to figures 15, 16 and 17, it was also noted that more this ratio increases, the both curves (analytical and experimental) are converged, on the other hand when this bar ratio decreases the analytical approach over-estimated the contribution of strengthening in term of flexional capacity. In the Esfahani study 2007 it is concluded that the design guidelines of ACI.2R-02 and ISIS Canadian overestimate the effect of CFRP in increasing flexural strength of beams with a small reinforcement bars ratio compared to the maximum bars ratio.

Moreover this over-estimate partially due to inappropriate calculation methods which does not consider the debonding phenomenon (premature failure) of the laminate, also in analytical model it's assumed a perfect adhesion laminate-concrete, therefore an optimal exploitation of CFRP whereas in reality it is not.

In the light of the results of the second part that of repair, it proved that a undamaged beam strengthened by CFRP had a quasi-similar behaviour to that damaged to 100 % and then strengthened, which makes uses of CFRP like means of strengthening or repair very effective and very significant.

Table 1 Mixture of different concrete component

Component	Rapport E/C	Cement	Sand	Water	Gravel 4/8	Gravel 8/12	Gravel 12/25	Sika Concrete-smooth	Air occlude	Slump	Density
Unit	/	Kg/m^3	Kg/m^3	L	Kg/m^3	Kg/m^3	Kg/m^3	L	%	mm	Kg/m^3
Volume	0.67	280	730	190	125	255	770	0.5	2.3	120	2340

Table 2 Characteristics of steel bars

Nuances	E_s(N/mm^2)	R_m(N/mm^2)	A_{gt} min (%)
FeE500	500	550	2.5

E_s: elastic modulus, R_m: tensile strength A_{gt}: extension

Table 3 Geometrical and mechanical characteristics of CFRP plates and sheets used

Type of PRFC	Section (mm^2)	Width (mm)	Thickness (mm)	Elasticity Modulus (N/mm^2)	Tensile strength (N/mm^2)	Elongation at Break (%)
Sika® Carbodur® S512*(plates)	60	50	1.2	165000	3100	>1.70
SikaWrap® -230 C/45(sheets)	-	300	1.76	234 000	4300	>1.80

3. Experimental program

3.1 Introduction

The experimental program was performed to reproduce the real conditions of older structures across the world, made using a low strength concrete.

3.2 Materials characteristics

3.2.1 Concrete

Table 1 summarizes the concrete mixture proportion of different constituents; the composition is selected to give a compressive strength lower than 20 MPa. For each casting series, the specified compressive strength was measured by testing 6 concrete cylinder specimens, with results indicating a compressive strength of 17 MPa from standard compressive test. The tensile strength was determined from the standard splitting test, and was 1.9 MPa.

3.2.2 Steel reinforcement

The specimens are reinforced with different sizes of tensile longitudinal bar sections, as follows: one beam using 8 mm, two beams using 14 mm, three beams using 16mm, and two beams using 20 mm diameter reinforcement bars. The characteristics of these bars are given in Table 2.

3.2.3 CFRP plates strengthening

In addition, the external strengthening used was made by the SIKA Company; where the CFRP

Table 4 mechanical characteristics of resin used for bonding SikaWrap® 230

Type of resin	Elasticity Modulus flexural (N/mm^2)	Elasticity Modulus tension (N/mm^2)	Tensile strength (N/mm^2)	Elongation at Break (%)
Sikadur® -330	3800	4500	30	>0.9

Table 5 mechanical characteristics of resin used for Sika® CarboDur® plates

Type of resin	Résistance en Compression (N/mm^2)	Résistance en cisaillement (N/mm2)	Résistance en traction (N/mm2)	Elasticity Compression Modulus (N/mm^2)	Elasticity Tensile Modulus (N/mm^2)
Sika® Carbodur® S512*(plates)	85-95	16-19	26-31	9600	11200

plates were provided in roll form to allow the cutting of desired lengths. These plates were bonded to the lower faces of the beams using an adhesive consisting of two components that are mixed together. Details of the material proprieties of the CFRP are given in Table 3. Likewise the characteristics of used resin are given in Tables 4 and 5.

3.3 Preparation of specimens

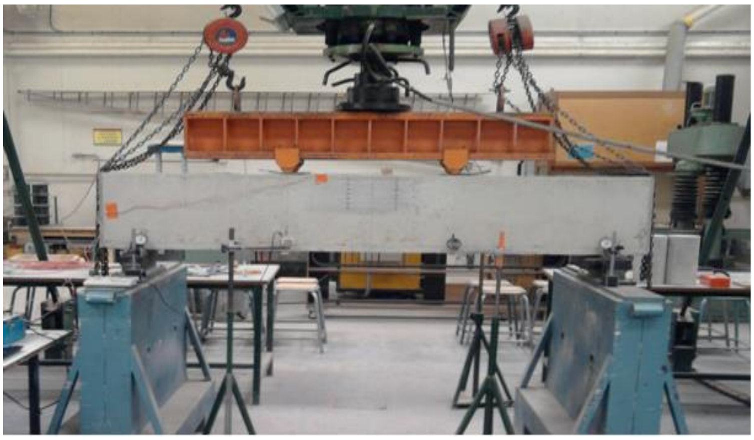
Eight beams measuring 400 mm in height, 200 mm in width and 2300 mm in length were tested in four-point bending. The tensile longitudinal bottom bars measure 8, 14, 16 and 20 mm in diameter, while10-mm diameter bars were used for the upper part of all the beams. The tension and compression bars were tied together using 8 mm stirrups with spacing appropriate to regain shear.

Before concreting, a mould made of wood was prepared and soaked in oil to prevent adhesion between the mould and the poured concrete during release of the mould. During concreting, a product called “smooth-concrete” was added to obtain specimens with smooth faces. Vibrating of concrete prevents any segregation. The specimens were kept in the moulds and protected by a plastic film for 48 hours.

The specimens were released from their moulds; and the bottom and side faces of the beams were cleaned using a sander and compressed air to remove the remaining particles and adjust the surfaces in order to ensure perfect adhesion of concrete plates. Further, four beams were strengthened externally on the bottom (tensile) part with CFRP plates (two plates per beam), three other beams were considered as control beams, and two beams were strengthened after they underwent 50 and 100% damage of their flexural capacities.

Each beam has been designed by for letters and tow numbers, the first is either CB or SB, indicating wither it is control or strengthened beam. The second is a number (1, 2, 3 or 4) characterises the ratio of tensile longitudinal bars (0.13 to 0.86%). The third is a number (1 or 2) specifies the concrete compressive strength (1 for 17MPa, 2 for 35MPa). The forth is a letter (U, P or W) refers the ration and the nature of strengthening (U: unstrengthened, P: plate, W: wrap). The fifth is a letter (N, d or D) indicating the degree of damaged (N: no damaged, d: 50% damaged, D: 100% damaged).

The procedure for external strengthening is initialised by the cutting of the strip, the preparation of the resin bond after attaching the plate onto the beam and a waiting period of one week for curing of the resin (following the manufacturer’s recommendations). The use of CFRP sheets as lateral strengthening helps prevent the plates from peeling-off at the ends of the specimens.



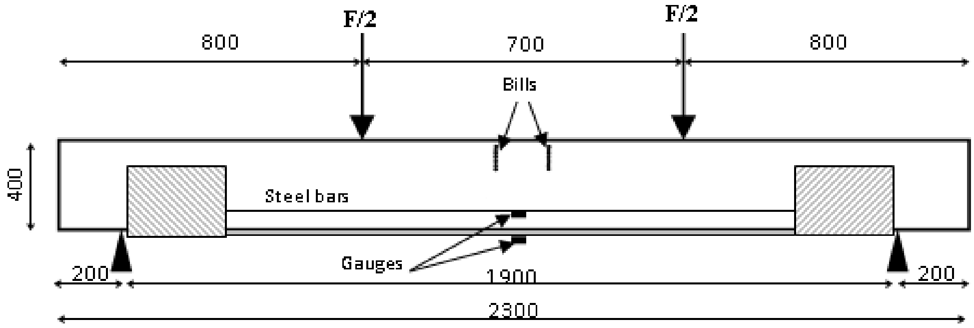


Fig. 1 Establishment of a strengthened beam in CFRP plate and submitted to simple bending (four points)

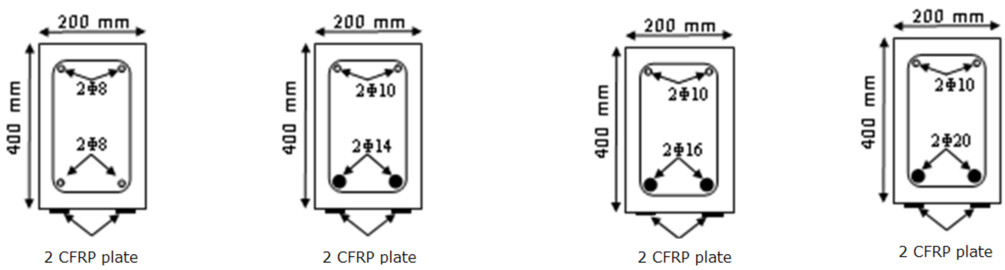


Fig. 2 Beam sections strengthened using CFRP plates with different rates of longitudinal reinforcement tension

As for instrumentation of the measurement systems, the specimens were tested in four-point bending using a hydraulic press with a maximum load capacity of 1000 KN linked to a computer. The deflection in mid-span was measured using an LVDT (Linear Variable Differential Transducer), presented in Fig.3.a, the deflection at the point of load application was measured using digital indicators. The movement of the two supports was measured and their influence on deflection was taken into account. The width of the crack was measured using an LVDT see Fig. 3(b).

(a) LVDT for displacement measuring (b) LVDT for measuring the crack

Fig. 3 Test instrumentation in LVDT

Fig. 4 Installing gauges in bars and plates

Fig. 5 Establishment of the balls for measuring the deformation of the concrete

Four electrical strain gauges were attached at mid-span of each beam; two on the reinforcing bars and two on the CFRP plates. The output data were recorded using a computer data acquisition system, as shown in Fig. 4.

Two series of 7 balls were installed parallel to the middle of each beam (the distance is 200 mm between two parallel balls) on the lateral side of the upper part of the beam. The balls were spread over a distance of 170 mm from top to bottom, in 25 mm increments, to determine the position of the neutral axis, as well as to record the deformation of the most compressed fibre in concrete using an extensometer, as presented in Fig. 5.

Table 4 Summary table showing the different results obtained

Beam	F_y (KN)	F_u (KN)	Gain obtain (%)	$\delta_{\max}$ (μm)	σ_u (N/mm^2)	ε_f (μstrain)	ε_{bc} (μstrain)	M_u (KN.m)	X (mm)	Failure mode
SB.1.1.P.N	100	140	0	8064	08	3337	720	42	90	SY/DL
CB.2.1.U.N	120	200	0	20133	11	/	1053	60	70	SY
SB.2.1.P.N	180	280	40	9785	14	4754	1211	84	116	SY/DL
CB.3.1.U.N	140	240	0	31652	12	/	1124	72	82.7	SY/CC
SB.3.1.P.N	200	400	67	12204	20	6811	1806	120	118	SY/DL
SB.3.1.P.d	220	375	56	10546	17	4588	1317	112.5	118	SY/DL
SB.3.1.P.D	/	355	48	12088	18	5013	1093	106.5	101	SY/DL
CB.4.1.U.N	240	390	0	43442	18	/	1396	117	119	SY/CC
SB.4.1.P.N	300	509	31	11215	25	5603	1881	152	123	SY/DL/CC

CC: concrete crushing, SY: steel yielding, DL: FRP delamination

4. Presentation and discussion of results

4.1 Flexural capacity and failure mode

The overall results obtained are indicated in Table 4, these results reveal the effect of strengthening on the flexural capacity of beams.

The failure mode of control beams SB.4.1.U.N and CB.3.1.U.N was characterized by the steel yielding and the concrete being crushed (Fig. 6(a)), unlike beam CB.2.1.U.N where failure occurs by steel yielding only. Regarding the five strengthened beams, the failure is produced by steel yielding and by delamination of CFRP plates at mid-span (see Figs. 6 (b)-(c)-(d)), this delamination propagates to the end of the plates and sometimes causes tearing of the lateral strengthening (see Fig. 6(c)). Furthermore, beam SB.4.1.P.N failed particularly in concrete crushing, steel yielding and delamination of CFRP compared to other beams. However, the strengthened beams never reached the ductility of control beams. The delamination phenomenon is caused primarily by the cracking of the concrete, which is located at mid-span.

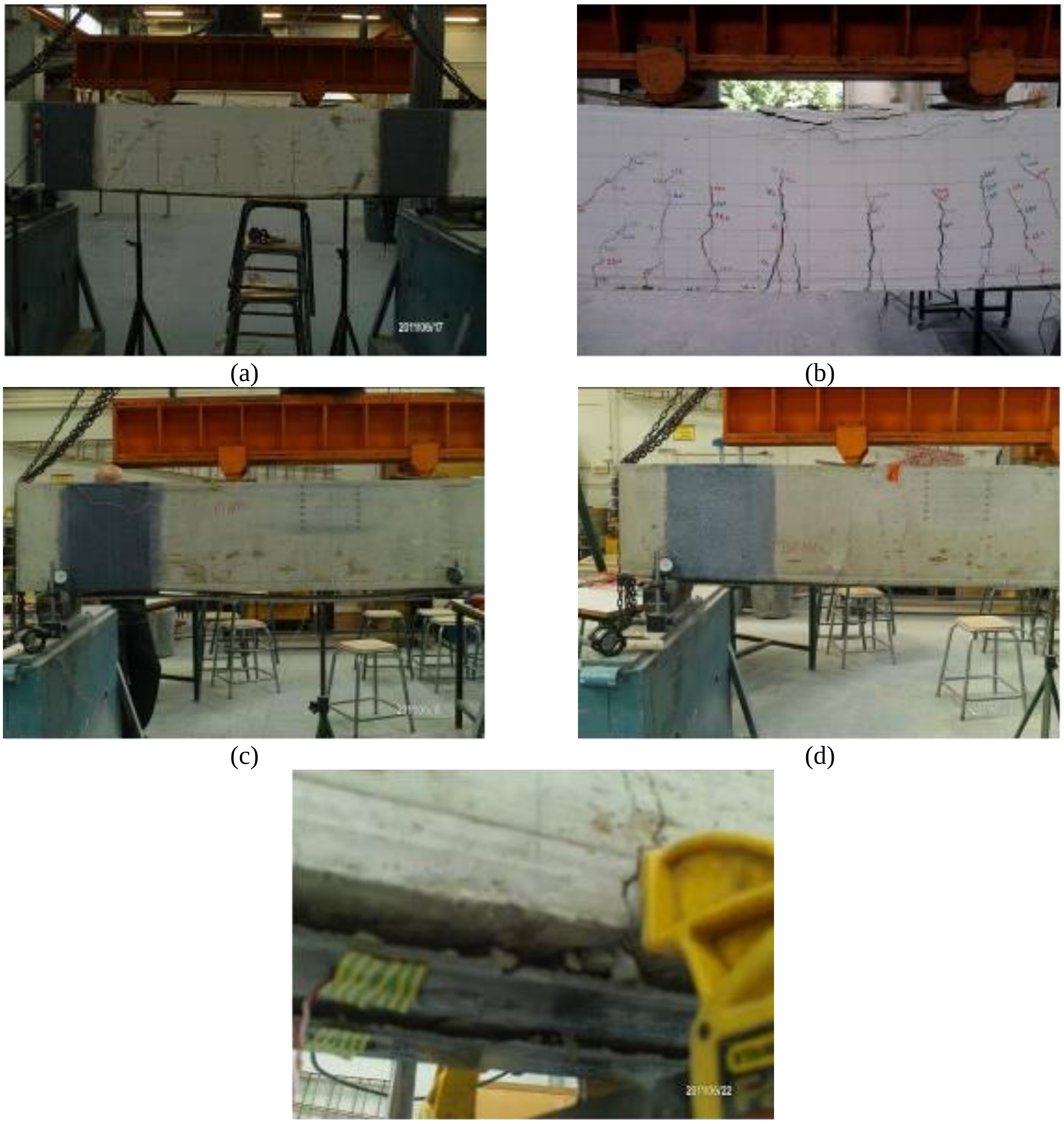
Comparison of the strengthened beams with different bar sections varying from ρ_s = 0.13 to 0.86%, shows that beam flexural capacity is proportional to bar ratio.

In the second stage (the repaired beams), the results show that the beams subjected to a rate of damage of 50 and 100% (P_u of SB.3.1.P.d is 375KN and P_u SB.3.1.P.N.D is 355KN) behaved almost like the undamaged strengthened beam (P_u of SB.3.1.P.N is 400KN).The gain in flexural capacity of beams SB.3.1.P.N.d, SB.3.1.P.N.D and SB.3.1.P.N, in relation to the control beam, is respectively 56, 48 and 67%. It should be noted that the stiffness of repaired beams would be lower than the strengthened beams without damage.

The strengthening of the beams directly affected the position of the neutral axis by increasing their values compared with controls beams.

4.2 Load versus displacement and moment versus curvature relationship

Displacement increases along with load until the appearance of the first crack, as shown in Fig. 7(a) (60 KN to CB.2.1.U.N, 80KN to CB.3.1.U.N, 80KN to CB.4.1.U.N) where the curve changes



(a) (b) (c) (d) (e)

Fig. 6 Presentation of different failure modes

its shape until the point corresponds to the yielding steel point. Beyond this point, the load stabilizes and displacement continues until collapse, which means that the beam behaves in a ductile manner.

Furthermore, the diameter of the longitudinal bar beams grew; the stiffness, the ultimate load and the maximum displacement values were greater.

The curves in Fig. 7(c) indicate that the displacement decreases while the load and stiffness increase significantly. The absence of a plate beyond (the point corresponds to the yielding steel point) shows a non-ductile behaviour of these strengthened beams.

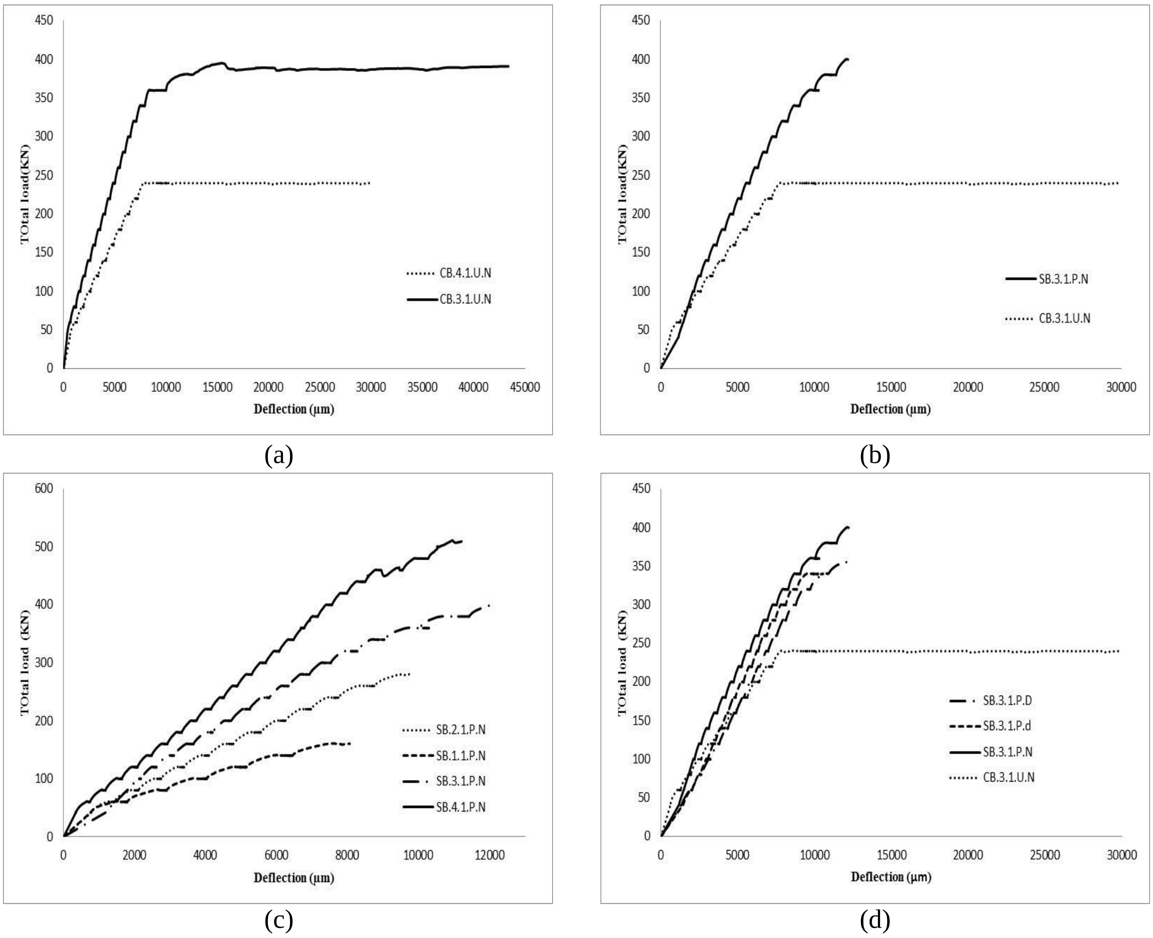


Fig. 7 Load versus displacement curves of different specimens

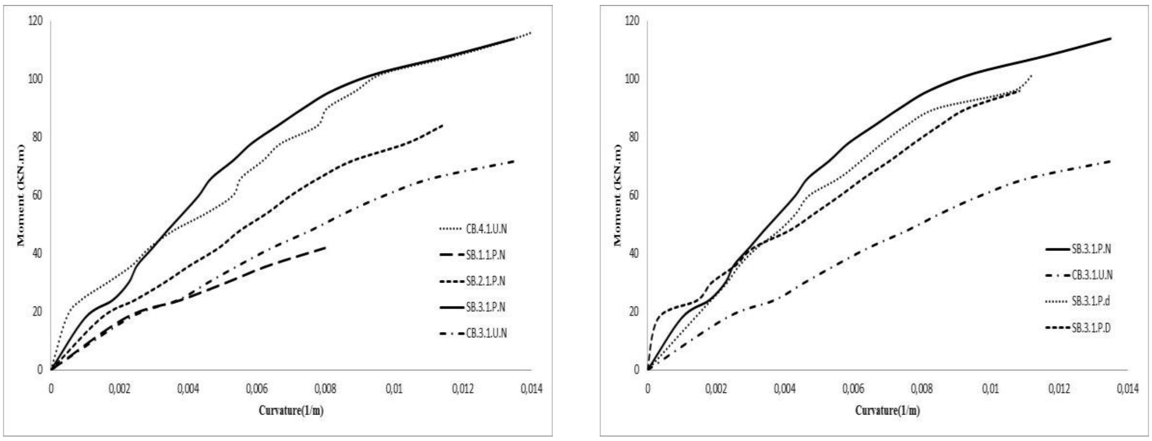


Fig. 8 Moment versus curvature response

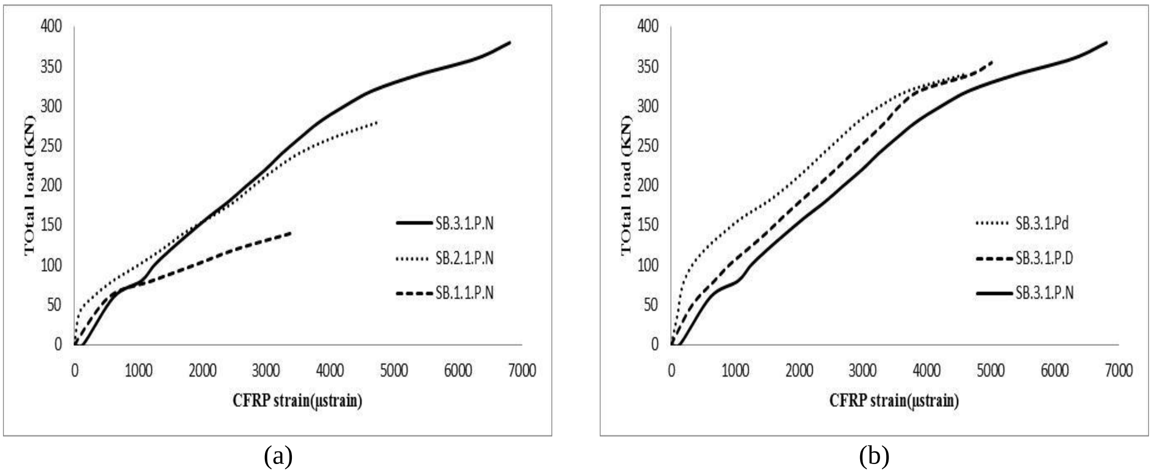


Fig. 9 Load versus strain CFRP response

The response of load versus displacement curves, shown in Fig. 7(d), corresponds to the three pre-loaded beams, and reveals that the curves have the same shape and were almost superimposed.

The diagram shown in Fig. 8 reveals that the curvature grows along with the moment, thus the strengthened beams have greater flexural capacity than the control beams.

4.3 Load versus CRFP strain response

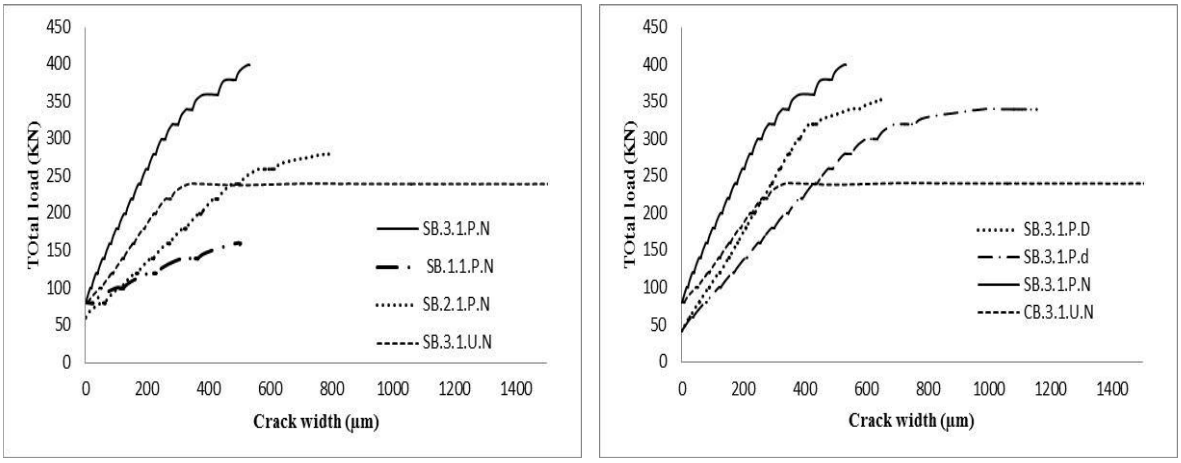
Fig. 9(a) presents the load versus CFRP plates strain curve for different bar sections. It is found that each curve consists of three parts, the first is the load which appears in the first crack (the first crack of CB.3.1.U.N appears at 80 KN), the second part is before the point corresponding to the yielding steel point (for SB.3.1.P.N, it is 200 kN, and for SB.2.1.P.N it is 280 kN), while beyond this point, the third part begins. Further, it was found that the greater the diameter of longitudinal bars, the greater the increase in ultimate load and the more the strain in CFRP plates increases.

In the case of pre-loaded then repaired beams presented in Fig. 9(b), it was found that the behaviour of damaged and repaired beams was similar to undamaged ones, so repairing damaged beams using CFRP makes them behave like strengthened beams.

4.4 Evaluation of the main crack width

The crack spacing acts as a constraint on the anchoring force transferred by bond along the beam, and therefore it plays a major role in the beam failure mechanism. The failure mode by concrete cover rip-off as well as the failure mode by FRP debonding are influenced by crack pattern also (Aprile and Feo 2007, Anil *et al.* 2012).

The relationship between load versus crack width, as illustrated in Fig. 10(a), reveals that the more the load increases, the more the width of the crack is enlarged. It should be noted that the width corresponding to beam SB.3.1.P.N is very limited and does not exceed (534 μm) compared with beam CB.3.1.U.N which has a very significant width that progressed and reached a value of (1545 μm). Strengthening directly affects crack width and prevents it from evolving, although there was an increase in ultimate load.



(a) (b)

Fig. 10 Load versus crack width response

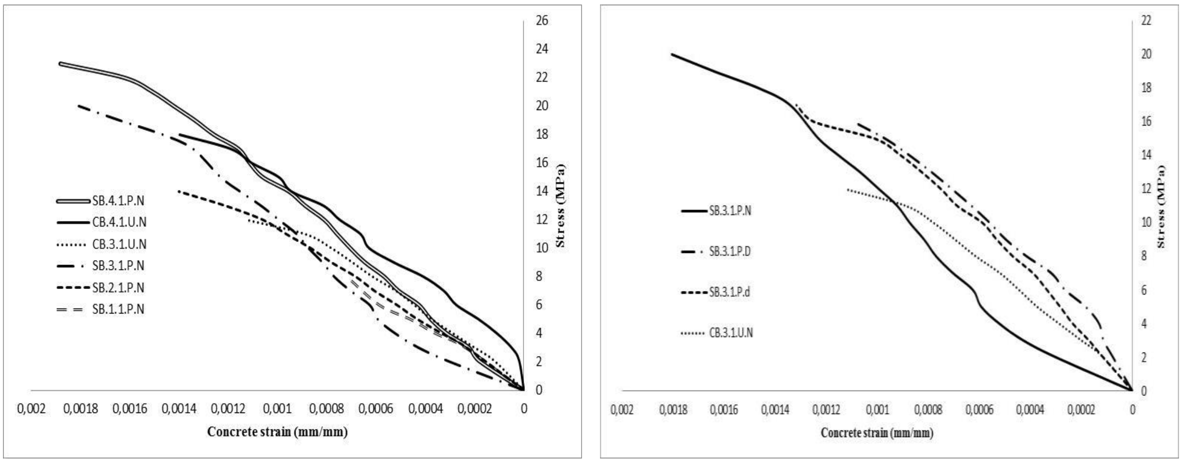


Fig. 11 Stress versus strain response of concrete

Otherwise, regarding the influence of longitudinal reinforcement bars on strengthened beams, it was found that SB.3.1.P.N (534 μm), SB.1.1.P.N (505 μm) and SB.2.1.P.N (800 μm) maintained nearly the same crack width. The responses of pre-loaded beams represented in Fig. 10(b), show almost the same width for the three beams (SB.3.1.P.N, SB.3.1.P.d and SB.3.1.P.D).

4.5 Stress versus concrete strain

It was noticed in Fig. 11, which represents stress versus concrete strain relationship, that strengthened beams have a deformation (ε_c) greater than that of control beams; this deformation was even more significant when the diameter of longitudinal bars was greater.

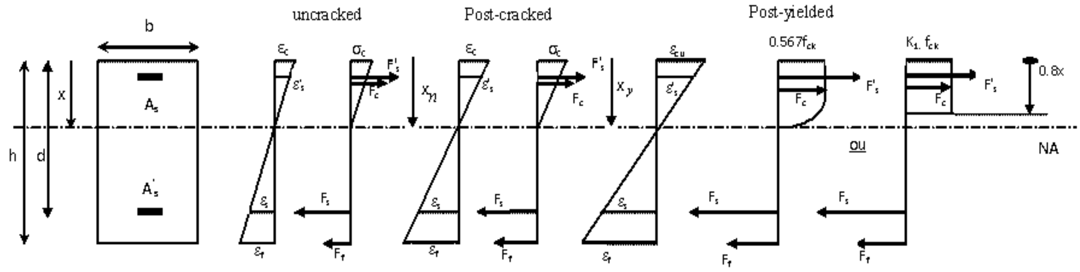


Fig. 12 Cross-sectional strains, stresses and forces distribution in the uncracked, post-cracked and post-yield stages

5. Analytical model

5.1 Introduction

Various analytical methods are available to analyze beams strengthened with bonded FRP laminate. An analytical approach based on cross-sectional analysis is an easy and accurate method to calculate the failure load of the strengthened beam. Also, this approach is applicable to the design of beams strengthened with FRP laminate (Akbarzadeh and Maghsoudi 2010). Therefore, an iterative analytical model was developed to predict the flexural behaviour of the low strength concrete beams strengthened with FRP composites. This analytical model uses the principles of strain compatibility, forces equilibrium, and constitutive material relations of the concrete, steel and FRP to predict flexural behaviour (Toutanji *et al*. 2006).

The flexural analysis of concrete sections with externally bonded tensile FRP reinforcement is based on the following assumptions:

• Plane sections before deformation remain plane at all times;
• There is no slip between the steel or FRP reinforcement and concrete;
• The contribution of the adhesive layer to flexural capacity is negligible.

The constitutive laws used for the concrete and steel are those adopted by the Eurocode2. As for the FRP, the elastic model was adopted.

5.2 Concrete design model

The bending moment capacity of the concrete sections, shown in Fig. 12, can be approximately evaluated using the equivalent parabolic-rectangular stress-block diagram for concrete in compression, which in this paper is in line with the Eurocode 2.

5.3 Steel reinforcement

Steel is commonly considered in design as a perfectly elasto-plastic material defined by its yield strength f_y, and elastic modulus E_s shown Fig. 13.

$$f_s = E_s \cdot \varepsilon_s \qquad 0 \leq \varepsilon_s \leq \varepsilon_y$$

$$f_s = \varepsilon_y \qquad \varepsilon_y \leq \varepsilon_s \leq \varepsilon_u$$

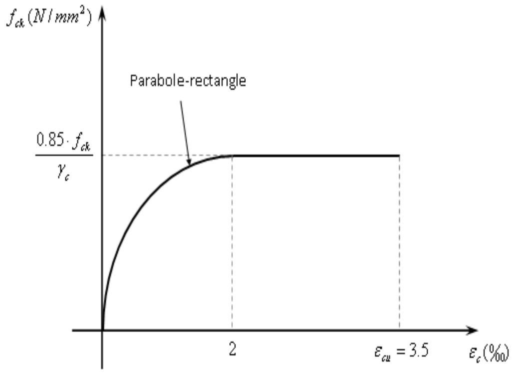


Fig. 13 Concrete stress versus strain model

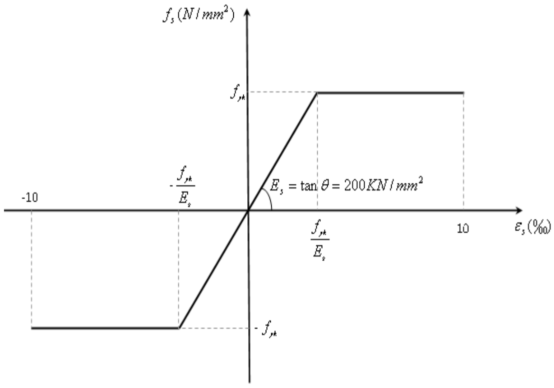


Fig. 14 Steel stress versus strain model

5.4 FRP reinforcement

For FRP materials, shown Fig. 14, an ideal elastic behaviour is assumed until failure, and the uniaxial tensile stress-strain relation is simply given by: $f_f = E_f \cdot \varepsilon_f \quad 0 \leq \varepsilon_f \leq \varepsilon_{fk}$

5.5 Cross-sectional relation

The relations between the neutral axis depth "*x*", the strains in steel and FRP reinforcement, and the maximum compressive strain in concrete are governed by the following compatibility equations

$$\varepsilon_s = \left(\frac{d-x}{x}\right) \cdot \varepsilon_c \tag{1}$$

$$\varepsilon'_s = \left(\frac{x-d'}{x}\right) \cdot \varepsilon_c \tag{2}$$

$$\varepsilon_f = \left(\frac{h + t/2 - x}{x}\right) \cdot \varepsilon_c \tag{3}$$

5.6 Modeling of moment curvature curve

Fig. 15 reveals that the moment curvature curve can be schematically divided into three straight lines, as follows:

When the moment increases from 0 to cracking moment M_{cr} (uncracked stage)

$$M_{cr} = \frac{2 \quad I_g \quad f_t}{h} \tag{4}$$

$$\varphi_{cr} = \frac{M_{cr}}{E_c \quad I_g} \tag{5}$$

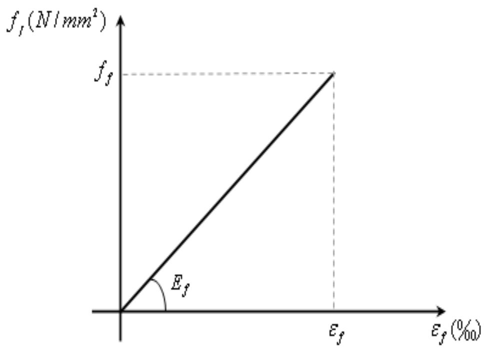


Fig. 15 FRP stress versus strain model

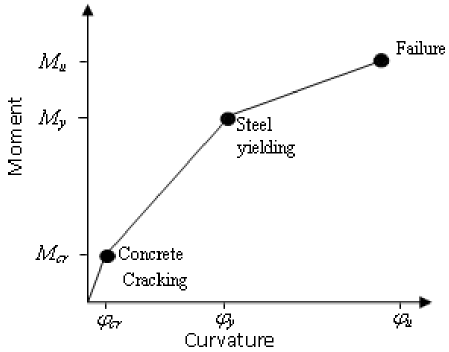


Fig. 16 Moment versus curvature

Where I_g is the moment of inertia of an uncracked section

$$I_g = \frac{b \quad h^3}{12} \tag{6}$$

f_t is the tensile strength of concrete in which

$$f_t = 0.3 f_c^{\frac{2}{3}} \tag{7}$$

$$E_c = 22000 \left[\frac{f_{cm}}{10} \right]^{0.3}, \quad f_{cm} = f_{ck} + 8MPa \quad \text{[Eurocode2]} \tag{8}$$

When the maximum moment increases from M_{cr} to moment corresponding to steel yielding M_y (post-cracked stage).

At this stage, the contribution of concrete in tension is ignored in the derivation; the total compressive force given by the concrete is

$$F_c = K_1 \cdot f_{ck} \cdot A_c(x), \quad \text{in which } K_1 = 0.567 \text{ (Whitney)} \tag{9}$$

$\varepsilon_s = \varepsilon_y$, the strains in compression steel, FRP and concrete at tensile yielding are given by

$$\varepsilon'_s = \left(\frac{x - d'}{d - x} \right) \cdot \varepsilon_y \tag{10}$$

$$\varepsilon_c = \left(\frac{x}{d - x} \right) \cdot \varepsilon_y \le 3.5‰ \tag{11}$$

$$\varepsilon_f = \left(\frac{h + t/2 - x}{d - x} \right) \cdot \varepsilon_y \tag{12}$$

Thus the yielding moment $M_{y/cog}$ (regarding the gravity center of the stress block of concrete) is given by

$$M_{y/cog} = F_s \cdot (0.4x - d') + F_y (d - 0.4x) + F_f (h + t/2 - 0.4x) \tag{13}$$

The curvature at mid-span at steel yielding φ_y is calculated by

$$\varphi_y = \frac{M_y}{E_c \quad I_{cr}} \tag{14}$$

The calculation of the moment of inertia I_{cr} is based on the transformed area method, thus

$$I_{cr} = \frac{1}{3} b x^3 + \frac{E_f}{E_c} A_f (h - x)^2 + \frac{E_s}{E_c} A_s (d - x)^2 + \frac{E_s}{E_c} A'_s (x - d')^2 \tag{15}$$

When the maximum moment increases from yielding moment M_y to ultimate moment M_u (post-yielded stage)

$$M_{u/cog} = F_s \cdot (0.4x - d') + F_y (d - 0.4x) + F_f (h + t/2 - 0.4x) \tag{16}$$

And the curvature is

$$\varphi_u = \frac{M_u}{E_c \quad I'_{cr}} \tag{17}$$

To simplify the problem and consider a conservative estimation for ultimate curvature, the contribution of steel stiffness after yielding is ignored (Toutanji *et al.* 2006). The transformed section method is still applicable for FRP and concrete in the compression zone. Thus the moment of inertia of the cracked section without tension steel I'_{cr} can be given by

$$I'_{cr} = \frac{1}{3} b x^3 + \frac{E_f}{E_c} A_f (h - x)^2 + \frac{E_s}{E_c} A'_s (x - d')^2 \tag{18}$$

Figs. 17-18 show a comparison between the experimental and analytical study on the moment versus curvature relationship of SB.3.1.P.N, and SB.4.1.P.N, It reveals an almost identical behaviour of two curves, which validated this theoretical demonstration. Unlike in Fig. 19 of SB.2.1.P.N, it was found that the analytical approach has over-estimated the effect because of the perfect adhesion that concrete-FRP assumed in the theoretical approach, so that it does not consider the delamination phenomenon.

The analytical approaches proposed by ACI440.2R-02 and ISIS Canada design guidelines are more appropriate when reinforcement bar ratio was increased to the maximum. In order to limit FRP laminate strain ACI440.2R-02 applies limit values of the ultimate FRP strain and ISIS Canada uses the reduction factors φ for concrete, steel and FRP but all this is not enough for a beams with a small bar ratio.

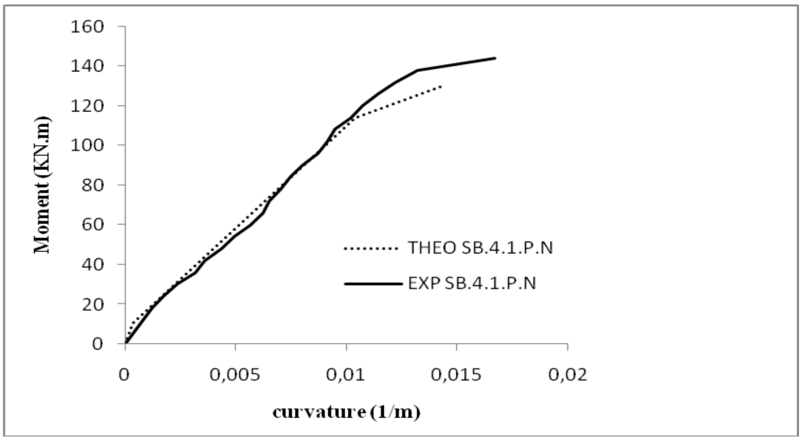


Fig. 17 Analytical and experimental moment versus curvature response for beam SB.4.1.P.N.

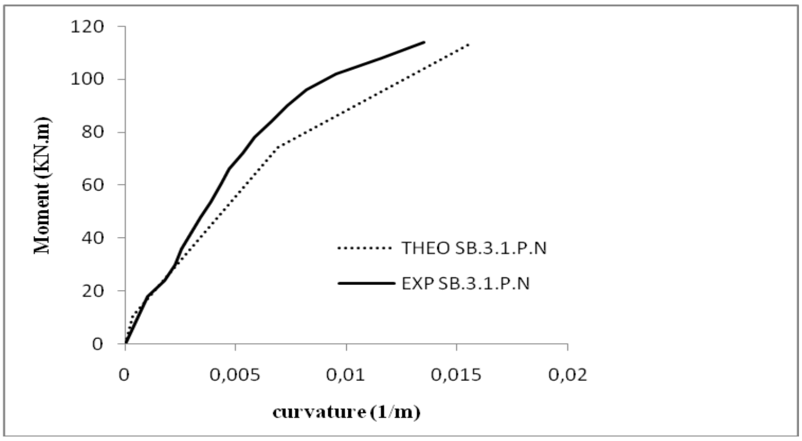


Fig. 18 Analytical and experimental moment versus curvature response for beam SB.3.1.P.N.

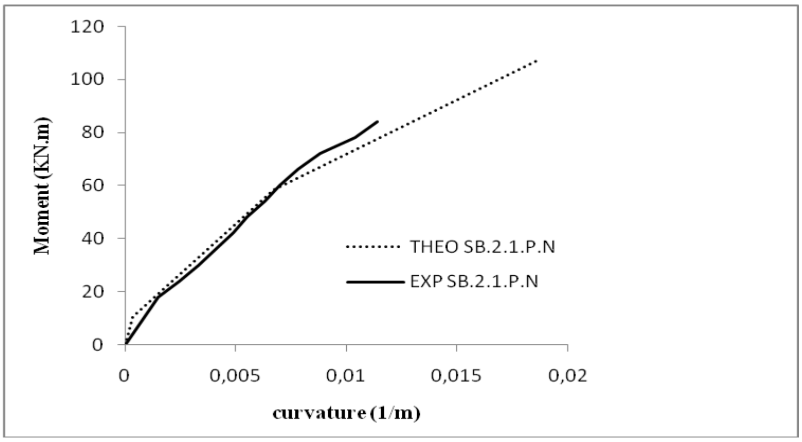


Fig. 19 Analytical and experimental moment versus curvature response for beam SB.2.1.P.N.

6. Comparison with previous studies

Many previous studies have been conducted on flexural behaviour of reinforced concrete beams strengthened by CFRP. Ceroni (2010), Toutanji *et al.* (2006), Esfahani *et al.* (2007), studied and compared the strengthened effect of CFRP. Ceroni (2010) studied the strengthening effect of CFRP laminate and Near Surface Mounted NSM bars under monotonic and cyclic load. Toutandji *et al.* (2006) studied the strengthening effect of CFRP sheets with different CFRP layers bonded by inorganic epoxy. Esfahani *et al.* (2007) studied the strengthening effect of CFRP with three different reinforcing ratios ρ and varied the width, length and number of CFRP layers. Pelà *et al.* (2012) studied a thorough experimental program concerning the RC deck of real old viaduct damaged at the extrados mainly due to environmental agents. In this study the authors replace the deteriorated concrete cover with a new concrete then it is strengthened in flexure. Obaidat *et al.* (2011) studied the behaviour of structurally damaged full-scale RC beams retrofitted by CFRP laminates in shear and flexure, it was found that the efficiency of the strengthening technique in flexure varied depending on the length of CFRP. Benjeddou *et al.* (2012) studied on damaged reinforced concrete beams repaired by external bonding of different amount of CFRP laminates. ZHANG *et al.* (2006) illustrated the behaviour RC beam strengthened by CFRP laminates at different levels of preloaded. Li *et al.* (2013) showed the contribution of U-shaped strips to the flexural capacity of low-strength reinforced concrete beams strengthened with carbon fibre composite sheets to avoid the CFRP debonding phenomena in the end of beams. Choi *et al.* (2013) present an experimental and analytical study aimed at investigating debonding of CFRP and GFRP sheet. The table below Table 5 summarizes a comparative study of different authors.

Comparisons have shown that the flexural strength and stiffness of strengthened beams increased, unlike the ductility, which was greatly reduced compared to control specimens.

Many kinds of failure mode occurred on the FRP-strengthened beams in these previous studies: flexural failure developed in all control beams, CFRP delamination in the majority of the strengthening beams. Furthermore the concrete cover interface separation occurred in Pelà *et al.* (2012) and CFRP rupture in Zhang *et al.* (2006).

Table 5 Summary table showing the different results of previous studies

Other Studies	designation	b (mm)	l (mm)	h (mm)	Main longitudinal Steel top	Main longitudinal Steel bottom	P_u (KN)	δ_{max} (mm)	Failure mode
M.R. Esfahani	B1-12D-0L	150	2000	200	157mm^2	226.08mm^2	49.46	23.34	SY/DL
	B2-12D-1L15				157mm2	226.08mm^2	61.45	10.20	SY
	B5-16D-0L				157mm2	401.92mm^2	75.94	15.52	SY/DL
	B7-16D-1L15				157mm2	401.92mm^2	94.92	11.01	SY/CC
	B9-20D-0L				157mm2	628.00mm^2	96.42	16.24	DL
	B11-16D-1L15				157mm2	628.00mm^2	108.91	19.14	SY/DL
F. Ceroni	A1	100	1000	180	100.48mm2	157.00mm^2	29.6	23.3	CC/SY
	A2				100.48mm2	157.00mm^2	37.4	19.2	CC/DL
	B1				100.48mm2	226.08mm^2	41.6	21.6	SY/CC
	B2				100.48mm2	226.08mm^2	49.2	18.6	DL
H. Toutanji	CB	108	1800	158	56.00mm2	142.00mm^2	37.1	34.3	CC/SY
	3L-1				56.00mm2	142.00mm^2	52.9	12.4	DL

Table 5 Continued

L. Pelà	Slab1	500	2000	200	-	452.00mm^2	109	34.3	Flexure
	Slab2	500	2000	230	-	904.00mm^2	240	34.3	Shear
	Slab3	500	2000	240	-	339.00mm^2	310	34.3	DL
	Slab4	500	2000	230	-	791.00mm^2	358	34.3	DL
Y.T. Obaidat	CB	500	2000	240	157mm^2	509.00mm^2	118	12	CC/SY
	RF1				157mm^2	509.00mm^2	166	8	DL
	RF2				157mm^2	509.00mm^2	142	6	DL
	RF3				157mm^2	509.00mm^2	128	7	DL
A. Zhang	AC	150	2500	250	157mm^2	226.19mm^2	45.4	19.6	CC/SY
	A10				157mm^2	226.19mm^2	62.5	24.3	DL
	A13				157mm^2	226.19mm^2	62.0	21.1	DL/CC
	A18				157mm^2	226.19mm^2	63.8	24 .4	DL
	BC				157mm^2	402.12mm^2	68.7	17.2	CC/SY
	B10				157mm^2	402.12mm^2	82.4	21.0	DL
	B13				157mm^2	402.12mm^2	91.3	21	RL/CC
	B18				157mm^2	226.19mm^2	90.1	28.0	RL/CC
O. Benjeddou	CB	120	2000	150	100mm^2	157.00mm^2	21.4	12.9	SY
	RB1				100mm^2	157.00mm^2	40.1	9.02	DL
	RB2				100mm^2	157.00mm^2	37.7	8.65	DL
	RB4				100mm^2	157.00mm^2	30.75	9.5	DL
E. Choi	C	150	550	150	-	-	37.9	0.79	DL
	G				-	-	18.2	0.66	DL
	CC				-	-	34.1	0.9	DL
	CG				-	-	30.25	0.64	DL
	GC				-	-	25.45	0.65	DL
	GG				-	-	24.1	0.65	DL
X. Li	B10	150	2600	300	56.54mm^2	226.19mm^2	40.5	38.4	F
	B11	152	2600	298	56.54mm^2	226.19mm^2	63.3	19.7	DL
	B12	152	2600	299	56.54mm^2	226.19mm^2	72.9	25.1	DL
	B20	150	2600	301	56.54mm^2 307.87mm^2		50.2	17.0	F
	B21	149	2600	299	56.54mm^2 307.87mm^2		76.3	13.3	DL
	B22	149	2600	302	56.54mm^2 307.87mm^2		88.9	23.6	DL

CC: concrete crushing, SY: steel yielding, DL: FRP delamination RL: FRP rupture F: Flexure

7. Conclusions

In this paper investigating the flexural behaviour of reinforced concrete beams strengthened using CFRP laminates, the main conclusions can be summarized as follows:

• Strengthening using CFRP plates is positive and effective even in the case of low strength concrete beams.

• The failure of strengthened beams generally occurs suddenly. This due to debonding between the CFRP plates and concrete.

• The displacement (deflection) of the strengthened beams is very limited compared to the control beams (38%).

• The depth of the neutral axis of strengthened beams is greater than that of control beams (SB.3.1.P.N has a neutral axis of 118.3 mm) (CB.3.1.U.N has a neutral axis of 82.7 mm).

• The crack width of beams strengthened with CFRP plates is very limited compared to those not strengthened (35%).

• The gains in terms of ultimate load and stiffness were very significant for the strengthened beams compared to those of the control beams.

• The behaviour of beams that were 50 and 100% damaged and then repaired is similar to undamaged strengthened beams.

• The diameter of tensile longitudinal bars directly affects the flexural capacity of the beam in a positive manner.

• The theoretical model becomes more appropriate when the reinforcing bar ratio increases, Esfahani *et al.* (2007) reveal that the prediction using ACI and ISIS Canada codes have overestimate the effect of strengthening, also more the bar ratio increase, the equations proposed by ACI and ISIS Canada become more appropriate.

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References

Abdessemed, M., Kenai, S., Bali, A. and Kibboua, A. (2011), "Dynamic analysis of a bridge repaired by CFRP: experimental and numerical modeling", *construction and building materials,* **25,** 1270-1276.

Ahmed, A. and Kodur, V.K.R. (2011), "Effect of bond degradation on fire resistance of FRP strengthened reinforced concrete beams", *Composites part B*, **42,** 226-237.

Akbarzadeh, H. and Maghsoudi, A.A. (2010), "Experimental and analytical of reinforced high strength concrete continuous beams strengthened with fiber reinforced polymer", *ELSEVIER Materials and Design,* **31,** 1130-1147.

Almusallam, T.H., Elsanadedy, H.M., Al-salloum, Y.A. and Alsayed, S.H. (2013), "Experimental and numerical investigation for the flexural strengthening of RC beams using near-surface mounted steel or GFRP bars", *Construction and Building Materials,* **40**, 145-161.

Anil, O., Bulut, N. and Ayhan, M. (2012), "Strain distribution between CFRP strip and concrete at strengthened RC beam against shear", *Steel and Composite Structure,* **41**(4), 509-525.

An, W., Saadatmanesh, H. and Ehsani, M.R. (1991), "RC Beams strengthened with FRP plates II : analysis and parametric study", *Journal of Structural Engineering*, **11**(117), 3434-3455.

Aprile, A. and Benedetti, A. (2004), "Coupled flexural-shear design of RC beams strengthened with FRP", *Composites Part B*, **35,** 1-25.

Aprile, A. and Feo, L. (2007), "Concrete cover rip-off RC beams strengthened with FRP composites", *Composites Part B,* **38,** 759-771.

Atadero, R.A. and Karbhari, V.M. (2008), "Calibration of resistance factors for reliability based design of externally-bonded FRP composites", *Composites Part B.*, **39** 665-679.

Barros, J.A.O., Dias, S.J.E. and Lima, J.L.T. (2007), "Efficacy of CFRP-based techniques for the flexural and shear strengthening of concrete beams", *Cement & Concrete Composites*, **29,** 203-217.

Bencardino, F., Colotti, V., Spadea, V.G. and Swamy, R.N. (2006), "Holistic design of RC beams and slabs strengthened with externally bonded FRP laminates", *Cement and Concrete Composites,* **28,** 832-844.

Benjeddou, O., Ouezdou, M.B. and Bedday, A. (2007), "Damaged RC beams repaired by bonding of CFRP laminates", *ELSEVIER Construction and Building Materials*, **21**, 1301-1310.

Capozucca, R. and Nilde Cerri, M. (2006), "Static and dynamic behaviour of RC beam model strengthened by CFRP-sheets", *Construction and Building Materials*, **16**, 91-99.

Ceroni, F. (2010), "Experimental performances of RC beams strengthened with FRP materials", *ELSEVIER Construction and Building Materials*, **24,** 1547-1559.

Choi, E., Utui, N. and Kil, H.S. (2013), "Experimental and analytical investigations on debonding of hybrid FRPsfor flexural strengthening of RC beams", *Composites Part B*, **45**, 248-256.

Esfahani, M.R., Kianoush, M.R. and Tjari, A.R. (2007), "Flexural behavior of reinforced concrete beams strengthened by CFRP sheets", *ELSEVIER Engineering Structures*, **29,** 2428-2444.

Ghosh. K.K. and Karbhari, V.M. (2010), "Use of infrared thermography for quantitative non-destructive evaluation in FRP strengthened bridge systems", *RILEM Materials and Mtructures*.

Hollaway, L.C. (2010), "A review of the present and future utilization of FRP composites in the civil infrastructure with reference to their important in-service properties", *ELSEVIER Construction and Building Materials,* **24,** 2419-2445.

Ibell, T., Darby, A. and Denton, S. (2009), "Research issues related to the appropriate use of FRP in concrete structures", *ELSEVIER Construction and Building Materials,* **23**, 1521-1538.

Jean, R. (2009), *Pratique de l'Eurocode 2*, afnor editions Paris, France.

Kantar, E. and Anil, O. (2012), "Low velocity impact behavior of concrete beam strengthened with CFRP strip", *Steel and Composite Structure,* **12**(3), 207-230.

Li, X., Gu, X.L., Song, X.B., Ouyang, Y. and Feng, Z.L. (2013), "Contribution of U-shaped strips to the flexural capacity of low-strength reinforced concrete beams strengthened with carbon fiber composite sheets", *Composites Part B,* **45** 117-126.

Maalej, M. and Leong, K.S. (2005), "Effect of beam size and FRP thickness on interfacial shear stress concentration and failure mode of FRP-strengthened beams", *Composite Science and Technology,* **65.** 1148-1158.

Massenzio, M., Jacqueli, E. and Ovigne, P.A. (2005), "Natural frequency evaluation of a cracked RC beam with or without composite strengthening for a damage assessment", *RILEM Materials and Structures,* **38,** 865-873.

Obeidat, Y.T., Heyden, S., Dalhblom. O. and Abu-Farsakh G. (2011), "Retrofitting of reinforced concrete beams using composite laminates", *ELSEVIER Construction and Building Materials,* **25,** 591-597.

Pelà, L., Aprile, A. and Benedetti, A. (2012), "Experimental study of retrofit solutions for damaged concrete bridge slabs", *Composites Part B,* **43**, 2471-2479.

Pesic, N. and Pilakoutas, K. (2005), "Flexural analysis and design of reinforced concrete beams with externally bonded FRP reinforcement", *RILEM Materials and Structure,* **38**, 183-192.

Rahimi, H. and Hutchinson, A. (2001), "Concrete beams strengthened with externally bonded FRP plates", *Journal of Composites for Construction ASCE,* **1**(5), 44-56.

Shehata, I.A., Shehata, L.C.D., Santos, E.W.F. and Simaoes, M.L.F. (2008), "strengthening of reinforced concrete beams in flexure by partial jacketing", *RILEM Materials and Structures,* **42,** 495-504.

Teng, J.G., Chen, J.F., Smith, S.T. and Lam, L. (2002), *Strengthened RC structures*, Wiley, New York, USA.

Toutanji, H., Zhao, L. and Zhang, Y. (2006), "Flexural behavior of reinforced concrete beams externally strengthened with CFRP sheets bonded with an inorganic matrix", *ELSEVIER Engineering Structures*, **28,** 557-566.

Xiang, K. and Wang, G.H. (2013), "Calculation of flexural strengthening of fire-damaged reinforced concrete beams with CFRP Sheets", *ELSEVIER Procedia Engineering ,* **52** 446-452.

Zhang, A., Jin, W.L. and Li, G.B. (2006), "Behavior of preloaded RC beams strengthened with CFRP laminates", *Journal of Zhejiang University Science,* **A7**(3), 436-444.