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Deflection prediction of inflatable flat panels under arbitrary conditions

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Abstract. Inflatable panels made of modern and new textile materials can be inflated at high pressure to have a high mechanical strength. This paper is based on the finite element method as a general solution to determine the characteristics of deformed inflatable panels at high pressure in various end and loading conditions. Proposed method is based on the construction of weak form of formulation and application of Reduced Integration Element method (RIE) to solve the numerical problem of shear locking. The numerical results are validated as an outcome of comparison with other published results.

Keywords: inflatable panel, high pressure, deformable structures, textile material, finite element method

1. Introduction

Inflatable structures are widely used in many engineering applications, such as: space antenna, temporary buildings, cofferdams, etc. Moreover, mentioned structures have many interesting properties like light weight and small volume when folded. Additionally, they can come back to their initial position after unloading because of their reversible behavior after instability. In general, they are used in low pressure but it is an important advantage to have a high mechanical strength by using a high pressure (critical load is proportional to the applied pressure). These structures play an important role in the design of inflatable structures and their behavior has to be understood. Clearly, they can very well have a long term role several applications because of the technology's potential for high mechanical-packaging efficiency, variable stowed geometry and deployment reliability. So it is necessary to predict their behavior and a general solution for achieving this aim needs to be introduced.

Unfortunately, there are few results on the deflections of inflatable structures at high pressure especially numerical results. Comer and Levy (1963), Main *et al*. (1994), have studied inflatable tubes and calculated the values of the deflections of cantilever inflatable structures by using the usual beam theory. However, the solution given by Comer and Levy (1963) was dependent to the internal pressure. The literature on the inflatable structures at high pressure was started by Wielgosz and Thomas (2002, 2004), Le van and Wielgosz (2007). They have studied inflatable panels and tubes respectively, and calculated the values of deflection of these structures by using the Timoshenko beam theory. Experimental and theoretical results were compared to show that

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these results can be applied to inflatable structures at high pressure. A special finite element method is suggested to obtain the values of deflection of inflatable tubes at high pressure (Wielgosz and Thomas 2003, 2004, Thomas and Wielgosz 2004).

Among various numerical methods, finite element method (FEM) is more popular but the literature on the finite element analysis of the inflatable structures is sparse. There are some references about pressurized membrane structures that can be used and applied to inflatable structures. The papers presented by Main *et al*. (1994), Kawabata and Ishii (1994) can be mentioned for finite element analysis of the pressurized membrane structures. Bending and the buckling of inflatable beams have been studied by Le van and Wielgosz (2005). Bonet (2000), Jil and Bonet (2006) presented a finite element analysis of closed membrane structures that contain a constant mass fluid such as air. These formulations were obtained by using the isoparametric finite element plane as a material reference configuration. General finite element membrane analysis is presented by Argyris *et al*. (1997), Gruttman and Taylor (1992), De Souza *et al*. (1995), Oden and Sato (1967). Inflated axisymmetrical pressurized membrane is also concerned and the construction of the axisymmeric finite element model is devoted by Bouzidi *et al*. (2003). Because of the important roles of inflatable structures, a numerical solution needs to be presented to analyze the behavior of these structures in different cases. More recent publications can be found in Davids and Zhang (2008), Coda (2009), Lampani and Gaudenzi (2010), Apedo *et al*. (2010), Lee *et al*. (2011), Brayley *et al*. (2012).

Despite the extensive field experiences and research activities in the area of inflatable structures, most of the applications of these methods, presented so far are limited to analysis with membrane theory and considering the inflatable structures as the elastic beam. In fact, the research on inflatable structures has concentrated too much on deflection analysis and/or experimental investigations with traditional theories and too little on using the numerical methods such as FEM.

In this paper, the finite element method is used to develop a numerical solution for inflatable flat panels inflated at high pressure. The modeling of textile structures by the finite element method is an approach based on the combination of geometric and mechanical models which appears the stiffness matrix. The application of this method first requires a mathematical formulation of the problem and then a mesh of the basic elements. This finite element model can be used as a general form for finding the deflections of inflatable panels in different cases of loading and supporting conditions. These structures can not be viewed as ordinary plates or beams, because their deformation pattern is quit different. The influence of the shear stress can not be neglected and so the Timoshenko's beam theory has to be used.

In the first section of this paper, the main analytical formulations will be considered and used to obtain the weak form of equations. The second section is devoted to the construction of the finite element model of inflatable panel. The stiffness matrix is then obtained by using linear shape functions. In the third section, various end conditions will be analyzed using the proposed numerical method. The FEM procedure and the code as developed here can be used for other cases of supporting conditions and loadings. The code calculates the deformations due to the applied loads. Numerical results are compared with analytical and experimental ones on inflatable flat panels inflated at high pressure to show a very good accuracy of the present solution and theory. Finally, it can be concluded that the proposed method can be used for predicting the behavior of inflatable panel inflated at high pressure with a good accuracy in different cases.

2. Inflatable panels at high pressure- Weak formulation

The theory of inflatable fabric panels (Fig. 1(a)) is defined in (Wielgosz and Thomas 2002) and used to build a finite element model devoted to predicting the deformations of inflatable panels. The finite element model of inflatable panels is obtained by use of their equilibrium equations and the constitutive law of the fabrics, and kinematical assumptions on their deformation pattern. This finite element method can be developed in the all cases of supported panel like clamped, simply supported, etc. A small element of the panel (Fig. 1(aa), 1(bb)) with length *dx* and width *t* and height *h* is considered. Equilibrium equations are written for this element in its deformed state.

The bending shape of the panel depends on the internal pressure. The free end points of the panel remain almost straight for the low internal pressure. However, when the pressure is higher, straight parts are shorter and a wider curved zone appears. Here, the Timoshenko beam theory is used because this theory is based on the assumption that the plane cross-sections remain plane but not perpendicular to the longitudinal axis after bending. So, the transverse shear strain can not be zero.

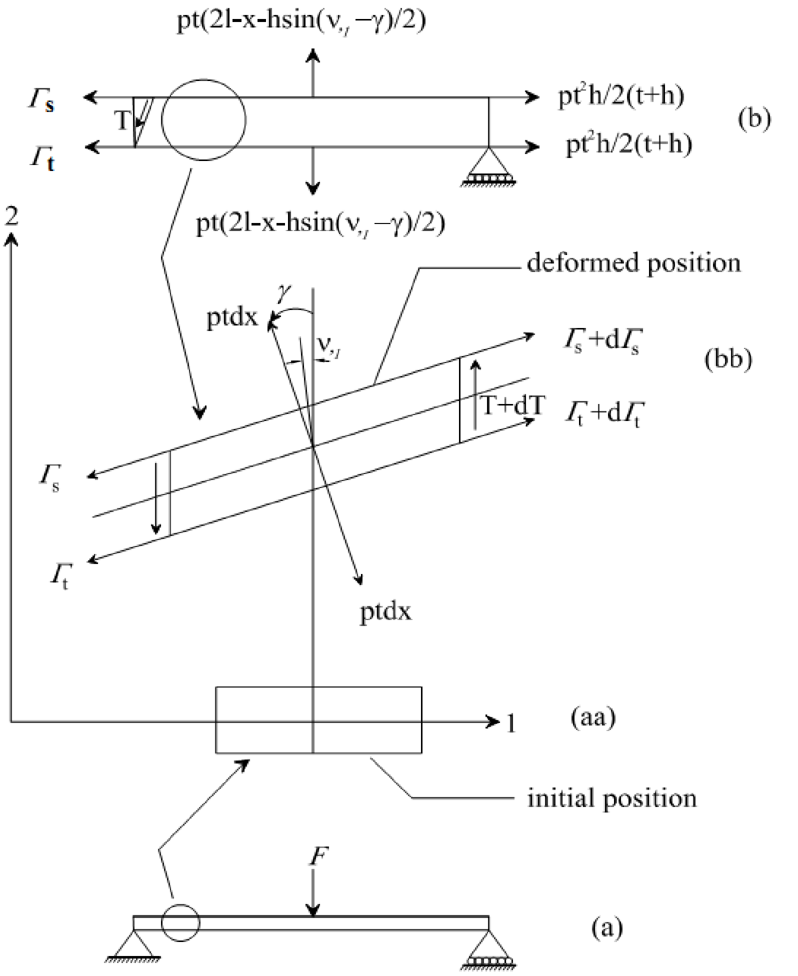


Fig. 1(a) The studied panel in initial position, (aa) Small element of inflatable panel in initial position, (bb) Small element of inflatable panel in deformed position, (b) Loads on an inflatable panel element

Let *u* and *v* be the axial and transverse displacements of the inflatable panel, respectively. The panel deflection is *v*, the slope of the panel is $v_{,1}$, the section rotation is γ and the transverse shear is $v_{,1}$-γ. The element stiffness matrix can be obtained from the equilibrium equations. They are given by Comer and Levy (1963)

$$(\Gamma_t + \Gamma_s)_{,1} + T_{,1}(v_{,1} - \gamma) = 0 \tag{1}$$

$$T_{,1} = 0 \tag{2}$$

$$\frac{h(\Gamma_t - \Gamma_s)_{,1}}{2} - pht(v_{,1} - \gamma) + T = 0 \tag{3}$$

where Γ_s and Γ_t are as resultant stresses in the lower and upper membrane. Also, T is as a shear force. The bending moment can be written in form of

$$M = \frac{h(\Gamma_t - \Gamma_s)}{2} \tag{4}$$

Substituting Eq. (4) in Eq. (3) leads to

$$M_{,1} - pht(v_{,1} - \gamma) + T = 0 \tag{5}$$

The constitutive laws of the panel are given by two equations. Here, the young modulus and the shear modulus are denoted by *E* and *G*, respectively. μ_s, is the shear coefficient and *A* is the area of cross-section.

$$T = \mu_s GA(v_{,1} - \gamma) \tag{6}$$

$$M = EI\gamma_{,1} \tag{7}$$

Now, Eq. (2) and Eq. (3) can be written in the other form by substituting of Eq. (6) and Eq. (7)

$$\mu_s GA(v_{,1} - \gamma)_{,1} = 0 \tag{8}$$

$$EI\gamma_{,11} + (pht + \mu_s GA)(v_{,1} - \gamma) = 0 \tag{9}$$

The weak form of Eq. (8) and Eq. (9) over an element is derived by multiplying of these equations with weight functions φ_1 and φ_2 and integrating over the element length

$$\int_{x_k^e}^{x_{k+1}^e} \varphi_1\{\mu_s GA(v_{,1} - \gamma)_{,1}\}dx = 0 \tag{10}$$

$$\int_{x_k^e}^{x_{k+1}^e} \varphi_2\{EI\gamma_{,11} + (pht + \mu_s GA)(v_{,1} - \gamma)\}dx = 0 \tag{11}$$

Each integral is obtained by integrating by parts

$$\left[\mu_s GA\varphi_1(v_{,1} - \gamma)\right]_{x_k^e}^{x_{k+1}^e} - \int_{x_k^e}^{x_{k+1}^e} \{\mu_s GA\varphi_{1,1}(v_{,1} - \gamma)dx\} = 0 \tag{12}$$

$$\left[EI\varphi_2\gamma_{,1}\right]_{x_k^e}^{x_{k+1}^e} - \int_{x_k^e}^{x_{k+1}^e}\{EI\varphi_{2,1}\gamma_{,1} + (pht + \mu_s GA)\varphi_2(\gamma - v_{,1})\}dx = 0 \tag{13}$$

The weight functions φ_1 and $\varphi_{,2}$ must have the physical interpolations that give $T\varphi_1$ and $M\varphi_2$ units of work. Clearly, φ_1 must be equivalent to (the variation of) the transverse deflection *v*, and φ_2 must be equivalent to (the variation of) the rotation function *γ*.

$$\varphi_1 \sim v$$

$$\varphi_2 \sim \gamma$$

The shear forces and the bending moments at the end points of the element are denoted by the expression

$$Q_1^e \equiv -\mu_s GA(v_{,1} - \gamma)\Big|_{x_k^e} \tag{14}$$

$$Q_2^e \equiv -EI\gamma_{,1}\Big|_{x_k^e} \tag{15}$$

$$Q_3^e \equiv \mu_s GA(v_{,1} - \gamma)\Big|_{x_{k+1}^e} \tag{16}$$

$$Q_4^e \equiv EI\gamma_{,1}\Big|_{x_{k+1}^e} \tag{17}$$

The weak form statements in Eq. (12) and Eq. (13) can be written in final form of

$$\int_{x_k^e}^{x_{k+1}^e}\{\mu_s GA\varphi_{1,1}(v_{,1} - \gamma)\}dx \; - \varphi_1(x_k^e)Q_1 - \varphi_1(x_{k+1}^e)Q_3 = 0 \tag{18}$$

$$\int_{x_k^e}^{x_{k+1}^e}\{EI\varphi_{2,1}\gamma_{,1} + (pht + \mu_s GA)\varphi_2(\gamma - v_{,1})\}dx - \varphi_2(x_k^e)Q_2 - \varphi_2(x_{k+1}^e)Q_4 = 0 \tag{19}$$

3. Finite element model

The variables *v* and *γ* can be interpolated, in general, with different degrees of interpolation. Here, Lagrange interpolation is considered and *γ* and *v* are written in form of

$$v = \sum_{j=1}^{m} v_j \psi_j^{(1)}$$

$$\gamma = \sum_{j=1}^{n} \gamma_j \psi_j^{(2)} \tag{20}$$

where $\psi_j^{(1)}$ and $\psi_j^{(2)}$ are the Lagrange interpolation function of degree $m-1$ and degree $n-1$, respectively. In general, *m* and *n* can be independent of each other, although $m = n$ is most common. When linear interpolation is used, the derivative of *v* can be written in form of

$$v_{,1} = \frac{\varphi_2^e - \varphi_1^e}{l_e} \tag{21}$$

For thin beams the transverse shear deformation is negligible. So, $v_{,1} = \gamma$ and then

$$\alpha_1^e \frac{x_{k+1}^e - x}{l_e} + \alpha_2^e \frac{x - x_k^e}{l_e} = \frac{\varphi_2^e - \varphi_1^e}{l_e} \tag{22}$$

By equating like coefficient on both sides

$$\alpha_1^e x_{k+1}^e - \alpha_2^e x_k^e = \varphi_2^e - \varphi_1^e \tag{23}$$

$$\varphi_2^e - \varphi_1^e = 0 \tag{24}$$

which, in turn requires

$$\alpha_1^e = \alpha_2^e = \frac{\varphi_2^e - \varphi_1^e}{l_e} \tag{25}$$

This implies that $\gamma(x)$ is a constant

$$\gamma(x) = \alpha_1^e \frac{x_{k+1}^e - x}{l_e} + \alpha_2^e \frac{x - x_k^e}{l_e} = \alpha_1^e = \alpha_2^e \tag{26}$$

However, a constant state of γ is not admissible, because the bending energy of the element would be zero.

$$\int_{x_k^e}^{x_{k+1}^e} \frac{EI}{2} (v_{,1})^2 dx = 0 \tag{27}$$

This numerical problem is known as shear locking which is a problem with all fully integrated, first-order, solid elements. Shear locking affects the performance of fully integrated, linear elements subjected to bending loads and causes the elements to be too stiff in bending. Here, for solving this problem, Reduced Integration Element Method (RIE) is used (Reddy 1993). Reduced-integration elements use one fewer integration point in each direction than the fully integrated elements. In this study the same interpolations for γ and v are used and integration is performed by using the Gaussian Quadrature method (Reddy 1993).

$$\int_a^b F(x_i) dx_i = \int_{-1}^1 \hat{F}(\xi) d\xi \approx \sum_{I=1}^{r} \hat{F}(\xi_I) w_I \tag{28}$$

By substituting Eq. (20) for γ and v and also, $\varphi_1 = \psi_i^{(1)}$ and $\varphi_2 = \psi_i^{(2)}$ into the weak forms (18) and (19), the finite element equations are obtained

$$0 = \sum_{j=1}^{m} k_{ij}^{11} v_j + \sum_{j=1}^{n} k_{ij}^{12} \gamma_j - Q_{2i-1} \tag{29}$$

$$0=\sum_{j=1}^{m}k_{ij}^{21}v_j+\sum_{j=1}^{n}k_{ij}^{22}\gamma_j-Q_{2i} \tag{30}$$

where

$$k_{ij}^{11}=-\mu_s GA\int_{x_k^e}^{x_{k+1}^e}\psi_{i,1}^{(1)}\psi_{j,1}^{(2)}dx$$

$$k_{ij}^{12}=\mu_s GA\int_{x_k^e}^{x_{k+1}^e}\psi_{i,1}^{(1)}\psi_{j}^{(2)}dx$$

$$k_{ij}^{21}=(\mu_s GA+pht)\int_{x_k^e}^{x_{k+1}^e}\psi_{j,1}^{(1)}\psi_{i}^{(2)}dx$$

$$k_{ij}^{22}=-\int_{x_k^e}^{x_{k+1}^e}\{EI\psi_{i,1}^{(2)}\psi_{j,1}^{(2)}+(pht+\mu_s GA)\psi_{i}^{(2)}\psi_{j}^{(2)}\}dx \tag{31}$$

These equations can be written in matrix form as

$$\begin{pmatrix}[k^{11}] & [k^{12}]\\ [k^{21}] & [k^{22}]\end{pmatrix}\begin{Bmatrix}\{v\}\\ \{\gamma\}\end{Bmatrix}=\begin{Bmatrix}\{Q^1\}\\ \{Q^2\}\end{Bmatrix} \tag{32}$$

The boundary conditions for this panel can be written as form of (Fig. 1(b))

$$\mathrm{T}_1=F_1-pht(v_{,1}-\gamma)$$

$$\mathrm{T}_2=F_2-pht(v_{,1}-\gamma) \tag{33}$$

It can be noted that the end condition for each element is illustrated in Fig. 2.

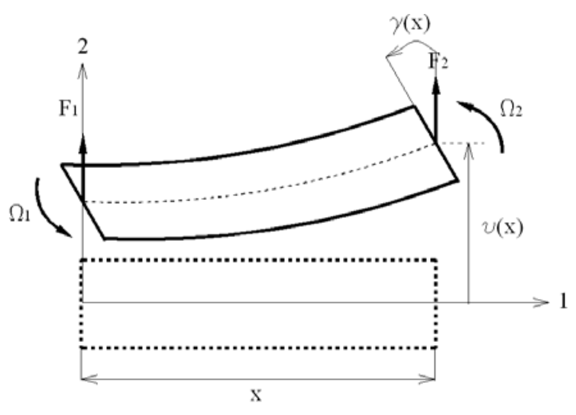


Fig. 2 End conditions in each element

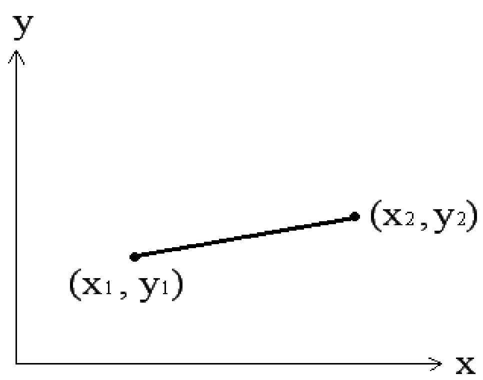


Fig. 3 An inflatable panel element in middle plane

So, it is obvious that

$$Q_1 = -F_1 \frac{\mu_s GA}{\mu_s GA + pht}$$

$$Q_2 = -\Omega_2$$

$$Q_3 = -F_2 \frac{\mu_s GA}{\mu_s GA + pht}$$

$$Q_4 = -\Omega_2 \tag{34}$$

The element is defined between the nodes 1 and 2 in the (x, y) plane, and has a unit width along the z-axis. The co-ordinates are named (x_1, y_1) and (x_2, y_2) (Fig. 3).

It should be noted that all sub-matrices in (32) are of the same order because of using equal interpolation approximation. However, one-point Gaussian quadrature integration method is used for evaluation of all element coefficient matrices, except the second part of k_{ij}^{22} that is evaluated using Reduced Integration Element (RIE). For constant values of $\mu_s GA$, EI and pht, and linear interpolation functions the sub-matrices can be written as explicit values

$$\left[k^{11}\right] = \frac{-\mu_s GA}{l_e}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$\left[k^{12}\right] = \frac{\mu_s GA}{2}\begin{bmatrix} -1 & -1 \\ 1 & 1 \end{bmatrix}$$

$$\left[k^{21}\right] = \frac{pht + \mu_s GA}{2}\begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix}$$

$$\left[k^{22}\right] = -\frac{EI}{l_e}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} - \frac{pht + \mu_s GA}{4} l_e \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \tag{35}$$

The element equations are

$$\begin{pmatrix} H_1 & H_2 & -H_1 & H_2 \\ H_2 & H_3 & -H_2 & H_4 \\ -H_1 & -H_2 & H_1 & -H_2 \\ H_2 & H_4 & -H_2 & H_3 \end{pmatrix} \begin{Bmatrix} v_1 \\ \gamma_1 \\ v_2 \\ \gamma_2 \end{Bmatrix} = \begin{Bmatrix} -F_1 \\ -\Omega_1 \\ -F_2 \\ -\Omega_2 \end{Bmatrix} \tag{36}$$

where

$$H_1 = -\frac{\mu_s GA + pht}{l_e}$$

$$H_2 = -\frac{\mu_s GA + pht}{2}$$

$$H_3 = -(\frac{EI}{l_e} + \frac{pht + \mu_s GA}{4} l_e)$$

$$H_4 = \frac{EI}{l_e} - \frac{pht + \mu_s GA}{4} l_e \tag{37}$$

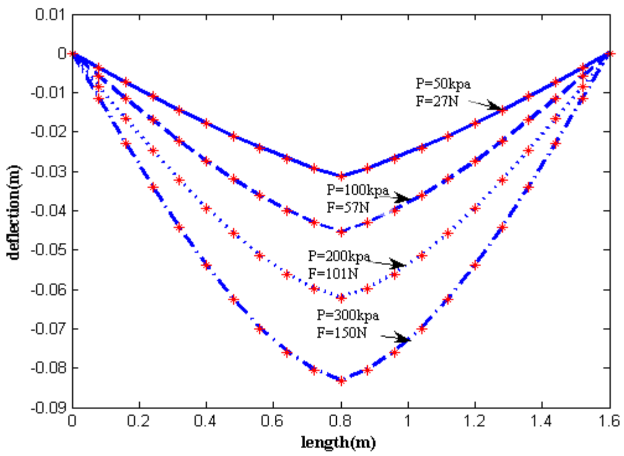


Fig. 4 Theoretical and numerical values of the deflection of simply supported panel

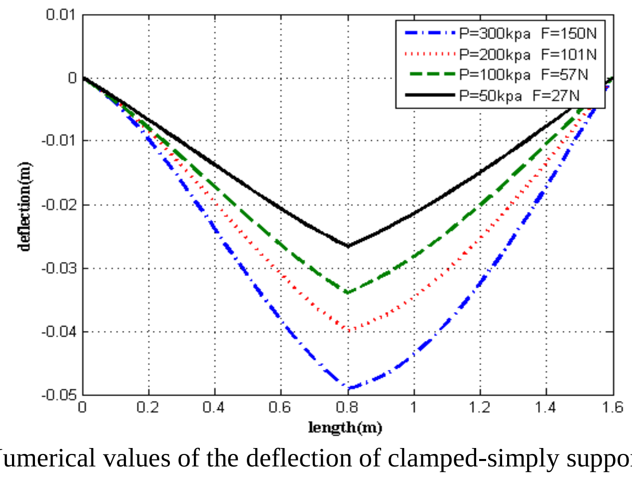


Fig. 5 Numerical values of the deflection of clamped-simply supported panel

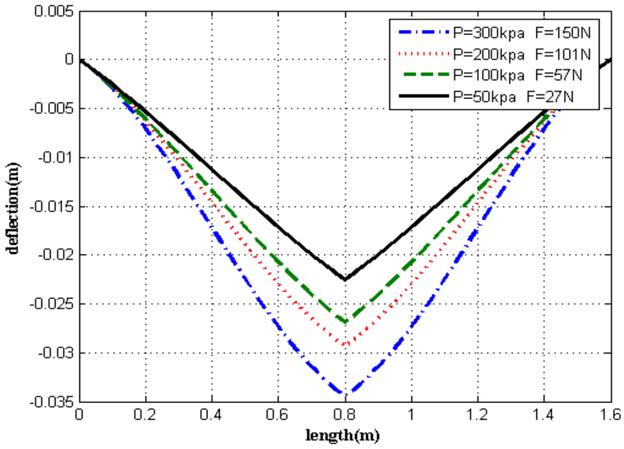
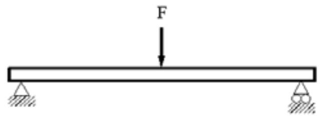
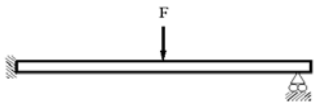
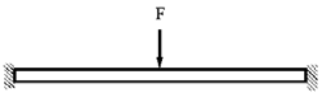


Fig. 6 Numerical values of the deflection of clamped-clamped panel

Table 1 Maximum deflections of various panels (m)

	F=150N P=300kpa	F=101N P=200kpa	F=57N P=100kpa	F=27N P=50kpa	Cases
Analytical Bouzidi *et al.* (2003)	-0.0833	-0.0622	-0.0455	-0.0314	
Proposed solution	-0.0832	-0.0621	-0.0454	-0.0313	
Error (%)	0.12	0.16	0.22	0.32	
Proposed solution	-0.04902	-0.0399	-0.03395	-0.0267	
Proposed solution	-0.0344	-0.0293	-0.0269	-0.0226	

4. Comparisons between theoretical and numerical results

The finite element analysis will be validated by comparing the results with those obtained from analytical solutions (Wielgosz and Thomas 2002). The proposed method was used to model the upper and lower layer of the panel in three arbitrary cases of supporting conditions such as simply-simply, clamped-simply and clamped-clamped supported in order to evaluate the deflections of inflatable panel. The panel has been submitted to different values of bending load in the middle of panel and uniform pressure to check the accuracy of presented study in mechanical analysis of inflatable panel at high pressure. Numerical computations were done using a membrane stiffness modulus equal to E = 650000*Pam*, evaluated by an inflation test Wielgosz and Thomas (2002). The distance between the supports was 1.6 m. The height and width of cross-section were h =

0.055 m and t = 0.2 m, respectively. Mentioned properties are common in all cases. Fig. 4 and Table 1 show the results of comparison for the simply supported panel between the numerical model and the analytical solution for three levels of pressures and three values of the loads. It can be clearly seen that the values of internal pressure have an important effect on the behavior of inflatable panel that it is appeared in stiffness matrix of inflatable panels. The maximum error in this case is less than 0.35%. As a result of this comparison, the accuracy of the proposed finite element model convinces us to extend the presented method for the other cases of boundary conditions like clamped-clamped and clamped-simply supporting (Fig. 5 and Fig. 6).

In order to show the convergence of proposed solution, the mesh sensitivity of inflatable panel in all three cases is illustrated in the Fig. 7, Fig. 8 and Fig. 9 for F = 150N and P = 300KP. The mesh generations in this study have a low effect in the values of deflection. As it can be clearly seen, differences between maximum deflections with 8 and 50 finite elements are about 0.001 m in simply-simply panel, 0.0013 m in clamped-simply panel and 0.0013 m in clamped-clamped panel that it shows the rate of convergence is acceptable.

The comparison between this numerical solution and the results obtained from experiment (Bonet 2000) also confirms the accuracy of mentioned method. It shows that the presented method is powerful and accurate for analysis of inflatable panels.

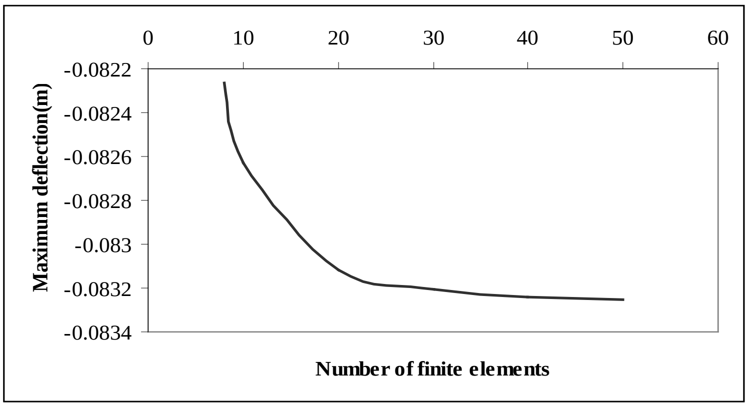


Fig. 7 Mesh sensitivity on maximum deflection in simply-simply panel (F = 150N, P = 300kpa)

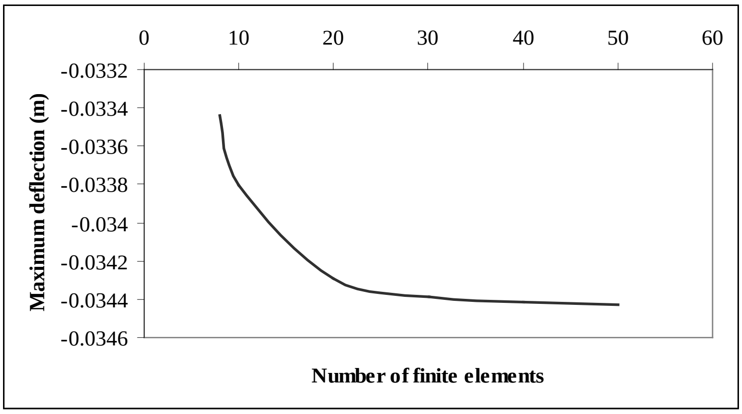


Fig. 8 Mesh sensitivity on maximum deflection in clamped-simply panel (F = 150N, P = 300kpa)

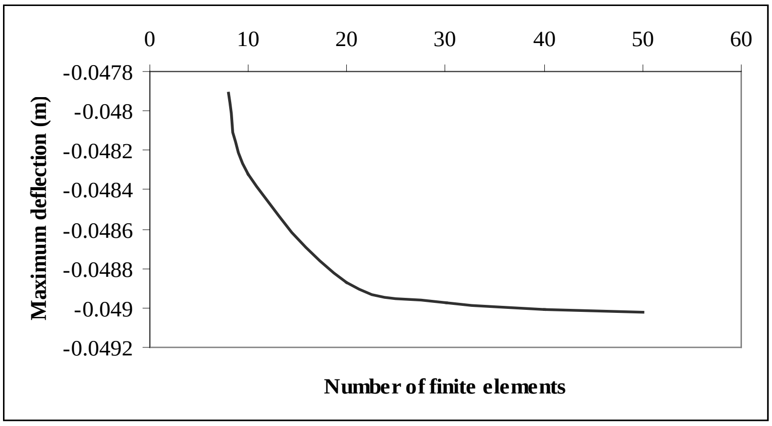


Fig. 9 Mesh sensitivity on maximum deflection in clamped-clamped panel (F = 150N, P = 300kpa)

5. Conclusions

In this paper finite element method was described for problem of inflatable panels. The paper deal with the general numerical results to determine characteristics of deformed inflatable panels. The main results were relative to the deflections under bending loads. Equilibrium equations were written in the deformed state to take into account the geometrical stiffness and the following forces. In fact, inflatable structures can not be viewed as ordinary plates or beams because their deformation pattern is quit different. The influence of the shear stress cannot also be neglected and Timoshenko’s beam theory has to be used. The modeling of textile structures by the finite element method is a new approach based on the combination of geometric and mechanical models which appears the stiffness matrix. Obviously, the stiffness matrix depended explicitly on the inflation pressure. The problem of shear locking was solved by use of Reduced Integration Element Method (RIE) to disappear the singularity of stiffness matrix of these structures. Then various end conditions were analyzed using proposed numerical method.

To sum up, the special feature of the presented paper was to develop a new solution for prediction of the behavior of inflatable structures. This paper shows that the finite element method as presented in the paper can be considered as an appropriate method to determine the deformation of inflatable structures. The finite element procedure is developed for inflatable panels under various supporting conditions and loadings. The code is validated by comparing the numerical results with earlier published results. It is now possible to figure out the mechanical behavior of inflatable panels and the method can be used for predicting the behavior of inflatable panels at high pressure with a good accuracy in different status.

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