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An efficient response surface method considering the nonlinear trend of the actual limit state

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Abstract. In structural reliability analysis, the response surface method is a powerful method to evaluate the probability of failure. However, the location of experimental points used to form a response surface function must be selected in a judicious way. It is necessary for the highly nonlinear limit state functions to consider the design point and the nonlinear trend of the limit state, because both of them influence the probability of failure. In this paper, in order to approximate the actual limit state more accurately, experimental points are selected close to the design point and the actual limit state, and consider the nonlinear trend of the limit state. Linear, quadratic and cubic polynomials without mixed terms are utilized to approximate the actual limit state. The direct Monte Carlo simulation on the approximated limit state is carried out to determine the probability of failure. Four examples are given to demonstrate the efficiency and the accuracy of the proposed method for both numerical and implicit limit states.

Keywords: response surface method; structural reliability; nonlinear trend of the limit state; adaptive experimental points; probability approach

1. Introduction

In structural reliability analysis, Monte Carlo simulation (MCS) is an accurate technique to estimate the probability of failure. Although it gives the exact solution, it is time-consuming for the large and complex structures with low probabilities of failure and implicit limit state functions. To reduce the number of structural analyses, the first-order reliability method (FORM) and the second-order reliability method (SORM) were developed (Hasofer and Lind 1974, Rackwitz and Fiessler 1978, Kiureghian *et al*. 1987, Liu and Kiureghian 1991, Ditlevsen and Madsen 1996, Lemaire 2009). However, FORM and SORM are also difficult as the actual implicit limit state function (LSF) usually cannot be easily expressed explicitly. In some cases, FORM and SORM may suffer convergence problems (Wang and Grandhi 1996). Therefore, approximation methods are desirable to perform the reliability analysis, such as the response surface method (Khuri and Cornell 1997, Myers and Montgomery 1995). In the response surface method (RSM), the LSF is

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replaced by the response surface function (RSF) of basic random variables. The RSF is then used instead of the actual LSF for the estimation of the failure probability. The RSM approximates the actual LSF by using experimental points and explicit mathematical functions (typically quadratic polynomials). As the approximated LSF is explicit, FORM or SORM can be applied to estimate the probability of failure directly. Alternatively, MCS can be used efficiently since the evaluation of the RSF requires very little computational effort.

Several researchers proposed improvements of the RSM in order to evaluate efficiently the failure probability of complex structures. Bucher and Bourgund (1990) proposed a quadratic polynomial response surface without mixed terms. A saturated experimental design with $2n+1$ experimental points is built, where n is the number of random variables. Rajashekhar and Ellingwood (1993) proposed some ideas to improve the response surface obtained from Bucher's algorithm, in which more iterations are repeated until the convergence parameter becomes very small or zero. Kim and Na (1997) proposed to arrange the experimental points in order to bring them close to the actual LSF by using the gradient projection technique. Linear RSFs are utilized and the reliability indices are evaluated by the Rackwitz-Fiessler algorithm. Gayton *et al*. (2003) proposed a RSM named CQ2RS (Complete Quadratic Response Surface with ReSampling). The method takes into account the knowledge of the engineer and the statistical resampling technique is used to determine the design point. Wong *et al*. (2005) suggested to choose $2n+1$ axial point designs and to select the parameter f as a decreasing function of the coefficient of variation of the random variables. Kaymaz and Chris (2005) utilized the sign evaluation for the random variables to form a reduced design space, the experimental points are also selected from the region where the design point is most likely to exist. Duprat and Sellier (2006) proposed a quadratic polynomial response surface with mixed terms. In this scheme, points efficiently positioned with respect to the design point are reused in the next iteration. Gavin and Yau (2008) presented the use of higher order polynomials for response surface approximations. The authors proposed to use a polynomial without a fixed degree in order to fit the LSF better. According to this approach high order terms may not be necessary. The degree of the polynomial is determined on the basis of a statistical analysis of the polynomial coefficients. Nguyen *et al*. (2009) proposed an adaptive RSM based on a double weighted regression technique. The main features of this method are the choice of the response surface expression, for the first iteration, a linear response surface is chosen, for the following iterations, a quadratic response surface with mixed terms is considered according to complementary points. Kang *et al*. (2010) proposed an efficient RSM applying the moving least squares approximation instead of the traditional least squares approximation. The linear RSF and the quadratic RSF are formed using the axial experimental points. The RSF is updated successively by adding the most probable failure point to the previous set of experimental points. Allaix and Carbone (2011) discussed the locations of the experimental points used to evaluate parameters of the response surface. The locations of the experimental points are chosen according to the importance sensitivity of each random variable. The response surface is then built after rotating the coordinate system. Basaga *et al*. (2012) proposed an improved RSM. In the algorithm, a quadratic approximate function is formed and design point is determined with FORM, a point close to the LSF is searched using the design point, vector projected method is used to generate the sample points, and SORM is performed to obtain reliability index and probability of failure. Li *et al*. (2013) compared three collocation point methods associated with the odd order stochastic RSM in a systematical and quantitative way. The results indicate that the origin often used as a collocation point is not absolutely necessary.

As seen from above short literature review, most of the proposed improvements to the RSM

focus on the location of experimental points and the design point, so that a quadratic polynomial can better approximate the actual LSF in the neighbourhood of the design point. However, a quadratic polynomial cannot fit highly nonlinear LSF over a large region accurately. There is the difference between the RSF and actual LSF outside the region of the experimental points resulting in calculation errors of failure probabilities, especially for highly nonlinear LSFs and problems with low probabilities of failure.

In this study, a different approach is proposed to approximate the actual LSF. Three aspects of the RSM are discussed in the following: the location of the experimental points, the degree of polynomial of the RSF and the estimation of the failure probability.

Concerning the first aspect, since the region around the design point gives the main contribution to the probability of failure, experimental points should be selected close to the design point so that the RSF can approximate the LSF near the design point accurately. On the other hand, for a highly nonlinear LSF, the accuracy of the approximation of the failure probability does not depend very much upon the region around the design point. In this case, experimental points should be selected close to the design point and consider the nonlinear trend of the LSF, so that the RSF can approximate the LSF over a larger region containing the design point. In fact, for complicated and large structural system, the nonlinear degree of the LSF is unknown generally. Thus, it is necessary for the nonlinear LSF to consider the design point and the nonlinear trend of LSF because both of them influence the probability of failure.

Concerning the second aspect, the degree of polynomial of the RSF should be a compromise between accuracy and efficiency. Considering the accuracy, a high order polynomial should be chosen. However, the high order polynomial involves time-consuming, and the rate of improvement of the approximation decreases with the degree of polynomial (Allaix and Carbone 2011). Thus, the high order terms of polynomial may not be necessary (Gavin and Yau 2008). With respect to the efficiency, the choice of the degree of the polynomial aims generally to reduce the computational effort. Thus, the degree of polynomial should be as low as possible. For these reasons, quadratic polynomial functions without mixed terms are generally used (Allaix and Carbone 2011). Thus, in this paper, linear, quadratic and cubic polynomials without mixed terms are utilized to approximate the actual LSF.

Concerning the third aspect, FORM and SORM are perfectly adequate for linear LSFs and slightly nonlinear LSFs. However, these methods are not accurate enough for highly nonlinear LSFs. The MCS can give the exact solution regardless of the complexity of the system or the limit state. When the probability of failure is estimated by using the MCS considering the RSF, the accuracy of the result becomes more important than the efficiency of the simulation method because the RSF is the explicit expression. Thus, in the proposed method, the values of the adjusted R^2, indicating the accuracy of the approximation, are calculated from the statistical data of each RSF. The RSF in terms of the best adjusted R^2 is used to estimate the failure probability by using the direct MCS.

2. Classical response surface method

In the RSM, the actual LSF $G(\mathbf{X})$ is replaced by a polynomial function $\bar{G}(\mathbf{X})$, typically a quadratic polynomial function without mixed terms, given as

$$\bar{G}(\mathbf{X}) = a + \sum_{i=1}^{n} b_i x_i + \sum_{i=1}^{n} c_i x_i^2 \tag{1}$$

where n is the number of random variables $\mathbf{X}$, and a, b_i, and c_i are the $2n+1$ unknown coefficients. The unknown coefficients are obtained from discrete evaluations of the implicit LSF, such as through evaluation of the finite element method.

It is seen that Eq. (1) does not contain mixed terms, hence the function $\bar{G}(\mathbf{X})$ basically represents the original function $G(\mathbf{X})$ along the coordinate axes. The experimental points required to obtain $\bar{G}(\mathbf{X})$ are chosen to be the mean values $\bar{x}_i$ and $x_i = \bar{x}_i \pm f\sigma_i$, $i = 1, 2, \ldots, n$, in which f is an arbitrary factor and σ_i are the standard deviations of x_i, respectively. Next, using the $2n+1$ function values of $G(\mathbf{X})$ at these points, the parameters a, b_i, and c_i are obtained from a set of linear equations. If there are more points than $2n+1$ coefficients in Eq. (1), a least squares or similar methods may need to be employed to approximate the surface.

The original LSF cannot be properly represented by the RSF evaluated using the information obtained at the experimental points chosen in the vicinity of the mean values of basic random variables. To improve the accuracy of the RSM, Bucher and Bourgund (1990) suggested an alternative process of selecting the experimental points. In the first step of this algorithm, the mean vector is selected as the center point. Then the RSF obtained is used to find an estimate of the design point $\mathbf{X}_D$. In the next step, the new center point $\mathbf{X}_M$ is chosen on a straight line from the mean vector $\bar{\mathbf{X}}$ to $\mathbf{X}_D$ so that $G(\mathbf{X})$ =0 at the new center point $\mathbf{X}_M$ from linear interpolation, i.e.

$$\mathbf{X}_M = \bar{\mathbf{X}} + (\mathbf{X}_D - \bar{\mathbf{X}}) \frac{G(\bar{\mathbf{X}})}{G(\bar{\mathbf{X}}) - G(\mathbf{X}_D)} \tag{2}$$

Next, the same interpolation is repeated to find an updated RSF $\bar{G}(\mathbf{X})$, as described above, by using $\mathbf{X}_M$ as the new center point.

In order to compare the results of the proposed method with those of the classical RSM and FORM, the convergence criteria of the classical RSM and FORM are checked by the following two convergence criteria at the same time. First, the relative difference between the reliability indices, β, in two subsequent iterations is small.

$$\left| \frac{\beta^{i+1} - \beta^i}{\beta^{i+1}} \right| < \varepsilon \tag{3}$$

Second, the value of the LSF at the design point $\mathbf{X}_D$ should be close to zero. For simplicity, it is required that the ratio between the values of the LSF at the current iteration and at the point of mean value is small.

$$\left| G(\mathbf{X}_D) / G(\bar{\mathbf{X}}) \right| < \varepsilon \tag{4}$$

where the constant ε is assumed to be equal to 0.001.

3. Improvement of the response surface

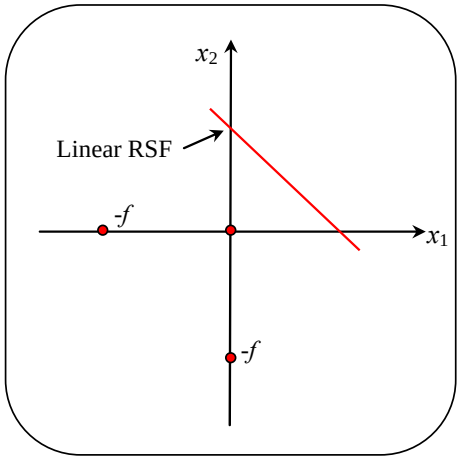

Fig. 1 Step 1 of the proposed method

3.1 Selection of the experimental points

In the proposed method, the experimental points are close to the actual design point and the actual limit state, and consider the nonlinear trend of the LSF. The location of the experimental points is discussed as following.

Step 1 Determine the linear RSF

1.1 Select (n+1) experimental points along the negative direction of each coordinate axis.

1.2 Evaluate the LSF with respect to the experimental points selected in sub-step 1.1.

1.3 Obtain the linear RSF, as described by Eq. (8), by using the (n+1) experimental points selected in sub-step 1.1.

The experimental points of step 1 in the two-dimensional standard normal space are shown in Fig. 1, where f =3.0.

Step 2 Determine the approximate design point

2.1 Determine the design point using FORM for the linear RSF obtained by step 1, the design point is denoted by $\mathbf{X}_{DL}$.

2.2 Bring the $\mathbf{X}_{DL}$ close to the actual limit state by using successive linear interpolation, i.e.

$$\mathbf{X}_{M_{i+1}} = \mathbf{X}_{M_i} + (\mathbf{X}_{M_{i-1}} - \mathbf{X}_{M_i}) \frac{G(\mathbf{X}_{M_i})}{G(\mathbf{X}_{M_i}) - G(\mathbf{X}_{M_{i-1}})} \tag{5}$$

For the first iteration, the Eq. (5) becomes the Eq. (2). Check the convergence, if the convergence is not achieved, repeat Eq. (5) for a new iteration. The convergence of the Eq. (5) is checked by the following convergence criterion.

$$\left| G(\mathbf{X}_{M_{i+1}}) \Big/ G(\bar{\mathbf{X}}) \right| < \varepsilon \tag{6}$$

where the constant ε is assumed to be equal to 0.001.

For convenience, the final experimental point satisfying with Eq. (6) is also denoted by $\mathbf{X}_D$. In this way, the approximate design point $\mathbf{X}_D$ lies on the original limit state surface and does not

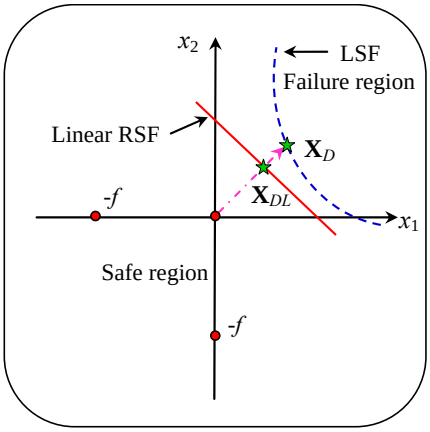

Fig. 2 Step 2 of the proposed method

deviate from the actual design point excessively.

The experimental points of step 2 in the two-dimensional standard normal space are shown in Fig. 2.

Step 3 Select experimental points considering the nonlinear trend of the LSF

3.1 Obtain the tangent hyperplane by using the vector from $\bar{\mathbf{X}}$ to $\mathbf{X}_D$ and $\mathbf{X}_D$. The expression of the tangent hyperplane is given by

$$(\mathbf{X}-\mathbf{X}_D)^T\boldsymbol{\omega}=0 \tag{7}$$

where $\boldsymbol{\omega}$ is the unit vector from $\bar{\mathbf{X}}$ to $\mathbf{X}_D$.

It is noted that $\mathbf{X}_D$ is the approximate design point in this step, thus the vector from $\bar{\mathbf{X}}$ to $\mathbf{X}_D$ is the approximate normal vector of the LSF in $\mathbf{X}_D$. Therefore, the tangent hyperplane obtained by using the vector from $\bar{\mathbf{X}}$ to $\mathbf{X}_D$ and $\mathbf{X}_D$ is the approximate tangent hyperplane to the LSF in $\mathbf{X}_D$.

3.2 Solve for the coordinate of each intersection between the tangent hyperplane with each coordinate axis.

3.3 Select one experimental point on each straight line from $\mathbf{X}_D$ to each intersection obtained by sub-step 3.2, respectively. The total number of experimental points selected by this sub-step is n. The experimental points of this sub-step in the two-dimensional standard normal space are shown in Fig.3, where k=2-4.

3.4 Evaluate the LSF with respect to the experimental points selected in sub-step 3.3. If $G(\mathbf{X}_i)>0$, select one experimental point along the positive direction of the vector of $\boldsymbol{\omega}$. If $G(\mathbf{X}_i)<0$, select one experimental point along the negative direction of the vector of $\boldsymbol{\omega}$. The total number of experimental points selected by this sub-step is n. The experimental points of this sub-step in the two-dimensional standard normal space are shown in Fig. 4.

3.5 In order to improve the fitting precision of the RSF to the LSF around the region of design point, the midpoints between the experimental points obtained by sub-step 3.4 and $\mathbf{X}_D$ are selected as new experimental points. The total number of experimental points selected by this sub-step is n. The experimental points of this sub-step in the two-dimensional standard normal space are shown in Fig. 5.

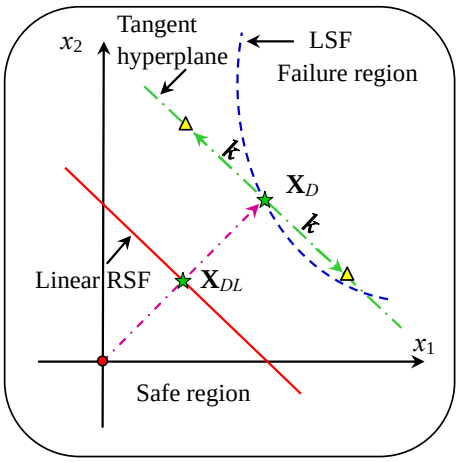

Fig. 3 Sub-step 3.3 of the proposed method

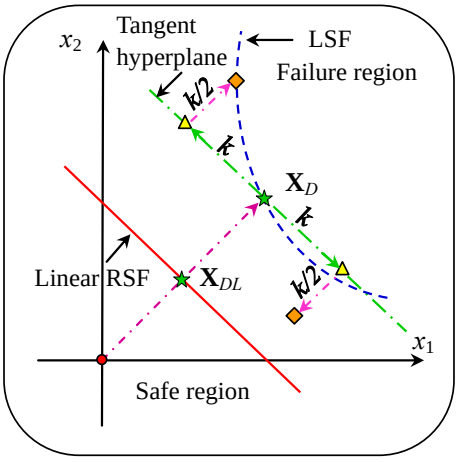

Fig. 4 Sub-step 3.4 of the proposed method

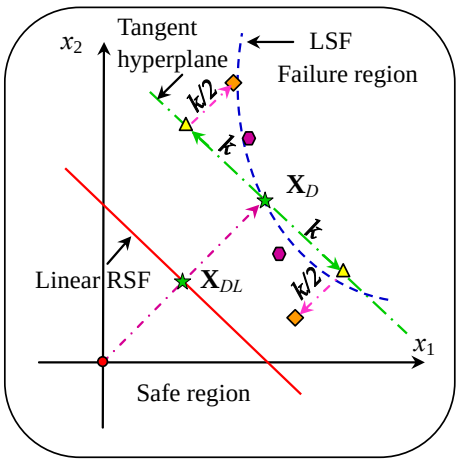

Fig. 5 Sub-step 3.5 of the proposed method

The proposed technology can be divided into 3 steps with respect to the experimental points. The number of experiment points of step 1 is n+1. The number of experiment points of step 2 is equal to the number of successive linear interpolation +1, where 1 expresses the LSF evaluation with respect to $\mathbf{X}_{DL}$ obtained by linear RSF. The number of experiment points of step 3 is 3n. Hence, the total number of LSF evaluations of the proposed method is (n+1) + (the number of successive linear interpolation+1) +3n = 4n+3+the number of successive linear interpolation.

3.2 Selection of the function of the RSF

In this paper, linear, quadratic and cubic polynomials without mixed terms are used to approximate the actual LSF, as follows

$$\overline{G}_1(\mathbf{X}) = a + \sum_{i=1}^{n} b_i x_i \tag{8}$$

$$\bar{G}_2(\mathbf{X}) = a + \sum_{i=1}^{n} b_i x_i + \sum_{i=1}^{n} c_i x_i^2 \tag{9}$$

$$\bar{G}_3(\mathbf{X}) = a + \sum_{i=1}^{n} b_i x_i + \sum_{i=1}^{n} c_i x_i^2 + \sum_{i=1}^{n} d_i x_i^3 \tag{10}$$

where n is the number of random variables $\mathbf{X}$, and a, b_i, c_i and d_i are unknown coefficients.

As seen from the above analysis, the experimental points of steps 2-3 in section 3.1 are close to actual limit state and actual design point, and these experimental points consider the trend of the LSF. Thus, the experimental points of steps 2-3 in section 3.1 are used to solve for the unknown coefficients of Eqs. (8), (9) and (10). In addition, the values of the adjusted R^2, indicating the accuracy of the approximation, are calculated from the statistical data of each RSF (Gavin and Yau 2008, Nguyen *et al*. 2009). The definition of the adjusted R^2 is given by

$$\text{Adjusted } R^2 = 1 - \frac{(P-1)\sum_{i=1}^{P}\left(\bar{G}(\mathbf{X}_i) - G(\mathbf{X}_i)\right)^2}{(P-N-1)\sum_{i=1}^{P}\left(\bar{G}(\mathbf{X}_i) - \frac{1}{P}\sum_{j=1}^{P} G(\mathbf{X}_j)\right)^2} \tag{11}$$

where P is the total number of experimental points, N is the total number of coefficients.

The adjusted R^2 is bounded by the interval [0, 1]. If its value tends to 1, the RSF is close to the actual LSF at the experimental points. Since three RSFs are utilized to approximate the actual LSF, three adjusted R^2 can be obtained. Thus, in this paper, the RSF is used in terms of the best adjusted R^2.

3.3 Estimation of the failure probability

In this paper, the direct MCS is performed to assess the accuracy of the failure probability obtained by the RSF. Then 1,000,000 simulations are performed by using the LSF and the RSF respectively. From this comparison it is possible to understand if the RSF is close to the actual LSF. A parametric analysis is performed with respect to the parameter k of the proposed method. The values 2, 3 and 4 are considered. In addition, the forward finite difference is applied in FORM to compute the first order partial derivatives of the LSF with respect to the random variables even if the LSF is expressed in the explicit form. The probabilities of failure obtained by FORM are given by $P_f = \Phi(-\beta)$, where Φ is the standard normal cumulative distribution function.

4. Numerical examples

4.1 Example 1: a cubic LSF

A hypothetical cubic LSF with two independent standard normal variables is considered

$$G(\mathbf{X}) = 0.16(x_1 - 1)^3 - x_2 + 4 - 0.04\sin(x_1 x_2) \tag{12}$$

For this example, if the coefficient before $\sin(x_1x_2)$ is large, the nonlinear of the LSF will be very high. Thus, the sine term represents the small effects of higher order terms (Gavin and Yau

Table 1 Comparison of analysis results of example 1

Method	Failure probability	Adjusted R^2	LSF eval.
Monte Carlo	3.249×10^{-2}	-	1,000,000
Classical RSM(f=3)	5.106×10^{-2}	-	36
FORM	0.621×10^{-2}	-	13
Improved method(k=2)	3.257×10^{-2}	0.9996	15
Improved method(k=3)	3.214×10^{-2}	0.9999	15
Improved method(k=4)	3.213×10^{-2}	0.9999	15

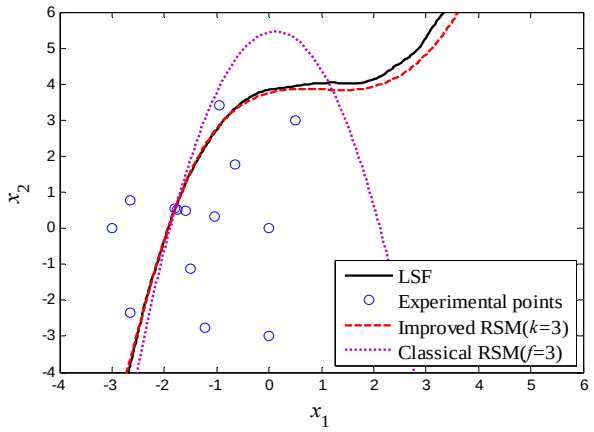

Fig. 6 Example 1-Response surface approximations

2008). For the example, the adjusted R^2 of the cubic polynomial is always the best when k=2, 3 and 4. Thus, the cubic polynomial is used to approximate the actual LSF. The numerical results are listed in Table 1 and also shown graphically in Fig. 6. As seen from the results in Table 1, the proposed method yields better results in terms of the probability of failure, regardless of the values of the parameter k. As seen from Fig. 6, since the experimental points of the proposed method consider the trend of the LSF, the proposed method is able to reach a better approximation of the LSF over a large region than the classical RSM.

4.2 Example 2: a quadratic LSF

A quadratic LSF with a mixed term is expressed as

$$G(\mathbf{X}) = 20 - (x_1 - 0.2x_2)^2 - 5x_2 \tag{13}$$

where x_1 and x_2 are independent standard normal variables, respectively.

Since the proposed method and the classical RSM do not contain mixed terms, the example is used to show the influence of mixed terms on two methods. For the example, the adjusted R^2 of the cubic polynomial is the best when k=2 and 3. The adjusted R^2 of the quadratic polynomial is the best when k=4. The numerical results are listed in Table 2 and also shown graphically in Fig. 7. As seen from Table 2 and Fig. 7, the result of proposed method is better then that of the classical RSM. The improvements in terms of the failure probability and the approximation of the LSF are

Table 2 Comparison of analysis results of example 2

Method	Failure probability	Adjusted R^2	LSF eval.
Monte Carlo	5.41×10^{-4}	-	1,000,000
Classical RSM(f=3)	6.45×10^{-4}	-	30
FORM	3.96×10^{-4}	-	22
Improved method(k=2)	5.45×10^{-4}	0.9660	12
Improved method(k=3)	5.24×10^{-4}	0.9112	12
Improved method(k=4)	5.23×10^{-4}	0.9088	12

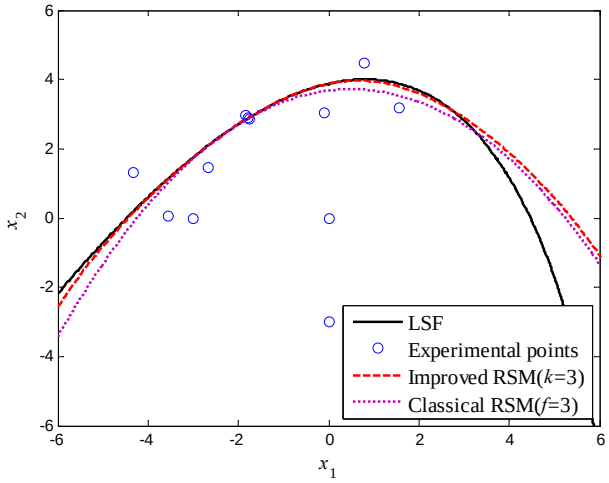

Fig. 7 Example 2-Response surface approximations

due to the location of the experimental points considering the trend of the LSF.

4.3 Example 3: a nonlinear LSF

This example has been reported in a number of papers (Nguyen *et al*. 2009, Kaymaz and Chris 2005, Elegbede 2005). The LSF is expressed as

$$G(\mathbf{X}) = \exp[0.4(x_1 + 2) + 6.2] - \exp[0.3x_2 + 5] - 200 \tag{14}$$

where x_1 and x_2 are assumed to be independent and have a standard normal distribution with zero mean and unit standard deviation.

For the example, the adjusted R^2 of the cubic polynomial is always the best when k=2, 3 and 4. The numerical results are listed in Table 3 and also shown graphically in Fig. 8. As seen from the results in Table 3 and Fig. 8, the proposed method yields a better approximation of the LSF over a large region then the classical RSM.

4.4 Example 4: a finite element problem

In order to illustrate the accuracy of the proposed method for dealing with complex finite

Table 3 Comparison of analysis results of example 3

Method	Failure probability	Adjusted R^2	LSF eval.
Monte Carlo	3.656×10^{-3}	-	1,000,000
Classical RSM(f=3)	3.586×10^{-3}	-	30
FORM	3.365×10^{-3}	-	19
Improved method(k=2)	3.665×10^{-3}	0.9979	12
Improved method(k=3)	3.664×10^{-3}	0.9992	12
Improved method(k=4)	3.650×10^{-3}	0.9998	12

Table 4 Comparison of analysis results of example 4

Method	Failure probability	Adjusted R^2	LSF eval.
Monte Carlo	0.1394	-	1,000,000
Classical RSM(f=3)	0.1393	-	88
FORM	0.0862	-	45
Improved method(k=2)	0.1367	0.9253	46
Improved method(k=3)	0.1371	0.9363	46
Improved method(k=4)	0.1386	0.9445	46

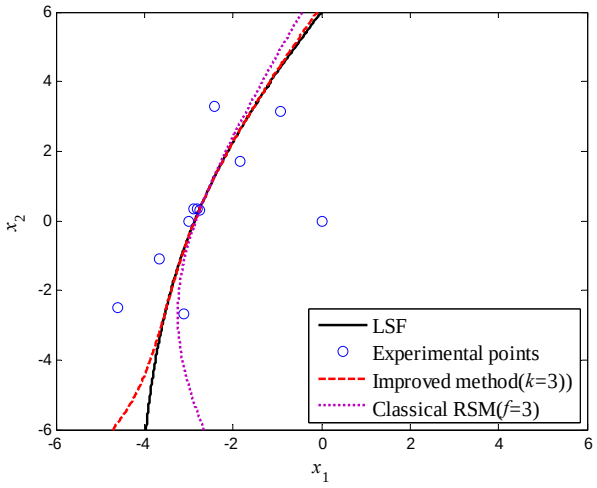

Fig. 8 Example 3-Response surface approximations

element problems, a 10 bar truss structure (Chowdhury *et al*. 2009) is considered as illustrated in Fig. 9. Young's modulus of the material is 10^7psi or 68947.573MPa. Two concentrated forces of 10^5lb or 444.822kN are applied at nodes 2 and 3. The cross-sectional area x_i, i =1, 2, ..., 10, for each bar follows normal distribution and has the mean μ_x=2.5in^2 or 1.6129×10^{-3}m^2 and standard deviation σ_x = 0.5in^2 or 3.2258×10^{-4}m^2. According to the loading condition, the maximum vertical displacement occurs at node 3, where a permissible displacement is limited to $u_{\max}$ =18in or 0.4572m. Hence, the LSF is

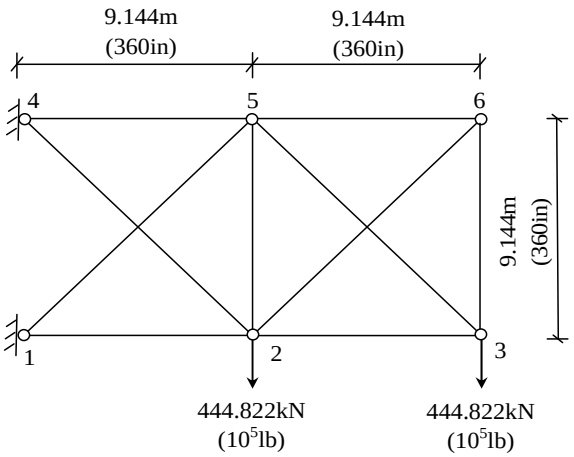

Fig. 9 10-bar truss structure

$$G(\mathbf{X}) = u_{\max} - u_3(\mathbf{X}) \tag{15}$$

For the example, the adjusted R^2 of the quadratic polynomial is always the best when k=2, 3 and 4. The numerical results are listed in Table 4. As seen from the results in Table 4, the results obtained by using the proposed method can be reasonably accepted in terms of the number of LSF evaluations. Through this example, it can be seen that the proposed method gives promising results even for problems of high dimension with a large number of random variables and implicit performance functions. It is noted that the results obtained by FORM are not the same as by Chowdhury *et al*. (2009) because the convergence criterion of FORM is not given in the study of Chowdhury *et al*. (2009). The different convergence criteria result in the different the probabilities of failure and the number of LSF evaluations.

5. Conclusions

In the paper, the classical RSM is improved by considering the significance of design point and the nonlinear trend of the LSF simultaneously. The proposed method can yield a better approximation of the actual LSF over a large region than the classical RSM. Numerical examples show that the proposed method can give a better evaluation of the failure probability in terms of the number of experimental points. The proposed method also gives an indication of the accuracy of the estimated failure probability by using the adjusted R^2. Moreover, unlike the classical RSM, the failure probabilities obtained by using the proposed method do not show any significant dependence on the parameter k.

However, since the approximate design point is used to reduce the computational effort, estimation results may become more inaccurate if the approximate design point is far from the actual design point, especially for multiple design points. In addition, if the adjusted R^2 is small, the fitting precision of the RSF to the LSF at selected experimental points will be bad. However, the experimental points with the big fitting errors may be far from the limit state surface. Thus, if

the adjusted R^2 is small, the accuracy of proposed method may be incredulous. Therefore, the further investigation of the proposed method should be required without increase in the computational effort.

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References

Allaix, D.L. and Carbone, V.I. (2011), "An improvement of the response surface method", *Struct. Safety*, **33**(2), 165-172.

Basaga, H.B., Bayraktar, A. and Kaymaz, I. (2012), "An improved response surface method for reliability analysis of structures" , *Struct. Eng. Mech.*, **42**(2), 175-189.

Bucher, C.G. and Bourgund, U. (1990), "A fast and efficient response surface approach for structural reliability problems", *Struct. Safety*, **7**(1), 57-66.

Chowdhury, R., Rao, B.N. and Prasad, A.M. (2009), "High-dimensional model representation for structural reliability analysis", *Commun. Numer. Meth. Eng.*, **25**(4),301-337.

Ditlevsen, O. and Madsen, H.O. (1996), *Structural Reliability Methods*, Wiley, Chischester, UK.

Duprat, F. and Sellier, A. (2006), "Probabilistic approach to corrosion risk due to carbonation via an adaptive response surface method", *Prob. Eng. Mech.*, **21**(3), 207-216.

Elegbede, C. (2005), "Structural reliability assessment based on particles swarm optimization", *Struct. Safety*, **27**(2), 171-186.

Gavin, H.P. and Yau, S.C. (2008), "High-order limit state functions in the response surface method for structural reliability analysis", *Struc. Safety*, **30**(2), 162-179.

Gayton, N., Bourinet, J.M. and Lemaire, M. (2003), "CQ2RS: a new statistical approach to the response surface method for reliability analysis", *Struct. Safety*, **25**(1), 99-121.

Hasofer, A.M. and Lind, N.C. (1974), "Exact and invariant second-moment code format", *J. Eng. Mech., ASCE,* **100**(1),111-121.

Kang, S.C., Koh, H.M. and Choo, J.F. (2010), "An efficient response surface method using moving least squares approximation for structural reliability analysis", *Prob. Eng. Mech.*, **25**(4), 365-371.

Kaymaz, I. and Chris, A.M. (2005), "A response surface method based on weighted regression for structural reliability analysis", *Prob. Eng. Mech.,* **20**(1), 11-17.

Khuri, A.I. and Cornell, J.A. (1997), *Response Surfaces: Design and Analyses*, Marcel and Dekker, New York.

Kim, S. and Na, S. (1997), "Response surface method using vector projected sampling points", *Struct. Safety*, **19**(1), 3-19.

Kiureghian, D.A., Lin, H.Z. and Hwang, S.J. (1987), "Second order reliability approximations", *J. Eng. Mech., ASCE*, **113**(8), 1208-1225.

Lemaire, M. (2009), *Structural reliability*, John Wiley & Sons, New York, USA.

Li, D.Q., Jiang, S.H., Cheng, Y.G. and Zhou, C.B. (2013), "A comparative study of three collocation point methods for odd order stochastic response surface method", *Struct. Eng. Mech.*, **45**(5), 595-611.

Liu, P.L. and Kiureghian, D.A. (1991), "Optimization algorithms for structural reliability", *Struct. Safety*, **9**(3), 161-177.

Myers, R.H. and Montgomery, D.C. (1995), *Response Surface Methodology: Process and Product*

Optimization using Designed Experiments, Wiley, Chischester, UK.
Nguyen, X.S., Sellier, A., Duprat, F. and Pons, G. (2009), "Adaptive response surface method based on a double weighted regression technique", *Prob. Eng. Mech.*, **24**(2),135-143.
Rackwitz, R. and Fiessler, B. (1978), "Structural reliability under combined random loadsequences", *Comput. Struct.*, **9**(5),489-494.
Rajashekhar, M.R. and Ellingwood, B.R. (1993), "A new look at the response surface approach for reliability analysis", *Struct. Safety*, **12**(3), 205-220.
Wang, L. and Grandhi, R.V. (1996), "Safety index calculation using intervening variables for structural reliability analysis", *Comput. Struct.*, **59**(6), 1139-1148.
Wong, S.M., Hobbs, R.E. and Onof, C. (2005), "An adaptive response surface method for reliability analysis of structures with multiple loading sequences", *Struct. Safety*, **27**(4), 287-308.