Structural Engineering and Mechanics, Vol. 46, No. 1 (2013) 1-17 1
DOI: http://dx.doi.org/10.12989/sem.2013.46.1.001

Free vibration analysis of asymmetric shear wall-frame buildings using modified finite element-transfer matrix method

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(Received June 3, 2012, Revised January 25, 2013, Accepted February 19, 2013)

Abstract. In this study, the modified finite element- transfer matrix methods are proposed for free vibration analysis of asymmetric structures, the bearing system of which consists of shear wall-frames. In the study, a multi-storey structure is divided into as many elements as the number of storeys and storey masses are influenced as separated at alignments of storeys. The shear walls and frames are assumed to be flexural and shear cantilever beam structures. The storey stiffness matrix is obtained by formulating the governing equation at the center of mass for the shear walls and the frames in the i.th floor. The system transfer matrix is constructed in the dimension of 6×6 by transforming the obtained stiffness matrix. Thus, the dimension, which is 12n×12n in classical finite elements, is reduced to the dimension of 6×6. To study the suitability of the method, the results are assessed by solving two examples taken from the literature.

Keywords: modified finite element-transfer matrix; vibration; asymmetric; wall-frame

1. Introduction

It is observed in the studies conducted on past earthquakes that the buildings having even a small amount of shear wall survive with little damage. The structural performance analysis of buildings subject to free vibrations can clearly identify the strong and weak aspects of building structures. There are many methods proposed for static and dynamic analysis of structures, the bearing system of which employs shear wall-frames.

One of them is the method in which the continuum system calculation model is used. The continuum system calculation model is frequently used in particular at the pre-dimensioning stage and there exists many studies carried out intended for the method in the literature (Rosman 1964, Heidebrecht and Stafford Smith 1973, Basu *et al*. 1979, Bilyap 1979, Balendra *et al*. 1984, Stafford Smith and Crowe 1986, Nollet and Stafford Smith 1993, Zalka 1994, Li and Choo 1996, Toutanji 1997, Miranda 1999, Mancini and Savassi 1999, Hoenderkamp and Snijder 2000, Kuang and Ng 2000, Ng and Kuang 2000, Wang *et al*. 2000, Hoenderkamp 2001, Swaddiwudhipong *et al*. 2001, Zalka 2001, Hoenderkamp 2002, Miranda and Reyes 2002, Zalka 2002, Potzta and Kollar 2003, Zalka 2003, Aksogan *et al*. 2003, Tarjan and Kollar 2004, Savassi and Mancini 2004, Boutin *et al*. 2005, Miranda and Taghavi 2005, Reinoso and Miranda 2005, Taghavi and Miranda 2005, Georgoussis 2006, Michel *et al*. 2006, Rafezy *et al*. 2007, Kaviani *et al*. 2008, Rafezy and

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Howson 2008, Laier 2008, Meftah and Tounsi 2008, Lee *et al*. 2008, Bozdogan 2009, Savassi and Mancini 2009, Kuang and Ng 2009, Zalka 2009, Yang *et al*. 2010, Rahgozar *et al*. 2010, Malekinejad and Rahgozar 2011a, b, Kazaz and Gulkan 2012, Bozdogan and Ozturk 2012, Wdowicki and Wdowicka 2012, Tekeli *et al*. 2012, Jahanshahia and Rahgozar 2012). Ng and Kuang (2000) recommended a method for dynamic analysis of structures, the bearing system of which consists of shear wall-frames. In this study, the angular frequencies are computed by solving the governing differential equations which are formulated at the the center of bending rigidity idealizing the multi-storey structure as a constant system. It is also assumed that the structure attributes are fixed throughout the height of structure and eccentricity between the center of shear rigidity and the center of bending rigidity is at negligible levels. Rafezy and Howson (2008) obtain the governing differential equations at the center of bending stiffness for the vibration analysis of structures which consist of shear wall-frames and construct the storey dynamic stiffness matrix by solving the differential equation system. The dynamic stiffness matrix method is used and the mass of storeys is assumed to be disrubuted uniformly. Bozdogan and Ozturk (2012) suggested transfer matrix method for free vibration analysis of asymmetric wall-frame structures. This study allows for step changes properties along the height of structures but eccentricity between the center of shear rigidity and the center of bending rigidity are ignored.

In our study, the modified finite element- transfer matrix is proposed for free vibration analysis of asymmetric structures the bearing system of which consists of shear wall-frames. The storey masses are applied at the alignments of the storeys in the proposed method. In the study, with a rigid diaphragm assumption, the shear walls and the frames are treated as flexural and shear cantilever beam structures respectively and the contribution of bending on the shear walls, the local bending on the frames and axial deformations on the columns are ignored.

2. Physical model and method

Fig. 1 shows a typical floor plan of asymmetric, three dimensional wall-frame structures (Ng and Kuang 2000). If shear deformations on the wall and the axial deformations on columns and beams are ignored, the wall-frame structures exhibit the shear- flexure-torsion coupled beam behaviour. In the method, first of all, the storey stiffness matrices are obtained at the center of mass and then the element stiffness matrix is formulated for the i. storey by adding these matrices. Then, the system matrix is constructed using the resulting storey stiffness matrix by means of the known transformations utilized in the literature. The frequency equation is formulated using the boundary conditions.

3. Storey stiffness matrix

3.1 Storey stiffness matrix of shear wall

Under horizontal excitations, the governing equations of the ith storey for shear walls can be written as

$$(EI)_{xi} \frac{d^4 u_i(z_i)}{dz_i^4} - (EI)_{xi} y_c \frac{d^4 \theta_i(z_i)}{dz_i^4} = 0 \tag{1}$$

$$(EI)_{yi}\frac{d^4 v_i(z_i)}{dz_i^4}+(EI)_{yi}x_c\frac{d^4\theta_i(z_i)}{dz_i^4}=0 \tag{2}$$

$$-(EI)_{xi}y_c\frac{d^4 u_i(z_i)}{dz_i^4}+(EI)_{yi}x_c\frac{d^4 v_i(z_i)}{dz_i^4}+(EI)_{wi}\frac{d^4\theta_i(z_i)}{dz_i^4}=0 \tag{3}$$

where u_i and v_i are the lateral deflections of the geometric center, respectively, θ_i is the torsional rotation of the floor plan about geometric center at the given height, z_i is the vertical axis of each storey.

$(EI)_{xi}$ and $(EI)_{yi}$ are the equivalent flexural stiffness of the storey for walls in x and y directions and can be calculated as follows (Ng and Kuang 2000, Rafezy and Howson 2008)

$$(EI)_{yi}=\sum_j EI_{yi,j} \quad (EI)_{xi}=\sum_j EI_{xi,j} \tag{4}$$

$(EI)_{wi}$ are the warping stiffness of i[th] storey and can be calculated as follows (Ng and Kuang 2000)

$$(EI)_{wi}=\sum_j[(\bar{y}_j-\bar{y}_c)^2(EI)_{xi,j}+(\bar{x}_j-\bar{x}_c)^2(EI)_{yi,j}] \tag{5}$$

where $\bar{y}_j$ and $\bar{x}_j$ are the coordinates at the location of the geometric center of the j-th bent at i-th storey in coordinate system $(\bar{y}_j,\bar{x}_j)$.

where y_c and x_c represent the distance from the geometric center to flexural center and can be calculated as follows (Ng and Kuang 2000, Rafezy and Howson 2008)

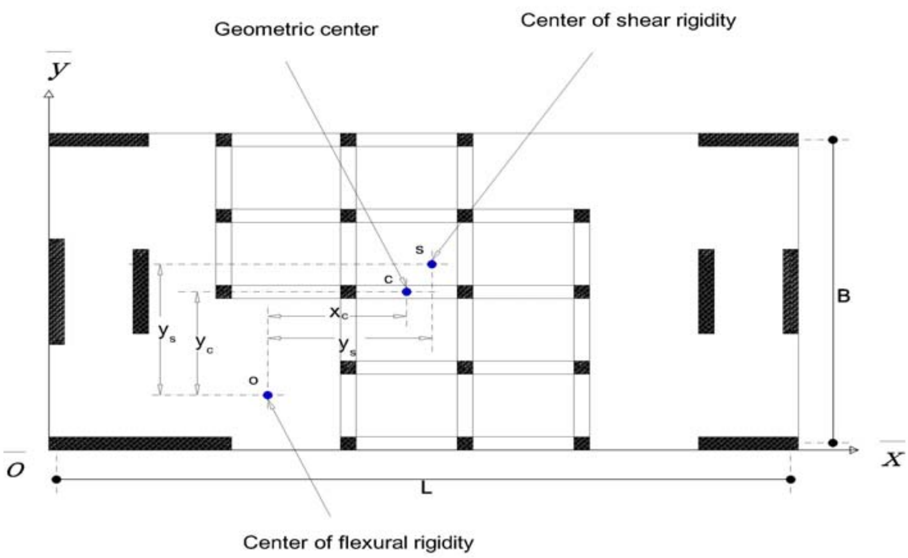

Fig. 1 Typical wall-frame system

$$y_c = \overline{y}_c - \overline{y}_o \qquad x_c = \overline{x}_c - \overline{x}_o \qquad (6, 7)$$

$\overline{y}_o$ and $\overline{x}_o$ are the coordinate of flexural center and can be calculated as follows (Ng and Kuang 2000)

$$\overline{y}_o = \frac{\sum_j \overline{y}_j (EI)_{xj}}{\sum_j (EI)_{xj}} \qquad (8)$$

$$\overline{x}_o = \frac{\sum_j \overline{x}_j (EI)_{yj}}{\sum_j (EI)_{yj}} \qquad (9)$$

When Eqs. (1), (2) and (3) are solved with respect to the z_i, $u_i(z_i)$ and $v_i(z_i)$ and $\theta_i(z_i)$ can be obtained as follows

$$u_i(z_i) = c_4 z_i^3 + c_3 z_i^2 + c_2 z_i + c_1 \qquad (10)$$

$$v_i(z_i) = c_8 z_i^3 + c_7 z_i^2 + c_6 z_i + c_5 \qquad (11)$$

$$\theta_i(z_i) = c_{12} z_i^3 + c_{11} z_i^2 + c_{10} z_i + c_9 \qquad (12)$$

where c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8, c_9, c_{10}, c_{11}, c_{12} are integral constants.

By using Eqs. (10), (11) and (12), the rotation angles in x and y direction (u_i', v_i'), the rate of twist (θ_i'), bending Moments in x and y directions (M_{xi}, M_{yi}) and bi-moment (M_{wi}), shear forces in x and y direction (V_{xi}, V_{yi}) and torque (T_i) for i.th storey can be obtained as follows

$$u_i'(z_i) = 3c_4 z_i^2 + 2c_3 z_i + c_2 \qquad (13)$$

$$v_i'(z_i) = 3c_8 z_i^2 + 2c_7 z_i + c_6 \qquad (14)$$

$$\theta_i'(z_i) = 3c_{12} z_i^2 + 2c_{11} z_i + c_{10} \qquad (15)$$

$$M_{xi}(z_i) = (EI)_{xi} \frac{d^2 u_i}{dz_i^2} - (EI)_{xi} y_c \frac{d^2 \theta_i}{dz_i^2} = (EI)_{xi}(2c_3 + 6c_4 z_i - y_c 2c_{11} - y_c 6c_{12} z_i) \qquad (16)$$

$$M_{yi}(z_i) = (EI)_{yi} \frac{d^2 v_i}{dz_i^2} + (EI)_{yi} x_c \frac{d^2 \theta_i}{dz_i^2} = (EI)_{yi}(2c_7 + 6c_8 z_i + x_c 2c_{11} + x_c 6c_{12} z_i) \qquad (17)$$

$$M_{wi}(z_i) = -y_c (EI)_{xi} \frac{d^2u_i}{dz_i^2} + x_c (EI)_{yi} \frac{d^2v_i}{dz_i^2} + (EI)_{wi} \frac{d^2\theta_i}{dz_i^2}$$

$$= (EI)_{wi}(2c_{11} + 6c_{12}z_i) - (EI)_{xi} y_c (2c_3 + 6c_4 z_i) + (EI)_{yi} x_c (2c_7 + 6c_8 z_i) \tag{18}$$

$$V_{xi}(z_i) = (EI)_{xi} \frac{d^3u_i}{dz_i^3} - (EI)_{xi} y_c \frac{d^3\theta_i}{dz_i^3} = (EI)_{xi}[6c_4 - y_c 6c_{12}] \tag{19}$$

$$V_{yi}(z_i) = (EI)_{yi} \frac{d^3v_i}{dz_i^3} + x_c (EI)_{yi} \frac{d^3\theta_i}{dz_i^3} = (EI)_{yi}[6c_8 + x_c 6c_{12}] \tag{20}$$

$$T_i(z_i) = -y_c (EI)_{xi} \frac{d^3u_i}{dz_i^3} + x_c (EI)_{yi} \frac{d^3v_i}{dz_i^3} + (EI)_{wi} \frac{d^3\theta_i}{dz_i^3}$$
$$= (EI)_{wi} 6c_{12} - y_c (EI)_{xi} 6c_4 + x_c (EI)_{yi} 6c_8 \tag{21}$$

For $z_i = 0$ and $z_i = h_i$, using Eqs. (10), (11), (12), (13), (14) and (15) the following matrix equation can be written

$$\begin{bmatrix} u_i(0) \\ v_i(0) \\ \theta_i(0) \\ u_i'(0) \\ v_i'(0) \\ \theta_i'(0) \\ u_i(h_i) \\ v_i(h_i) \\ \theta_i(h_i) \\ u_i'(h_i) \\ v_i'('h_i) \\ \theta_i'(h_i) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & h_i & h_i^2 & h_i^3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & h_i & h_i^2 & h_i^3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & h_i & h_i^2 & h_i^3 \\ 0 & 1 & 2h_i & 3h_i^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 2h_i & 3h_i^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2h_i & 3h_i^2 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \\ c_9 \\ c_{10} \\ c_{11} \\ c_{12} \end{bmatrix} = A_i \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \\ c_9 \\ c_{10} \\ c_{11} \\ c_{12} \end{bmatrix} \tag{22}$$

For $z_i = 0$ and $z_i = h_i$, using Eqs. (16), (17), (18), (19), (20) and (21) the following matrix equation can be written

$$\begin{bmatrix} V_{xi}(0) \\ V_{yi}(0) \\ T_i(0) \\ M_{xi}(0) \\ M_{yi}(0) \\ M_{wi}(0) \\ -V_{xi}(h_i) \\ -V_{yi}(h_i) \\ -T_i(h_i) \\ -M_{xi}(h_i) \\ -M_{yi}(h_i) \\ -M_{wi}(h_i) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 6EI_x & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -6y_cEI_x \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 6EI_y & 0 & 0 & 0 & 6x_cEI_y \\ 0 & 0 & 0 & -6y_cEI_x & 0 & 0 & 0 & 6x_cEI_y & 0 & 0 & 0 & 6EI_w \\ 0 & 0 & 2EI_x & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -y_c2EI_x & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2EI_y & 0 & 0 & 0 & x_c2EI_y & 0 \\ 0 & 0 & -y_c2EI_x & 0 & 0 & 0 & x_c2EI_y & 0 & 0 & 0 & 2EI_w & 0 \\ 0 & 0 & 0 & -6EI_x & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 6y_cEI_x \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -6EI_y & 0 & 0 & 0 & -6x_cEI_y \\ 0 & 0 & 0 & 6y_cEI_x & 0 & 0 & 0 & -6x_cEI_y & 0 & 0 & 0 & -6EI_w \\ 0 & 0 & -2EI_x & -6h_iEI_x & 0 & 0 & 0 & 0 & 0 & 0 & y_c2EI_x & y_c6h_iEI_x \\ 0 & 0 & 0 & 0 & 0 & 0 & -2EI_y & -6h_iEI_y & 0 & 0 & -x_c2EI_y & -x_c6h_iEI_y \\ 0 & 0 & y_c2EI_x & y_c6h_iEI_x & 0 & 0 & -x_c2EI_y & x_c2EI_y & 0 & 0 & -2EI_w & -6h_iEI_w \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \\ c_9 \\ c_{10} \\ c_{11} \\ c_{12} \end{bmatrix} = B_i \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \\ c_9 \\ c_{10} \\ c_{11} \\ c_{12} \end{bmatrix} \tag{23}$$

When vector c is solved by implementing Eq. (22) and substituted in Eq. (23), then Eq. (24) would be obtained.

$$\begin{bmatrix} V_{xi}(0) \\ V_{yi}(0) \\ T_i(0) \\ M_{xi}(0) \\ M_{yi}(0) \\ M_{wi}(0) \\ -V_{xi}(h_i) \\ -V_{yi}(h_i) \\ -T_i(h_i) \\ -M_{xi}(h_i) \\ -M_{yi}(h_i) \\ -M_{wi}(h_i) \end{bmatrix} = B_i * A_i^{-1} \begin{bmatrix} u_i(0) \\ v_i(0) \\ \theta_i(0) \\ u_i'(0) \\ v_i'(0) \\ \theta_i'(0) \\ u_i(h_i) \\ v_i(h_i) \\ \theta_i(h_i) \\ u_i'(h_i) \\ v_i'(h_i) \\ \theta_i'(h_i) \end{bmatrix} = k_{wi} * \begin{bmatrix} u_i(0) \\ v_i(0) \\ \theta_i(0) \\ u_i'(0) \\ v_i'(0) \\ \theta_i'(0) \\ u_i(h_i) \\ v_i(h_i) \\ \theta_i(h_i) \\ u_i'(h_i) \\ v_i'(h_i) \\ \theta_i'(h_i) \end{bmatrix} \tag{24}$$

k_{wi} represents the storey stiffness matrix of shear walls.

3.2 Storey stiffness matrix of frame

Under the horizontal excitation governing equations of the ith storey for frame can be written as

$$(GA)_{xi}\frac{d^2u_i}{dz_i^2}-(GA)_{xi}y_s\frac{d^2\theta_i}{dz_i^2}=0 \tag{25}$$

$$(GA)_{yi}\frac{d^2v_i}{dz_i^2}+(GA)_{yi}x_s\frac{d^2\theta_i}{dz_i^2}=0 \tag{26}$$

$$-(GA)_{xi}y_s\frac{d^2u_i}{dz_i^2}+(GA)_{yi}x_s\frac{d^2v_i}{dz_i^2}+(GJ)_i\frac{d^2\theta_i}{dz_i^2}=0 \tag{27}$$

where u_i and v_i are the lateral deflections of the geometric center, respectively, θ_i is the torsional rotation of the floor plans about geometric center at the given height, z_i is the vertical axis of each storey.
where y_s, x_s represent the distance from the geometric center to shear rigidity center and can be calculated as follows (Rafezy *et al.* 2007, Kuang and Ng 2009)

$$y_s=\bar{y}_s-\bar{y}_c \tag{28}$$

$$x_s=\bar{x}_s-\bar{x}_c \tag{29}$$

$\bar{y}_s$ and $\bar{x}_s$ are the coordinates of shear rigidity center and can be calculated as follows (Kuang and Ng 2009)

$$\bar{y}_s=\frac{\sum_j \bar{y}_j(GA)_{xj}}{\sum_j (GA)_{xj}} \tag{30}$$

$$\bar{x}_s=\frac{\sum_j \bar{x}_j(GA)_{yj}}{\sum_j (GA)_{yj}} \tag{31}$$

$(GA)_{xj}$ and $(GA)_{yj}$ are the equivalent shear rigidity of the storey for framework in x and y directions. For frame elements which consists of n columns and n-1 beams, GA can be calculated as follows (Baikov and Sigalov 1983, Stafford Smith and Crowe 1986)

$$(GA)_j=\frac{12E}{h_i[1/\sum_1^n I_c/h_i+1/\sum_1^{n-1} I_g/l)} \tag{32}$$

where $\sum I_c/h_i$ represents the sum of moments of inertia of the columns per unit height in ith storey of frame j, and $\sum I_g/l$ represents the sum of moments of inertia of each beam per unit span across one floor of frame j.

For i^{th} storey, j framework in x direction and m framework in y direction $(GA)_{xi}$ and $(GA)_{yi}$ can be calculated as follows

$$(GA)_{xi} = \sum_{t=1}^{m} (GA)_t \tag{33}$$

$$(GA)_{yi} = \sum_{t=1}^{m} (GA)_t \tag{34}$$

$(GJ)_i$ are the St. Venant torsion stiffness of i^{th} storey and can be calculates as follows (Kuang and Ng 2009)

$$(GJ)_i = \sum_j [(\bar{y}_j - \bar{y}_c)^2 (GA)_{xj} + (\bar{x}_j - \bar{x}_c)^2 (GA)_{yj}] \tag{35}$$

where $\bar{y}_j$ and $\bar{x}_j$ are the coordinates at the location of the geometric center of the j-th bent at i-th storey in coordinate system ($\bar{y}_j, \bar{x}_j$).

When Eqs. (25) ,(26) and (27) are solved with respect to the z_i, $u_i(z_i)$ and $v_i(z_i)$ and $\theta_i(z_i)$ can be obtained as follows

$$u_i(z_i) = c_1 + c_2 z_i \tag{36}$$

$$v_i(z_i) = c_3 + c_4 z_i \tag{37}$$

$$\theta_i(z_i) = c_5 + c_6 z_i \tag{38}$$

where c_1, c_2, c_3, c_4, c_5, and c_6 are integral constants.

For $z_i = 0$ and $z_i = h_i$, using Eqs. (36), (37) and (38) the following matrix equation can be written

$$\begin{bmatrix} u_i(0) \\ v_i(0) \\ \theta_i(0) \\ u_i(h_i) \\ v_i(h_i) \\ \theta_i(h_i) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & h_i & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & h_i & 0 & 0) \\ 0 & 0 & 0 & 0 & 1 & h_i \end{bmatrix} * \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \end{bmatrix} = A_i * \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \end{bmatrix} \tag{39}$$

By using Eqs. (36), (37) and (38), the shear force in x and y direction, and torsion moment can be obtained as follows

$$V_{xi}(z_i) = (GA)_{xi} \frac{du_i}{dz_i} - (GA)_{xi} y_c \frac{d\theta_i}{dz_i} = (GA)_{xi} c_2 - (GA)_{xi} y_s c_6 \tag{40}$$

$$V_{yi} = (GA)_{yi} \frac{dv_i}{dz_i} + (GA)_{yi} x_c \frac{d\theta_i}{dz_i} = (GA)_{yi} c_4 + (GA)_{yi} x_s c_6 \tag{41}$$

$$M_{ti}(z_i) = -(GA)_{xi} y_c \frac{du_i}{dz_i} + x_c \frac{dv_i}{dz_i^2} + (GJ)_i \frac{d\theta_i}{dz_i} =$$

$$-(GA)_{xi} y_s c_2 + (GA)_{yi} x_s c_4 + (GJ)c_6 \tag{42}$$

For $z_i = 0$ and $z_i = h_i$, using Eqs. (40), (41) and (42) the following matrix equation can be written

$$\begin{bmatrix} V_{xi}(0) \\ V_{yi}(0) \\ M_{ti}(0) \\ -V_{xi}(h_i) \\ -V_{yi}(h_i) \\ -M_{ti}(h_i) \end{bmatrix} = \begin{bmatrix} 0 & GA_{xi} & 0 & 0 & 0 & -y_s GA_{xi} \\ 0 & 0 & 0 & GA_{yi} & 0 & x_s GA_{yi} \\ 0 & -y_s GA_{xi} & 0 & x_s GA_{yi} & 0 & GJ_i \\ 0 & -GA_{xi} & 0 & 0 & 0 & y_s GA_{xi} \\ 0 & 0 & 0 & -GA_{yi} & 0 & -x_s GA_{yi} \\ 0 & y_s GA_{xi} & 0 & -x_s GA_{yi} & 0 & -GJ_i \end{bmatrix} * \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \end{bmatrix} = B_i * \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \end{bmatrix} \tag{43}$$

When vector c is solved by implementing Eq. (39) and substituted in Eq. (43), then Eq. (44) would be obtained.

$$\begin{bmatrix} V_{xi}(0) \\ V_{yi}(0) \\ M_{ti}(0) \\ -V_{xi}(h_i) \\ -V_{yi}(h_i) \\ -M_{ti}(h_i) \end{bmatrix} = B_i * A_i^{-1} * \begin{bmatrix} u_i(0) \\ v_i(0) \\ \theta_i(0) \\ u_i(h_i) \\ v_i(h_i) \\ \theta_i(h_i) \end{bmatrix} = f_i * \begin{bmatrix} u_i(0) \\ v_i(0) \\ \theta_i(0) \\ u_i(h_i) \\ v_i(h_i) \\ \theta_i(h_i) \end{bmatrix} \tag{44}$$

In order add the walls and frames stiffness matrices, the frames matrix must be enlarged The storey stiffness matrix of frame can be obtained as

$$\begin{bmatrix} V_{xi}(0) \\ V_{yi}(0) \\ T_i(0) \\ M_{xi}(0) \\ M_{yi}(0) \\ M_{wi}(0) \\ -V_{xi}(h_i) \\ -V_{yi}(h_i) \\ -T_i(h_i) \\ -M_{xi}(h_i) \\ -M_{yi}(h_i) \\ -M_{wi}(h_i) \end{bmatrix} = t * f_i * t^T \begin{bmatrix} u_i(0) \\ v_i(0) \\ \theta_i(0) \\ u_i'(0) \\ v_i'(0) \\ \theta_i'(0) \\ u_i(h_i) \\ v_i(h_i) \\ \theta_i(h_i) \\ u_i'(h_i) \\ v_i'(h_i) \\ \theta_i'(h_i) \end{bmatrix} = k_{fi} * \begin{bmatrix} u_i(0) \\ v_i(0) \\ \theta_i(0) \\ u_i'(0) \\ v_i'(0) \\ \theta_i'(0) \\ u_i(h_i) \\ v_i(h_i) \\ \theta_i(h_i) \\ u_i'(h_i) \\ v_i'(h_i) \\ \theta_i'(h_i) \end{bmatrix} \tag{45}$$

Where t are the transformation matrix and can be calculated as

$$t = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{46}$$

3.3 Total storey stiffness matrix

Storey stiffness matrix pertaining to shear wall-frame system can be written as below as the sum of stiffness matrices of shear wall and frames.

$$k_i = k_{wi} - k_{fi} \tag{47}$$

4. Transfer relationship of modified finite element transfer matrix method

Numerical methods are more suitable for the solution of the initial value problems than the boundary value problems (Xue 1994, Choi 2003, Rong *et al.* 2011). For this reason, the generalized stiffness equation, which relates the force vector to the displacement vector on the output end of the i.th strip, is introduced by Xue and Choi (Xue 1994, Choi 2003, Rong *et al.* 2011, Ozturk *et al.* 2012).

$$F_{(i,i+1)} = T_i d_{(i,i+1)} \qquad (i \geq 1) \tag{48}$$

For i^{th} storey the following matrix equation can be written (Rong *et al.* 2011)

$$S_i \begin{bmatrix} d_{i,i-1} \\ d_{i,i+1} \end{bmatrix} = \begin{bmatrix} s_1 & s_2 \\ s_3 & s_4 \end{bmatrix} \begin{bmatrix} d_{i,i-1} \\ d_{i,i+1} \end{bmatrix} = \begin{bmatrix} -F_{i,i-1} \\ F_{i,i+1} \end{bmatrix} \tag{49}$$

Where S_i is the dynamic stiffness matrix of i^{th} storey and can be calculated as

$$S_i = k_i - \omega^2 * M_i \tag{50}$$

k_i is the i.th storey stiffness matrix and M_i is the mass matrix of i^{th} storey and can be calculated as follows

$$M_i = \begin{bmatrix} 0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&m_i&0&0&0&0&0 \\ 0&0&0&0&0&0&0&m_i&0&0&0&0 \\ 0&0&0&0&0&0&0&0&m_i r_m^2&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0 \end{bmatrix} \tag{51}$$

where, m_i is the mass of i^{th} storey and r_{mi} is the inertial radius of gyration of the i^{th} storey.
Matrix Eq. (49) can be written in two matrix equation as

$$s_1 d_{i,i-1} + s_2 d_{i,i+1} = -F_{i,i-1} \tag{52}$$

$$s_3 d_{i,i-1} + s_4 d_{i,i+1} = F_{i,i+1} \tag{53}$$

Using Eq. (48), Eq. (52) can be written as

$$s_1 d_{i,i-1} + s_2 d_{i,i+1} = -T_{i-1} d_{i,i-1} \tag{54}$$

Eq. (54) can yield

$$d_{i,i-1} = (T_{i-1} + k_1)^{-1} k_2 d_{i,i+1} \tag{55}$$

Combining Eqs. (48) and (53) the following equation can be obtained

$$s_3 d_{i,i-1} + s_4 d_{i,i+1} = T_i d_{i,i+1} \tag{56}$$

Combining Eqs. (55) and (56) can yield

$$s_3 (T_{i-1} + s_1)^{-1} s_2 d_{i,i+1} + s_4 d_{i,i+1} = T_i d_{i,i+1} \tag{57}$$

From Eq. (57), the following recursion relationship can be obtained

$$T_i = -s_{3,i}(s_{1,i} + T_{i-1})^{-1} s_{2,i} + s_{4,i} \qquad (i \geq 2) \tag{58}$$

At the base, the lateral deflections of the geometric center (u, v), the torsional rotation (θ_i), the rotation angles in x and y direction (u_i', v_i') , the rate of twist (θ_i') are zero. According to this boundary conditions

1) $u_{base} = 0$ 2) $v_{base} = 0$ 3) $\theta_{base} = 0$ 4) u'base = 0 5) $v'_{base} = 0$ 6) $\theta'_{base} = 0$

For the first segment (storey), applying the boundary condition at the base, T_1 can be obtained as follows

$$T_1 = s_{4,i} \tag{59}$$

For the last segment Eq. (48) can be written as

$$F_{(n,n+1)} = T_n d_{n,n+1} \tag{60}$$

The size of system matrix (T_n) is 6×6. In the classical finite element when applying the boundary condition the dimension of system matrix is equal the 3×storey number.

In the top of structures the shear force in x and y direction, the bending moment in x and y direction, warping torsion and St. Venant torsion moment should be zero. According to this boundary conditions, the eigenfrequency equation can be obtained:

$$|T_n| = 0 \tag{61}$$

The values of ω which set the determinant to zero, are the angular frequencies of the building

5. Numerical examples

A numerical example has been solved by a program written in MATLAB to verify the proposed method in this part of the study. The results are then compared with those given in the literature.

5.1 Numerical example 1

A typical asymmetric wall-frame structure (Fig. 1) is analyzed as an example. The structure has 30 storeys with total height H = 90 m, and floor dimensions L = 42 m and B = 24 m. The structure consists of eight walls of 0.25-m thick and the multibent frames. An elastic modulus $E=20\times10^6$ kN/m^2 and the density of floor slabs ρ = 2.350 kg/m^3. The structural properties are given in Table 1. The natural frequencies calculated by this method are compared with the results in the reference (Ng and Kuang 2000). The results are presented in Table 2, Table 3 and Table 4.

In the method presented in the literature, the storey masses are considered as uniformly distubuted and the eccentricity between the center of shear rigidity and bending rigidity is not taken into account in the calculations. On the other hand, in this study, storey masses are assumed as separated at storey alignments in the presented method and the eccentricity between the center of shear rigidity and the center of mass is also taken into account in the calculations. Therefore, the results obtained by the method presented in this article yielded much closer results to the results obtained by ETABS.

Table 1 Structural property of asymmetric wall-frame structures (Example 1)

Structural Properties	
$(EI)_x$	990.70×10^6 kNm^2
$(EI)_y$	574.53×10^6 kNm^2
$(EI)_w$	356.940×10^9 kNm^4
$(GA)_x$	274.29×10^3 kN
$(GA)_y$	297.14×10^3 kN
(GJ)	77.633×10^6 kNm^2
x_c	7.81 m
y_c	−7.63 m
x_s	6.57 m
y_s	−5.32 m
m	355.41 kNsn^2/m
r_m	13.964 m

Table 2 Comparison of first three natural frequencies in Example 1 (rd/s)

Natural frequencies of the second three modes (rd/s)								
Proposed Method			Ng and Kuang			ETABS (Ng and Kuang 2000)		
ω_1	ω_2	ω_3	ω_1	ω_2	ω_3	ω_1	ω_2	ω_3
1.199	1.548	2.323	1.163	1.587	2.437	1.197	1.539	2.299

Table 3 Comparison of second three natural frequencies in Example 1 (rd/s)

Natural frequencies of the second three modes (rd/s)								
Proposed Method			Ng and Kuang			ETABS (Ng and Kuang 2000)		
ω_4	ω_5	ω_6	ω_4	ω_5	ω_6	ω_4	ω_5	ω_6
5.694	7.430	11.888	5.799	7.655	12.348	5.898	7.313	11.642

Table 4 Comparison of third three natural frequencies in Example 1 (rd/s)

Natural frequencies of the second three modes (rd/s)								
Proposed Method			Ng and Kuang			ETABS (Ng and Kuang 2000)		
ω_7	ω_8	ω_9	ω_7	ω_8	ω_9	ω_7	ω_8	ω_9
14.898	19.648	28.705	15.317	20.265	33.108	14.775	19.455	31.350

5.1 Numerical example 2

A typical asymmetric 10 storeys wall-frame structure (Fig. 2) is analyzed as an example. The structural properties are given in Table 5. The natural frequencies calculated by this method are compared with the results in the reference (Rafezy and Howson 2008). The results are presented in Table 6.

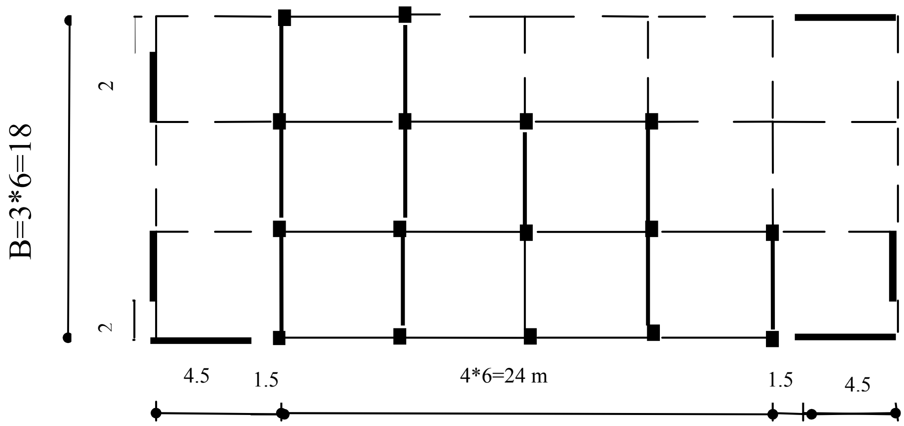

Fig. 2 Wall frame system (Example 2)

Table 5 Structural property of asymmetric wall-frame structures (Example 2)

Structural Properties	
$(EI)_x$	91.13×10^6 kNm^2
$(EI)_y$	64×10^6 kNm^2
$(EI)_w$	28.114×10^9 kNm^4
$(GA)_x$	564.7×10^3 kN
$(GA)_y$	517.6×10^3 kN
(GJ)	63.289×10^6 kNm^2
x_c	6 m
y_c	3 m
x_s	2.727 m
y_s	2.5 m
m	237.8 kNsn^2/m
r_m	11.619 m

Table 6 Comparison of frequencies (Hz) in Example 2

Frequency No	Proposed Method	Rafezy and Howson	ETABS (Rafezy and Howson 2008)
1	0.8602	0.9337	0.8703
2	1.0106	1.1085	1.0283
3	1.2924	1.4082	1.2981

6. Conclusions

In this article, a method is proposed for determining free vibration periods of structures, the bearing system of which contains shear wall-frames. In this method, first, the storey stiffness matrix is obtained summing up the stiffness matrices of shear walls and frames and then the system transfer matrix is constructed by means of the calculated storey stiffness matrices and a widely used transformation method in the literature. The angular frequencies are computed by applying the boundary conditions in the system transfer matrix and formulating the frequency equation.

The results are assessed by solving the samples taken from the literature to justify the suitability of the method at the end of the study. The assessment suggests that the results of presented method are closer to the results obtained by ETABS in comparison to the results obtained by other methods proposed in the literature. The presented method calculates the angular frequencies quite fast and can be easily programmed due to the fact that it predicates on the transfer matrix. The dimension which is 12n×12n in classical finite elements, is reduced to 6×6.

References

Aksoğan, O., Arslan, H. M. and Choo, B.S. (2003), "Forced vibration analysis of stiffened coupled shear walls using continuous connection method", *Eng. Struct.*, **25**, 499-506.

Baikov, V. and Sigalov, E. (1983), *Reinforced Concrete Structures Volume 2*, Mir Publisher, Moscov.

Balendra, T., Swaddiwudhipong, S. and Quek, S.T. (1984), "Free vibration of asymmetric shear wall-frame buildings", *Earth. Eng. Struct. Dyn*., **12(**5), 629-650.

Basu, A., Nagpal, A.K., Bajaj, R.S. and Guliani, A. (1979), "Dynamic characteristics of coupled shear walls", *J. Struct. Eng. -ASCE*, **105**, 1637-1651.

Bilyap, S. (1979), "An approximate solution for high- rise reinforced concrete panel buildings with combined diaphragms", *Int. J. Housing Sci.*, **3**(6), 477-481.

Boutin, C., Hans, S., Ibraim, E. and Roussilon, P. (2005), "In situ experiments and seismic analysis of existing buildings, part II: seismic integrity treshold", *Earth. Eng. Struct. Dyn.,* **34**(12), 1531-1546.

Bozdogan, K.B. (2009), "An approximate method for static and dynamic analyses of symmetric wall-frame buildings", *Struct. Des. Tall Spec. Build.*, **18**(3), 279-290.

Bozdogan, K.B. and Ozturk, D. (2012), "Vibration analysis of asymmetric shear wall-frame structures using the transfer matrix method", *Iranian J. Sci. Tech. Transaction B Engineering,* **36**(C1), 1-12.

Choi, M.S. (2003), "Free vibration analysis of plate structures using finite element transfer stiffness coefficient method", *J. Mech. Sci. Tech.*, **17**(6), 805-815.

Georgoussis, K.G. (2006), "A simple model for assessing and modal response quantities in symmetrical buildings", *Struct. Des. Tall Special Build*., **15**(2), 139-151.

Heidebrecht, A.C. and Stafford Smith, B. (1973), "Approximate analysis of tall wall-frame buildings", *J. Struct. Eng., ASCE*, **99** (2), 199-221.

Hoenderkamp, D.C.J. and Snijder, H.H. (2000), "Approximate analysis of high-rise frames with flexible connections*", Struct. Des. Tall Build.*, **9**(3), 233-248.

Hoenderkamp, D.C.J. (2001), "Elastic analysis of asymmetric tall buildings", *Struct. Des. Tall Build.*, **10**(4), 245-261.

Hoenderkamp, D.C.J. (2002), "A simplified analysis of high-rise structures with cores", *Struct. Des. Tall Build.*, **11**(2), 93-107.

Jahanshahia, M.R. and Rahgozar, R. (2012), "Free vibration analysis of combined system with variable cross section in tall buildings", *Struct. Eng. Mech.,* **42**(5), 715-728.

Kazaz, I., Gulkan, P. (2012), "An alternative frame –shear wall model: continuum formulation", *Struct. Des. Tall Special Build.*, DOI:10.1002/tal.626.

Kuang, J.S. and Ng, S.C. (2000), "Coupled lateral vibration of asymmetric shear wall structures", *Thin Walled Struct.*, **38**(2), 93-104.

Kuang, J.S. and Ng, C. (2009), "Lateral shear –st. venant torsion coupled vibration of asymmetric –plan frame structures", *Struct. Des. Tall Special Build.*, **18**(6), 647-656.

Kaviani, P., Rahgozar, R. and Saffari, H. (2008), "Approximate analysis of tall buildings using sandwich

beam models with variable cross-section", *Struct. Des. Tall Special Build.*, **17**(2), 401-418.
Laier, J.E. (2008), "An improved continuous medium technique for structural frame analysis", *Struct. Des. Tall Special Build.*, **17**(1), 25-38.
Lee, J., Bang, M. and Kim, J.Y. (2008), "An analytical model for high- rise wall-frame structures with outriggers", *Struct. Des. Tall Special Build.*, **17**(4), 839-851.
Li, G.Q. and Choo, B.S. (1996), "A continous-discrete approach to the free vibration analysis of stiffened pierced walls on flexible foundations", *Int. J. Solids Struct.*, **33**(2), 249-263.
Malekinejad, M. and Rahgozar, R. (2011a), "An analytical approach to free vibration analysis of multi-outrigger-belt truss-reinforced tall buildings", *Struct. Des. Tall Build.*, **8**(2), 155-170.
Malekinejad, M. and Rahgozar, R. (2011b), "Free vibration analysis of tall buildings with outrigger-belt truss system", *Earth. Struct.,* **2**(1), 89-107.
Mancini, E. and Savassi, W. (1999), "Tall buildings structures unified plane panels behavior", *Struct. Des. Tall Build.,* **8**(2), 155-170.
Meftah, S.A. and Tounsi, A. (2008), "Vibration characteristics of tall buildings braced by shear walls and thin-walled open-section structures", *Struct. Des. Tall Special Build.*, **17**(1), 203-216.
Michel, C., Hans, S., Guegen, P. and Boutin, C. (2006), "In situ experiment and modeling of rc structure using ambient vibration and timoshenko beam", *First European Conference on Earthquake Engineering And Seismology,* Geneva-Switzerland.
Miranda, E. (1999), "Approximate lateral drift demands in multi-story buildings subjected to earthquakes", *J. Struct. Eng. ASCE*, **125**(4), 417-425.
Miranda, E. and Reyes, J.C. (2002), "Approximate lateral drift demands in multi-story buildings with nonuniform stiffness", *J. Struct. Eng., ASCE*, **128**(7), 840-849.
Miranda, E. and Taghavi, S. (2005), "Approximate floor acceleration demands in multistorey buildings I formulation", *J. Struct. Eng. ASCE*, **131**(2), 203-211.
Ng, S.C. and Kuang, J.S. (2000), "Triply coupled vibration of asymmetric wall-frame structures", *J. Struct. Eng., ASCE*, **128**(7), 840-849.
Nollet, J.M. and Stafford Smith, B. (1993), "Behavior of curtailed wall-frame structures", *J. Struct. Eng., ASCE*, **119**(10), 2835-2853.
Ozturk, D., Bozdogan, K. and Nuhoglu, A. (2012), "Modified finite element-transfer matrix method for the static analysis of structures", *Struct. Eng. Mech.,* **43**(6), 761-769.
Potzta, G. and Kollar, L.P. (2003), "Analysis of building structures by replacement sandwich beams", *Int. J. Solids Struct.*, **40**, 535-553.
Rafezy, B., Zare, A. and Howson, W.P. (2007), "Coupled lateral –torsional frequencies of asymmetric, three dimensional frame structures", *Int. J. Solids Struct*., **44(**1), 128-144.
Rafezy, B. and Howson, W.P. (2008), "Vibration analysis of doubly asymmetric, three dimensional structures comprising wall and frame assemblies with variable cross section", *J. Sound Vib.*, **318**(1-2), 247-266.
Rahgozar, R., Ahmadi, A. and Sharifi, Y. (2010), "A simple mathematical model for approximate analysis of tall buildings", *J. App. Math. Modelling*, **34**, 2437-2451.
Reinoso, E. and Miranda, E. (2005), "Estimation of floor acceleration demands in high rise buildings during earthquakes", *Struct. Des. Tall Special Build*., **14**(2), 107-130.
Rong, B., Rui, X.T. and Wang, G.P. (2011), "Modified finite element transfer matrix method for eigenvalue problem of flexible structures", *J. App. Mech.*, **78**(2), 021016.
Rosman, R. (1964), "Approximate analysis of shear walls subject to lateral loads", *Proc. Am. Concr. Inst*., **61**(6), 717-734.
Savassi, W. and Mancini, E. (2004), "One-dimensional finite element solution for tall building structures unified plane panels formulation", *Struct. Des. Tall Special Build.,* **13**(4), 315-333.
Savassi, W. and Mancini, E. (2009), "One-dimensional finite element solution for non-uniform tall building structures and loading", *Struct. Des. Tall Special Build.*, **18**(4), 441-453.
Stafford Smith, B. and Crowe, E. (1986), "Estimating periods of vibration of tall buildings", *J. Struct. Eng., ASCE,* **112**(5), 1005-1019.

Swaddiwudhipong, S., Lee, L.S. and Zhou, Q. (2001), "Effect of the axial deformation on vibration of tall buildings", *Struct. Des. Tall Buil.,* **10**(2), 79-91.

Taghavi, S. and Miranda, E. (2005), "Approximate floor acceleration demands in multistorey buildings II: applications", *J. Struct. Eng., ASCE*, **131**(2), 212-220.

Tarjan, G. and Kollar, P.L. (2004), "Approximate analysis of building structures with identical stories subjected to earthquakes", *Int. J. Solids Struct.*, **41**(5-6), 1411-1433.

Tekeli, H., Atimtay, E. and Turkmen, M. (2012), "A simplified method for determining sway in reinforced concrete dual buildings and design applications", *Struct. Des. Tall Special Build*., DOI:10.1002/tal.761.

Toutanji, H. (1997), "The effect of foundation flexibility on the interaction of walls and frames", *Eng. Struct*., **19**(12), 1036-1042.

Xue, H.Y. (1994), "A combined dynamic finite element-ricatti transfer matrix method for solving non-linear eigenproblems of vibrations", *Comp. Struct.*, **53**(6), 1257-1261.

Wang, Y., Arnaouti, C. and Guo, S. (2000), "A simple approximate formulation for the first two frequencies of asymmetric wall-frame multi-storey building structures", *J. Sound and Vib.*, **236** (1), 141-160.

Wdowicki, J. and Wdowicka, E. (2012), "Analysis of shear wall structures of variable cross section", *Struct. Des. Tall Special Build*., **21**(1),1-15.

Yang, D., Pan, J. and Li, G. (2010), "Interstory drift ratio of building structures subjected to near-fault ground motions based on generalized drift spectral analysis", *Soil Dyn. Earthq. Eng*., **30,** 1182-1197.

Zalka, K.A. (1994), "Mode coupling in the torsional flexural buckling of regular multistorey buildings", *Struct. Des. Tall Build*., **3**(4), 227-245.

Zalka, K.A. (2001), "A simplified method for calculation of natural frequencies of wall-frame Buildings", *Eng. Struct.,* **23**(12),1544-1555.

Zalka, K.A. (2002), "Buckling analysis of buildings braced by frameworks, shear walls and cores", *Struct. Des. Tall Build.***,** **11**(3), 197-219.

Zalka, K.A. (2003), "Hand method for predicting the stability of regular buildings, using frequency measurements", *Struct. Des. Tall Special Build.*, **12**(4), 273-281.

Zalka, K.A. (2009), "A simple method for the deflection analysis of tall-wall-frame building structures under horizontal load", *Struct. Des. Tall Special Build.*, **18**(3), 291-311.