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An improved particle swarm optimizer for steel grillage systems

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Abstract. In this paper, an improved version of particle swarm optimization based optimum design algorithm (IPSO) is presented for the steel grillage systems. The optimum design problem is formulated considering the provisions of American Institute of Steel Construction concerning Load and Resistance Factor Design. The optimum design algorithm selects the appropriate W-sections for the beams of the grillage system such that the design constraints are satisfied and the grillage weight is the minimum. When an improved version of the technique is extended to be implemented, the related results and convergence performance prove to be better than the simple particle swarm optimization algorithm and some other meta-heuristic optimization techniques. The efficiency of different inertia weight parameters of the proposed algorithm is also numerically investigated considering a number of numerical grillage system examples.

Keywords: optimum structural design; particle swarm optimization; minimum weight; search technique; combinatorial optimization; grillage systems

1. Introduction

Designing the steel structures optimally has always been the aim of structural designers. In recent years, structural optimization has witnessed an emergence of novel and innovative design techniques that strictly avoid gradient-based search to counteract with challenges that traditional optimization algorithms have faced for years. The basic idea behind each of these stochastic search techniques rests on simulating the paradigm of a biological, chemical, or social system such as *survival of the fittest in genetic algorithm* (Goldberg 1989), *shortest path to food source in ant colony algorithm* (Dorigo *and* Stützle 2004), *best harmony of instruments in the harmony search algorithm* (Geem and Kim 2001), *immune system* (Dasgupta 1999), *evolution* (Xie and Steven 1997), *and annealing process* (Van Laarhoven and Aaarts 1998), that is automated by nature to achieve the task of optimization of its own.

The long-standing research on computational efficiencies of meta-heuristic search techniques has clearly evinced that the design algorithms developed using these methods are suitable for obtaining rapid and precise solutions to optimum structural design problems as they can handle

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both the continuous and discrete design problems equally well. Furthermore, apart from requiring the derivatives of the objective function and constraints, they use probabilistic transition rules instead of deterministic rules. A large number of optimum structural design algorithms developed in recent years is based on these robust techniques (Adeli and Kumar 1995, Leite and Topping 1999, Luh and Chueh 2004, Camp and Barron 2004, Lee and Geem 2005).

Particle swarm optimization (PSO) is a relatively new heuristic approach utilized for optimization problems due to its simple principle and ease of implementation. This stochastic search method is based on swarm intelligence (Bonabeau *et al.* 1999). In this meta-heuristic technique, there are implicit rules each member of bird flock and fish school has to abide by so that they can move in a synchronized way without colliding, where each individual in a flock maintains optimum distance from the neighboring individuals so that there is no collision within the flock. Particle swarm optimizer is a simulator of social behavior that is used to realize the movement of a birds' flock, which is population based optimization algorithm. Its population is called a swarm, and each individual in the swarm is called a particle. Each particle flies through the problem space to search for the optimum way out. In the original particle swarm optimization technique (Kennedy *et al.* 1995), continuous design variable assumption is made. Then, researchers have applied this assumption in most of the applications of particle swarm optimization algorithm to the structural optimization problems in the literature (He *et al.* 2004). Such an assumption can not be made in the optimum design problem of steel frames where the steel sections for their beams and columns are to be selected from a steel profile list which consists of discrete values. There exist two different approaches in the literature, which convert the integer numbers to continuous ones; first suggested by Kennedy and Eberhart (1997) and the second applied by Kaveh and Talatahari (2009). The former used the binary numbers are to achieve a discrete set; whereas the latter rounds of the real number to the nearest integer number in the each iteration. The rounding off method is implemented in the present study due to its simplicity. However, it is realized that this technique raise convergence problems in the optimization process. In the present study, an improved version of particle swarm optimization is employed to overcome this problem where minimum weight design problem of grillages is carried out using this IPSO algorithm, and the results are compared with the ones obtained with classic particle swarm optimizer (CPSO), harmony search (HSO), simple genetic algorithm (sGA) and charged system search (CSS) based design algorithms.

2. Discrete particle swarm optimizer

Particle swarm optimization (PSO) algorithm is one of the recent additions to the meta-heuristic search techniques of combinatorial optimization problems which are based on the social behavior of animals such as insect swarming, fish schooling and birds flocking as mentioned before. This social behavior is concerned with grouping by social forces that depend on both the memory of each individual as well as the knowledge gained by the swarm. The procedure involves a number of particles which represents the swarm are initialized randomly in the search space of an objective function. Each particle in the swarm represents a candidate solution of the optimum design problem. Originally particle swarm optimizer is developed for continuous design variables. To be able to use the method for discrete design variables, some adjustments are required. The basic steps of the particle swarm optimization for a general discrete optimization problem can be outlined as follows (Fourie and Groenwold 2002, Perez and Behdinan 2007).

Step 1. Initializing Particles: A swarm is composed of a pre-selected number of particles called as swarm size (μ). A design vector **I** and a velocity vector **v** are two set of components that each particle should have (Eq. (1)). The positions of variables are retained by the position vector **I**, while the velocity vector **v** is used to change positions during the search. Random initialization is used to set up each particle in the swarm such that all initial positions $I_i^{(0)}$ and velocities $v_i^{(0)}$ are assigned from Eqs. (7)-(8)

$$\mathbf{P} = (\mathbf{I}, \mathbf{v}), \quad \mathbf{I} = \left[I_1, I_2, ..., I_{N_d}\right] \quad , \quad \mathbf{v} = \left[v_1, v_2, ..., v_{N_d}\right] \tag{1}$$

$$I_i^{(0)} = I_{\min} + r(I_{\max} - I_{\min}), \quad i = 1,.., N_d \tag{2}$$

$$v_i^{(0)} = \frac{I_{\min} + r(I_{\max} - I_{\min})}{\Delta t}, \quad i = 1, .. N_d \tag{3}$$

In Eqs. (2)-(3), $I_{\min}$ and $I_{\max}$ are the sequence numbers of the first and last standard steel sections in the profile list, respectively, r represents a random number between 0 and 1; Δt is referred to as the time step increment.

Step 2. Evaluating particles: The analysis of the structure is carried out with the potential designs represented by each particle. The objective function values (f_j) are evaluated using the design space positions.

Step 3. Updating the particle's best and the global best: Particle's best (Pbest) refers to the particle's best position which the best design is having minimum objective function during iterations so far. Each particle has a vector **B** containing the particle's best. Another vector **G** stores the best feasible design obtained by any particle since the beginning of the process (Eq. (4)), which is the global best position (gbest). Both the particles' bests and the global best are updated at the current iteration k.

$$\mathbf{B}^{(k)} = \left[B_1^{(k)}, ... B_i^{(k)} .., B_{N_d}^{(k)}\right] \qquad \mathbf{G}^{(k)} = \left[G_1^{(k)}, .. G_i^{(k)} .. G_{N_d}^{(k)}\right] \tag{4}$$

Step 4. Updating a Particle's Velocity Vector: The velocity vector of each particle is updated using Eq. (5) considering the particle's current position, the particle's best position and global best position.

$$v_i^{(k+1)} = w v_i^{(k)} + c_1 r_1 \left(\frac{G_i^{(k)} - I_i^{(k)}}{\Delta t}\right) + c_2 r_2 \left(\frac{B_i^{(k)} - I_i^{(k)}}{\Delta t}\right) \tag{5}$$

Where, w is the inertia of the particle which controls the exploration properties of the algorithm; r_1 and r_2 are random numbers between 0 and 1; and c_1 and c_2 are the trust parameters, indicating how much confidence the particle has in itself and in the swarm, respectively.

Step 5. Updating a Particle's Position Vector: Using the updated velocity vector, the position vector of each particle is updated (Eq. (6)), which is rounded to nearest integer value for discrete variables.

$$I_i^{(k+1)} = I_i^{(k)} + v_i^{(k+1)} \Delta t \tag{6}$$

Step 6. Termination: The steps 2 through 5 are repeated until the termination criterion which is the pre-selected maximum number of cycles (N_{ite}) is reached.

3. Improved particle swarm optimizer

Particle swarm optimization technique based optimum design algorithm necessitates updating the positions of all the particles using Eqs. (7)-(8). During the procedure, particles' velocities and positions change, and these changes lead to revisions of particle and global bests. Numerical applications indicate that when the velocities are updated through the use of Eq. (4), the current and best positions of all the particles in the swarm are eventually dragged to the position identified by the global best position. Hence, the current and best positions the particles become identical to the global best resulting in almost zero velocity vectors. In the present study, the following equations are reported to tackle with this problem in IPSO.

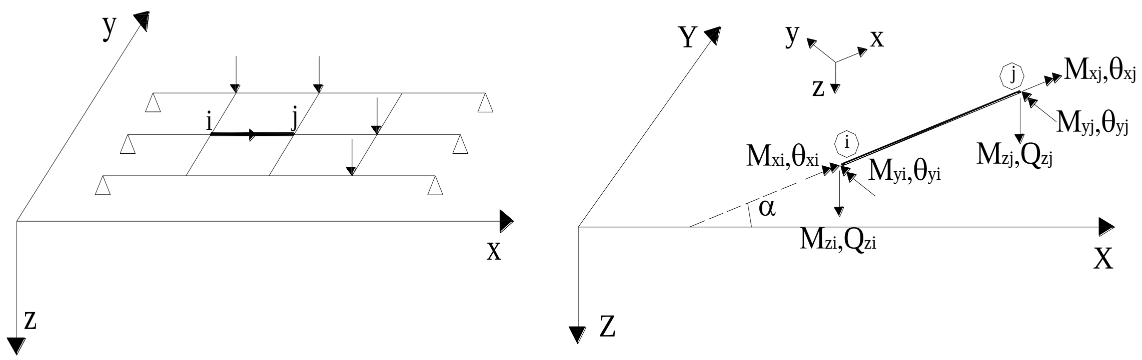
$$v_i^{(k+1)} = wv_i^{(k)} + c_1 r_1 \left(\frac{G_i^{(k)} - I_i^{(k)}}{\Delta t} \right) + c_2 r_2 \left(\frac{B_i^{(k)} - I_i^{(k)}}{\Delta t} \right) + \hbar_i r_3 \frac{\sqrt{N_s}}{\Delta t} \tag{7}$$

$$\hbar_i = \begin{cases} 1 & if \;\; r \le \dfrac{1}{2N_d} \\ 0 & f \;\; r > \dfrac{1}{2N_d} \end{cases} \tag{8}$$

Where, r_3 is a random number between 0 and 1; N_s is referred to as the number of steel sections in the profile list; and $\hbar_i$ is 0-1 heaviside function implemented by sampling a random number r between 0 and 1. In IPSO, the role of additional term in Eq. (7) is to facilitate flying of the particles when the swarm is collected as a whole in the same region of the design domain. Through this term, some particles are enforced to move randomly in certain directions to keep the mobility the swarm alive to carry on an effective search process. Probabilistically speaking, Eq. (8) implies that the each iteration of the half of the particles in the swarm is given a random velocity in one direction. A verification of the value $1/2N_d$ in Eq. (8) has been conducted using a number of test problems. It has been found that higher values, such as $1/N_d$ or $2/N_d$ might turn the search into a randomized process, whereas lower values (e.g., $1/4N_d$) would be insufficient or less effective to prevent stagnation of the algorithm.

The additional term in PSO serves to provide communication between particles in order to accelerate the convergence rate of the algorithm. Therein, a particle is encouraged to modify its velocity based on the position of another particle in the swarm. The new formulated equation has been observed to eliminate the aforementioned drawback and greatly improve the efficiency of the technique. The improvements in the technique are demonstrated by several numerical examples that are explained in section 5.

Constraint handling: In this study, fly-back mechanism is used for in handling the design constraints, which is proven to be effective in Arumugam *et al*. (2008). Once all particle positions (I^i) are generated, the objective functions are evaluated for each of these and the constraints in the problem are then computed with these values to find out whether they violate the design constraints. If one or a number of the particle gives infeasible solutions, these are discarded and new ones are re-generated. If a particle is slightly infeasible, then such particles are kept in the solution. These particles having one or more slightly infeasible constraints are utilized in the design process which might provide a new particle that may be feasible. This is achieved by using



(a) Simple steel grillage (b) End forces and displacements of a grillage member

Fig. 1 Typical grillage system

larger error values initially for the acceptability of the new design vectors. Later, this value could be gradually reduces during the design cycles. Finally, an error value of 0.001 or any value that is required to be selected for the permissible error term towards the end of iterations could be used. This adaptive error strategy is found quite effective in handling the design constraints in large design problems.

4. Optimum design of grillage systems

Grillage systems are widely used in structures to cover large areas such as in bridge decks, airplane wings, ship hulls and floors. The design of these systems is one of the problems of steel structures that practicing engineers have to deal with. Optimum design of a typical grillage system shown in Fig. 1 aims at finding the cross sectional properties of transverse and longitudinal beams. In this case, the response of the system under the external loading should be within the allowable limits described in a code of practice while the weight or the cost of the system is the minimum. In one of the previous studies on grillage systems, the optimum design problem is formulated by treating the moment of inertias of the beams and joint displacements as design variables (Saka 1987). Stiffness, stress, displacement and size constraints are included in the design formulation. The effect of warping is taken into account in the computation of the stresses in the grillage members. The nonlinear programming problem is solved by the approximating programming method (Saka 1981). The formulation of the same design problem is carried out only by treating the cross-sectional areas of members in the grillage system in (Saka 1981, Kaveh and Talatahari 2010) where the warping and shear effects are also taken into account in the computation of the response of the system. Displacements, stress and size limitations are included in the design formulation according to ASD-AISC (Allowable Stress Design – American Institute of Steel Construction) code. The solution of the optimum design problems is obtained using optimality criteria approach. In (Saka and Erdal 2009), harmony search algorithm is used to determine the optimum wide flange beam sections (W) for the members of grillage system from the set of LRFD-AISC sections. The deflection limitations and the allowable stress constraints are considered in the formulation of the design problem.

4.1 Optimum design problem to LRFD-AISC

If the design variables are selected from steel sections from W-sections list of LRFD-AISC, optimum design problem of a grillage steel structure, consisting of N_k members that are collected in N_d design variables, according to LRFD-AISC (1999) code yields the following discrete programming problem. In order to find a vector of integer values **I** representing the sequence numbers of steel sections assigned to N_d member groups, the following formulae are applied to minimize the weight of the grillage system.

$$\mathbf{I}^T = \left[I_1, I_2, ..., I_{N_d}\right] \tag{7}$$

$$Minimum \quad W = \sum_{k=1}^{N_d} m_k \sum_{i=1}^{N_k} l_i \tag{8}$$

Where m_k is the unit weight of grillage belonging to group k to be selected from W-sections list of LRFD-AISC, N_t is the total number of groups in the grillage system. l_i is the length of member i. The grillage system members are subjected to following behavioral constraints.

$$\delta_j / \delta_{ju} \le 1 \qquad and \qquad j = 1,2,........, p \tag{9}$$

$$M_{ur} /(\phi_b M_{nr}) \le 1 \qquad and \qquad r = 1,2,......., nm \tag{10}$$

$$V_{ur} /(\phi_v V_{nr}) \le 1 \qquad and \qquad r = 1,2,........, nm \tag{11}$$

Where δ_j in Eq. (9) is the displacement of joint j and δ_{ju} is its upper bound. The joint displacements are computed using the matrix displacement method for grillage systems. Eq. (10) represents the strength requirement for laterally supported beam in load and resistance factor design according to LRFD-F2. In this inequality $\emptyset_b$ is the resistance factor for flexure which is given as 0.9, M_{nr} is the nominal moment strength and M_{ur} is the factored service load moment for member r. Eq. (11) represents the shear strength requirement in load and resistance factor design according to LRFD-F2. In this inequality $\emptyset_v$ represents the resistance factor for shear given as 0.9, V_{nr} is the nominal strength in shear and V_{ur} is the factored service load shear for member r. The details of obtaining nominal moment strength and nominal shear strength of a W-section according to LRFD are given in the following.

4.2 Grillage analysis

The structural analysis of the grillage system that is required to determine its response under the external loads is carried out using the matrix displacement method. The joint displacements vector of a grillage element r which connects joints i and j is related to the vector of joint loading in global coordinate system as $\{P\}_r = [K]_r\{D\}_r$ where $[K]_r$ is the stiffness matrix of the grillage member r in global coordinates. This matrix is obtained by carrying out triple matrix multiplication $[K]_r = [B]_r^T [k] [B]_r$ where $[B]_r$ is the displacement transformation matrix and $[k]_r$ is the stiffness matrix of the grillage member r in local coordinates. The stiffness matrix of member r in global coordinates has the following form.

$$[K]=\begin{bmatrix} a & b & c & . & d & e & -c \\ b & f & g & . & e & h & -g \\ c & g & p & . & c & g & -p \\ . & . & . & . & . & . & . \\ d & e & c & . & a & b & -c \\ e & h & g & . & b & f & -g \\ -c & -g & -p & . & -c & -g & p \end{bmatrix} \tag{12}$$

where

$$a=\frac{GJ}{L}\cos^2\alpha+\frac{4EI}{L}\sin^2\alpha \qquad b=\left(\frac{GJ}{L}-\frac{4EI}{L}\right)\cos\alpha\sin\alpha$$

$$c=-\frac{6EI}{L^2}\sin\alpha \qquad d=-\frac{GJ}{L}\cos^2\alpha+\frac{2EI}{L}\sin^2\alpha$$

$$e=\left(-\frac{GJ}{L}-\frac{2EI}{L}\right)\cos\alpha\sin\alpha \qquad f=\frac{GJ}{L}\sin^2\alpha+\frac{4EI}{L}\cos^2\alpha \tag{13}$$

$$g=\frac{6EI}{L^2}\cos\alpha \qquad h=-\frac{GJ}{L}\sin^2\alpha+\frac{2EI}{L}\cos^2\alpha$$

$$p=\frac{12EI}{L^3}$$

in which E is the modulus of elasticity, G is the shear modulus, I is the moment of inertia about major axis , J is the torsional moment of inertia of the cross section and L is the length of grillage element. α is the angle between the local x axis of the element and the global X axis. The overall stiffness matrix of the grillage system is obtained by collecting together the stiffness matrices of each member in global coordinates. The solution of the stiffness equations $\{P\} = [K]\{D\}$ yields to the joint displacements. Once the joint displacements are obtained the vector of member end forces for each member is then computed from $\{F\}_r = [k][B]_r\{D\}_r$ where $\{F\}_r$ represents the vector of member end forces for member r shown in Fig. 1(b).

4.3 Classification of cross sections and shear for laterally supported rolled beams

In the computation of the nominal flexural strength M_n of a laterally supported beam, it is

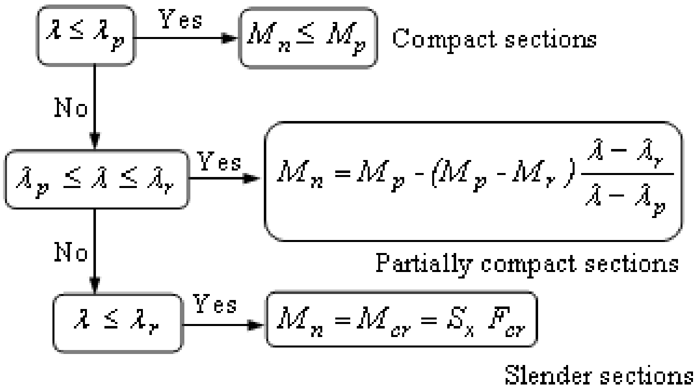


Fig. 2 Flowchart for determining nominal flexural strength

Table 1 Limiting width to thickness ratios for I-beams

Type of Element	λ_p	λ_r
Outstand Element of Compression Flange	$0.38\sqrt{\dfrac{E}{F_y}}$	$0.83\sqrt{\dfrac{E}{F_y - F_r}}$
For web, with neutral axis at mid-depth	$3.76\sqrt{\dfrac{E}{F_y}}$	$5.70\sqrt{\dfrac{E}{F_y}}$

necessary first to determine the classification of cross sections. In compact sections, local buckling of the compression flange and the web do not occur before the plastic hinge develops in the cross section. On the other hand in practically compact section, the local buckling of compression flange or web may occur after the first yield is reacted at the outer fiber of the flanges. The flowchart for checking procedure is given in Fig. 3 according to LRFD-AISC specification.

In this figure, $\lambda = b_f/(2t_f)$ is the slenderness parameter for I-shaped member flanges, in which b_f and t_f represent the width and the thickness of the flange respectively, similarly; $\lambda = h/t_w$ is the same for beam web, in which $h = d - 2k$ plus allowance for undersize inside fillet at compression flange for rolled I-shaped sections. d is referred to as the depth of the section and k is the distance from outer face of flange to web toe of fillet. t_w denotes the web thickness. h/t_w values are readily available in W-section properties table. λ_p and λ_r are given in Table 1 according to LRFD-B5.1 provision.

In which, E is the modulus of elasticity and F_y is the yield stress of steel. Besides, F_r represents the compressive residual stress in flange which is given as 69 MPa for rolled shapes in the code. It is apparent that M_n is computed for the flange and for the web separately by using corresponding λ values. The smallest amongst all is taken as the nominal moment strength of the *W* section under consideration. According to LRFD-AISC-F2.2, nominal shear strength of a rolled compact, semi-compact and slender *W* section is given in Fig. 3. Where, F_{yw} represents the yield stress of web steel. V_n is computed from one of the expressions of Fig. 4 depending upon the value of h/t_w of the *W* section under consideration.

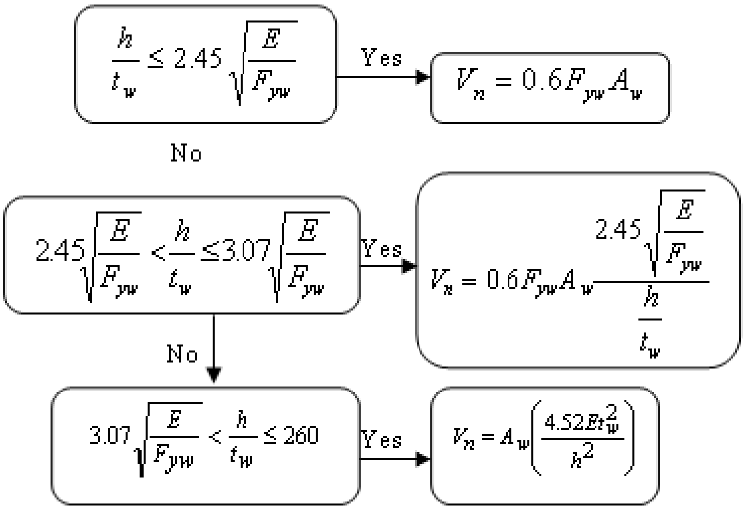


Fig. 3 Shear capacity of un-stiffened webs

5. Design examples

In this section, improved particle swarm based optimum design algorithm presented in the previous sections is used to design of three different grillage systems to test and compare the numerical performance of this proposed technique with other heuristic search methods according to LRFD-AISC code provisions. Sections are assumed to be made up of A36 mild steel, which has the yield stress of 250MPa. The values of 205kN/mm^2 and 81kN/mm^2 are used for the modulus of elasticity and the shear modulus, respectively. The discrete set from which the design algorithm selects the sectional designations for grillage members is considered to be the complete set of 272 W-sections starting from W100×19.3 to W1100×499mm as given in LRFD-AISC. The sequence number of each section in the set is used as design variable.

5.1 Cantilever grillage system with 14-members

The 14-member cantilever simple grillage system shown in Fig. 4 is selected as the first numerical design example to solve the optimum design algorithm developed. The dimensions, member grouping and the external loading of the system are also shown in Fig. 5. The upper bound imposed on vertical deflections of joints 7 and 8 is restricted to 20 mm while the strength constraints (2) and (3) are implemented from LRFD-AISC. The system is designed by collecting the grillage members in two different groups. The longitudinal beams are considered to be group 1 and the transverse beams are taken as group 2 as shown in Fig. 4.

After 32 cycles, the sectional designations that correspond to the sequence numbers given in the first row are W530×74 for group 1 and W200×15 for groups 2 which yield to a grillage system with a weight of 890.6 kg. The analysis of the system with these sections resulted in 14.4 mm maximum vertical displacements at joints 5, 6, 7 and 8. The strength ratios of (2) computed for these sections are 0.89 for member 1. These values clearly indicate that improved particle swarm optimizer should be continued to determine even a better combination because the safety margin on strength constraints is large. The minimum weight design of the cantilever grillage system

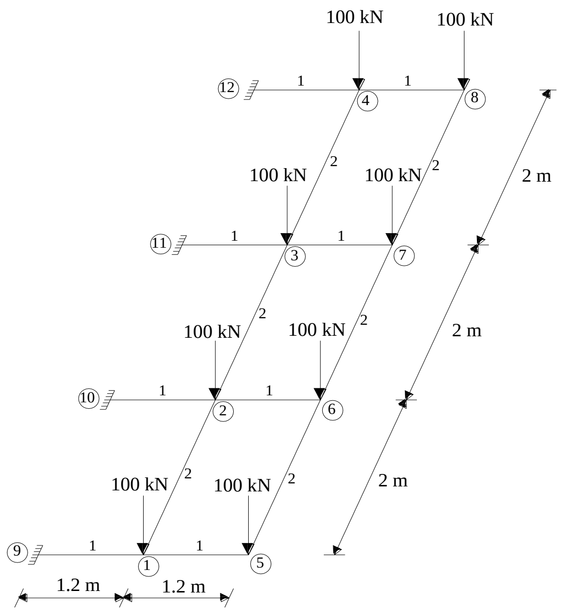


Fig. 4 Cantilever grillage system with 14-members

Table 2 The parameter data set of particle swarm algorithm

Search Method	Optimum W-Section		δ_{max} (mm)	Maximum Strength Ratio	Minimum Weight (kg)
	Group 1	Group 2			
IPSO	W460×74	W150×13.5	17.8	0.98	872.23
CPSO	W530×74	W200×15	14.4	0.89	890.58
ECSS	W530×72	W150×13.5	7.39	0.91	847.20

obtained after 90 steps is given in Table 2. The sectional designations for this combination are W460×74 and W150×13.5 which yield to a grillage system with a weight of 872.2 kg. With these sections the vertical displacements of joints 5, 6, 7 and 8 are 17.8 mm, and maximum strength ratio is 0.97 for member in group 1. This result indicates that strength constraints are more dominant from displacement in the design problem. Further use of particle swarm method with more than 10000 iterations produces the same combination. Accordingly, as compared to the solution of the standard algorithm (CPSO), which is 890.58 kg, a much better final design weight of 872.23 kg is located by the improved PSO method. The improved and classical PSO results are also compared to those of enhanced charged system search (Kaveh and Nikaeen 2013). Consequently the solution found in Table 2 represents the optimum solution for IPSO which corresponds to the grillage system with sections W460×74 selected for longitudinal members and W150×13.5 chosen for transverse members. ECSS approach found the minimum weight of the 14-member grillage as 847.2 kg, which is only 2.87% lighter than IPSO algorithm.

5.2 Simply supported grillage system with 40-members

Fig. 5 shows the geometry of a 40-member grillage which are collected in four different groups.

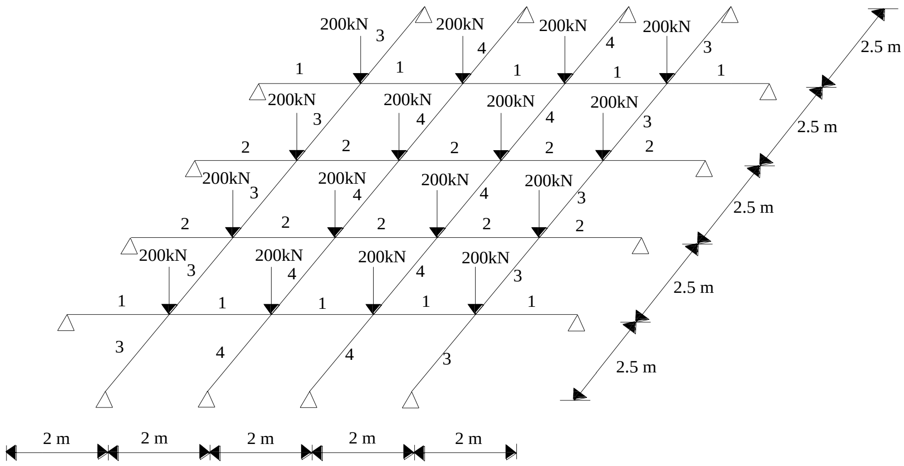


Fig. 5 Grillage system with 40-members

The outer and inner longitudinal beams are considered to be group 1 and 2 respectively while the outer and inner transverse beams are taken as group 3 and 4.

The vertical displacements of joints 6, 7, 10 and 11 are restricted to 25mm for this 40-member grillage system. Considering stochastic nature of the PSO method, 40-member grillage is separately solved with both improved and standard versions of particle swarm algorithms. The results are compared to those of the harmony search, charged system search and genetic algorithm to demonstrate the efficiency of the present approach. The size of the initial population and the maximum number of generations are kept the same in the harmony search method, charged system search and genetic algorithm. The parameterization of the improved method is conducted in line with the recommendations of the former studies (Shi and Eberhart 1998), and hence the following parameter value set is used in solving the problem: population size of NPT=20, maximum search number of N_s=10.000, control parameters of c_1 and c_2=1.0, inertia weight of w=0.08, the system parameter of V_{max}=2 and time step value of Δt=2. The minimum weight designs of the grillage obtained from the improved and standard algorithms of PSO and other three stochastic search techniques are given in Table 3 with sectional designations attained for all member groups used in the design problem.

Accordingly, as compared to the solution of the standard algorithm (CPSO), which is 7198.2 kg, a much better final design weight of 7138.04 kg is located by the improved particle swarm algorithm (IPSO). Furthermore, it is noticed that the maximum vertical displacement is 24.6 mm while the maximum value of the strength ratio is 0.99 which is almost upper bound. This clearly reveals both the strength and geometric constraints are dominant in IPSO algorithm. The design history curve for these solutions is plotted in Fig. 6, which displays the variation of the feasible best design obtained so far during the search versus the number of designs sampled. It is clear from this figure that the IPSO algorithm performs the best convergence rate toward the optimum solution. In an effort to compare the solution of IPSO algorithm with those of other metaheuristic techniques, these techniques yield the following design weights for the same problem: 7168.0 kg with charged system search (Kaveh and Talatahari 2010), 7198.2 kg with harmony search method (Saka and Erdal 2009), and 8087.91 kg with simple genetic algorithm (Saka 1998).

Table 3 The parameter data set of particle swarm algorithm

Search Method	Optimum W-Section		δ_{max} (mm)	Maximum Strength Ratio	Minimum Weight (kg)
	Group No	Designation			
IPSO	1 2 3 4	W150×13.5 W150×13.5 W760×147 W840×176	24.6	0.99	7138.04
CSS	1 2 3 4	W410×46.1 W460×52 W150×13.5 W1000×222	23.7	0.99	7168.05
CPSO	1 2 3 4	W410×46.1 W460×52 W200×15 W1000×222	23.2	0.99	7198.21
HSO	1 2 3 4	W410×46.1 W460×52 W200×15 W1000×222	23.2	0.99	7198.21
sGA	1 2 3 4	W150×13.5 W610×92 W410×46.1 W840×226	24.7	0.84	8087.91

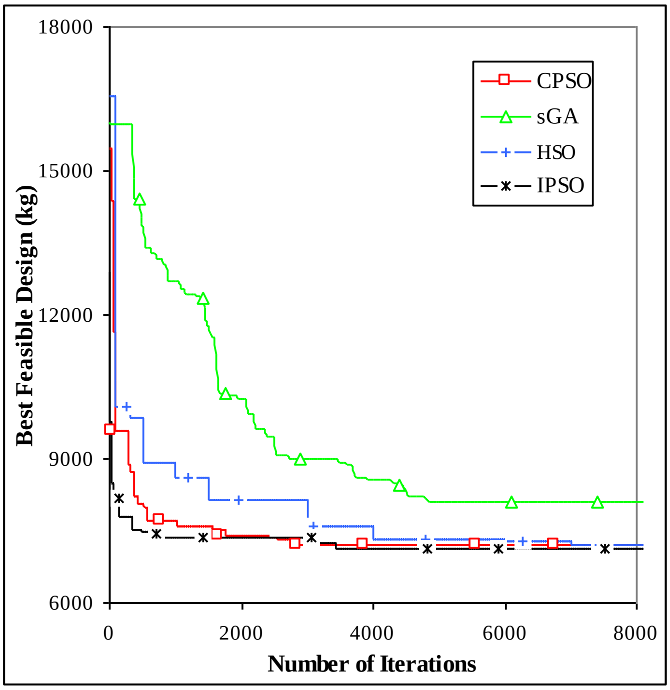


Fig. 6 Design history graph for 40-member grillage system

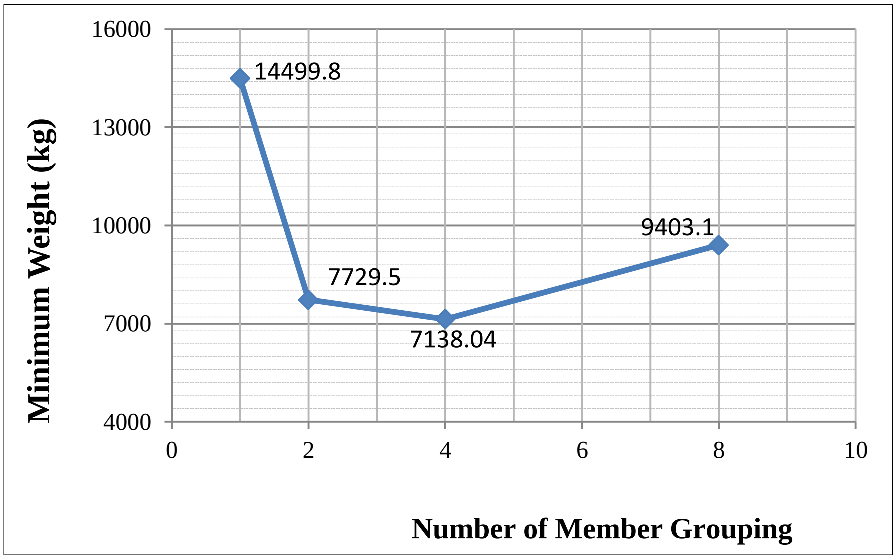


Fig. 7 Variation of weight versus member grouping using IPSO

Table 4 The Variation of best designs with member grouping for 40-member grillage

Number of Groups	Maximum Displacement Ratio	Maximum Strength Ratio	Minimum Weight (kg)
1	0.97	0.73	14499.8
2	0.98	0.81	7729.5
4	0.89	0.99	7138.1
8	0.86	1.00	9403.2

5.2.1 The effect of member grouping to minimum weight

Member grouping in the optimum design of grillage systems has a considerable effect on the minimum weight and it is more appropriate to consider parameters as a design variable if a better design is looked for. In order to demonstrate this effect, 40-member grillage system is designed several times by considering different member groupings. The variation of the minimum weight with the member grouping using IPSO method is plotted in Fig. 7. It is clear from this figure that when the optimum design problem is carried out considering only one member group, the minimum weight of the system turns out to be 14499.8kg. While the longitudinal members are considered as one group and the transverse ones are collected in another member group, the minimum weight drops down almost by half to 7729.5kg. Further reduction is possible if longitudinal members are collected in two groups and transverse members are considered as another group. It is apparent from Table 5 that consideration of four member groups represents the optimum grouping for 40-member grillage system with 7138.04kg. Finally, the number of groups is increased from 4 to 8 in both directions. It is interesting to notice that when all the members are allowed to have separate groups, the minimum weight of the grillage system also increases from 7138.04kg to 9403.1kg. Furthermore, it is also evident from Table 4 that for the larger number of groups, the strength constraints becomes dominant in the design problem, while for the cases where less number of

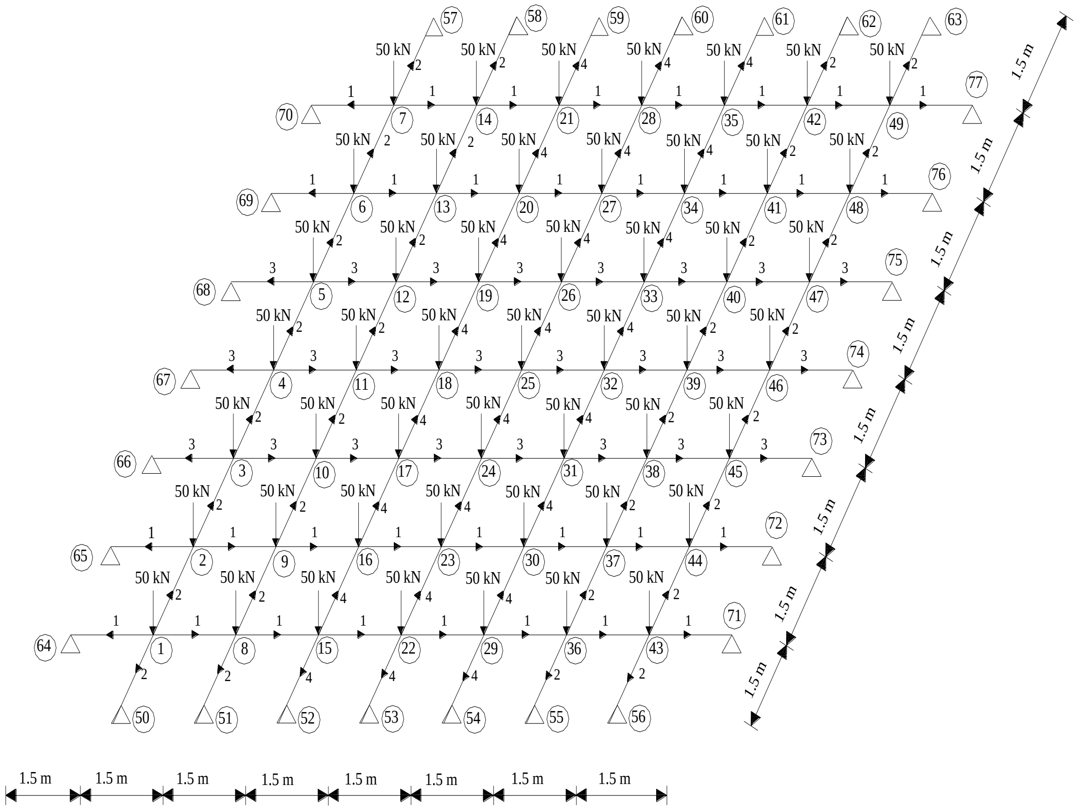


Fig. 8 Steel grillage system with 112-members

groups is considered, the displacement constraints become dominant.

5.2 Simply supported grillage system with 112-members

The grillage system shown in Fig. 8 has 112 members which are firstly collected in two different groups. The longitudinal beams are considered to be group 1 while the transverse beams are taken as group 2. The external loading distributed to the joints of the system as point loads is also shown in the same figure. The vertical displacements of joints 18, 19, 25 and 26 are restricted to 25mm for this system. The 112-member grillage is separately solved by improved and standard version of PSO. The results are compared to those of the HSS and CSS techniques to demonstrate the efficiency of the suggested algorithm. The result of the sensitivity analysis, carried out to determine the appropriate values of the PSO is given with more detail in Shi and Eberhart (1998).

The values of 10 for *NPT*, 1.0 for c_1 and c_2, 1 for w and 2 for V_{max} and Δt produce the least weight design for this grillage. Optimum design obtained with these parameters is given in Table 5.

IPSO algorithm selects W150×13.5 for the first group and W760×147 for the second group in the optimum design where the minimum weight of the system is 13465.0kg. Furthermore, the

Table 5 Best designs of 112-member grillage system for the case of two groups

Search Method	Optimum W-Section		δ_{max}	Maximum	Minimum
	Group No	Designation	(mm)	Strength Ratio	Weight (kg)
IPSO	1 2	W150×13.5 W760×147	23.9	0.49	13464.9
CSS	1 2	W150×13.5 W770×147	24.3	0.45	13519.2
CPSO	1 2	W150×13.5 W760×161	24.1	0.50	14664.1
HSO [36]	1 2	W200×15 W690×192	23.2	0.48	17363.4

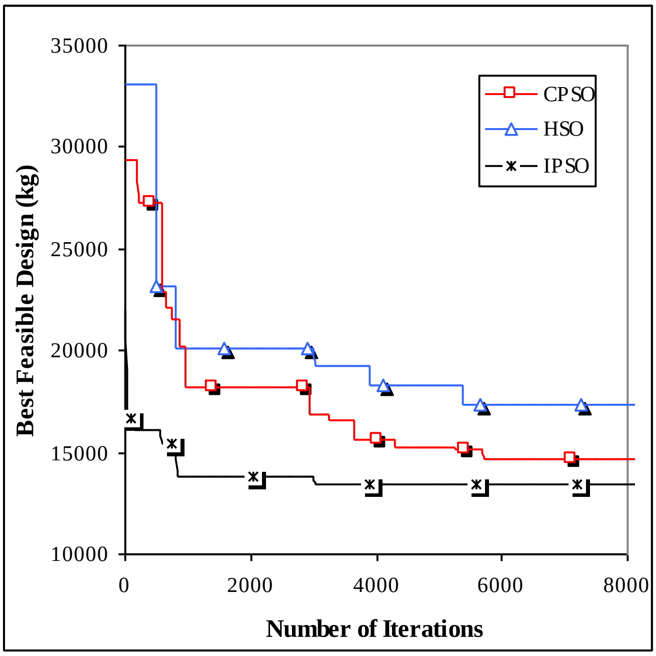


Fig. 9 Design history graph for 112-member grillage with two groups

maximum vertical displacement is 23.9 mm while the maximum value of the strength ratio is 0.49 for IPSO method. This clearly reveals the fact that the displacement constraints are dominant for this design problem. In the case where the beams of the grillage system are collected in two groups these produced optimum design weight 8.9% lighter than a design weight of 14664.1kg obtained with standard PSO algorithm. These designs are shown in Table 5 with sectional designations attained for two groups used in the problem. Fig. 9 shows the design history graph obtained for these two solutions. The attempts to optimize this grillage with other metaheuristic techniques yield higher final design weights of 13519.0 kg with charged system search and 17363.4 kg with harmony search algorithm methods, which are also tabulated in Table 5 for comparison purposes.

Later, the members of the 112-member grillage are collected in four groups to get a better design for the system. Both IPSO and CPSO algorithms are performed independently using the same parameter value set employed in the previous part. IPSO algorithm exhibited a rapid and

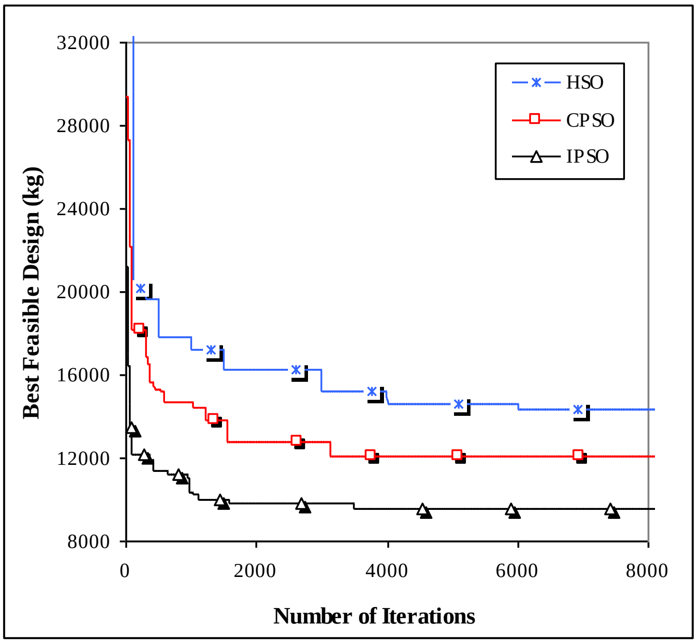


Fig. 9 Design history graph for 112-member grillage with four groups

Table 6 Best designs of 112-member grillage system for the case of four groups

Search Method	Optimum W-Section		δ_{max} (mm)	Maximum Strength Ratio	Minimum Weight (kg)
	Group No	Designation			
IPSO	1	W310×23.8	24.5	0.99	9980.2
	2	W250×17.9			
	3	W310×23.8W			
	4	920×201			
CSS	1	W150×13.5	24.4	0.75	11548.0
	2	W840×176			
	3	W150×13.5			
	4	W300×21			
CPSO	1	W250×80	25.0	0.69	12057.3
	2	W250×32.7			
	3	W360×79			
	4	W840×193			
HSO	1	W200×15	22.8	0.99	14362.0
	2	W200×15			
	3	W460×192			
	4	W610×307			

linear convergence towards the optimum in the early stages of the optimization process and produced lightest design where the minimum weight of the system is 9980.2 kg, which is 20.8% lighter than the same system with two groups. CSS approach found the minimum weight of the 112-member grillage as 11548.0 kg, which is 15.7% heavier than the previous one. The other minimum weights obtained by CPSO and HS methods are 20.8% and 43.9% heavier, respectively than the minimum weight achieved by IPSO. In this design example HS method has attained the heaviest design. The design history curve of 112-member grillage collected in four groups for PSO and HS techniques is plotted in Fig. 10. The optimum designs obtained by all methods for this case are tabulated in Table 6 with section designations attained for each member group, and are considered to be the optimum solution of the problem reached in the present study.

6. Conclusions

An improved version of particle swarm based minimum weight design algorithm developed produces satisfactory results for the optimum design problem of grillage systems where the design constraints are implemented as in LRFD-AISC provisions. The algorithm of the presented method is mathematically quite simple but robust in finding the solutions of combinatorial optimization problems. Convergence problem encountered during the iteration process is overcome by employing an improved version of the suggested algorithm. The efficiency of the proposed algorithm is numerically tested with various design examples on size optimum design of grillage systems. The design history graphs generated for these problems using improved and standard PSO algorithms clearly evidence a significant performance improvement achieved with the former. Comparison of the optimum designs attained by improved particle swarm optimizer with other search techniques also clearly demonstrates that the improved version of the method outperforms the latter in the selected design examples. It produces lighter optimum designs and requires less number of structural analyses, which makes it computationally more efficient. However, it should be stated that performance of the algorithm depends on the selection of appropriate values for its parameter similar to the cases of other meta-heuristic techniques. Furthermore, it is not possible to generalize the robustness of the algorithm for all design problems just based upon its performance in the optimum design of few grillage systems considered in this study. It is necessary to carry out a more detailed comparative study before such a conclusion can be drawn.

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