Structural Engineering and Mechanics, *Vol. 47, No. 4 (2013) 495-511*
DOI: http://dx.doi.org/10.12989/sem.2013.47.4.495

Structural optimization with teaching-learning-based optimization algorithm

Tayfun Dede[*1] and Yusuf Ayvaz[2a]

[1]*Department of Civil Engineering, Karadeniz Technical University, 61080, Trabzon, Turkey*
[2]*Department of Civil Engineering,Yıldız Technical University, 34220, Esenler, İstanbul, Turkey*

(Received December 12, 2012, Revised July 31, 2013, Accepted August 7, 2013)

Abstract. In this paper, a new efficient optimization algorithm called Teaching–Learning-Based Optimization (TLBO) is used for the least weight design of trusses with continuous design variables. The TLBO algorithm is based on the effect of the influence of a teacher on the output of learners in a class. Several truss structures are analyzed to show the efficiency of the TLBO algorithm and the results are compared with those reported in the literature. It is concluded that the TLBO algorithm presented in this study can be effectively used in the weight minimization of truss structures.

Keywords: teaching-learning-based optimization; truss structure; weight minimization

1. Introduction

Until now, a lot of optimization algorithms have been developed to find minimum weight or volume of the structural system for many engineering problems. All of these methods purpose to obtain an optimum set of discrete or continuous design variables with no violation of certain constraints. Among these optimization algorithms genetic algorithm (GA), ant colony optimization (ACO), particle swarm optimization (PSO), harmony search (HS) and simulated annealing (SA) are the most popular optimization algorithms.

GA which is a search strategy that models mechanism of genetic evolution, was first described by John Holland in the 1960s and further developed by Holland and his students and colleagues at the University of Michigan in the 1960s and 1970s (Holland 1975, Goldberg 1989). Minimum weight design of truss structures (Rajeev and Krishnamoorthy 1997, Krishnamoorthy *et al*. 2002, Dede *et al.* 2011, Bekiroğlu *et al*. 2009, Toğan and Daloğlu 2008), optimization of concrete structures (Lee and Ahn 2003, Leps and Sejnoha 2003, Catallo 2004, Castilho 2006) and design of frame structures with GA (Pezeshk 2000, Cao 1996) are well documented in the literature. Ant colony optimization inspired by the foraging behavior exhibited by real ant colonies was proposed by Dorigo (1991) for the solution of hard combinatorial optimization problems. Yaseen and Al-Slamy (2008) used ACO to solve the traveling salesman problem, Capriles *et al.* (2006) and Christodoulou (2005) presented their studies on design of truss structures by using ACO. The

∗Corresponding author, Ph.D., E-mail: tayfundede@gmail.com
[a]Ph.D., E-mail: yusufayvaz@gmail.com

http://www.techno-press.org/?journal=sem&subpage=8 ISSN: 1225-4568 (Print), 1598-6217 (Online)

particle swarm optimization which is based on the behavior of animals was first developed by Kennedy and Eberhart (1995). This algorithm simulates a simplified social model. Fish schooling, physical movement of birds to avoid predators, and seeking food of insect are example of social sharing of information of animals. By using PSO algorithm, design of truss structures are presented by Oliveira and Gomes (2010), Schutte and Groenwold (2003), Gomes (2011), Kaveh and Talatahari (2008) in the literature.

Harmony search developed by Geem *et al.* (2001) as an optimization algorithm is based on natural musical performance processes that take place when a musician searches for a better state of harmony (Kaveh and Abadi 2011). Optimization of truss structures (Lee *et al.* 2005, Lee and Geem 2004), optimization of reinforced cantilever retaining walls (Kaveh and Abadi 2011), and design of steel frames (Değertekin 2008) are well documented in the literature for application of HS. Simulated annealing was produced independently by Kirkpatrick *et al.* (1983) and Cerny (1985). This algorithm simulates the annealing process of metals to solve optimization problems. Design of truss structures (Hasançebi and Erbatur 2002, Sesok *et al*. 2010) and grillages (Lamberti 2008) are the applications of the SA for optimization problems. The extensive review papers on the optimization algorithms mentioned above has been presented by Lamberti (2008) and Saka (2007).

This paper proposes a new structural optimization algorithm, Teaching-learning-based optimization (TLBO) which has been recently developed by Rao *et al.* (2011) for the continuous optimization of truss structures. Several truss examples under multiple loading conditions with stress and displacement constraints are presented to demonstrate the effectiveness of the new method.

2. The TLBO Algorithm

Teaching-learning-based optimization was firstly used for constrained mechanical design optimization problems by Rao *et al.* (2011). They obtained better results of their studies as compared to the results given in the literature. The general flow chart of TLBO is given in Fig. 1. Like the other optimization algorithm, TLBO also uses a randomly generated initial population. This population consists of an even number of students which are any solutions in TLBO. These students consist of a number of design variables (X_i).

A new population is obtained as a result of two phase called as teacher phase and student phase in TLBO algorithm. In the teacher phase, the student having minimum objective function (*f*) value is assigned as a teacher. The other students in the current population are modified as neighborhood of the teacher. This modification is carried out by using the following equations.

$$student_i = \left[\mathrm{X}_{\mathrm{i,1}} \quad \mathrm{X}_{\mathrm{i,2}} \quad ... \quad \mathrm{X}_{\mathrm{i,Dn}} \right], \qquad i = 1, 2, ..., Pn \tag{2.1}$$

$$mean = \left[\mathrm{mean}(\mathrm{X}_1) \quad \mathrm{mean}(\mathrm{X}_2) \quad ... \quad (\mathrm{X}_{\mathrm{Dn}}) \right] \tag{2.2}$$

$$student_{new\_i} = student_i + r.*(teacher - TF * mean) \tag{2.3}$$

Where, *Dn* is number of design variables, *Pn* is size of population, *r* is a vector created randomly in the range [0,1] and TF is the teaching factor and TF can be either 1 or 2. It should be noted that the size of r must be equal to size of the student for the scalar multiplication given in Eq. (2.3). If the objective function of modified student is greater than the objective function of old student, the new student is not taken into account. The teaching phase is carried out by the hope that the level of students will be updated to the level of teacher. In student phase, all modified students are compared with each other to increase their knowledge. Implementation of this comparison is as follows.

$$
\begin{array}{l}
for \quad i = 1 : Pn \\
randomly\ select\ student_j\ ,\ i \neq j \\
if\ \ f\,(student_i) < f\,(student_j) \\
difference = student_i - student_j \\
else \\
difference = student_j - student_i \\
end\ if \\
student_{new\_i} = student_i + r.*\,difference \\
end\ \ for
\end{array}
$$

As noted in the teacher phase, the new student obtained from student phase is not taken into account if its objective function is not better. At the end of the last iteration, the student whose objective function is minimum in the population is the best solution of optimization problem. Extensive details about the TLBO algorithm and its implementation had been presented by Rao *et al.* (2011). Also, the papers given below are related to the TLBO algorithm: Rao and Patel (2012a, b), Rao *et al*. (2012a, b), Rao and Kalyankar (2012a, b), Waghmare (2012), Rao and Kalyankar (2013a, b), Rao and Patel (2013a, b, c), Pawar and Rao (2013), Toğan (2013).

3. Objective function of problem

One of the most important factors in the structural design is the total structural weight. In this study, truss structures are designed to be the minimum weight. For this aim, the objective function for the truss structures is formulated as

$$min W = \sum_{k=1}^{ng} A_k \sum_{i=1}^{nm} \rho_i L_i \tag{3.1}$$

Where, W is the objective function which is also the minimum weight of the structure, ρ is the density of materials, A is the cross-section areas of the each member, nm is the number of member of the truss structures and ng is the number of group. For this problem, displacement, stress and stability constraints are given as

$$\delta_i \leq \delta_u\,, \qquad c_i = \frac{\delta_i}{\delta_u} \;\rightarrow\; i = 1, 2, ..., p \tag{3.2}$$

$$\sigma_j \leq \sigma_u\,, \qquad c_j = \frac{\sigma_j}{\sigma_u} \;\rightarrow\; j = 1, 2, ..., nm \tag{3.3}$$

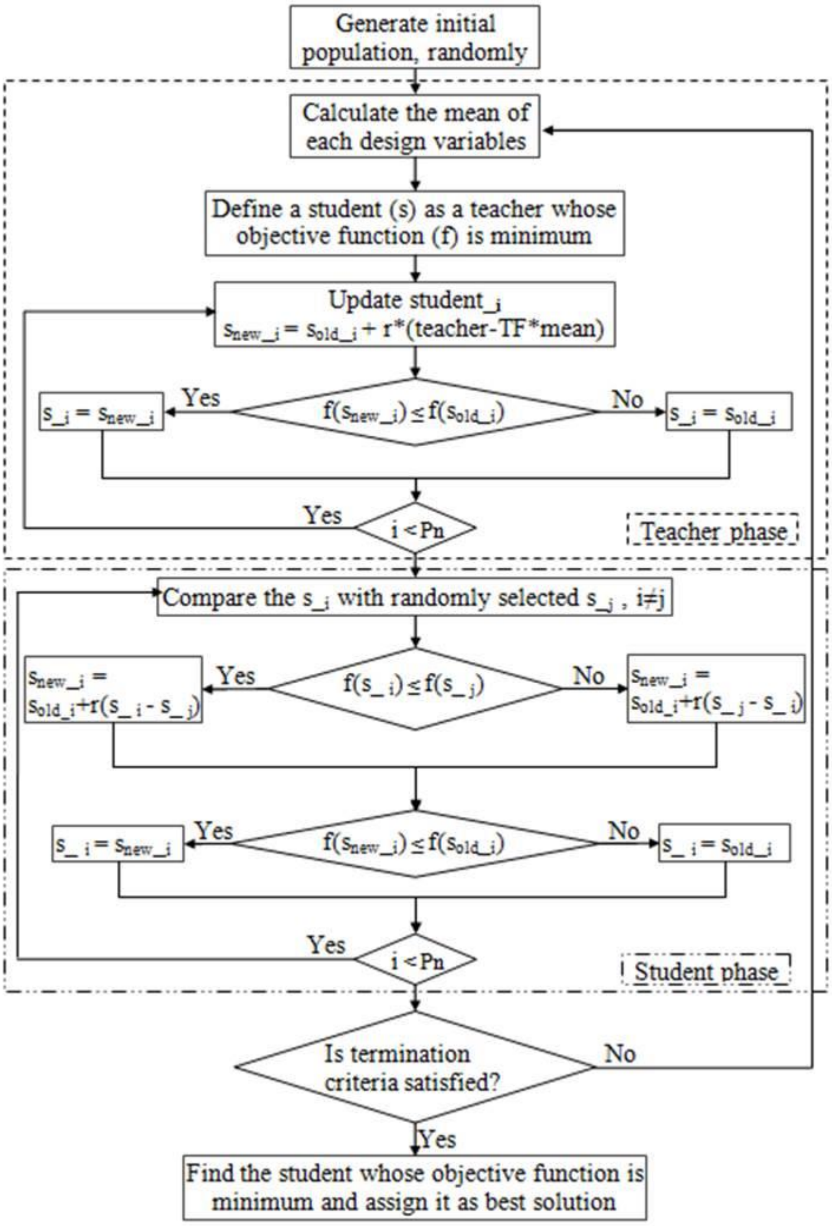


Fig. 1 Flow chart for TLBO

Where, c is the value of each constraints, δ_i and δ_u are the calculated and allowable displacement for point i, respectively. p is the number of points with restricted displacements. σ_j and σ_u are the calculated and allowable stress for member j, respectively. nm is the number of members in truss structure.

The objective function must be changed as independent of constraints. For this aim, a penalty function calculating value of violation of constraints is determined. By means of this function, the

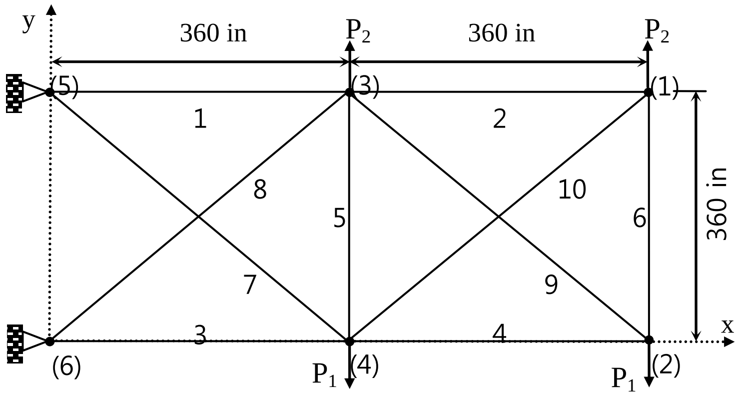


Fig. 2 10-bar truss system

objective function is changed to a function including constraints. Penalty function is given as

$$C = \sum_{i=1}^{m} c_i \tag{3.4}$$

Where, m is the number of the constraints. Objective function is changed to penalized objective function by adding penalty function to it. The penalized objective function, Φ, can be formulated as

$$\Phi(x) = W(x)[1 + P.C] \tag{3.5}$$

Where, P is a positive constant which is a variable for each problem. This constant can be determined by the user to take into account of the constraints.

4. Numerical examples

In this study, four design examples are considered to demonstrate the application of the TLBO algorithm. These are 10-bar plane truss structure, 25-bar space truss structure, 72-bar space truss structure, and 200-bar plane truss structure.

4.1 Ten bar plane truss structure

Ten bar truss structure considered and shown in Fig. 2 was previously solved by several researchers such as Lee and Geem (2004), Sönmez (2011), Renwei and Peng (1986), Li *et al.* (2007), Khan *et al.* (1979), Ali *et al.* (2010), Schmit and Farshi (1974), Venkayya (1971), Dobbs and Nelson (1976), Rizzi (1976).

Modulus of elasticity is 10000 ksi and density of material 0.1 lb/in3. The allowable stress for all members is set to ±25 ksi and allowable displacement for all free nodes is set to 2 in for the x and y directions. The minimum cross sectional area is set to 0.1 in2. This structure is analyzed for two independent cases. P1=100 kips and P2=0 for the case 1 and P1=150 kips and P2=50 for the case 2.

For case 1, the best solution vector is [30.5253, 0.1000, 23.2070, 15.1940, 0.1000, 0.5463, 7.4568, 21.0340, 21.5480, 0.1000] in2 and the minimum weight of structure is obtained as 5060.8688 lb. Convergence history of the minimum weight for 10-bar plane truss for case I and case II are given in Fig. 3 and Fig. 4, respectively. As seen from Fig. 3, the minimum weight of structure was firstly obtained as 5060.9744 lb in the 788th iteration. After this iteration, the algorithm was continued to find better solution to compare the solutions given in literature. In the same way, the best solution vector is [23.3432, 0.1000, 25.3021, 14.4234, 0.1000, 1.9703, 12.3763, 12.8504, 20.4082, 0.1001] in2 and the minimum weight is 4677.0462 lb, for case II.

The results obtained from this study and the results given in literature are given in Table 1 and in Table 2 for the case 1 and case 2, respectively. As seen from these tables, the solutions obtained from this study with no violations are better than the results given in literature.

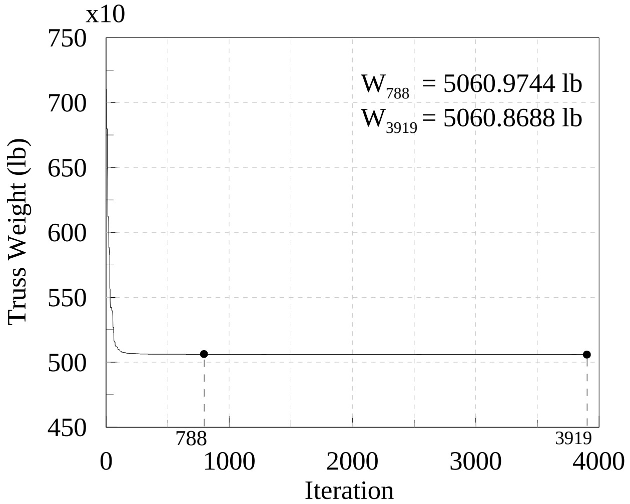


Fig. 3 Convergence history of the minimum weight for 10-bar plane truss for case I

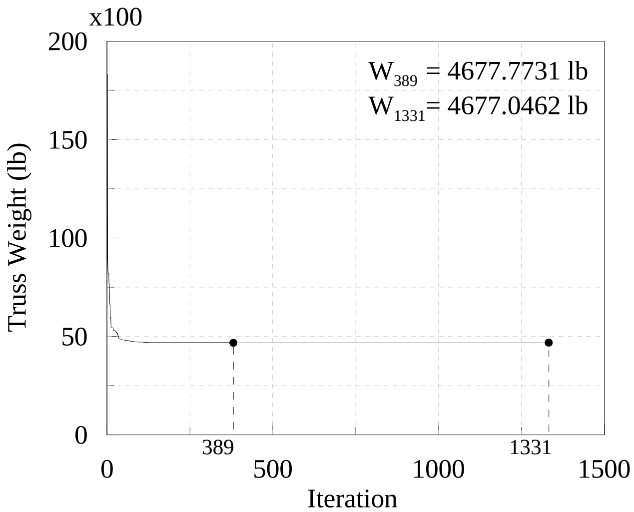


Fig. 4 Convergence history of the minimum weight for 10-bar plane truss for case II

Table 1 Optimal design comparison for the 10-bar planar truss (Case 1)

Design Variables (area)		Optimal cross-sectional areas (in^2)											
		Renwei & Peng(1986)	Li *et al.* (2007)	Lee & Geem (2008)	Sönmez (2011)	Khan *et al.* (1979)	Ali *et al.* (2010) Classic ABC	Ali *et al.* (2010) MABC	Schmit & Farsh (1974)	Venkayya (1971)	Dobbs & Nelson (1976)	Rizzi (1976)	Present Study Using TLBO
1	A1	30.5900	30.7040	30.1500	30.5480	30.9800	34.3057	30.6573	33.4300	30.4200	30.5000	30.7300	30.5253
2	A2	0.1000	0.1000	0.1020	0.1000	0.1000	0.1000	0.1000	0.1000	0.1280	0.1000	0.1000	0.1000
3	A3	23.2700	23.1670	22.7100	23.1800	24.1700	20.6728	23.0429	24.2600	23.4100	23.2900	23.9300	23.2070
4	A4	15.1900	15.1830	15.2700	15.2180	14.8100	14.5074	15.2821	14.2600	14.9100	15.4300	14.7300	15.1940
5	A5	0.1000	0.1000	0.1020	0.1000	0.1000	0.1000	0.1000	0.1000	0.1010	0.1000	0.1000	0.1000
6	A6	0.4600	0.5510	0.5440	0.5510	0.4060	0.6609	0.5626	0.1000	0.1010	0.2100	0.1000	0.5463
7	A7	7.5000	7.4600	7.5410	7.4630	7.5470	7.8696	7.4721	8.3880	8.6960	7.6490	8.5420	7.4568
8	A8	21.0700	20.9780	21.5600	21.0580	21.0500	20.3461	21.0084	20.7400	21.0800	20.9800	20.9500	21.0340
9	A9	21.4800	21.5080	21.4500	21.5010	20.9400	22.0232	21.5094	19.6900	21.0800	21.8200	21.8400	21.5480
10	A10	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000	0.1860	0.1000	0.1000	0.1000
W (lb)		5062.17	5060.92	5057.88	5060.88	5066.98	5095.33	5060.97	5089.00	5084.90	5080.00	5076.66	5060.8688
C		None	None	0.000907	None	None	None	None	None	None	None	None	None

Table 2 Optimal design comparison for the 10-bar planar truss (Case 2)

Design Variables (area)		Optimal cross-sectional areas (in^2)										
		Dobbs & Nelson (1976)	Li *et al.* (2007)	Lee & Geem (2008)	Sönmez (2011)	Ali *et al.* (2010) Classic ABC	Ali *et al.* (2010) MABC	Schmit & Farshi (1974)	Venkayya (1971)	Rizzi (1976)	Khan *et al.* (1979)	Present Study using TLBO
1	A1	25.8100	23.3530	23.2500	23.4692	24.8143	23.6383	24.2900	25.1900	23.5300	24.7200	23.3432
2	A2	0.1000	0.1000	0.1020	0.1005	0.1000	0.1000	0.1000	0.3630	0.1000	0.1000	0.1000
3	A3	27.2300	25.5020	25.7300	25.2393	26.0480	25.3237	23.3500	25.4200	25.2900	26.5400	25.3021
4	A4	16.6500	14.2500	14.5100	14.3540	14.8772	14.4108	13.6600	14.3300	14.3700	13.2200	14.4234
5	A5	0.1000	0.1000	0.1000	0.1001	0.1000	0.1001	0.1000	0.4170	0.1000	0.1080	0.1000
6	A6	2.0240	1.9720	1.9770	1.9701	2.0055	1.9707	1.9690	3.1440	1.9700	4.8350	1.9703
7	A7	12.7800	12.3630	12.2100	12.4128	12.4467	12.3781	12.6700	12.0800	12.3900	12.6600	12.3763
8	A8	14.2200	12.8940	12.6100	12.8925	12.6835	12.7739	12.5400	14.6100	12.8300	13.7800	12.8504
9	A9	22.1400	20.3560	20.3600	20.3343	18.8669	20.2678	21.9700	20.2600	20.3300	18.4400	20.4082
10	A10	0.1000	0.1010	0.1000	0.1000	0.1000	0.1000	0.1000	0.5130	0.1000	0.1000	0.1001
W (lb)		5059.70	4677.29	4668.81	4677.07	4691.07	4677.06	4691.84	4895.60	4676.92	4792.52	4677.0463
C		None	$0.25*10^{-5}$	$3.561*10^{-3}$	None	None	None	$1.38*10^{-5}$	$5.269*10^{-5}$	$6.34*10^{-5}$	0.0026	None

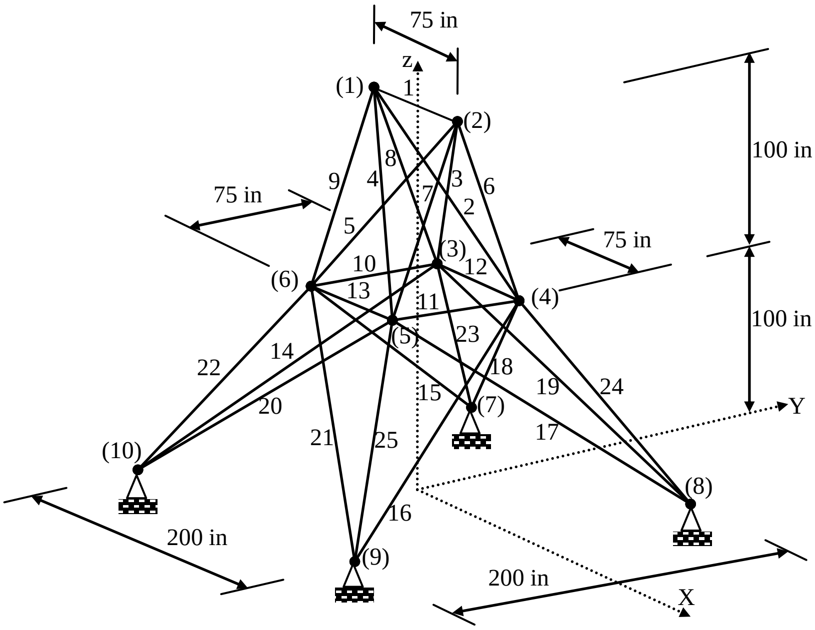


Fig. 5 25-bar space truss structure

Table 3 Multiple loading (kip) conditions for the 25-bar space truss

Case	Node	*Fx*	*Fy*	*Fz*
1	1	1.0	10.0	-5.0
	2	0.0	10.0	-5.0
	3	0.5	0.0	0.0
	6	0.5	0.0	0.0
2	1	0.0	20.0	-5.0
	2	0.0	-20.0	-5.0

4.2 25-Bar space truss structure

Configuration of 25-bar space truss structure is given in Fig. 5. This system is designed by several researchers. Some of these researchers are Cao (1996), Lee and Geem (2004), Lamberti (2008), Sönmez (2011), Li *et al*. (2001), Schmit and Farshi (1974), Venkayya (1971), Adeli and Kamal (1986), Saka (1990), and Champ (2007). This structure is subjected to multiple loading conditions given in Table 3.

Modulus of elasticity is 10000 ksi and density of material 0.1 lb/in3. The allowable stresses for all members are given in Table 4 and allowable displacement is 0.35 in at nodes 1 and 2. The minimum cross sectional area is 0.01 in2. Members of this structure are categorized into 8 groups. This grouping can be seen from the first column of the Table 5.

The best solution vector obtained in this study is [0.0100, 1.9870, 2.9924, 0.0100, 0.0100, 0.6887, 1.6771, 2.6564] under the multiple loading conditions. Convergence history of the minimum weight for 25-bar space truss under multiple load cases is given in Fig. 6.

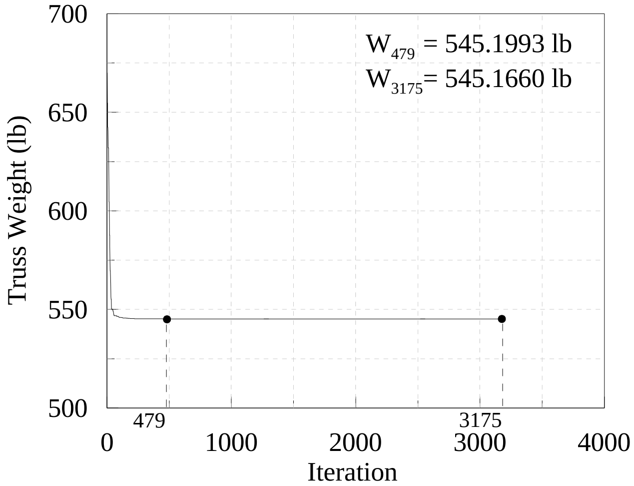


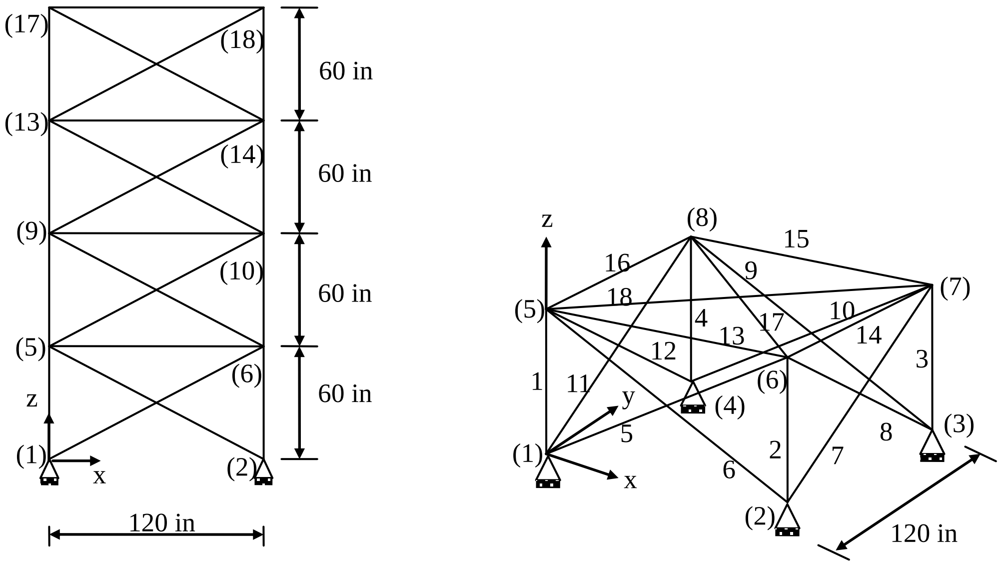
Fig. 6 Convergence history of the minimum weight for 25-bar space truss under multiple load cases

Table 4 Allowable stresses (ksi) for the 25-bar space truss

Members	Compression	Tension
A1	35.092	40
A2~A5	11.59	40
A6~A9	17.305	40
A10~A11	35.092	40
A12~A13	35.092	40
A14~A17	6.759	40
A18~A21	6.959	40
A22~A25	11.082	40

Table 5 Optimal design comparison for the 25-bar space truss under multiple load case

Design Variables (area)	Optimal cross-sectional areas (in^2)										
	Lee & Geem (2004)	Li *et al.* (2007)	Lamberti (2008)	Schmit & Farshi (1974)	Venkayya (1971)	Adeli & Kamal (1986)	Saka (1990)	Cao (1996)	Champ (2007)		Present Study using TLBO
									Phase 1	Phase 2	
A1	0.0470	0.0100	0.0100	0.0100	0.0280	0.0100	0.0100	0.0100	0.0100	0.0100	0.0100
A2~A5	2.0220	1.9700	1.9870	1.9640	1.9640	1.9860	2.0850	2.0119	2.0920	2.0920	1.9870
A6~A9	2.9500	3.0160	2.9940	3.0330	3.0810	2.9610	2.9880	2.9493	2.9640	2.9640	2.9924
A10~A11	0.0100	0.0100	0.0100	0.0100	0.0100	0.0100	0.0100	0.0100	0.0100	0.0100	0.0100
A12~A13	0.0140	0.0100	0.0100	0.0100	0.0100	0.0100	0.0100	0.0295	0.0100	0.0100	0.0100
A14~A17	0.6880	0.6940	0.6940	0.6700	0.6930	0.8060	0.6960	0.6838	0.6890	0.6890	0.6887
A18~A21	1.6570	1.6810	1.6810	1.6800	1.6780	1.6800	1.6700	1.6798	1.6010	1.6010	1.6771
A22~A25	2.6630	2.6430	2.6430	2.6700	2.6270	2.5300	2.5920	2.6759	2.6860	2.6860	2.6564
W (lb)	544.38	545.19	545.16	545.22	545.49	545.66	545.23	545.80	545.48	545.38	545.166
C	0.0122	None	0.0012	None	None	None	None	$5.269*10^{-5}$	-	-	None



(a) profile view (b) element and node numbering system

Fig. 7 72-bar truss structure (a) profile view, (b) element and node numbering system

Table 6 Multiple loading (kip) conditions for the 72-bar space truss

Case	Node	*Fx*	*Fy*	*Fz*
1	17	0.0	0.0	-5.0
	18	0.0	0.0	-5.0
	19	0.0	0.0	-5.0
	20	0.0	0.0	-5.0
2	17	5.0	5.0	-5.0

As seen from this figure, the minimum weight is obtained as 545.1993 lb in the 479th iteration and 545.1660 lb after the 3175 iterations. These results are compared with the results given in the literature in Table 5. As seen from this table, the solution with no violations obtained from this study is better than the others.

4.3 72-Bar space truss structure

Configuration of 72-bar space truss structure is given in Fig. 7. In this figure, the numbers in the parenthesis show the node number and the others show element numbers. This structure was previously solved by several researchers such as Cao (1996), Schmit and Farshi (1971), Venkayya (1971), Champ (2007), Camp and Bichon (2004), Chao *et al.* (1984), Erbatur *et al.* (2000), Gellatly (1971), Renwei (1987).

Table 6 shows multiple loading conditions for this structure. Modulus of elasticity is 10000 ksi and density of material 0.1 lb/in^3. The allowable stress for all members is ±25 ksi and allowable displacement is ±0.25 in at nodes 17, 18, 19, and 20. The minimum cross sectional area is 0.1 in2.

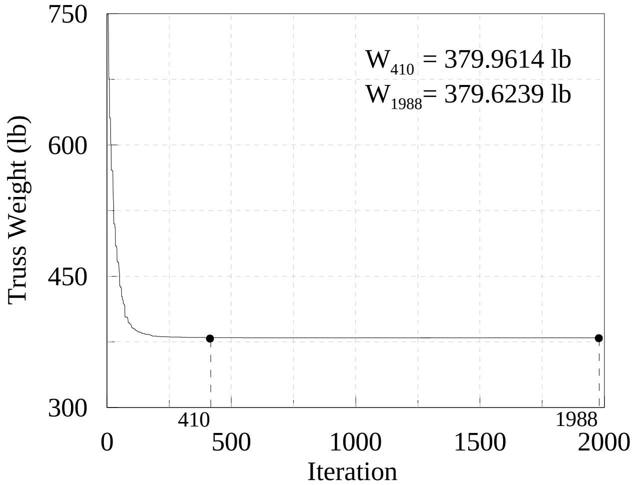


Fig. 8 Convergence history of the minimum weight for 72-bar space truss under multiple load cases

Table 7 Optimal design comparison for the 72-bar space truss under multiple load case

Design Variables	Optimal cross-sectional areas (in^2)										
	Cao (1996)	Camp & Bichon (2004)	Champ (2007) Phase 1	Champ (2007) Phase 2	Chao *et al.* (1984)	Gellatly (1971)	Renwei (1987)	Schmit & Farshi (1974)	Erbatur *et al.* (2000) GAOS 1	Erbatur *et al.* (2000) GAOS 2	Present Study using TLBO
A1~A4	1.8562	1.9480	1.9004	1.8577	1.8321	0.1492	0.1641	0.1585	0.1550	0.1610	1.8788
A5~A12	0.4933	0.5080	0.5252	0.5059	0.5119	0.7733	0.5552	0.5936	0.5350	0.5440	0.5155
A13~A16	0.1000	0.1010	0.1000	0.1000	0.1000	0.4534	0.4187	0.3414	0.4800	0.3790	0.1000
A17~A18	0.1000	0.1020	0.1000	0.1000	0.1000	0.3417	0.5758	0.6076	0.5200	0.5210	0.1000
A19~A22	1.2830	1.3030	1.3134	1.2476	1.2521	0.5521	0.5327	0.2643	0.4600	0.5350	1.2753
A23~A30	0.5028	0.5110	0.4801	0.5269	0.5241	0.6084	0.5256	0.5480	0.5300	0.5350	0.5118
A31~A34	0.1000	0.1010	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000	0.1200	0.1030	0.1000
A35~A36	0.1000	0.1000	0.1000	0.1012	0.1000	0.1000	0.1000	0.1509	0.1650	0.1110	0.1000
A37~A40	0.5177	0.5610	0.5254	0.5209	0.5127	1.0235	1.2893	1.1067	1.1550	1.3100	0.5168
A41~A48	0.5227	0.4920	0.5267	0.5172	0.5289	0.5421	0.5201	0.5793	0.5850	0.4980	0.5167
A49~A52	0.1000	0.1000	0.1016	0.1004	0.1000	0.1000	0.1000	0.1000	0.1000	0.1100	0.1000
A53~A54	0.1049	0.1070	0.1253	0.1005	0.1000	0.1000	0.1000	0.1000	0.1000	0.1030	0.1000
A55~A58	0.1557	0.1560	0.1558	0.1565	0.1565	1.4640	1.9173	2.0784	1.7550	1.9100	0.1566
A59~A66	0.5501	0.5500	0.5456	0.5507	0.5493	0.5207	0.5207	0.5034	0.5050	0.5250	0.5462
A67~A70	0.3981	0.3900	0.4314	0.3922	0.4061	0.1000	0.1000	0.1000	0.1050	0.1220	0.4085
A71~A72	0.6749	0.5920	0.5231	0.5922	0.5550	0.1000	0.1000	0.1000	0.1550	0.1030	0.5667
W (lb)	380.32	380.24	380.46	379.85	379.62	395.970	379.66	388.63	385.76	383.12	379.62

Members of this structure are categorized into 16 groups. This grouping can be seen from the first column of Table 7.

Under the multiple loading conditions, the best solution vector obtained in this study is [1.8788, 0.5155, 0.1000, 0.1000, 1.2753, 0.5118, 0.1000, 0.1000, 0.5168, 0.5167, 0.1000, 0.1000, 0.1566, 0.5462, 0.4085, 0.5667]. Convergence history of the minimum weight for this space truss under multiple load cases is given in Fig. 8.

As seen from this figure, the minimum weight is obtained as 379.9614 lb in the 410th iteration

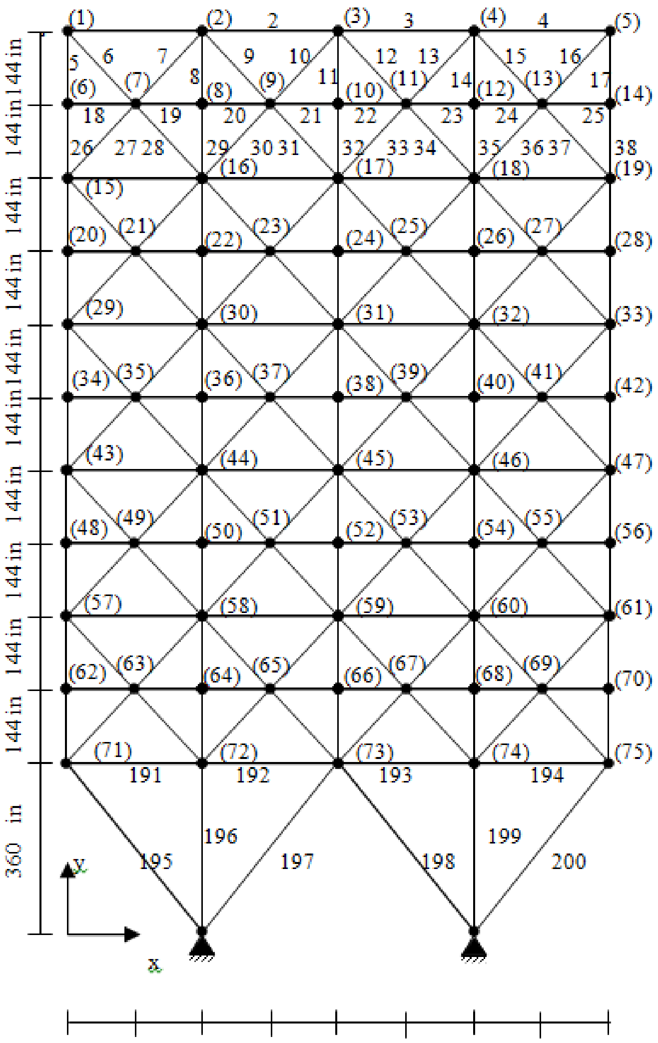


Fig. 9 Configuration of 200-bar plane truss structure

and 379.6239 lb with no violations after the 1988 iterations. These results are compared with the results given in the literature in Table 7. As seen from this table, Chao *et al*. (1984) gives the weight of this structure as 379.62 lb, but their cross sectional areas violates the constraints approximately 5.2404×10−4.

4.4 200-Bar plane truss structure

Configuration of 200-bar space truss structure is given in Fig. 9. All element numbers are not given for the clarity of figure. This truss structure is designed by using different types of constraints under different number of design variables in the literature.

In this study, the members of this structure are categorized into 29 groups as in Toğan and Daloğlu (2008), Lee and Geem (2004), Lamberti (2008), Xu (2010), and Coello (2000). The

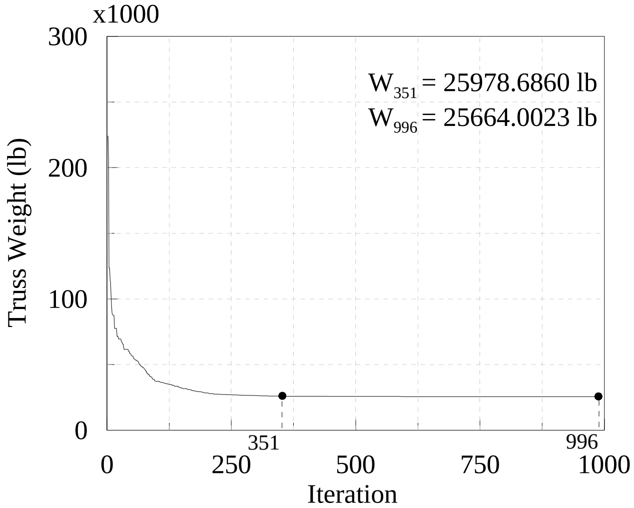


Fig. 10 Convergence history of the minimum weight for 200-bar truss under multiple load cases

details of grouping is given in Table 8. Material properties and constraints used in this study are as follows: Modulus of elasticity is 30000 ksi and density of material is 0.283 lb/in 3. The allowable stress for all members is ±10 ksi and there is no limitation for displacement of free nodes. This structure is subjected to 3 different load conditions and they are as follows:

Load case 1: 1 kip acting in the positive x direction at nodes 1, 6, 15, 20, 29, 34, 43, 48, 57, 62, and 71

Load case 2: 10 kips acting in the negative y direction at nodes 1, 2, 3, 4, 5, 6, 8, 10, 12, 14, 15, 16, 17, 18, 19, 20, 22, 24, 26, 28, 29, 30, 31, 32, 33, 34, 36, 38, 40, 42, 43, 44, 45, 46, 47, 48, 50, 52, 54, 56, 57, 58, 59, 60, 61, 62, 64, 66, 68, 70, 71,72, 73,74, and 75

Load case 3: cases 1 and 2 are combined.

Fig. 10 shows the Convergence history of the minimum weight for 200-bar plane truss under multiple load conditions.

As seen from this figure, the minimum weight is obtained as 25978.6860 lb in the 351th iteration and 25664.0023 lb after the 996 iterations with no violations. These results are compared with the results given in literature in Table 8. As seen from this table, the solution with no violations obtained from this study is better than the other results given in literature.

5. Conclusions

A recently proposed new optimization algorithm called TLBO is implemented in this paper for the continuous optimization of truss structures. Like other nature-inspired algorithms, TLBO is also a population based method which uses a population of solutions to proceed to the global solution. For TLBO, the population is considered as a group of learners or a class of learners. The process of working of TLBO is divided into two parts. The first part consists of ‘Teacher Phase’ and the second part consists of ‘Learner Phase’. The ‘Teacher Phase’ means learning from the teacher and the ‘Learner Phase’ means learning through the interaction between learners.

Table 8 Optimal design comparison for the 200-bar planar truss under multiple load case

Design Variables	Group members	Optimal cross-sectional areas (in^2)						
		Coello (2000)	Lee & Geem (2004)	Xu (2010)		Lamberti (2008)	Toğan & Daloğlu (2008)	Present Study using TLBO
				Convent	Present			
1	1,2,3,4	-	0.1253	0.1260	0.2870	0.1468	0.347	0.113546
2	5,8,11,14,17	-	1.0157	1.3620	1.2020	0.9400	1.081	0.948427
3	19,20,21,22,23,24	-	0.1069	0.1140	0.1500	0.1000	0.100	0.107798
4	18,25,56,63,94,101,132,139,170,177	-	0.1096	0.1870	0.2260	0.1000	0.100	0.100009
5	26,29,32,35,38	-	1.9369	2.0120	2.3730	1.9400	2.142	1.934462
6	6,7,9,10,12,13,15,16,27,28,30,31,33	-	0.2686	0.3100	0.4050	0.2962	0.347	0.288872
7	34,36,37	-	0.1042	0.3320	0.1000	0.1000	0.100	0.211586
8	39,40,41,42	-	2.9731	3.4040	3.4200	3.1042	3.565	3.090253
9	43,46,49,52,55	-	0.1309	0.2590	0.1060	0.1000	0.347	0.103114
10	57,58,59,60,61,62	-	4.1831	5.0530	4.2150	4.1042	4.805	4.090254
11	64,67,70,73,76	-	0.3967	0.8490	0.7350	0.4034	0.440	0.450150
12	44,45,47,48,50,51,53,54,65,66,68,69	-	0.4416	0.4260	0.6530	0.1912	0.440	0.100707
13	71,72,74,75	-	5.1873	6.7610	7.3330	5.4284	5.952	5.479848
14	77,78,79,80	-	0.1912	0.1210	0.1140	0.1000	0.347	0.101144
15	81,84,87,90,93	-	6.2410	9.9610	8.2680	6.4284	6.572	6.479849
16	95,96,97,98,99,100	-	0.6994	0.9870	0.9930	0.5734	0.954	0.532949
17	102,105,108,111,114	-	0.1158	0.2020	0.4300	0.1327	0.347	0.132492
18	82,83,85,86,88,89,91,92,103,104,106	-	7.7643	9.6120	9.7820	7.9717	8.525	7.944450
19	107,109,110,112,113	-	0.1000	0.2420	0.1840	0.1000	0.100	0.100486
20	115,116,117,118	-	8.8279	11.7500	10.6600	8.9717	9.300	8.944437
21	119,122,125,128,131	-	0.6986	1.3000	1.2490	0.7049	0.954	0.701077
22	133,134,135,136,137,138	-	1.5563	3.9170	4.5240	0.4196	1.764	1.377693
23	140,143,146,149,152	-	10.9806	13.9100	13.7100	10.8636	13.300	11.239401
24	120,121,123,124,126,127,129,130,141	-	0.1317	0.4260	0.3980	0.1000	0.347	0.228718
25	142,144,145,147,148,150,151	-	12.1429	14.6900	14.6200	11.8606	13.300	12.239392
26	153,154,155,156	-	1.6373	3.6180	3.9120	1.0339	2.142	1.684935
27	157,160,163,166,169	-	5.0023	7.9690	7.9450	6.6818	4.805	4.913586
28	171,172,173,174,175,176	-	9.3545	17.8200	17.9000	10.8113	9.300	9.718956
29	178,181,184,187,190	-	15.0919	19.9200	18.0900	13.8404	17.170	15.021916
	W (lb)	36167.73	25447.100	38605.000	38104.800	25447.528	28554.140	25664.002
	Constraint violation	-	0.40023	None	None	0.00310	None	None

Some plane and space truss structures from the literature are analyzed in this paper to demonstrate the efficiency of the TLBO algorithm. The TLBO method has shown better performance with less computational effort for the considered problems attempted by previous researchers. It is concluded that the TLBO algorithm presented in this study can be effectively used in the weight minimization of truss structures. This method can be easily extended for the optimization of other structural design applications.

Acknowledgements

The authors thank Prof. R. Venkata Rao, Department of Mechanical Engineering, S.V. National Institute of Technology, Surat-395 007, India, for his help and comments.

References

Adeli, H. and Kamal, O. (1986), "Efficient optimization of space trusses", *Comput. Struct.*, **24**(3), 501-511.

Bekiroğlu, S., Dede, T. and Ayvaz, Y. (2009), "Implementation of different encoding types on structural optimization based on adaptive genetic algorithm", *Finite Elements in Analysis and Design*, **45**, 826-835.

Camp, C.V. (2007), "Design of space trusses using big-bang crush optimization", *J. Struct. Eng.*, **133**(7), 999-1108.

Camp, C.V. and Bichon, B.J. (2004), "Design of space trusses using ant colony optimization", *J. Struct. Eng.*, **130**(5), 741-751.

Cao, G. (1996), "Optimized design of framed structures using a genetic algorithm", Ph.D. Thesis, The University of Memphis.

Capriles, P.V.S.Z., Foncesa, L.G., Barbosa, H.J.C. and Lemonge, A.C.C. (2006), "Rank-based ant colony algorithms for truss weight minimization with discrete variables", *Communications in Numerical Methods in Engineering*, **23**, 553-575.

Castilho, V.C.D., Debs, M.E.E. and Nicoletti, M.D.C. (2006), "Using a modified genetic algorithm to minimize the production costs for slabs of precast prestressed concrete joists", *Engineering Applications of Artificial Intelligence*, **20**, 519-530.

Catallo, L. (2004), "Genetic anti-optimization for reliability structural assessment of precast concrete structures", *Comput. Struct*., **82**, 1053-1065.

Cerny, V. (1985), "Thermodynamical approach to the travelling saleman problem: an efficient simulation algorithm", *J. Optim. Theory Appl*., **45**, 41-51.

Chao, N.H., Fenves, S.J. and Westerberg, A.W. (1984), *Application of reduced quadratic programming techniques to optimal structural design*, New directions in optimum structural design, (Eds. Atrek, E., Gallagher, R. H., Radgsdell, K. M. and Zienkiewicz, O. C.), Wiley, New York.

Christodoulou, S. (2005), "Optimal truss design using ant colony optimization", *5th Gracm International congress on Computational Mechanics*, Limassol.

Coello, C.A. and Christiansen, A.D. (2000), "Multiobjective optimization of trusses using genetic algorithms", *Comput. Struct*., **75(**6), 647-660.

de Oliveira, C.J.P. and Gomes, H.M. (2010), "A particle swarm optimization algorithm for truss optimization on shape and size with dynamic constraints", *2nd international conference on engineering optimization*, Lisbon, Portugal.

Dede, T., Bekiroğlu, S. and Ayvaz, Y. (2011), "Weight minimization of trusses with genetic algorithm", *Applied Soft Computing*, **11**, 2565-2575.

Değertekin, S.O. (2008), "Optimum design of steel frames using harmony search algorithm", *Struct. Multidisc. Optim.*, **36**, 393-401.

Dobbs, M.W. and Nelson, R.B. (1976), "Application of optimality criteria to automated structural design", *AIAA J*., **14**(10), 1436-43.

Dorigo, M. (1991), "Ant colony optimization, new optimization techniques in engineering", by Onwubolu, G.C. and Babu, B.V., Springer-Verlag, Berlin, Heidelberg, **101**.

Erbatur, F., Hasançebi, O., Tütüncü İ. and Kılıç, H. (2000), "Optimal design of planar and space structures with genetic algorithm", *Comput. Struct.*, **75**, 209-224.

Geem, Z.W., Kim, J.H. and Loganathan, G.V. (2001), "A new heuristic optimization algorithm: harmony search", *Simulation*, **76**(2), 60-68.

Gellatly, R.A. and Berke, L. (1971), "Optimal structural design", AFFDL-TR, 70-165

Gomes, H.M. (2011), "Truss optimization with dynamic constraints using a particle swarm algorithm", *Expert Systems with Application*, **38**, 957-968.

Goldberg, D.E. (1989), *Genetic Algorithms in Search, Optimization and Machine Learning*, New York.

Hadidi, A., Azad, S.K. and Azad, S.K. (2010), "Structural optimization using artificial bee colony algorithm", *2nd International Conference on Engineering Optimization*, Lisbon, Portugal September.

Hasançebi, O. and Erbatur, F. (2002), "Layout optimization of trusses using simulated annealing", *Advances*

in Engineering Software, **33**, 681-696.
Holland, J.H. (1975), *Adaptation in Natural and Artificial Systems*, The University of Michigan Press, Ann Arbor, Mich.
Kaveh, A. and Abadi, A.S.M. (2011), "Harmony search based algorithms for the optimum cost design of reinforced concerete cantilever retaining walls", *International Journal of Civil Engineering*, **9**(1), 1-8.
Kaveh, A. and Talatahari, S. (2008), "A hybrid particle swarm and ant colony optimization for design of truss structures", *Asian Journal of Civil Engineering*, **9**(4), 329-348.
Kennedy, J. and Eberhart, R. (1995), "Particle swarm optimization", *Proc. 1995 IEEE Int. Conf. Neural Networks*, **4**, 1942-1948.
Khan, M.R., Willmert, K.D. and Thornton, W.A. (1979), "An optimality criterion method for large scale structures", *AIAA Journal*, **17**(7), 753-761.
Kirkpatrick, S., Gelatt, C.D. and Vecchi, M.P. (1983), "Optimization by simulated annealing", *Science*, **220**, 671-680.
Krishnamoorthy, C.S., Venkatesh, P.P. and Sudarshan, R. (2002), "Object-oriented framework for genetic algorithms with application to space truss optimization", *J. Comp. in Civil Eng*., **16(**1), 66-75.
Lamberti, L. (2008), "An efficient simulated annealing algorithm for design optimization of truss structures", *Computer and Structures*, **86**, 1936-1953.
Li, L., Huang, Z.B., Liu, F. and Wu, Q.H. (2007), "A heuristic particle swarm optimizer for optimization of pin connected structures", *Computers and Structures*, **85**, 340-349.
Lee, C. and Ahn, J. (203), "Flexural design of reinforced concrete frames by genetic algorithm", *J. Struct. Eng.*, **129**(6), 762-774.
Leps, M. and Sejnoha, M. (2003), "New approach to optimization of reinforced concrete beams", *Comp. Struct*., **81**, 1957-1966.
Lee, K.S., Geem, Z.W., Lee, S.H. and Bae, K.W. (2005), "The harmony search heuristic algorithm for discrete structural optimization", *Engineering Optimization*, **37**(7), 663-684.
Lee, K.S. and Geem, Z.W. (2004), "A new structural optimization method based on the harmony search algorithm", *Computers and Structures*, **82**, 781-798.
Pawar, R. and Rao, R.V. (2013), "Parameter optimization of machining processes using teaching-learning-based optimization algorithm", *International Journal of Advanced Manufacturing Technology*, **67**(5-8), 995-1006.
Pezeshk, P., Camp, C.V. and Chen, D. (2000), "Design of nonlinear framed structures using genetic optimization", *J. Struct. Eng*., **126**(3), 382-388.
Rajeev, S. and Krishnamoorthy, C.S. (1987), "Genetic algorithms-based methodologies for design optimization of trusses", *J. Struct. Eng*., **123**(3), 350-358.
Rao, R.V., Savsani, V.J. and Vakharia, D.P. (2011), "Teaching-learning-based optimization: A new method for constrained mechanical design optimization problems", *Computer-Aided Design*, **4**,303-315.
Rao, R.V. and Kalyankar V.D. (2013a), "Multi-pass turning process parameters optimization using teaching-learning-based optimization algorithm", *Scientia Iranica*, **20**(3), 967-974.
Rao, R.V. and Kalyankar V.D. (2013b), "Parameter optimization of modern machining processes using teaching learning-based optimization algorithm", *Engineering Applications of Artificial Intelligence*, **26**(1), 524-531.
Rao, R.V. and Patel, V. (2013a), "Multi-objective optimization of two stage thermoelectric cooler using a modified teaching-learning-based optimization algorithm", *Engineering Applications of Artificial Intelligence*, **26**(1), 430-445.
Rao, R.V. and Patel, V. (2013b), "Multi-objective optimization of heat exchangers using a modified teaching-learning-based optimization algorithm", *Applied Mathematical Modelling*, **37**(3), 1147-1162.
Rao, R.V. and Patel, V. (2013c), "An improved teaching-learning-based optimization algorithm for solving unconstrained optimization problems", *Scientia Iranica*, **20**(3), 710-720.
Rao, R.V., Savsani, V.J. and Vakharia, D.P. (2012a), "Teaching-learning-based optimization: an optimization method for continuous non-linear large scale problems", *Information Sciences*, **183**(1), 1-15.
Rao, R.V., Savsani, V.J. and Balic, J. (2012b), "Teaching-learning-based optimization algorithm for

unconstrained and constrained real-parameter optimization problems", *Engineering Optimization*, **44(**12), 1447-1462.

Rao, R.V. and Patel, V. (2012a), "An elitist teaching-learning-based optimization algorithm for solving complex constrained optimization problems", *International Journal of Industrial Engineering Computations*, **3**(4), 535-560.

Rao, R.V. and Patel, V. (2012b), "Comparative performance of an elitist teaching-learning-based optimization algorithm for solving unconstrained optimization problems", *International Journal of Industrial Engineering Computations*, **4**(1), 29-50.

Rao, R.V. and Kalyankar V.D. (2012a), "Parameter optimization of machining processes using a new optimization method", *Materials and Manufacturing Processes*, **27**(9), 978-985.

Rao, R.V. and Kalyankar V.D. (2012b), "Multi-objective multi-parameter optimization of the industrial LBW process using a new optimization algorithm", *Journal of Engineering Manufacture*, **226**(6), 1018-1025.

Renwei, X. and Peng, L. (1986), "An efficient method for structural optimization", *Acta Mechanica Sinica*, **2**(4), 348-361.

Renwei, X. and Peng, L. (1987), "Structural optimization based on second order approximations of functions and dual theory", *Computer Methods in Applied Mechanics and Engineering*, **65**, 101-4.

Rizzi, P. (1976), "Optimization of multiconstrained structures based on optimality criteria", *AIAA/ASME/SAE 17th Structures, Structural Dynamics, and Materials Conference*, King of Prussia, PA.

Saka, M.P. (1990), "Optimum design of pin-jointed steel structures with practical applications", *J. Struct. Eng.*, **116**(10), 2599-620.

Saka, M.P. (2007), "Optimum design of steel frames", (Ed. Topping, B.H.V.), Civil Engineering Computations Tools and Techniques, Stirling: Saxe-Coburg Publications, **6**,105-48.

Schmit, Jr. L.A. and Farshi, B. (1974), "Some approximation concepts for structural synthesis", *AIAA J.*, **12**(5), 692-9.

Schutte, J.F. and Groenwold, A.A. (2003), "Sizing design of truss structures using particle swarms", *Struct. Multidisc. Optim.* **25**, 261-269.

Sesok, D., Mockus, J., Belevicius, R. and Kaceniauskas, A. (2010), "Global optimization of grillages using simulated annealing and high performance computing", *Journal of Civil Engineering and Management*, **16**(1), 95-101.

Sonmez, M. (2011), "Artificial bee colony algorithm for optimization of truss structures", *Applied Soft Computing*, **11**, 2406-2418.

Toğan, V. (2013), "Design of pin jointed structures using teaching-learning based optimization", *Structural Engineering and Mechanics*, **47**(2), 209-225.

Toğan, V. and Daloğlu, A. (2008), "An improved genetic algorithm with initial population strategy and self adaptive member grouping", *Computers and Structures*, **86,** 1204-1218.

Venkayya, V.B. (1971), "Design of optimum structures", *Comput & Structures*, **1**(1-2), 265-309.

Waghmare, G. (2012), "Comments on "A note on teaching-learning-based optimization algorithm", *Information Sciences*, **229**, 159-169.

Xu, T., Zuo, W. and Xu, T. (2010), "An adaptive reanalysis method for genetic algorithm with application to fast truss optimization", *Acta Mech Sin*, **26**, 225-234.

Yaseen, S.G. and Al-Slamy, N.M.A. (2008), "Ant colony optimization", *International Journal of Computer Science and Network Security*, **8**(6), 351-357.