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Approximate methods to evaluate storey stiffness and interstory drift of RC buildings in seismic area

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Abstract. During preliminary design of a RC building located in a seismic area, having quick but reliable analytical measurement of interstory drifts and storey stiffnesses might be helpful in order to check the fulfillment of damage limit state and stiffness regularity in elevation required by seismic design codes. This paper presents two approximate methods, strongly interrelated each other, and addressed to achieve each of these two purposes for frame buildings. A brief description of some already existing methods addressed to the same aims is included to compare the main differences in terms of general approaches and assumptions. Both new approximate methods are then applied to 9 'ideal' frames and 2 'real' buildings designed according to the Italian seismic code. The results are compared with the 'exact' values obtained by the code-based standard calculation, performed via FEM models, showing a satisfactory range of accuracy. Compared with those by the other methods from literature, they indicate the proposed procedures lead to a better approximation of the objective structural parameters, especially for those buildings designed according to the modern 'capacity design' philosophy.

Keywords: interstory drift; storey stiffness; regularity in elevation; shear type frames; flexural type frames

1. Introduction

In the last years the awareness about the importance of the buildings regularity and non-structural damage prevention for frame systems located in seismic areas has grown significantly, especially after the observation of disastrous effects of seismic events. Therefore, the most recent building codes (e.g., Eurocode 8 2003, Italian code OPCM3274 2003) discourage design of irregular buildings by not allowing the engineers to adopt some simplifications in terms of structural model and analysis method, generally valid for regular structures. Moreover, they impose larger magnitudes of lateral forces, compared to regular frame, through imposing a penalizing value of so called 'behavior factor' commonly used for linear analyses.

Actually the stiffness regularity in elevation, defined as a continuous variation of the storey stiffness along the height of the building without abrupt changes, reduces the likelihood of

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dangerous concentrations of plastic deformations in a few (more flexible) stories. One of the most common vertical structural irregularities is due to a large open space or car-parking (with less partition walls than the upper stories) located at the bottom of the building. The earthquake energy concentrates on highly demanded columns and walls at the more flexible stories where, as a consequence, the mechanism known as ‘soft-storey’ or ‘weak-storey’ may develop. New codes define the vertical stiffness regularity on the basis of the percentage variation between the lateral stiffness of two adjacent stories. For example, according to the OPCM3274 the variation between two adjacent stories has to be less than or equal to 30% when it represents a reduction of stiffness and less than or equal to 10% if it corresponds to an increase of stiffness.

Having large probability of occurrence during the life-cycle of a building, the most recent earthquake resistant design codes emphasize the importance of limiting damage under low to medium intensity seismic events. Owing to the fact that probable damage derives from frame displacements, limiting interstory drifts leads to limit expected losses. Besides the structural issues, non-structural components of buildings also play a significant role in performance-based earthquake engineering, especially in the case of critical facilities that provide vital emergency assistance to the communities when earthquakes happen. Thus, new codes emphasize the importance of limiting non-structural damage through limiting interstory drifts at given values, depending on the non-structural elements features and on the way they interact with structural deformations.

The present study explores the possibility to use expeditious methods to evaluate lateral interstory drifts and stiffnesses during the preliminary design of a given RC frame structure. Although these methods are approximate, they allow the designer to quickly check if the pre-dimensioned building fulfills code requirements about structural regularity and lateral displacements magnitude. Nowadays the available computer technology and structural analysis programs may strongly support the designer to refine structural models and to compute more accurate structural response, even under seismic actions. Nevertheless, authors strongly believe in the usefulness of quick tools for the performance assessment of large building stocks or the preliminary design of new buildings, possibly based on the use of simple spreadsheets. Actually, in both cases being able to obtain reliable estimates of response parameters given a small amount of input data is really crucial.

Throughout last decades the topic has been subject of many researches either appraising interstory drift ratio (IDR) or assessment of storey stiffness. About the preliminary evaluation of the IDR, Lepage (1996) estimates the maximum nonlinear displacement of RC buildings basing on the use of idealized linear response spectrum. Miranda (1999) and also Miranda and Reyes (2002) introduced a model to calculate the estimation of maximum roof displacement and maximum interstory drift ratios in buildings with uniform lateral stiffness implementing a continuum structural model derived from a combination of a flexural and a shear cantilever beams assumed to be connected by axially rigid members that transmit horizontal forces. Gupta and Krawinkler (2000) outlined a process estimates seismic roof and storey drift demand starting from the spectral displacement demand. Similar to what is pointed out in FEMA-356 (2000) where target displacement equalized by corrected spectral displacement, the spectral displacement demand associated with roof drift demand modified by correction factors to relate SDOF response to MDOF response, elastic demand to inelastic demand, individual storey drift to roof drift, to account P-Δ effect, and to take into consideration stiffness degradation and strength degradation. Gülkan and Akkar (2002) defined a simplified expression for the drift spectrum and showed that the shear beam fundamental mode and response spectrum concepts could be combined to the ground story drift within an error bound of 10 percent for shear frames with fundamental periods less than 2s under near-fault ground motions. Matamoros *et al.* (2003) presented a simplified procedure to estimate drift, based on relationship between spectral acceleration and spectral displacement based on the ratio of mass over stiffness of structure, where the mass is assumed proportional to the total floor area of building and the stiffness proportional to the sum of the effective area of columns and walls at the base storey. Akkar *et al.* (2004) proposed a procedure calculates the maximum ground storey drift ratio and maximum interstory drift ratio (MIDR) along the height of the structure for MRF behaviour based on modal analysis concept. The subject of study followed by some other complementary researches such as Erdogan (2007) modified the study done by Akkar *et al.* (2004) which is valid for uniform buildings, formula been proposed to modify equations by including stiffness distribution, soft storey factor, regular storey height and number of stories. By observing the variation in the maximum interstory drift ratio of inelastic building systems Ay and Akkar (2008) modifies the former version (Akkar *et al*. 2005) to estimate inelastic MIDR of frame type structures. Dinh and Ichinose (2005) proposed a probabilistic procedure to evaluate storey drift of RC buildings. As result of pushover analysis mean and standard deviation of seismic storey drift are acquired as superposition effect of elastic deformation and plastic deformation due to total and storey failure mechanism. Yong *et al*. (2007) outlined a methodology to assess the seismic drift demand of RC buildings with limited structural and geotechnical information. Knowing depth of soil above bedrock, height of building, estimated fundamental period of building and calibrated dynamic drift factor determined by response spectrum analyses leads to building seismic induced drift. Xie and Wen (2008) developed a method based on the continuous Timoshenko beam model to provide an estimate of maximum interstory drift demands for earthquake ground motions. Lam *et al*. (2010) developed a spreadsheet to support the early stage of the design process. This spreadsheet is able to perform dynamic time-history analyses of buildings with minimum input information, aiming to obtain estimates of dynamic response parameters useful for the design, among which the interstory drifts. Lin and Miranda (2010) introduced two approximate methods for the estimation of the maximum inelastic roof displacement of multi-story buildings based on the analysis of the equivalent first-mode elasto-plastic SDOF system. Takewaki and Tsujimoto (2011) showed how the prediction of the drift demand is crucial to rationally scale the design earthquake ground motions needed for the risk-based design of tall buildings.

As a matter of researches dedicated to evaluation of storey stiffness, Heidebrecht and Smith (1973) computed the horizontal force-deflection relationship for each column of the frame conventionally considering the reduction in stiffness related to degree of rotational restraint that derives by assuming points of contraflexure (i.e., of zero bending moment) located at mid-span of beams and at mid-height of columns. Likewise, Paulay and Priestley (1992) presented an approximate analysis of lateral stiffness of frames starting from that related to substructures individuated according to given assumption on the position of contraflexure points. Heidebrecht and Smith considered, as substructure, two mid-height columns (taken respectively above and below a given floor level) supported by two beams, whereas Paulay and Priestley focused their analyses on a full height column supported by four adjacent beams. Schulz (1992) presents a closed-form expression for approximating the lateral stiffness of stories in elastic rectangular frames. They assumed each storey isolated from the rest of the frame in such way girders and columns act with points of inflection at mid-length. Based on the idea of an equivalent single-bay single-storey frame module for every storey of the real multi-bay multi-storey frame, Hoseini and Imagh-e-Naiini (1999) presented a quick method to estimate lateral stiffness of building system in which main frame can be substituted by the one which consists of a given number of sub-frames or frame modules connected to each other by hinges. Ramasco (2000) proposed a simplified model in which planar frame are identified as single vertical cantilever, having the height of the building, in order to calculate the stiffness of storey. Each cantilever is restrained at each storey by rotational springs conventionally accounting for the bending stiffness of the beams located at that level. Points of zero bending moment are assumed to be located at the mid-height of the column at each storey, except for the ground floor where they are assumed to be positioned at 2/3 of the storey height. Eroglu and Akkar (2010) developed a rational methodology to adapt the lateral stiffness variation of discrete buildings to continuum models. It accounts for the changes in the boundary conditions along the building height and defines the flexural and shear components of total lateral stiffness at the story levels.

Alternative approaches to perform preliminary evaluation of storey stiffness and interstory drifts respectively are proposed herein. Their formulation started from these statements:

1. Lateral displacements and storey stiffness values for frames subjected to earthquakes are strongly related to each other; therefore, their calculation can be performed in series. That is, firstly assessing lateral stiffness of each storey, and then evaluating lateral drifts based on the former values.
2. Existing methods addressed to evaluate storey stiffness are typically based on the implicit assumption the frame behaves not so different from shear type structure. They do not always lead to well approximated results for new buildings designed according to the 'capacity design' philosophy that generally drives to 'strong column-weak beam' systems. The proposed procedure aims at giving satisfactory results also when a flexural type or an intermediate type of frame building is considered.

To verify the result and check the proficiency of the proposed technique the aforementioned methods are applied to 9 'ideal' frames and to 2 'real' buildings designed according to the Italian seismic code OPCM 3274 (2003). As a matter of involved parts the latter is consistent to Eurocode 8 (2003) provisions. Authors set as main goal of the study providing to the practitioners quick ways to predict some of the results the most commonly adopted types of analysis (i.e., linear) for low and medium-rise buildings would lead to. The consideration of plasticity and nonlinear effects is beyond the scope of this work.

The results are compared with those computed using the standard procedures defined by the above code as well as the ones derived using some of the above cited already existing approximate methods. To have a better view of how distinctive they are, in the following more details about the latter procedures have been described to make the reader be able to more easily compare them to newly proposed ones.

2. Approximate methods considered for comparison

As been mentioned in the prior section during recent years various methodologies have been suggested either to assess interstory drift or to estimate storey stiffness of frame buildings. Among them the authors selected some of those widely cited in the literature to calculate the storey stiffness (Heidebrecht *et al*. 1973, Paulay and Priestley 1992, Ramasco 2000) and also to attain the storey drift ratio (Miranda and Reyes 1999, Akkar *et al*. 2004). For the convenience of reader first the principal concept of the procedures are summarized. It has been tried to highlight the main aspects of procedures in attempt to manage their practical application. Then, they are adopted with reference to some case studies, together with the new methods proposed herein in order to make comparison among them and to draw interesting concluding remarks.

2.1 Procedures to estimate storey stiffness

Two opposite types of frame systems can be defined on the basis of beam-to-column stiffness. A 'shear type' frame is characterized by beams much stiffer than columns, vice versa as a 'flexural type' system. The lateral displacement shape and the bending moment distribution along the height of the building are very different in these two cases. Fig. 1 shows a shear type frame with its peculiar lateral displacement shape (a) and moment distribution on the columns (b); the points of contraflexure occur at mid-height of these elements. An example of a flexural type frame is then shown in Fig. 1(c) and Fig. 1(d). The behavior is like that of flexural vertical cantilever beams connected by axially rigid members at each level. As a matter of the fact frames generally have the combined behavior of shear and flexural cases. The point of contraflexure on the columns may not necessarily be positioned in the mid-height but in some specific cases they may also be out of the storey column, especially at the first storey (Fig. 1(e)).

The lateral stiffness k_i of a given i-th storey can be calculated as the sum of the lateral stiffness of columns belong to each storey level, that is $k_i = \sum_j k_{ij}$. The stiffness k_{ij} of the j-th column in the i-the storey can be generally expressed as follows

$$k_{ij} = \alpha \frac{12EI_{ij}}{h_i^3} \tag{1}$$

where E is the modulus of elasticity of the material, I_{ij} is the second moment of area of the cross section, h_i is the storey height and α is a real number ($0.25 \leq \alpha \leq 1$) that depends on the degree of rotational restraints given by adjacent beams to the column ends. Coefficient α tends to be 0.25 when a flexural type frame is considered while tends to be 1 for shear type structures (where both ends are fully restrained against rotations). In more general cases, this coefficient results included between these two extreme values as a function of the beam-to-column stiffness ratio. Approximate methods to calculate storey stiffness just aim to quickly define a likely value of α for each column at each level of the building, starting from given assumptions. Regarding the priory mentioned explanation about the engineering viewpoint toward storey stiffness, in following sections three already suggested methodologies are presented so that their analysis of the storey stiffness might be more apparent.

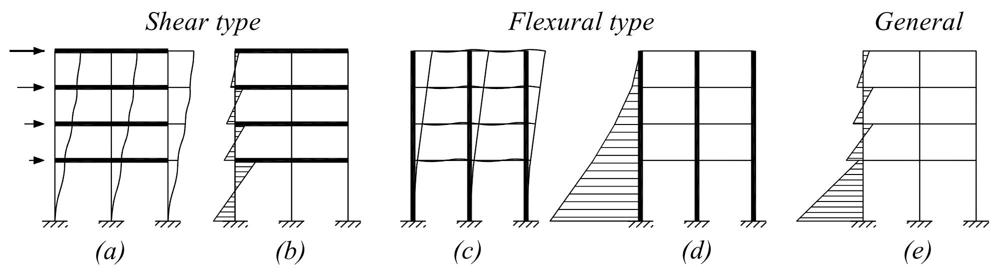

Fig. 1 Frame behavior types

2.1.1 Approximate storey stiffness by Heidebrecht and Stafford Smith

Heidebrecht and Stafford Smith (1973) considered the structure consists of flexural and shear vertical cantilever beams connected by axially rigid members able to transmit horizontal forces. The contribution of the single column to the equivalent shear-flexure beam is calculated assuming that points of contraflexure occur at mid-height of the columns and at mid-span of the beams. Therefore, the sub-frame individuated by the 4 points of zero bending moment in the surrounding of each beam-column joint was analyzed; that is, the half length of two adjacent beams and the half height of the columns above and below the joint are involved (Fig. 2(a)). Assuming that these two columns have the same second moment of area I_h and height h, authors provided the horizontal force-deflection relationship of the storey, corresponding to the following expression of the reduction coefficient α, where b_1 and b_2 are the total length of adjacent beams, I_{b1} and I_{b2}, their second moments of area.

$$\alpha = \left[1 + \frac{2I_h}{h\left(\frac{I_{b1}}{b_1} + \frac{I_{b2}}{b_2} \right)} \right]^{-1} \tag{2}$$

2.1.2 Approximate storey stiffness by Paulay and Priestley

Paulay and Priestley (1992), slightly modifying a similar method proposed by Muto (1965), presented an approximate analysis aiming to determine the share of each of columns of a given storey in resisting the total storey shear force. This analysis passes through the estimation of lateral stiffness of each column. They made the same assumptions as Heidebrecht and Stafford Smith (1973) about contraflexure points position on beams and columns, even if analyzed a different model constituted by the entire column at the storey of interest and half length of the four beams adjacent (Fig. 2(*b*); k_i is the relative stiffness of the i-th element, beam or column, that is the ratio I_i/l_i of the second moment of area of the cross section over the length of that member). Furthermore they assumed that relative stiffnesses of beams at the top and the bottom of a given storey are similar.

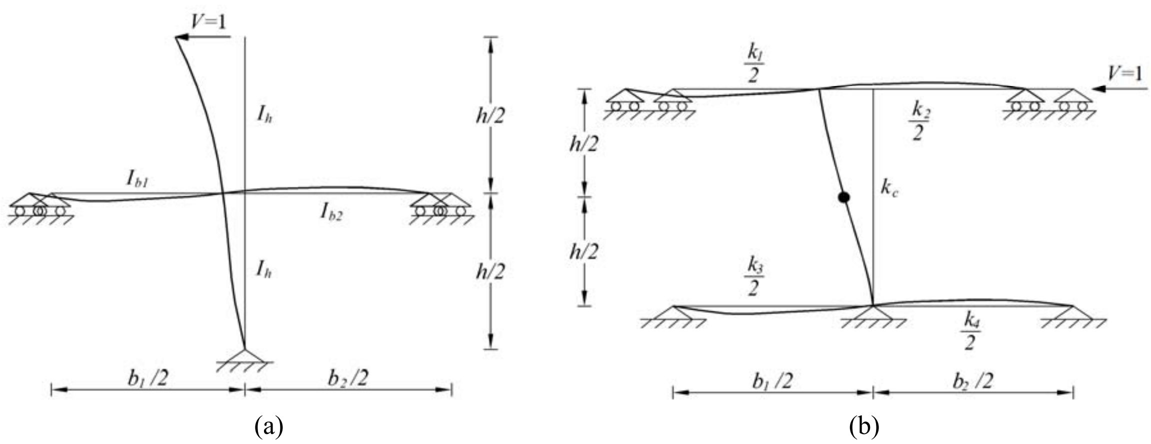

Fig. 2 Sub-frames considered for storey stiffness evaluation by (a)Heidebrecht *et al*., (b) Paulay *et al*.

Cited authors, once introduced the beam-to-column relative stiffness (i.e., the ratio $K = (k_1+k_2+k_3+k_4)/(2k_c)$ between the mean value of the beam stiffness over the column stiffness), defined the factor above named as α (refer to Eq. (1)), relating to the degree of restraint provided by the beams to the column ends, as the ratio $K/(K+2)$.

2.1.3 Approximate storey stiffness by Ramasco

Ramasco (2000) modelled the generic planar frame as an equivalent cantilever (Fig. 3) where at the i-th level (i = 1, 2, …, n where n is the total number of storeys) the second moment of area $(EI)'_i$ is equal to the sum of the moments of inertia of the columns in that level. The rotational restraint offered by the beams is modelled applying rotational springs to the cantilever, at the corresponding level. The spring at the i-th level is assumed to have a rotational stiffness $k'_{\varphi i}$ equal to $12\sum_j(EI/l)_j$ where the sum is extended to all the beams belong to i-th level. The ratio $(EI/l)_j$ represents the flexural stiffness of the j-th beam; E, I and l are the elastic modulus, second moment of area and span length of that beam respectively. This derives from the assumption that all joint rotations at the same level have the same value.

Ramasco assumed an approximate lateral force distribution is known and the contraflexure points occur at mid-height of columns above the first floor, at 2/3 of the height from the bottom for the first storey columns, leading to formulate the following expression of the coefficient α for the i-th storey

$$\alpha = \frac{1}{1+\dfrac{3(EI)_i}{h_i\, k'_{\varphi i}}\beta_i} \qquad \text{where} \quad \beta_i = \left[\left(\frac{V_{i+1}\cdot h_{i+1}}{V_i \cdot h_i}+1\right)+\left(\frac{V_{i-1}\cdot h_{i-1}}{V_i\cdot h_i}+1\right)\frac{k'_{\varphi\, i}}{k'_{\varphi\, i-1}}\right] \tag{3}$$

where h_i and V_i are interstory height and storey shear at the i-th level of the building. As for the expression of α proposed by other authors, this tends to be one for increasing values of the beam-to-column stiffness ratio.

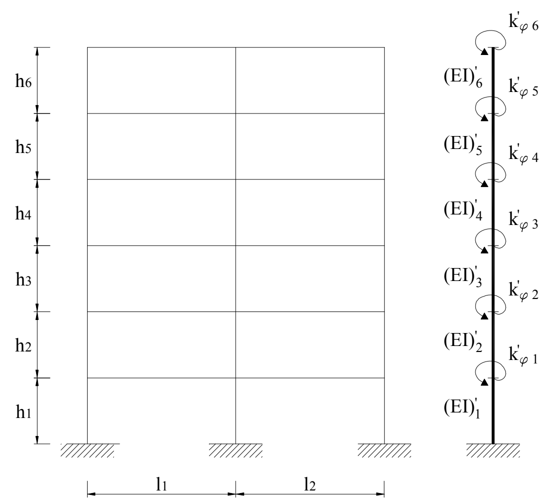

Fig. 3 Example of equivalent cantilever beam used by Ramasco (2000) to evaluate storey stiffness

2.2 Procedures to estimate lateral drifts

The interstory drift ratio (IDR) is defined as the ratio of the interstory horizontal relative displacement to the interstory height. Actually, according to the performance based design codes, damage states result from lateral displacement of building, generally to be within given limit values. As a matter of concern of this study which tries to draw a simplified approach to estimate interstory drifts, two existing methods aiming at solving the same problem are considered herein to have a suitable benchmark to compare the results.

2.2.1 Approximate lateral drifts by Miranda and Reyes

The method proposed by Miranda and Reyes (1999) assumes the simplified model introduced by Heidebrecht and Stafford Smith (1973), which is the combination of a flexural and a shear cantilever beams connected by axially rigid members able to transmit horizontal forces. Starting from this model, they showed a way to estimate the maximum roof displacement (u_{roof}) and the maximum interstory drift ratio (IDR_{max}) in buildings with uniform lateral stiffness. In 2002, Miranda extended the procedure also to structures with non-uniform lateral stiffness along the height. In both cases the contribution of the first mode is considered only and the distribution of masses is assumed to be uniform along the height of the building.

Cited authors expressed IDR_{max} as a function of the total height of the building (H), its fundamental period of vibration (T_1), a dimensionless parameter (α) that controls the degree of participation of overall shear and flexural deformation (the value of 1 for structural wall structures, 3.5 for dual systems, 10 for frame buildings is suggested), the number of stories (N), the height of each storey (h_i), and finally a dimensionless parameter (a) that controls the shape of lateral load (0.01 for triangular lateral load, 2000 for uniform lateral load, intermediate values for parabolic shapes).

A closed-form expression for the first mode shape is given as a function of a, α and H. The consequent approximate value of the first modal participation factor β_1 is multiplied by the spectral displacement evaluated at the fundamental mode of vibration $S_d(T_1)$ in order to obtain the maximum roof displacement (u_{roof}). A second closed-form expression is then given for the drift value at the generic height along the building, normalized by the roof drift ratio u_{roof}/H whose maximum value leads to calculate the desired value of IDR_{max}. The variation of lateral stiffness along the height is taking into account by Miranda (2002) involving besides two additional parameters have been taken in the procedure. The first parameter is the ratio of the lateral stiffness at the top and base of structure which the second one controls the shape of the stiffness variation along the height.

For the applications shown in the following, values of a and α equal to 0.01 (suggested for inverted triangular distribution of lateral forces) and 10 (frame buildings) are assumed respectively. Finally, the fundamental period of vibration is evaluated according to the simplified expression suggested by Eurocode 8 (2003), i.e., $T_1 = C_t\,(H)^{3/4}$ with $C_t = 0.075$ devised for reinforced concrete frame structures.

2.2.2 Approximate lateral drifts by Akkar et al.

Akkar *et al*. (2004) proposed a procedure to calculate the maximum ground storey drift ratio (GSDR) and maximum interstory drift ratio (IDR_{max}) along the height of the structure for MRF behaviour based on modal analysis concept. Using the first mode shape of a uniform, continuous shear beam, they found the maximum ground storey drift ratio of shear frames $GSDR_{sh}$ can be approximated as follows

$$GSDR_{sh} = 1.27\, sin\left(\frac{\pi h}{2H}\right)\frac{S_d\left(T_1, \xi\right)}{h} \tag{4}$$

where S_d (T_1, ξ) is the spectral displacement value at the fundamental period T_1 and damping ratio ξ; H and h are the frame and single storey height respectively. Two modification factors γ_1 and γ_2 multiply the $GSDR_{sh}$ value, so changing it first to the value GSDR corresponding to the frame type under examination, and then to the IDR_{max} value. These two factors, derived via regression analysis, depend on the fundamental period of the structure and the beam-to-column stiffness ratio ρ (the larger values of ρ imply shear behaviour and make γ_1 tending to 1; the smaller values of ρ indicate a frame behaviour tending to a flexural type one and make γ_1 assuming larger values). For the numerical applications, the fundamental period of vibration is evaluated according to the simplified expression suggested by Eurocode 8 (2003), as for the method discussed in section 2.2.1.

3. Proposed approximate methods

The already existing approximate procedures concerning the lateral storey stiffness are generally based on the implicit assumption that frame behaves like a shear type structure or that similar rotation of joints occurs at the same storey and/or at two consecutive levels. Actually the shear frame model is widely used for expeditious analysis to be done by hand or simple spreadsheets given the simplicity of the governing equilibrium equations and the ease with which these can be solved.

In the worldwide earthquake prone regions, recent building codes unanimously give to the so called ‘capacity design’ principle a crucial role within the structural design. According to this philosophy, some elements or components involved in the lateral force resisting system are first chosen and designed for energy dissipation under severe imposed deformations. These members are chosen among the others given their larger inherent ductility and ability to dissipate energy. The remaining potentially brittle regions or components are protected ensuring that their strength exceeds the demands originating from the plasticization of the first, ductile connected elements (Paulay and Priestley 1992). When designing a frame structure is concerned, this philosophy is also applied to make columns (typically more brittle than girders) to be stronger than beams which in many cases leads to define frames where columns are also stiffer than beams and the lateral behavior inclined to be quite far from shear type. An approximate method to calculate storey stiffness should be able to analyze also such a kind of structures where contraflexure points on columns can also be located outside the corresponding element, especially at the first storey of the building. Also the assumptions about similarity of joints rotations made for some methods seem to be not convenient when procedures are applied to identify eventual structural irregularities in height. This hypothesis, reasonably fulfilled for regular structures, however may alter the results in a non-conservative way.

The approximate procedure proposed herein to evaluate storey stiffness has devoted to achieve satisfactory results also when applied to a flexural or an intermediate type of frame building. Furthermore, this method is able to evaluate stiffness in an appreciably good manner also when inertia of elements, beam spans and storey heights are very inhomogeneous along the height of the building. Also, the relationship between storey stiffness and lateral displacement is exploited to estimate interstory drifts for a rapid check to fulfill seismic serviceability limit state requirements by code. Finally, supplementary numerical analyses applied on different frame buildings are shown to compare results given by proposed approximate procedures with those by a FEM analysis. The results obtained from the other approximate methods described above are compared to those from proposed procedure as well.

3.1 Estimating storey stiffnesses

As other existing methods the proposed one leads to define lateral storey stiffness starting from the analysis of a sub-structure ideally extracted from one of the planar frames composing the building. This sub-frame is isolated by the remaining structure disconnecting it at probable points of zero moment, so making each end solicited by shear force only (Fig. 4). Such a model is assumed as a reference to calculate lateral stiffness of the column having height h belonging to the generic storey of a given building. Adding the lateral stiffness of all columns of the same level, subjected to shear forces in a given direction, the storey stiffness along that direction can be obtained. The parameters h_1 and h_2 define the position of the contraflexure point on the columns above and below the considered one, respectively; b_1 and b_2 represent the span length of the beams at the two sides of the columns; I_c, I_{c1}, I_{c2} (columns) and I_{b1}, I_{b2}, I_{b3}, I_{b4} (beams) are the second moment of area relative to each of the seven elements composing the model, as shown in Fig. 4.

For the complete definition of the model, assumptions on contraflexure points have to be defined (Fig. 4). Similarly to existing methods, these points are assumed to be located at mid-span on beams which is the simplest as well as realistic assumption can be made. Actually they have a less effect on the storey stiffness value, the latter being much dependent on the position of the point of zero moment on the columns above and below the level under examination (that is on the value of the parameters h_1 and h_2 shown in Fig. 4). According to Muto (1965) these values are assumed from the results of a wide range of parametric analyses on different frame structures subjected to lateral loads. Muto presented the probable distance of the point of contraflexure on a given column from its base as a function of:

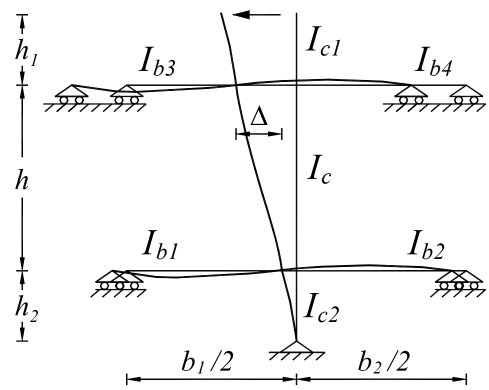

Fig. 4 Sub-frame model adopted for the proposed method

• the beams-to-columns stiffness ratio K, i.e., the dimensionless parameter defined above with reference to the method by Paulay and Priestley (1992) as the total sum of relative stiffnesses of beams above and below the column divided by two times that of the column;
• the total number of storey (n);
• the storey level the column under consideration belong to (i = 1, 2, …, n) ;
• the lateral force pattern (uniform or inverse triangular).

In particular, Muto (1965) provided the dimensionless ratio (η) of the above distance by the height h of the column (Fig. 5); relative stiffnesses are defined as said above about the Paulay and Priestley's method. Many analyses have been conducted by authors on different type of frames in order to assess the actual ability of the Muto's procedure in predicting contraflexure point position on columns, leading to very satisfactory results: approximate (Muto 1965) points of zero moment resulted to be always very close to the ones evaluated by a FEM analysis (performed with SAP2000NL 2011). Fig. 6 illustrates some of these frames and the results obtained (an inverse triangular load is considered for these cases). Two of these are tending to be frame with shear type behavior (a, d), two with flexural type (c, e) and the last one is intermediate frame type (b). The good approximation of the Muto's method can be observed, even when contraflexure points occur out of the element (e.g., first storeys of frames that tend to the flexural type).

The elastic analysis of the statically indeterminate sub-frame (Fig. 4) has been performed to derive the relation between shear and drift of the central column which is to get the lateral stiffness of the latter. The following expression of the coefficient α (above defined as the ratio of the actual stiffness by the value would correspond to shear type behaviour) has been obtained. Relative stiffness coefficients A and B are given by Eq. (6).

$$\alpha = \frac{1}{1-3\left(1+\frac{2h_1}{h}\right)\left[1-\left(1+\frac{EI_c\left(h+2h_2\right)}{A\left(2hh_1+h^2\right)}\right)\cdot\left(\frac{\frac{h}{EI_c}+\frac{1}{B}}{\frac{h}{EI_c}-\frac{EI_c}{ABh}}\right)\right]} \tag{5}$$

$$A=\left(\frac{EI_c}{h}+\frac{6EI_{b3}}{b_1}+\frac{6EI_{b4}}{b_2}\right);\qquad B=\left(\frac{EI_c}{h}+\frac{6EI_{b1}}{b_1}+\frac{6EI_{b2}}{b_2}\right) \tag{6}$$

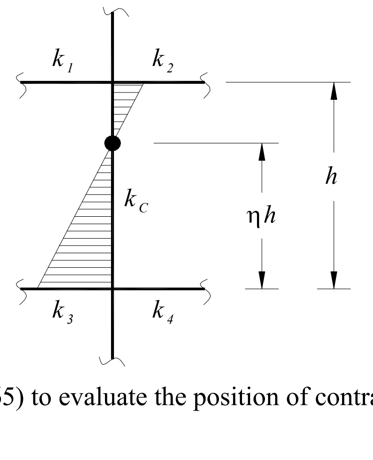

Fig. 5 Muto's method (1965) to evaluate the position of contraflexure point on columns

Regarding Eq. (5), consistently with its definition, α tends to be 1 when the beam-to-column stiffness tends to infinity (i.e., shear type frame). As for the existing methods, special comments about the ground and roof level stories have to be done, since they present peculiar issues compared to the intermediate ones. Thus, the model for the first storey is assumed as what is shown in Fig. 4 but the column is fully restrained against rotations at the base and have only two beams at its top (Fig. 7). This model leads to the expression of α (the index '1' stands for '1st storey') in Eq. (7).

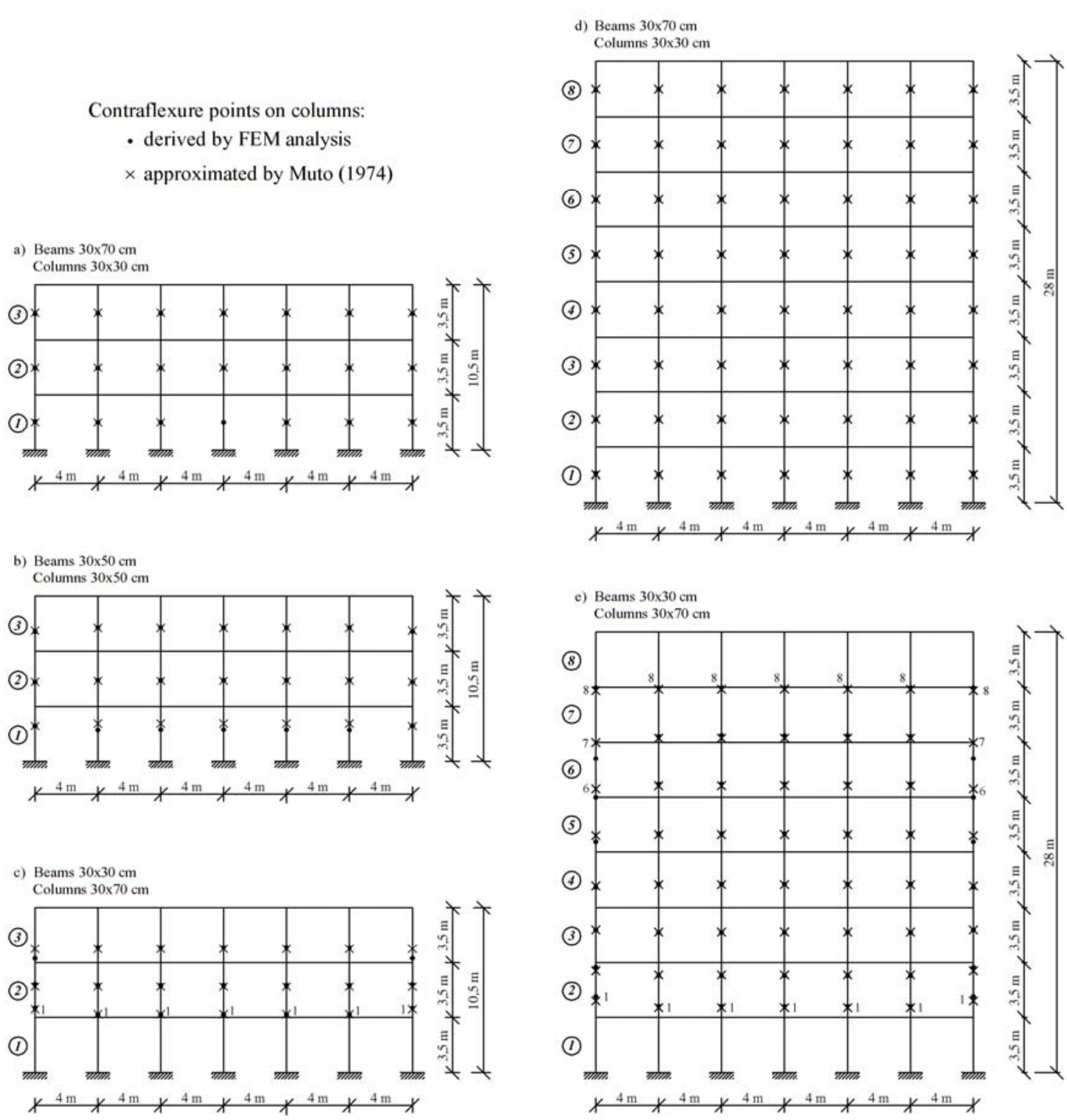

Fig. 6 Muto's method (1965) applied to exemplary structures to evaluate contraflexure points on columns: comparison with those from FEM analysis

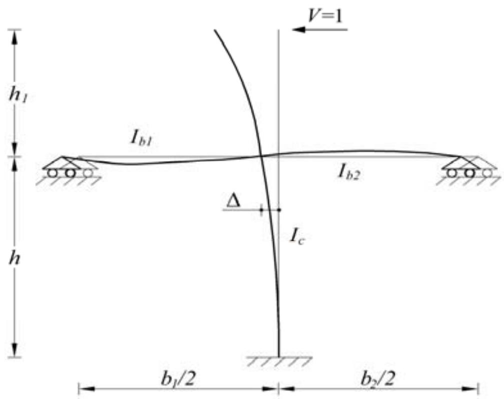

Fig. 7 1st storey sub-frame model

$$\alpha = \alpha_1 = \left[1 + \frac{1 + \frac{2h_1}{h}}{1 + \frac{h}{I_c}\left(\frac{I_{b1}}{b_1} + \frac{I_{b2}}{b_2} \right)} \right]^{-1} \tag{7}$$

where h and I_c are the height and moment of inertia of the 1st storey column, h_1 is the distance of the contraflexure point on the 2nd storey column measured from the first floor level, I_{b1}, I_{b2} and b_1, b_2 are respectively the second moment areas and span lengths of the two beams adjacent to the column under consideration. For the last storey the model in Fig. 4 has still to be assumed, as for the other levels (Eqs. (5) and (6) have to be used). It is only needed to consider an 'auxiliary' column that ideally extends the real one above the roof for a length h_1 equal to the last interstory height h minus the distance h_2 relative to the penultimate floor.

3.2 Estimating lateral drifts

Applying the above proposed procedure, a simple approach to calculate interstory drifts is suggested herein starting from the knowledge of storey stiffnesses. The process initiates by approximating the storey masses m_i of the i-th floor. Once the overall floor area is known, typical values of seismic mass per unit area which are usually considered for a preliminary design have to be used (e.g., 0.8-1.1 tons/m^2 is a reasonable range of values for reinforced concrete buildings; Ghersi 1986). Then the fundamental period T_1 of the building has to be estimated according to one of the simplified expressions suggested by codes (e.g., Eurocode 8, 2003) or even by literature (e.g., Paulay and Priestley 1992). Such information allows the designer to calculate approximate lateral force distribution would be used for a linear static analysis according to Eurocode 8 (2003) or other seismic design codes. Dividing the forces by the number of parallel planar frames composing the building along the push direction under exam, the quota F_i (i = 1, 2, …, n) of lateral force applied at the i-th floor of the generic frame is known; therefore, the storey shear V_i (Fig. 8) is derived as follows

$$V_i = \sum_{j=i}^{n} F_j \tag{8}$$

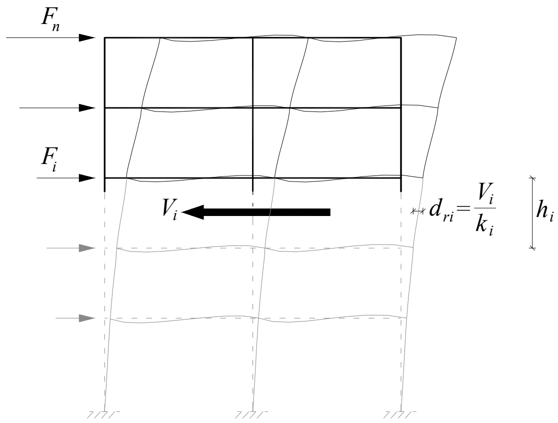

Fig. 8 Definition of storey shear V_i

Consequently, interstory drift d_{ri} at the i-th storey can be evaluated as the ratio of the storey shear V_i and the stiffness k_i. The corresponding interstory drift ratio (IDR_i) is determined by dividing the drift value (d_{ri}) by the storey height (h_i). Finally, the maximum interstory drift ratio (IDR_{max}) is calculated as follows

$$IDR_{max} = \max_i IDR_i = \max_i \left(\frac{d_{ri}}{h_i} \right) = \max_i \left(\frac{V_i}{k_i \, h_i} \right) \tag{9}$$

4. Numerical applications: evaluation of the proposed methods

Both approximate methods proposed herein have been applied to 9 'ideal' planar frames and to 2 'real' buildings. All the considered structures are assumed to be located in Italy, in the 1st seismic zone (peak ground acceleration equal to 0.35g), soil type C (OPCM 3274 2003; Eurocode 8 2003). The 'ideal' frames are generated such that all columns and all beams have the same cross section. Authors considered:

• three types of frame different for beams and columns cross sections (Table 1): type 1 is close to be shear type, having beams much stiffer than columns, type 2 corresponds to an intermediate case, and type 3 conversely tends to have a flexural behaviour;

• three types of overall heights (3, 8 and 15 storeys respectively) are then considered for each frame type,

for a total of 9 'ideal' frames. Interstory heights and span lengths have been assumed to be equal to 3.5 m and 4.0 m respectively in all cases (Fig. 9).

Table 1 'Ideal' frames: three different typologies

Frame type	Beams section (cm^2)	Columns section (cm^2)
1	30×70	30×30
2	30×50	30×50
3	30×30	30×70

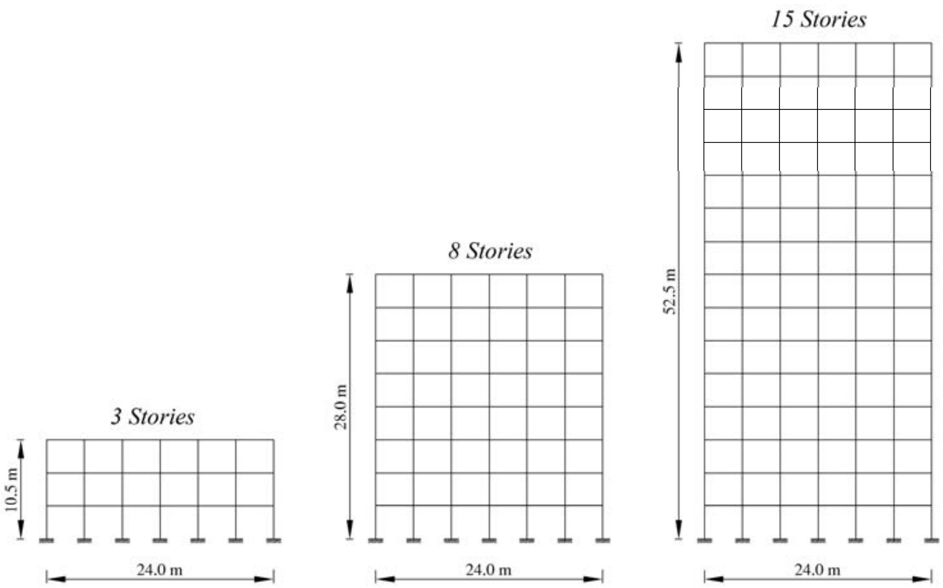

Fig. 9 Different height of 'ideal' frame structures

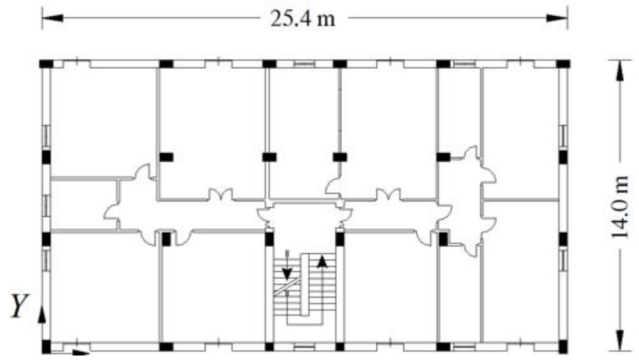

Fig. 10 Standard floor plan for 'real' buildings (Cosenza *et al*. 2005)

'Real' frames are two four-storey 3D reinforced concrete buildings having rectangular plan, designed according to the Italian seismic code OPCM 3274 (2003) - the latter being consistent for the involved parts to Eurocode 8 (2003) provisions - for two different ductility levels respectively (Cosenza *et al*. 2005). The first sample building designed for a high level (type 'A' according to the Italian design code, and type 'DCH' stated in the Eurocode 8), and the second one for a lower level (type 'B' and 'DCM' for the Italian and European codes, respectively). The storey height is 4.0 m at the first floor, and 3.2 m for the other three. Uncracked sectional stiffness has been considered for all the numerical applications, although the user may also make a different choice. Also the other methods from literature described in previous sections have been applied to the same structures for comparison.

4.1 Applications to the 'ideal' frames

The lateral stiffness of each storey of the 9 frames has been calculated according to the proposed method as well as applying the other above cited methods taken from literature for comparison. The results have been also compared with those coming from a FEM analysis (with SAP2000NL 2011) of the fully modelled structures according to the standard procedures given by the code called modal response spectrum analysis (MRSA) and linear static analysis (LSA). It is worth noting that these code-based methods generally lead to similar results even if non identical. This for well known reasons related to the fact that LSA takes into account only the fundamental vibration mode, giving to it a conventional lateral shape and a predetermined amount of participating mass, neglecting the effects of higher modes.

For the sake of brevity, in Fig. 11 only the comparisons for the three-storey frames are shown; however, the comparison extended to the all frames is carried out in Fig. 12. Here each point corresponds to a single storey of a given frame and has abscissa equal to the approximate value of its storey stiffness, ordinate equal to the same value calculated by MRSA. Fig. 12 is such that the adopted approximate method is more accurate as more points result to be aligned on or close to the bisector of 1st and 4th quadrant.

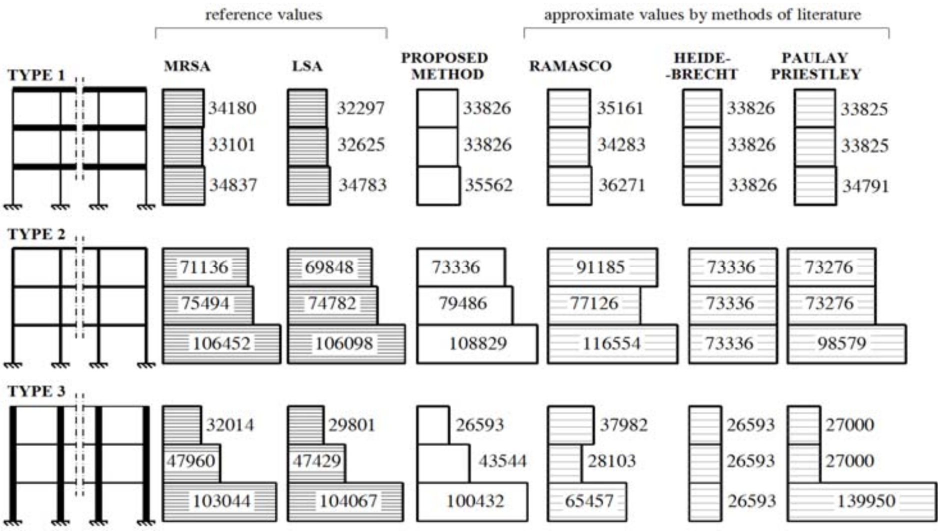

Fig. 11 Storey stiffnesses of the three storeys frames: comparisons (kN/m)

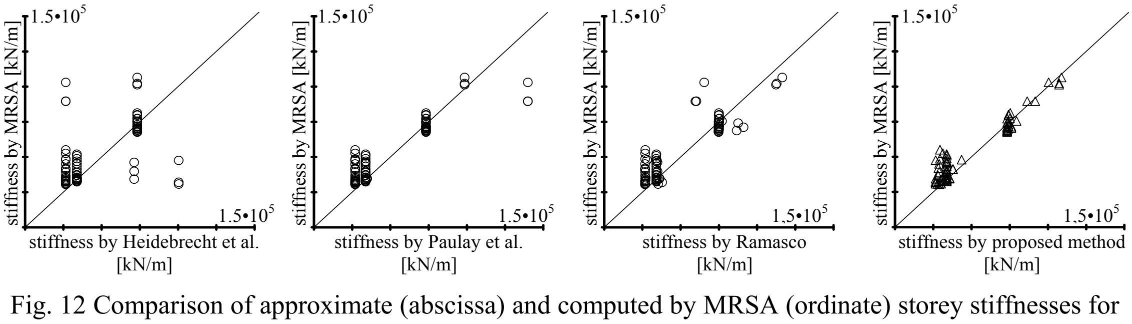

Fig. 12 Comparison of approximate (abscissa) and computed by MRSA (ordinate) storey stiffnesses for all the 'ideal' frames

The results show that all the investigated methods, as expected, leaded to a good approximation when the frame is closer to a shear type. Divergent results have been obtained from different methods as the frame approaches the flexural type. In such cases, the proposed method seems to be able to yield more accurate values. The good approximation of the method may also be derived by observing the reduced scattering around the ideal 1:1 line (Fig. 12(d)) compared with the other methods, especially for higher values of stiffness (lower stories).

As known, most of the seismic design codes require checking the stiffness regularity along the height of a building on the basis of the percentage variation between the lateral stiffness of two adjacent stories. According to these rules, a given structure can be defined 'regular' in elevation if the storey stiffness changes from a level to the upper one gradually, without abrupt variations. These variations have to be included in a given range acceptable for a regular lateral response of the structure. Therefore, more than the single values of storey stiffness, the designer who wants to check the regularity in elevation of a building as conceived with a preliminary design, is interested to evaluate the percentage variations of this value along the height of the structure. For this reason the results have been analyzed also in these terms (Fig. 13). The figure shows the comparison between approximate and evaluated by MRSA stiffness variations. Also in this case, the better results are those corresponding to the points as much as close to the bisector. One can notice that methods from literature often leaded to null or almost null variation of stiffness along the height even when variation measured by MRSA assume significant values. In some cases estimated variation of stiffness resulted to have opposite sign to the one by MRSA. Refer to Fig. 13(d), it is possible to notice a good agreement of approximate values of stiffness variation to the one derived by the complete FEM analysis. Indeed, comparing percentage variations of stiffness, the scattering of points around the bisector (Fig. 13(d)) seems to be reduced compared to the one referred to absolute storey stiffnesses (Fig. 12(d)).

The nine ideal frames have been analyzed also in terms of lateral displacements to define interstory drifts and to identify the maximum interstory drift ratio along the height of the building (IDR_{max}). Firstly this is done using a complete FEM model of the structures and performing a modal response spectrum analysis as well as a linear static analysis, whose results have been taken as reference. Then the approximate methods by Miranda, by Akkar *et al.* and the one proposed herein have been applied.

For the application of the proposed method, approximate storey masses have been assessed assuming typical values of seismic mass per unit area for residential RC frame structures (1 tons/m^2, 0.9 tons/m^2 for the roof). According to these assumptions each floor of the frames under consideration has a seismic mass equal to 98 tons, except the last floor where the mass is 88 tons.

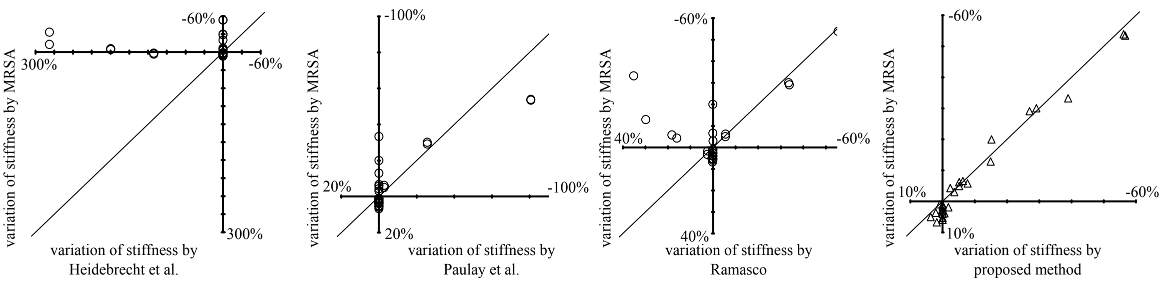

Fig. 13 Comparison of approximate (abscissa) and computed by MRSA (ordinate) percentage variations of storey stiffness along the height of the 'ideal' frames

It is worth noting that MRSA and LSA yielded similar results for all cases. The comparison of the maximum interstory drift ratio IDR_{max} for each frame calculated by MRSA, LSA and the approximate methods is shown in Table 2. The results produced by the method proposed in this paper are in good agreement with those obtained by MRSA and LSA as well as those deriving from the application of approximate methods from literature. This is further confirmed by the value of the correlation factors shown in the same table for each method, referred to each of the two code-based analyses, as well as the value of the mean ratio between ‘actual’ values and the approximate ones. All these values are not far from 1. In this way the good ability of the investigated approximate methods is demonstrated. The use of the proposed method can be suggested when storey stiffnesses are known; that is, when they have been previously assessed according to the procedure herein introduced.

4.2 Applications to the ‘real’ frame

‘Real’ frames are two four-storey 3D reinforced concrete buildings having rectangular plan. The storey height of both is 4.0 m for the first floor, 3.2 m for the other floors. They apparently have the same geometry, even if they have been designed in order to achieve two different ductility levels, ‘high’ (so named ‘A’ in the Italian code and ‘DCH’ in the Eurocode 8) and ‘medium’ (named ‘B’ or ‘DCM’) respectively. Two different levels of seismic demand for linear analyses have been considered accordingly (OPCM, 2003). In the following, these buildings will be referred to simply as ‘A’ and ‘B’ respectively.

Fig. 14 shows a generic floor plan reporting dimensions of bays and joist directions for the reinforced concrete slabs, valid for both the structures. Building A (the FEM model performed with SAP2000NL (2011) is shown in Fig. 15) framed with deep beams with 40×60 cm^2 cross section at the first level, 40×50 cm^2 at the upper ones. Columns are 40×65 cm^2 at the first and second stories, 40×55 cm^2 at the third storey, 40×50 cm^2 at the last one. Half of the beams at each level of building B are deep, half are shallow. The deep beams have the same cross sections of building A. The shallow ones are 115×22 cm^2 at the first two stories, 105×22 cm^2 at the two upper floors. Columns are 40×75 cm^2 at the first and second stories, 40×65 cm^2 above, up to the roof.

Table 2 Comparison of approximate and computed by MRSA and LSA maximum interstory drift ratio

No. of storeys	Frame type	MRSA	LSA	Miranda	Akkar *et al.*	Proposed method
3	1	0.0062	0.0058	0.0069	0.0068	0.0056
	2	0.0033	0.0033	0.0040	0.0039	0.0031
	3	0.0045	0.0047	0.0052	0.0050	0.0051
8	1	0.0067	0.0066	0.0069	0.0068	0.0062
	2	0.0045	0.0045	0.0045	0.0046	0.0042
	3	0.0062	0.0066	0.0065	0.0063	0.0066
15	1	0.0045	0.0039	0.0040	0.0040	0.0036
	2	0.0040	0.0040	0.0040	0.0041	0.0035
	3	0.0042	0.0043	0.0040	0.0039	0.0042
Correlation factor with data from MRSA				*0.95*	*0.95*	*0.92*
Correlation factor with data from LSA				*0.94*	*0.94*	*0.98*
Mean ratio approximate / MRSA data				*1.04*	*1.03*	*0.95*
Mean ratio approximate / LSA data				*1.05*	*1.05*	*0.96*

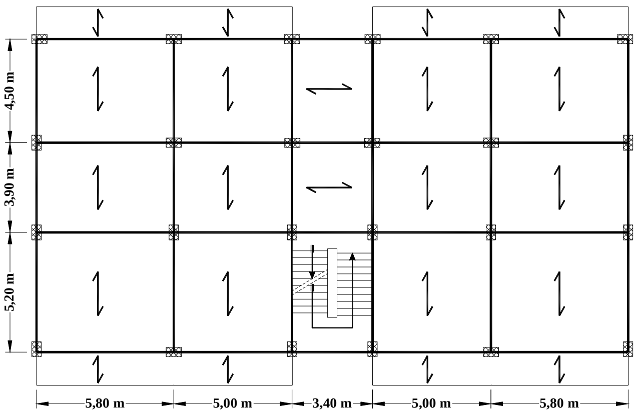

Fig. 14 Floor plan of buildings A and B: span lengths and direction of joists (courtesy of Cosenza *et al*. 2005)

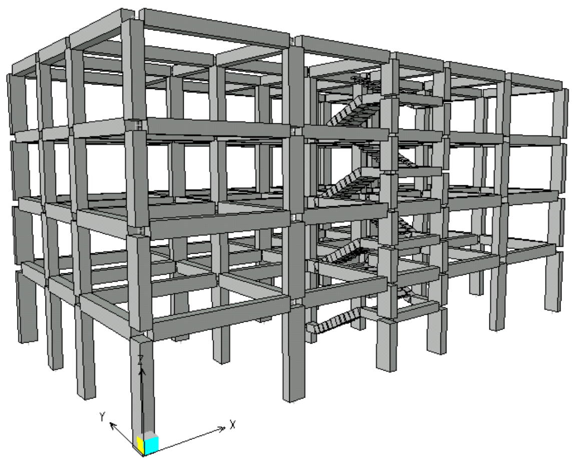

Fig. 15 3D view of the building A (courtesy of Cosenza *et al*. 2005)

Table 3 Storey weights and masses

Storey	Building A				Building B			
	W_i [kN]	m_i [tons]	A_i [m^2]	m_i / A_i [tons/m^2]	W_i [kN]	m_i [tons]	A_i [m^2]	m_i / A_i [tons/m^2]
1st	4132	421	356	1.18	4134	421	356	1.18
2nd	3834	391	356	1.10	3913	399	356	1.12
3rd	3800	387	356	1.09	3828	390	356	1.10
4th	3585	365	356	1.03	3590	366	356	1.03

A detailed analysis of seismic weights W_i (i = 1, 2, 3, 4) has been done for both structures which the results are reported in Table 3. Also masses m_i and areas in plan A_i for each level are given. The mass for unit of area m_i/A_i resulted to be consistent with the above cited range of values typically used for preliminary design of RC structures (0.8-1.1 tons/m^2, Ghersi 1986).

A modal analysis of the FEM model of these two structures has been performed. For both of them, the first mode resulted to be translational along X, and the second translational along Y. The first two periods of vibration have been assessed as 0.511 s and 0.473 s for building A; 0.548 s and 0.509 s for building B.

With the exception of the stairs, both are symmetric in both directions X and Y (Fig. 10); hence, they have been modelled for earthquake excitations along the X and Y directions as the sequence of all the planar frames parallel to the X and Y axes. All the planar frames are assumed to be connected by axially rigid horizontal elements able to transmit horizontal forces and assure the displacements compatibility due to the presence of a concrete slab at each floor.

Storey stiffnesses of the two buildings have been first analyzed, according to the standard methods given by codes and to the approximate methods, as done before for the ‘ideal’ frame structures (Tables 4 and 5). It is worth noting that the procedure by Heidebrecht *et al*. (1973) has not been applied since it is not suitable for structure having columns cross sections changing along the height. In order to allow a more direct comparison of approximate results with the MRSA outcomes (reference values), Fig. 16 reports the same data in a chart. A larger number of points close to the 45° line indicate a better approximation of the considered method. One can observe that values approximated by the proposed method are in all the cases very close to the one predicted by the MRSA standard procedure. Actually in Fig. 16 filled squares and circles corresponding to the proposed procedure are less scattered around the bisector than empty squares and circles related to the other methods.

Table 4 Storey stiffnesses of the building A (kN/m)

Direction	Storey	MRSA	LSA	Paulay-Priestley	Ramasco	Proposed method
X	1	480521	479317	357298	543521	488110
	2	431458	428584	456103	455147	427497
	3	357715	350662	352900	375535	352900
	4	317978	305509	328203	350931	328203
Y	1	455190	453854	339391	516240	489061
	2	417182	414458	432896	440423	419366
	3	346878	339916	346636	363610	346636
	4	307278	295054	322573	340438	322573

Table 5 Storey stiffnesses of the building B (kN/m)

Direction	Storey	MRSA	LSA	Paulay-Priestley	Ramasco	Proposed method
X	1	493998	492967	404930	522094	468947
	2	389196	385578	380377	390724	355415
	3	315328	308347	292894	319936	292872
	4	275983	259983	291213	321342	291213
Y	1	479831	478883	382771	518808	491365
	2	391478	387721	383608	391130	368330
	3	317188	310155	300651	317922	300622
	4	279340	264705	299089	319315	311857

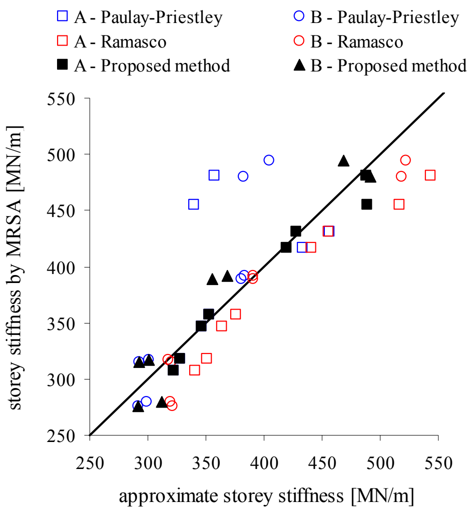

Fig. 16 Comparison of approximate and obtained by MRSA storey stiffness values for buildings A and B

Table 6 Buildings A and B: maximum interstory drift ratio

Building-Direction	MRSA	LSA	Miranda	Akkar *et al.*	Proposed method
A-X	0.0034	0.0034	0.0039	0.0039	0.0034
A-Y	0.0034	0.0035			0.0035
B-X	0.0036	0.0038	0.0039	0.0040	0.0042
B-Y	0.0036	0.0038			0.0041

After that, interstory drifts for both buildings have been calculated according to MRSA, LSA and the proposed approximate method as well as the methods from literature. As far as the proposed procedure is concerned, approximate storey masses have been assessed by assuming values of seismic mass per unit area equal to 1 tons/m^2 for the each storey, 0.9 tons/m^2 for the roof. These conventional values result to be consistent with those have been derived by the detailed analysis of seismic masses mentioned above (Table 3). Fundamental period calculated with the approximate formula given by cited codes (T_l=0.075$H^{3/4}$ with H equal to the total height of the building, equal to 13.6 m), results to be 0.53 s for both structures, close to the values given above from modal analysis. Interstory drift analysis has been done first along the X direction (considering all the planar frames parallel to the XZ plane, in the sense specified above) then along Y (planar frames parallel to the YZ plane) leading to two separate values. It is worth noting that, for a serviceability check of the building under seismic action, the larger of these two values has to be considered as an overall drift demand.

The comparison of the maximum interstory drift ratio IDR_{max} for each frame by MRSA, LSA and approximate methods is shown in Table 6. Values derived from the proposed procedure are in good agreement with those obtained by both MRS and LS analyses, also being comparable with those from approximate methods of literature. A good capability of all the investigated approximate methods in predicting the maximum lateral drift ratio can be inferred. The proposed method is suggested to be used when the storey stiffnesses are already estimated because previously evaluated according to the procedure given herein.

5. Conclusions

For a designer, approximate analyses may be useful to obtain estimates of building behavior during preliminary design or to verify the results of a more sophisticated computer analysis. Two approximate methods have been presented to estimate storey stiffnesses and maximum interstory drift in the first dominated vibration mode of multi-storey frames. It is believed that the proposed techniques can be helpful to quickly evaluate the vertical stiffness regularity and the fulfilment of the code requirements for limiting non-structural damage under moderate seismic action. Since lateral displacements and storey stiffnesses for frames subjected to earthquakes are strongly related one to each other, authors proposed to perform in series the two procedures above; that is, firstly assessing lateral stiffness of each storey, then evaluating lateral drifts basing on the former results.

Assumptions implicitly underlying many existing methods (e.g. shear type behavior of the frame or similar rotation of joints at two consecutive levels; they generally are equivalent to assume contraflexure points located at mid-length of each frame element) to calculate storey stiffness have been removed, considering them unlikely to be fulfilled for modern building, conceived according to the 'capacity design' philosophy. The Muto's procedure (1965) is suggested instead of assessing position of the points of zero moment on columns, even when (e.g. for flexural type frame) they are located far from the mid-height of elements. Interstory drifts are then obtained starting from approximate stiffness values, estimating storey masses on the basis of typical values per unit area usually adopted for preliminary design.

These methods are applied to 9 'ideal' frames and to 2 'real' buildings designed according to the Italian seismic code OPCM 3274 (2003) provisions, consistent with those from Eurocode 8 (2003). The results are compared with those from standard code methods (modal response spectrum – MRSA – and linear static analyses – LSA) as well as derived from the application of widely adopted existing approximate methods.

The results showed that all the investigated methods for assessing storey stiffness leaded to a good approximation when the frame is closer to a shear type. Divergent results have been obtained from different methods as the frame approaches the flexural type. In such cases, the proposed method resulted to be able to yield more accurate values. Comparing percentage variations of stiffness along the height (i.e. the reference values to check regularity in elevation of a given frame), the scattering of approximate values obtained by proposed procedure around the one by MRSA is even more reduced.

Interstory drift values deriving from the proposed procedure are in good agreement with those obtained by both MRSA and LSA, also being comparable with those from approximate methods of literature. New practice is suggested to be used when storey stiffnesses are already estimated because previously evaluated according to the procedure given herein.

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