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Estimation of semi-rigid joints by cross modal strain energy method

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Abstract. We present a semi-rigid connection estimation method by using cross modal strain energy method. While rigid or pinned assumptions are adopted for steel frames in traditional modeling via finite element method, the actual behavior of the connections is usually neither. Semi-rigid joints enable connections to be modeled as partially restrained, which improves the quality of the model. To identify the connection stiffness and update the FE model, a newly-developed cross modal strain energy (CMSE) method is extended to incorporate the connection stiffness estimation. Meanwhile, the relations between the correction coefficients for the CMSE method are derived, which enables less modal information to be used in the estimation procedure. To illustrate the capability of the proposed parameter estimation algorithm, a four-story frame structure is demonstrated in the numerical studies. Several cases, including Semi-rigid joint(s) on single connection and on multi-connections, without and with measurement noise, are investigated. Numerical results indicate that an excellent updating is achievable and the connection stiffness can be estimated by CMSE method.

Keywords: finite-element model; model updating; semi-rigid joints; modal frequency; mode shapes

1. Introduction

In modern analysis of structural dynamics, much effort is devoted to the derivation of accurate models. These accurate models are used in many applications of civil engineering structures like response prediction, damage detection and vibration control. A typical way to establish a mathematical model for a civil structure or mechanical system is via the use of the finite element (FE) method. The FE model of a structure is constructed on the basis of highly idealized engineering blueprints and designs that may or may not truly represent all the physical aspects of an actual structure (Cunha *et al*. 2008).

Structural modeling errors cannot be completely avoided in any analytical procedure that relies on finite-element models; these errors originate from many aspects such as simplified materials, boundary conditions, as well as connections. In the field of civil engineering, beam-to-column connections play an important role in dynamic and stability analysis of frame structures, where FE models are usually not modeled with sufficient accuracy (Hadianfard and Razani 2003, Hasan 2010). In modeling the connection, one of the basic assumptions of conventional finite element models is that joints are either perfectly rigid or idealized hinged. Although the adoption of such

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idealized joint behavior simplifies the modeling process, it by no means represents the actual behavior of the structure. These non-ideal connections are often referred to as flexible connections, semi-rigid connections, spring hinged joints, etc. That is to say, the connection stiffness is somewhere in between. Under such circumstances, errors may be present if this simplified model is used for further analysis such as response prediction, damage detection, and so on.

Common ways to consider the semi-rigid joints include analytical procedure and/or experimental study (Fan *et al*. 2012, Loureiro *et al*. 2010, Alfonsas and Kestutis 2008, Loureiro *et al*. 2012). In addition, model updating is usually used to obtain an accurate model which is more representative of the real structure when field dynamic measurements are available. Model updating is defined as the process of correcting the numerical values of individual parameters in a mathematical model using data obtained from an associated experimental model such that the updated model more correctly describes the dynamic properties of the subject structure.

A number of model updating methods have been proposed. In general, these methods can be classified into two major groups: non-iterative (direct matrix methods) and iterative method (Ewins 2000, Friswell and Mottershead 1995, Mottershead and Friswell 1993, Hu *et al*. 2007). The former was based on computing changes made directly to the mass and stiffness matrices. Such changes may have succeeded in generating modified models which had properties close to those measured in the tests, but do not generally maintain structural connectivity and the corrections are not always physically meaningful. The iterative methods are generally of indirect physical property adjustment methods which involve using the sensitivity of the parameters to update the model. And they are in many ways more acceptable in that the parameters which they adjust are much closer to physically realizable quantities.

Hu *et al*. (2006), Wang *et al*. (2007) developed cross modal strain energy (CMSE) method for damage localization and severity estimation. The method is so named because it involves solving a set of linear simultaneous equations for the physically meaningful correction coefficients, in which each equation is formulated based on the product terms from two same/different modes associated with the mathematical and experimental models, respectively. The advantages of the CMSE method include: (1) no need scaling and pairing of mode shapes; (2) being a non-iterative solution method; (3) requiring very few measured modes to implement the method.

The objective of this paper is for extending the CMSE method to incorporate the localization of semi-rigid connections and estimation of the connection stiffness. In particular, the relations among the correction coefficients for different kinds of semi-rigid connection cases are derived which enables less measured modes to be used in the updating. The rest of the paper is arranged as follows. In section 2, the stiffness matrix of a beam member with semi-rigid joints on both ends is formulated. Next, the CMSE method is illustrated in section 3. The relationships among the correction coefficients for different kinds of end connections are formulated in section 4. Meanwhile, relationships between the correction coefficients and the rotational connection springs are derived. Section 5 demonstrated a four-story frame structure for verifying the capability of the proposed method for model updating and connection estimation. Finally, a conclusion was drawn in section 6.

2. Beam member with semi-rigid joints

Semi-rigid joints can be modeled using a rotational spring which separates a member from its surrounding environment. To incorporate semi-rigid end connection, the effects of connection

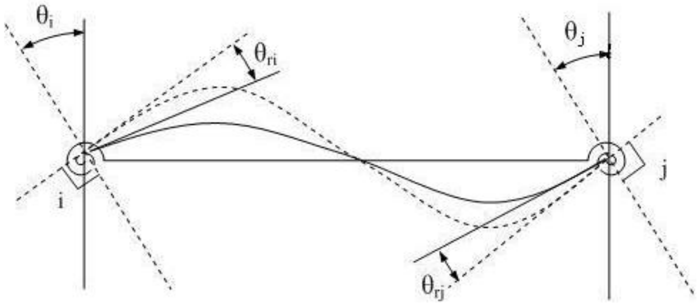

Fig. 1 Beam member with rotational springs

flexibility are modeled by attaching rotational springs at the two ends of a beam member, as shown in Fig. 1. A so-called end-fixity factor r defines the stiffness of the connection to the attached beam-column member (Monforton and Wu 1963). The end-fixity factor is defined as r=1/(1+3EI/kL) where k denotes for the rotational stiffness of the spring connections at either end i or j; L is the length of the beam-column member, I the moment of inertia, and E the Young's modulus. For pinned connections, the rotational stiffness of the connection is idealized as zero and thus the value of the end-fixity factor is zero (r = 0). For rigid connections, the end-fixity factor has a value of one (r = 1), because the connection rotational stiffness is taken to be infinite. A semi-rigid connection has an end-fixity factor between zero and one ($0 < r < 1$).

When a moment is applied to a beam-column connection, the connected beam and column rotate relative to each other by an amount of θ_r. The relationship between end-moments and end rotations of a beam can be written by replacing the end-rotations θ_i and θ_j by $\theta_i - \theta_{ri}$ and $\theta_j - \theta_{rj}$ respectively. Then the following stiffness matrix of a semi-rigid beam member with 6 degrees of freedom in local coordinates can be obtained, after considering the vertical displacements of the ends and the axial loading (Chen and Lui 1991, Wang 2013b).

$$k_r = \begin{bmatrix} EA/L & & & & & sym \\ 0 & (s_{ii}+2s_{ij}+s_{jj})EI/L^3 & & & & \\ 0 & (s_{ii}+s_{ij})EI/L^2 & s_{ii}EI/L & 0 & & \\ -EA/L & 0 & 0 & EA/L & & \\ 0 & -(s_{ii}+2s_{ij}+s_{jj})EI/L^3 & -(s_{ii}+s_{ij})EI/L^2 & 0 & (s_{ii}+2s_{ij}+s_{jj})EI/L^3 & \\ 0 & (s_{ij}+s_{jj})EI/L^2 & s_{ij}EI/L & 0 & -(s_{ij}+s_{jj})EI/L^2 & s_{jj}EI/L \end{bmatrix} \quad (1)$$

where s_{ii}=(4+12EI/Lk_j)/R^*, s_{ij}=2/R^*, s_{jj}=(4+12EI/Lk_i)/R^*, R^*=(1+4EI/Lk_i) (1+4EI/Lk_j) - (2EI)^2 /$L^2k_ik_j$ in which k_i and k_j are the rotational stiffness of the spring connections at ends i and j, respectively. A is the cross-sectional area of the member. Applying the known steps of the matrix displacement method, this matrix is obtained in global coordinates for each member and structural stiffness matrix can be constituted.

The stiffness matrix of Eq. (1) for $k_i = \infty$ and $k_j = \infty$ converges to the stiffness matrix corresponding to rigid connections at both ends. Similarly, Eq. (1) leads to the stiffness matrix associated with pinned connections at both ends for $k_i = 0$ and $k_j = 0$. Combinations ($k_i = \infty$, $k_j = 0$) or ($k_i = 0$, $k_j = \infty$) results in the stiffness matrix of an element with one end rigidly constrained and the other released. For $1 > k_i$ and/or $k_j > 0$, the stiffness matrix is algebraically a nonlinear function of k_i and/or k_j and **I**, which must be considered during parameter estimation.

When $k_i = \infty$ and $k_j = K_\Phi$, the stiffness matrix converges to the partially restrained frame (PRF) element (Sanayei 1999), as shown in Eq. (2).

$$k_s = \begin{bmatrix} \frac{EA}{L} & & & & & sym \\ 0 & \frac{12EI}{L^3}\frac{EI+LK_\Phi}{4EI+LK_\Phi} & & & & \\ 0 & \frac{6EI}{L^2}\frac{2EI+LK_\Phi}{4EI+LK_\Phi} & \frac{4EI}{L}\frac{3EI+LK_\Phi}{4EI+LK_\Phi} & 0 & & \\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & & \\ 0 & -\frac{12EI}{L^3}\frac{EI+LK_\Phi}{4EI+LK_\Phi} & -\frac{6EI}{L^2}\frac{2EI+LK_\Phi}{4EI+LK_\Phi} & 0 & \frac{12EI}{L^3}\frac{EI+LK_\Phi}{4EI+LK_\Phi} & \\ 0 & \frac{6EIK_\Phi}{L(4EI+LK_\Phi)} & \frac{2EIK_\Phi}{4EI+LK_\Phi} & 0 & -\frac{6EIK_\Phi}{L(4EI+LK_\Phi)} & \frac{4EIK_\Phi}{4EI+LK_\Phi} \end{bmatrix} \tag{2}$$

3. Cross modal strain energy method

This section briefly summarizes the cross modal strain energy (CMSE) method, to update an initial baseline model with measured information from actual structure which is assumed to have semi-rigid joint(s), and to estimate the rotational connection stiffness of semi-rigid joint(s).

It is assumed that the stiffness K and mass M matrices of the structure are obtained from a finite-element model, with rigid connections. It is intended to “correct” or “update” the stiffness K and mass M matrices by measured modal measurements, including a few mode shapes and their corresponding frequencies, from actual structure with semi-rigid joints.

The ith eigenvalue and eigenvector associated with K and M is expressed as

$$K\Phi_i = \lambda_i M\Phi_i \tag{3}$$

where λ_i and Φ_i denote the ith eigenvalue and eigenvector, respectively. Assume that the stiffness matrix K^* of the actual (experimental) model with semi-rigid joints is a modification of K to be formulated as

$$K^* = K + \sum_{n=1}^{N_K} \alpha_n K_n \tag{4}$$

where K_n is the stiffness sub-matrix chosen for correction; N_K is the total number of stiffness sub-matrix to be corrected; and α_n are the corresponding correction coefficients to be estimated. How to chose the sub-matrix K_n for semi-rigid joints updating will be presented in detail in section 4. For simplicity, it is assumed that the semi-rigid joints don’t change the mass matrix of the baseline.

Express the jth eigenvalue and eigenvector associated with $K^*$and M^* as

$$K^*\Phi_j^* = \lambda_j^* M^*\Phi_j^* \tag{5}$$

where λ_j^* and Φ_j* denote the *j*th eigenvalue and eigenvector measured from actual structures, respectively. Premultiplying Eq. (5) by Φ_i^T and substituting Eq. (4) into it yields

$$\sum_{n=1}^{N_K} \alpha_n \Phi_i^T K_n \Phi_j^* = \left(\frac{\lambda_j^*}{\lambda_i} - 1 \right) \Phi_i^T K \Phi_j^* \tag{6}$$

Define the structural cross modal strain energy C_{ij} and the corresponding elemental cross modal strain energy $C_{n,ij}$ between the *i*th mode of the baseline structure and the *j*th mode of the measured structure as

$$C_{ij} = \Phi_i^T K \Phi_j^*, \quad C_{n,ij} = \Phi_i^T K_n \Phi_j^* \tag{7}$$

After introducing a new index *m* to replace *ij*, Eq. (6) becomes

$$\sum_{n=1}^{N_K} \alpha_n C_{n,m} = b_m \tag{8}$$

where $b_m = (\lambda_j^*/\lambda_i - 1)\, \Phi_i^T K \Phi_j^*$. When N_i and N_j modes are available for the baseline structure and measured structure respectively, totally $N_q = N_i \times N_j$ equations can be formed from Eq. (8). Written in a matrix form, one has

$$C\alpha = b \tag{9}$$

in which C is an N_q-by-N_K matrix, α and b are column vectors of size N_K and N_q, respectively. When N_q is greater than N_K, a least-squares approach can be taken to solve for α. The estimate is written as

$$\alpha = \left(C^T C \right)^{-1} C^T b \tag{10}$$

After α is estimated, it can be substituted into Eq. (4) to obtain the updated stiffness matrix of actual structure with semi-rigid joints. It is worthy to mention that those N_i and N_j modes of the baseline and actual structures can be arbitrary modes in the sense that they are not required to start from the first mode. In practice, it is easy to obtain the analytical modes of the baseline structure, but difficult or expensive to extract the measured modes of the actual structure, therefore one may choose a much larger N_i than N_j. It should also be noted that spatially-complete mode shapes are required for conducting the CMSE method. When the measured mode shapes are spatially incomplete, model reduction or modal expansion techniques could be used to overcome the spatial incompleteness (Wang 2013a).

4. Estimation of rotational connection stiffness

In this section, the relationships among the correction coefficients for different types of end connections are formulated. Formulas of rotational stiffness of spring connections with correction coefficients are derived. These formulas can be used to estimate the rotational stiffness of the semi-rigid joints when the correction coefficients have been obtained by applying CMSE method.

4.1 Relationships of correction coefficients for different types of end connections

As mentioned earlier, the beam member is generally modeled as a frame element. For a beam member with rotational end connections, there are three categories, namely, connection at the left end, connection at right end or connections at both ends. The relationships among the correction coefficients for these three categories are formulated below. One generally updates an initial model which is often obtained via finite element method. Here we assume that all joints of the initial model are rigid connections, with stiffness matrix for an Euler-Bernoulli uniform beam element in 2-dimension as follows.

$$k_r = \begin{bmatrix} EA/L & 0 & 0 & -EA/L & 0 & 0 \\ 0 & 12EI/L^3 & 6EI/L^2 & 0 & -12EI/L^3 & 6EI/L^2 \\ 0 & 6EI/L^2 & 4EI/L & 0 & -6EI/L^2 & 2EI/L \\ -EA/L & 0 & 0 & EA/L & 0 & 0 \\ 0 & -12EI/L^3 & -6EI/L^2 & 0 & 12EI/L^3 & -6EI/L^2 \\ 0 & 6EI/L^2 & 2EI/L & 0 & -6EI/L^2 & 4EI/L \end{bmatrix} \quad (11)$$

Comparing Eq. (11) with Eq. (1), one can find that the connection stiffness changed the element entries corresponding to vertical and rotational degree of freedoms (DoFs). Therefore updating Eq. (11) requires focusing on these elements. One can select each individual K_n corresponding to one set of the connection stiffness. Denoting k_n as the counterpart of K_n in a 6 × 6 matrix form associated with the connection, one has the sub-matrices chosen as $k_1 = \begin{bmatrix} (2,2) & (2,5) \\ (5,2) & (2,2) \end{bmatrix}$, $k_2 = \begin{bmatrix} (2,6) & (5,6) \\ (6,2) & (6,5) \end{bmatrix}$, $k_3 = [(3,3)]$, $k_4 = \begin{bmatrix} & (3,6) \\ (6,3) & \end{bmatrix}$, $k_5 = \begin{bmatrix} (2,3) & (3,5) \\ (3,2) & (5,3) \end{bmatrix}$, $k_6 = [(6,6)]$. The digital pairs in the parentheses of each sub-matrix denote the associated entries which are chosen from the stiffness matrix as shown in Eq. (11). For example, (2, 2) in k_1 denotes the entry of row 2, column 2 of k_r. In this way, k_1 is chosen as

$$\mathbf{k}_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 12EI/L^3 & 0 & 0 & -12EI/L^3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -12EI/L^3 & 0 & 0 & 12EI/L^3 & 0 \\ 0 & 0^2 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (12)$$

and other sub-matrices are formed similarly. Hereby, one sets N_K=6 for semi-rigid corrections of a single beam. And it seems that one needs six correction coefficients to update the stiffness matrix of rotational connection at both ends. However, these six correction coefficients aren’t independent to each other, as shown in Task 1 of the numerical example.

By a theoretical analysis, one finds that the correction coefficients have the following relationships by comparing Eq. (11) with Eq. (1) via Eq. (4).

$$s_{ii}+2s_{ij}+s_{jj}=12(1+\alpha_1) \quad (13a)$$

$$s_{ij}+s_{jj}=6(1+\alpha_2) \quad (13b)$$

$$s_{ii} = 4(1+\alpha_3) \tag{13c}$$

$$s_{ij} = 2(1+\alpha_4) \tag{13d}$$

$$s_{ii}+s_{ij} = 6(1+\alpha_5) \tag{13e}$$

$$s_{jj} = 4(1+\alpha_6) \tag{13f}$$

From these equations, one finds there are only three independent coefficients. If α_1, α_2 and α_3 are chosen as the independent variables, α_4, α_5 and α_6 can be expressed as.

$$\alpha_4 = 6\alpha_1 - 3\alpha_2 - 2\alpha_3, \alpha_5 = 2\alpha_1 - \alpha_2,\ \alpha_6 = -3\alpha_1 + 3\alpha_2 + \alpha_3 \tag{14}$$

By the same token, one can formulate that only one independent correction coefficient is needed for single end rotational connection. If α_1 is chosen as the independent variable to be corrected, the relations are as follows

$$\alpha_2 = \frac{4}{3}\alpha_1, \alpha_3 = \frac{1}{3}\alpha_1,\ \alpha_4 = \frac{4}{3}\alpha_1,\ \alpha_5 = \frac{2}{3}\alpha_1, \alpha_6 = \frac{4}{3}\alpha_1 \tag{15}$$

for a right rotational connection case; and

$$\alpha_2 = \frac{2}{3}\alpha_1, \alpha_3 = \frac{4}{3}\alpha_1,\ \alpha_4 = \frac{4}{3}\alpha_1,\ \alpha_5 = \frac{4}{3}\alpha_1, \alpha_6 = \frac{1}{3}\alpha_1 \tag{16}$$

for a left rotational connection case. Eqs. (14), (15) and (16) can be used to reduce the number of correction factors, furthermore the measured modal information in the model updating procedure.

4.2 Estimation of the rotational connection stiffness

Once the stiffness sub-matrices are chosen and the correction coefficients for the stiffness matrix are correctly estimated, one can compute the stiffness of the rotational connection, after some manipulations. With Eq. (13c), Eq. (13d) and Eq. (13f), the left and right rotational connection stiffness yield Eq. (17a) and Eq. (17b), respectively, for the case with rotational connections at both ends.

$$\frac{s_{jj}}{s_{ij}} = 2 + \frac{6EI}{Lk_i} = \frac{2(1+\alpha_6)}{1+\alpha_4} \Rightarrow k_i = \frac{3EI}{L}\frac{1+\alpha_4}{\alpha_4-\alpha_6} \tag{17a}$$

$$\frac{s_{ii}}{s_{ij}} = 2 + \frac{6EI}{Lk_j} = \frac{2(1+\alpha_3)}{1+\alpha_4} \Rightarrow k_j = \frac{3EI}{L}\frac{1+\alpha_4}{\alpha_4-\alpha_3} \tag{17a}$$

For one end, either the left or the right, connection case, the rotational connection stiffness is equivalent and yields

$$\frac{EI+Lk_j}{4EI+Lk_j} = 1+\alpha_1 \Rightarrow k_j = -\frac{EI}{L}\frac{3+4\alpha_1}{\alpha_1} \tag{18}$$

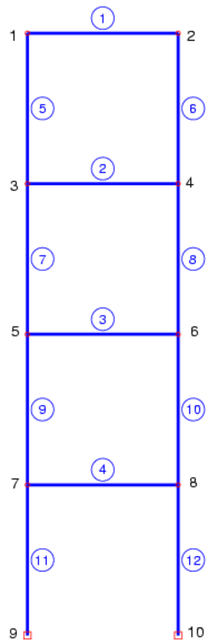

Fig. 2 Sketch of the four story frame structure

5. Numerical study

In this numerical example, the CMSE method for updating the model and estimating connection stiffness will be illustrated and verified on a four-story frame structure, as shown in Fig. 2, where each structural member is modeled as an Euler-Bernoulli uniform beam element. In Fig. 2, a circled number denotes a finite element while a number without a circle denotes a node. The essential geometrical and material properties of the frame structure are given below. The length of all members is 2 m. The cross-sectional area and the associated moment of inertia for all members are A=0.05m^2 and I=1.66×10^{4}m^4, respectively. The density and elastic modulus of the material of the members are ρ=7800 kg/m^3 and E=2.1×10^{11} N/m^2, respectively.

Finite element model with rigid connections is formed as the baseline. Modal analysis is carried out by developing a program in Matlab environment to get the FE frequencies and mode shapes. For simulating an “actual” structure with semi-rigid joint (connection stiffness), it is assumed that the selected beam element(s) is/are connected with the associated column(s) by rotational spring(s). Four tasks are considered in this example. Throughout this numerical example, the *baseline* model is defined as the finite element model with rigid connections. The “*actual*” structure is defined as the finite element model with semi-rigid joints at the end(s) of selected beam member(s).

5.1 Task 1-Right end of a single beam is connected with rotational spring

In task 1, the semi-rigid joint is assumed to be located at the right end of beam element 1. And the connection stiffness is assumed to be 1.0×10^7 N.m/rad. The modal analysis is again carried out to get the assumed “measured” modal parameters for the new model with this semi-rigid joint. The modal frequencies, relative errors, and modal assurance criterion (MAC) of the first six modes are shown in Table 1. It can be seen that the maximum error that appeared in frequency is 7.94% in the third mode and minimum value of MAC is 89.55% in the third mode as well.

When taking six correction coefficients, applying the CMSE method with any *three* measured modes from actual structure, one can obtain the correction coefficients. From the point of view of application, the lower modes of the first several orders may be preferable. The results are shown in Fig. 3 when the first three modes are utilized. The updating is exact, i.e., the frequencies and mode shapes of the updated model are identical to those measured from the "actual" structure with the semi-rigid joints and the MACs are unity. From Fig. 3, one can find that these six correction coefficients do relate to each other, as formulated in Eq. (13). For example, $\alpha_1=-0.6562$, $\alpha_2=\alpha_4=\alpha_6=4/3\alpha_1$, $\alpha_3=1/3\alpha_1=-0.2187$, $\alpha_5=2/3\alpha_1=-0.4375$. If only one correction coefficient α_1 is chosen as the independent variable to be corrected, only taking one measured mode can update the joint stiffness exactly. For task 1, the following three cases are investigated.

- Case 1: The exact location of the semi-rigid joint is known a priori.
- Case 2: The exact beam element which has the semi-rigid joint is known a priori.
- Case 3: The location information of the semi-rigid joint is totally unknown.

Table 1 Frequencies and MAC of the frame structure without and with flexible joint in task 1

Modes	Baseline(Hz)	With joint stiffness(Hz)	Relative error (%)	MAC(%)
1	8.0686	7.9949	0.9213	99.880
2	26.302	24.829	5.9343	95.896
3	49.185	45.569	7.9361	91.454
4	72.336	69.736	3.7276	93.362
5	121.62	119.62	1.6779	89.550
6	135.90	132.59	2.5005	94.322

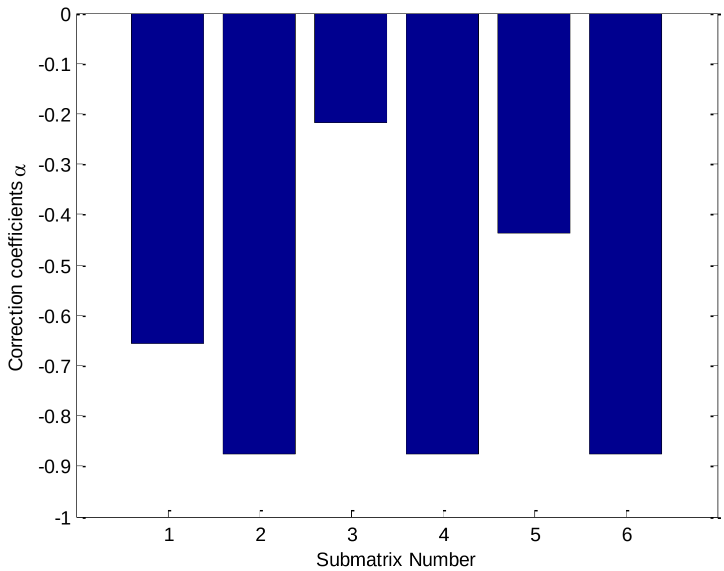

Fig. 3 The six correction coefficients for case 1 of task 1

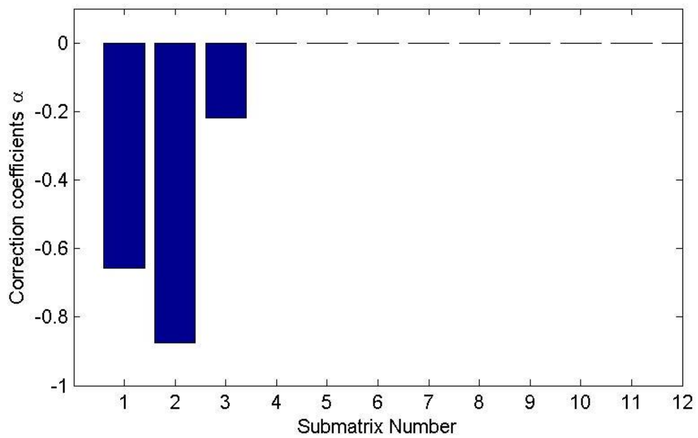

Fig. 4 The correction coefficients for case 3 of task 1

For case 1, if the exact location of the semi-rigid joint is known a priori, only one independent correction coefficient α_1 needs to be estimated. Using any single measured mode can obtain α_1= -0.6562 and the rotational spring can be estimated exactly, being k_j=1.0×10^7 N.m/rad.

In case 2, the exact location of the flexible joint isn't known beforehand. However, the beam element with this flexible joint is presumably known. In this case, one should assume that both ends of the beam member 1 may be semi-rigid connections. Therefore three independent correction coefficients (α_1, α_2, α_3) need to be estimated. Using any three measured modes can obtain $\alpha_1 = -0.6562$, $\alpha_2 = -0.875$, $\alpha_3 = -0.2187$. The rotational springs computed by using the estimated α are k_i = 6.2×10^{16} N.m/rad and k_j=1.0×10^7 N.m/rad, respectively. It is obvious that the left estimated stiffness is far larger than the linear stiffness EI/L and can be assumed to be rigidly connected (r_i=1). The right estimated joint spring is identical to the preset one.

For case 3, since one doesn't know the exact location of semi-rigid joint, all the beam elements are assumed to be subset whose ends are rotationally connected with springs. Therefore 12 correction coefficients, with each element three coefficients, are to be estimated. When the first two measured modes are utilized in the CMSE procedure, the correction coefficients are shown in Fig. 4. From Fig. 4, it is obvious that only the first element has been updated, with the correction coefficients identical to those estimated in case 2. Other correction coefficients are left to be zeros. The estimated connection stiffness are k_i = 4.2×10^{17} N.m/rad and k_j=1.0×10^7 N.m/rad, respectively. That is to say, one can still identify the connection location and estimate the spring stiffness exactly.

5.2 Task 2-Both ends of a single beam are connected with rotational springs

In task 2, both ends of a single beam (element 1) are connected with rotational springs, with the left rotational stiffness 5.0×10^7 N.m/rad and with right rotational stiffness 1.0×10^7 N.m/rad, respectively. The corresponding end-fixity factors are r_i=0.4878 and r_j=0.16, respectively. The modal frequencies, relative errors, and MAC of the first six modes are listed in Table 2. The maximum error that appeared in frequency is 11.245% in the third mode and minimum value of MAC is 87% in the 5th mode.

Table 2 Frequencies and MAC of the frame structure without and with flexible joints in task 2

Modes	Baseline (Hz)	With joint stiffness(Hz)	Relative error (%)	MAC(%)
1	8.0686	7.9533	1.4493	99.732
2	26.302	24.060	9.3186	92.542
3	49.185	44.213	11.245	88.105
4	72.336	69.061	4.7411	92.642
5	121.62	118.05	3.0314	87.005
6	135.90	131.44	3.3975	95.291

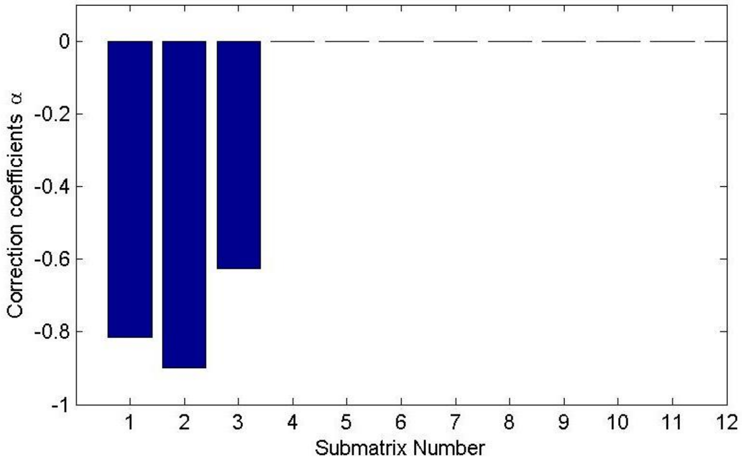

Fig. 5 The correction coefficients for case 6 in task 2

Similarly to task 1, three cases are studied too. For not confusing, they are ordered as cases 4 to 6.

• Case 4: Only one end, either the right or the left, of the beam is assumed to be spring connected.

• Case 5: Both ends of the beam element are assumed to be spring-connected.

• Case 6: All the beam elements in the model are assumed to spring connected for both ends.

For case 4, if only one end, either the right or the left, of the beam is assumed to be connected with a rotational spring, no correct results can be obtained. For example, if one assumes the right end of the beam to be spring-connected, the estimated results for the left and right rotational stiffness are $k_i = \infty$ and $k_j = -3.6833\times10^6$ N.m/rad, respectively.

For case 5, if both ends of the beam element are assumed to be spring-connected, one needs three independent correction coefficients to be estimated. By applying CMSE with any three measured modes, one estimates $\alpha_1 = -0.8149$, $\alpha_2 = -0.8985$, $\alpha_3 = -0.6269$. Then the left and right spring stiffness are computed as $k_i = 5.0\times10^7$ N.m/rad and $k_j = 1.0\times10^7$ N.m/rad. In this case, one can estimate the spring stiffness properly.

For case 6, all the beam elements are assumed to be subset whose ends are rotationally connected with springs. Therefore 12 independent correction coefficients, with each element three coefficients, are to be estimated and the results are shown in Fig. 5. From Fig. 5, it is obvious that only the first element has been updated, with the correction coefficients identical to those estimated in case 5. Other correction coefficients are left to be zeros. The estimated connection

Table 3 Frequencies and MAC of the frame structure without and with flexible joint in task 3

Modes	Baseline (Hz)	With joint stiffness(Hz)	Relative error (%)	MAC(%)
1	8.0686	7.7488	4.1267	99.477
2	26.302	23.053	14.093	95.237
3	49.185	45.333	8.4977	91.974
4	72.336	69.401	4.2292	93. 003
5	121.62	119.53	1.7556	88.941
6	135.90	131.28	3.5213	95.564

stiffnesses are k_i = 5.0×10^7 N.m/rad for left end and k_j = 1.0×10^7 N.m/rad for right end of beam 1, respectively. The estimated connection stiffnesses for beams 2 to 4 are very large numbers of magnitude 10^{18}. The corresponding end-fixity factors are all unity for these beams. From these results, it can obviously be seen that the rotational springs for both ends of beam 1 can be identified and estimated correctly.

5.3 Task 3-Different ends of two beams are connected with rotational springs

In task 3, semi-rigid joints located at different ends of two different beams are simulated and investigated. In this task, one flexible joint is located at the right end of element 1 with a connection stiffness 1.0×10^7 N.m/rad and the other is located at the left end of element 2, with a rotational stiffness 5.0×10^7 N.m/rad, respectively. The modal frequencies, relative errors, and MAC of the first six modes are listed in Table 3. The maximum error that appeared in frequency is 14.09% in the 2nd mode and minimum value of MAC is 88.94% in the 5th mode.
The following three cases are studied in task 3, named as cases 7 to 9.

• Case 7: Only one end, either the right or the left, of beam 1 or beam 2, is assumed to be spring connected.

• Case 8: Both ends of the beam elements 1 and 2 are assumed to be spring-connected.

• Case 9: All the beam elements in the model are assumed to be spring connected for both ends.

Case 7 investigates the case where only partial information of the flexible joints locations is known. Under this condition, no correct results can be estimated. For example, if only the right end of element 1 is assumed to be spring-connected, the estimated correction coefficients is $\alpha_1 = -0.7431$ and the rotational stiffness is k_j = 6.5115×10^7 N.m/rad for beam 1. However, if only the left end of element 1 is assumed to spring-connected, the estimated correction coefficients is $\alpha_1 = -0.8908$ and the rotational stiffness is k_j = −1.1063×10^7 N.m/rad for beam 1. The negative stiffness indicates that one sets wrong end of element 1.

In case 8, both ends of the beam elements 1 and 2 are assumed to be spring-connected. Six independent correction coefficients need to be estimated. The results are $\alpha_1 = -0.6562$, $\alpha_2 = -0.875$, $\alpha_3 = -0.2187$ for beam 1 and $\alpha_1 = -0.4375$, $\alpha_2 = -0.2917$, $\alpha_3 = -0.5833$ for beam 2, respectively. The identified spring stiffness are $k_i = \infty$ (r_i=1) and k_j= 1.0×10^7 N.m/rad for beam 1 and k_i = 5.0×10^7 N.m/rad and $k_j = \infty$ (r_j=1) for beam 2. Correct results could be obtained.

In case 9, all the beam elements in the model are assumed to be spring connected for both ends. Therefore 12 independent correction coefficients, with each element three coefficients, are to be estimated and the results are shown in Fig. 6. In Fig. 6, the first three correction coefficients are for beam 1 and the next three for beam 2. From Fig. 6, it is obvious that only elements 1 and 2 have

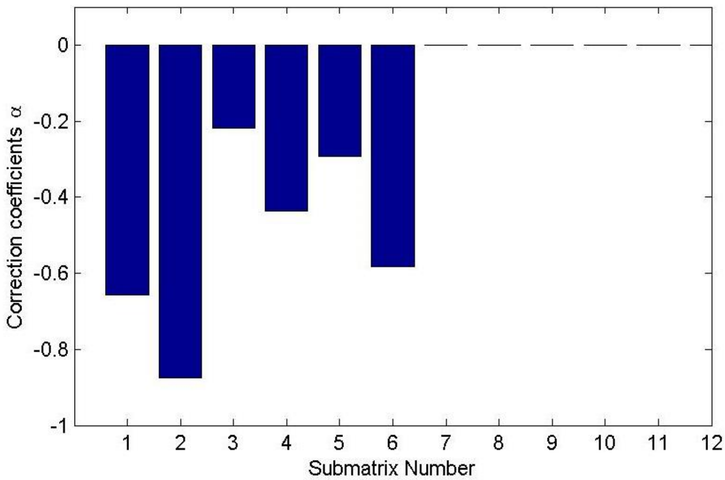

Fig. 6 The correction coefficients for case 9 in task 3

been updated, with the correction coefficients identical to those estimated in case 8. Other six correction coefficients for beams 3 and 4 are left to be zeros. The estimated connection stiffness are $k_i = \infty$ with end fixity factor r_i=1 and k_j =1.0×10^7 N.m/rad for beam 1, k_i = 5.0×10^7 N.m/rad and $k_j = \infty$ for beam 2, respectively. The estimated stiffnesses for beam 3 and 4 are all of magnitude of 10^{18}. From these results, it is obvious that the rotational springs for right end of beam 1 and left end of beam 2 can be identified and estimated exactly.

5.4 Task 4-investigation of noise effect on the estimation of semi-rigid joints

In practice, modes measured from experimental modal testing always contain noise or error. For studying the influence due to the measurement noise to the model updating and flexible joints estimation, the measured mode shapes are simulated by adding Gaussian noises to the noise-free modes. The measurement of the polluted *j*-th mode of the structure with flexible joints at the *k*-th DoF, has been simulated by adding a Gaussian random error to the corresponding true value, $\mathbf{\Phi}_{jk}$*

$$\hat{\Phi}^*_{jk} = \Phi^*_{jk}(1 + n\gamma_{jk}) \tag{19}$$

where n denotes a noise level, and γ_{jk} is a Gaussian random number with zero mean and unit standard deviation.

For space limitation, only the first case in task 1 is investigated for the noise-polluted measurements in task 4. In this case, a Gaussian noise of 0.5% noise level was added to the noise-free mode shapes. Using the first noise-polluted measured mode for model updating and flexible joints estimation, the first six modal frequencies and MAC of the updated model are shown in Table 4. Compared with Table 1, one can see that the maximum relative error has reduced to 0.12% in the third mode. And all the MACs have increased to near unity. The estimated flexible stiffness is k_j = 0.947×10^7 N.m/rad, with a 5.6% error compared to the true value k_j = 1.0×10^7 N.m/rad. When more modes are used in the CMSE method, a pretty good result can still be estimated for a higher noise level. For instance, the flexible stiffness is estimated to be k_i =

Table 4 Frequencies and MAC of the frame structure under 0.5% noise level

Modes	Baseline (Hz)	With joint stiffness(Hz)	Updated model(Hz)	MAC(%)
1	8.0686	7.9949	7.9935	100.00
2	26.302	24.829	24.801	99.999
3	49.185	45.569	45.513	99.998
4	72.336	69.736	69.703	99.999
5	121.62	119.62	119.58	99.998
6	135.90	132.59	132.54	99.999

1.062×10^{7} N.m/rad when using the first three modes for a 2% noise level. And the relative error is about 5.9%.

6. Conclusions

This article introduced the concept of semi-rigid joints where a beam is/are connected to column(s) by rotational spring(s), and developed an efficient cross modal strain energy (CMSE) method to update the initial baseline by solving a set of linear equations for the correction coefficients associated with the connection stiffness using a few of measured modes. The relationships between the correction coefficients and the connection stiffness for different cases are derived. The relationships could be used to reduce the correction coefficient numbers so that fewer measured mode shapes will be applied in the updating procedure. The estimated correction coefficients can be used to estimate the connection stiffness of the semi-rigid joints.

Numerical example related to a four story frame structure using synthesized data were carried out to illustrate the effectiveness of the CMSE solution procedure, as well as to demonstrate the achievability of locating the connection springs and estimating the stiffness. Four tasks are considered, in which different cases are investigated for each task. In task 1, only the right end of a single beam is connected to column with rotational spring. In task 2, both ends of a single beam are connected to columns with rotational springs. While tasks 1 and 2 focus on semi-rigid joints of single beam, semi-rigid joints located at different ends of two beams are considered in task 3. Task 4 investigates the influence of measurement noise on the updating and estimation of the semi-rigid joints.

From the four tasks, one can draw the following conclusions. If the exact location (end) of semi-rigid joint is known *a priori*, one can update the model correctly with one measured mode, and the estimated connection stiffness is exact. If the exact location of semi-rigid joints is not known, one should enlarge the subset suspectable to be semi-rigid joints. By applying the CMSE method with more measured modes, one can still identify the exact location of flexible joints and estimate the connection stiffness correctly.

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