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Static and free vibration analysis of shallow sagging inclined cables

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Abstract. Based on link-model, we conducted a static analysis and computation of a three-span suspended cable structure in the present paper, and obtained the static configuration and tension distribution of the cable. Using the link and beam model based on finite element method, we analyzed the vibration modal of three-span suspended cable structure, and compared with the results obtained from ANSYS using link and beam element. The vibration modals of shallow sagging inclined cables calculated from proposed method agrees well with ANSYS results, which validates the proposed method. As a result, the influence of bend stiffness on in-plane natural frequencies is much greater than that on out-of-plane natural frequencies of inclined cables.

Keywords: inclined cables; link-model; beam-model; static analysis; non-linear vibration

1. Introduction

Taking the advantage of low density, high strength and large flexibility, cables are widely used as the main bearing components in structures such as cable-stayed bridges. However, the large-amplitude vibration caused by the wind and rain, which mainly involves in-plane and out-of-plane modes, might cause problems in practice. The vibration of cables has been studied by many researchers (Kunihiro 2008, Vassilopoulou 2010, Canelas 2010). Hagedorn *et al.* (1980) derived the non-linear vibration of cables considering large-amplitude vibration. Perkins *et al.* (1977) and Hassan *et al.* (1987) studied the linear vibration theory of inclined cables, and discussed the influence of related parameters such as stiffness-to-weight ratio on the cables' natural frequencies. Nayfeh *et al.* (1989) reviewed the theoretical and experimental studies on the influence of the modal interactions on the nonlinear response of harmonically excited structures, and discussed the quadratic nonlinearities, which may lead to two-to-one and combination autoparametric resonances. Perkins (1992) derived the three-dimensional motion of travelling cables with arbitrary initial sag and arbitrary support eyelet elevations, and predicted the natural frequencies, mode shapes and the stability of equilibrium of cable. Lacarbonara *et al.* (2007, 2008) and EI-Attar *et al.* (2000) formulated the coupled non-linear vibration equations of cables due to

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the transverse and vertical multiple support excitations, and investigated the importance of phase differences between the support excitations at the cable ends. Wang *et al.* (2009) investigated the nonlinear response of a shallow suspended cable subjecting to the primary resonance excitation, and analyzed the effects of the excitation amplitude on the frequency–response curves of the cable. Srinil *et al.* (2004) presented a model analyzing three dimensional large-amplitude free vibrations of a suspended cable, and analyzed both the non-linear coupling between three- and two-dimensional motions, and the tension responses of non-linear cable. Yu *et al.* (1999) studied the vibration and control of cables considering damping effect. Desai *et al.* (1995, 1996) presented a finite element method with three-node, isoparametric cable element to successfully simulate field galloping records.

Previous researchers analyzed galloping of cables neglecting the influence of bending stiffness on free vibration and galloping. This paper will discuss the difference between the beam model with bending stiffness and link model without bending stiffness.

2. Link model of cables without bending stiffness

The cables are one dimensional elastic structure with bending and torsional stiffness.

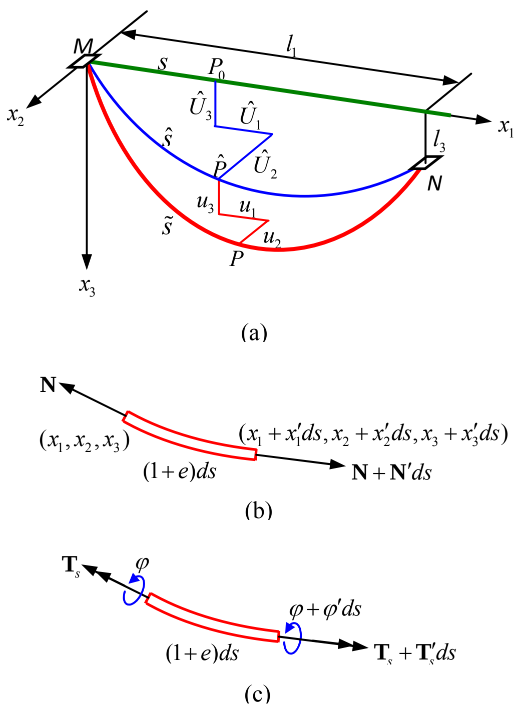

Fig. 1 The model of inclined cable: (a) The static configuration of inclined cable; (b) The displacement and tension of cable element; (c) The torsion angle and torque of cable element

The vibration model of inclined cables is shown in Fig. 1. Here, $x_1x_2x_3$ are Cartesian coordinate system; P_0 is the natural unstretched position; $\hat{P}(U_1,U_2,U_3)$ is the initial deformation position under gravity; $P(x_1, x_2, x_3)$ is the dynamic deformation position under dynamic loads. (u_1, u_2, u_3) are the displacement of position P in coordinate system $x_1x_2x_3$, φ is the torsion angle of cross section of cable in position P.

The displacement of cable in position P_0 can be written as follows

$$x_i = U_i + u_i, \quad i = 1,2,3 \tag{1}$$

Hamilton's principle is written as

$$\int_0^t (\delta K - \delta\Pi + \delta W_{nc})dt = 0 \tag{2}$$

where δK is the variation of total kinetic energy, $\delta\Pi$ is the variation of strain potential energy, δW_{nc} is the work done by external forces as a result of the variation.

Motion equations of inclined cables based on link-model can be written as follows

$$m_0\ddot{u}_i + c_i\dot{u}_i = \frac{\partial}{\partial s}\left[\frac{N}{1+e}(U_i' + u_i')\right] + f_i, \quad i = 1,2,3 \tag{3}$$

$$\mathrm{J}\ddot{\varphi} + c_4\dot{\varphi} = \frac{\partial}{\partial s}\left[\frac{GI}{1+e}\varphi'\right] + f_4 \tag{4}$$

where $()' = \partial()/\partial_S$.

And boundary conditions can be written as follows,

$$u_i \text{ or } \frac{N}{1+e}(U_i' + u_i') \text{ at } s = 0 \text{ and } L, \quad i = 1,2,3$$
$$\varphi \text{ or } \frac{N}{1+e}\varphi' \text{ at } s = 0 \text{ and } L$$

where

m — the mass per unit length of inclined cables;
J — the rotational inertia of inclined cables about longitudinal axis;
f_i — the distribution forces on cables;
c_i — structural damping coefficient in x_i;
I — the cross sectional polar inertia moment;
N — tension of the cables;
G — Shear modulus.

In order to get the static configuration of cables, we assume that $u_i = 0$, $\varphi = 0$, $f_4 = 0$, $f_i = \hat{f}_i$ and then Eq. (3) can be simplified as below

$$\frac{d}{ds}\left[\frac{\hat{N}}{1+\hat{e}}U_i'\right] + \hat{f}_i = 0, \quad i = 1,2,3 \tag{5}$$

where $\hat{N}$ is the static tension of cables, $\hat{e}$, the static strain of cables, $\hat{f}_i$, the static distribution force of cables.

Supposing that the static distribution force acting on cable is gravity only, and boundary conditions $U_1 = U_2 = U_3 = 0$ are applied at $s = 0$. Then the solution of Eq. (5) can be found as

$$U_1 = \frac{F_1 s}{EA_0} + \frac{F_1}{m_0 g}\left[\sinh^{-1}\left(\frac{F_3}{F_1}\right) - \sinh^{-1}\left(\frac{F_3 - m_0 g s}{F_1}\right)\right] \tag{6}$$

$$U_3 = \frac{2F_3 s - m_0 g s^2}{2EA_0} + \frac{1}{m_0 g}\left[\sqrt{F_1^2 + F_3^2} - \sqrt{F_1^2 + (F_3 - m_0 g s)^2}\right] \tag{7}$$

With boundary conditions $U_1 = l_1$, $U_2 = 0$, $U_3 = l_3$ at $s = L$, we can obtain

$$l_1 = \frac{F_1 L}{EA_0} + \frac{F_1}{m_0 g}\left[\sinh^{-1}\left(\frac{F_3}{F_1}\right) - \sinh^{-1}\left(\frac{F_3 - W}{F_1}\right)\right] \tag{8}$$

$$l_3 = \frac{2F_3 L - WL}{2EA_0} + \frac{L}{W}\left[\sqrt{F_1^2 + F_3^2} - \sqrt{F_1^2 + (F_3 - W)^2}\right] \tag{9}$$

where $W = m_0 g L$ is the total weight of the cables.

3. Beam model of cables

The Link model neglects the bending stiffness of cables. For more accurate approximation, the cable can be regarded as Euler-Bernoulli beam. Furthermore, the material of the cable can be regarded as isotropic and linear elastic if large deflections but small strains are considered in describing the motion of cables.

The three dimensional governing equation of motion based on Euler-Bernoulli beam theory can be presented as

$$\left[EAeT_{11} + \lambda_2 T_{21} + \lambda_3 T_{31}\right]' = m\ddot{u}_1 + c_1\dot{u}_1 - f_1 \tag{10}$$

$$\left[EAeT_{12} + \lambda_2 T_{22} + \lambda_3 T_{32}\right]' = m\ddot{u}_2 + c_2\dot{u}_2 - f_2 \tag{11}$$

$$\left[EAeT_{13} + \lambda_2 T_{23} + \lambda_3 T_{33}\right]' = m\ddot{u}_3 + c_3\dot{u}_3 - f_3 \tag{12}$$

$$\left(\frac{2GI\rho_1}{1+e}\right)' + f_4 = J\dot{\omega}_1 + c_4\ddot{\varphi} \tag{13}$$

$$\begin{aligned} \lambda_2 &= \frac{1}{1+e}\left[-(EI\rho_3)' + (2GI - EI)\rho_1\rho_2 + \frac{1}{2}J\dot{\omega}_3 - \frac{1}{2}J\omega_1\omega_2\right] \\ \lambda_3 &= \frac{1}{1+e}\left[(EI\rho_2)' + (2GI - EI)\rho_1\rho_3 - \frac{1}{2}J\dot{\omega}_2 - \frac{1}{2}J\omega_1\omega_3\right] \end{aligned} \tag{14}$$

where $[T]$ is the coordinate transformation matrix, ρ_1, ρ_2, ρ_3 are the curvature radius of cables, $\omega = (\omega_1, \omega_2, \omega_3)$ is the angular velocity vector of local coordinates. ρ_1, ρ_2, ρ_3 are much smaller compared with the length of inclined cables, so the terms containing $\rho_1 \rho_2$ and $\rho_1 \rho_3$ in Eq. (14) can be omitted. Otherwise, we can neglect the influence of rotational inertia on the transverse vibration, since inclined cables' galloping frequencies is very low and their galloping wavelength is much larger than the diameter of the cross section of the cable, and hence terms containing J in Eq. (14) can be omitted. Furthermore, $J\dot{\omega}_1$ in Eq. (13), which is the torsional inertia moment of inclined cables, can also be omitted since it is a small term. However, in order to keep the dynamic form of Eq. (13), we reserve the $\dot{\varphi}$ term in $\omega_1 = \dot{\varphi} - \dot{\psi}\sin\theta$, so that Eqs. (10)-(14) can be rewritten in the following forms

$$\left[EAeT_{11} + \lambda_2 T_{21} + \lambda_3 T_{31}\right]' = m\ddot{u}_1 + c_1\dot{u}_1 - f_1 \tag{15}$$

$$\left[EAeT_{12} + \lambda_2 T_{22} + \lambda_3 T_{32}\right]' = m\ddot{u}_2 + c_2\dot{u}_2 - f_2 \tag{16}$$

$$\left[EAeT_{13} + \lambda_2 T_{23} + \lambda_3 T_{33}\right]' = m\ddot{u}_3 + c_3\dot{u}_3 - f_3 \tag{17}$$

$$\left(\frac{2GI\rho_1}{1+e}\right)' = J\ddot{\varphi} + c_4\dot{\varphi} - (\hat{f}_4 + \tilde{f}_4) \tag{18}$$

$$\lambda_2 \approx -\frac{EI}{1+e}\rho_3', \quad \lambda_3 \approx \frac{EI}{1+e}\rho_2' \tag{19}$$

Eqs. (15)-(19) can be simplified by neglecting higher order terms with small quantities, and the simplified linear equations are

$$\begin{Bmatrix} EAeT_{11} + \lambda_2 T_{21} + \lambda_3 T_{31} \\ EAeT_{12} + \lambda_2 T_{22} + \lambda_3 T_{32} \\ EAeT_{13} + \lambda_2 T_{23} + \lambda_3 T_{33} \\ \left(\dfrac{2GI\rho_1}{1+e}\right) \end{Bmatrix}' = -D_0 - [D]u' \tag{20}$$

where

$$D_0 = -\begin{Bmatrix} \left(\sqrt{U_1'^2 + U_3'^2} - 1\right)\dfrac{EAU_1'}{\sqrt{U_1'^2 + U_3'^2}} \\ 0 \\ \left(\sqrt{U_1'^2 + U_3'^2} - 1\right)\dfrac{EAU_1'}{\sqrt{U_1'^2 + U_3'^2}} \\ 0 \end{Bmatrix}, \quad [D] = -\begin{bmatrix} D_{11} & 0 & D_{13} & 0 \\ 0 & D_{22} & 0 & 0 \\ D_{13} & 0 & D_{33} & 0 \\ 0 & 0 & 0 & D_{44} \end{bmatrix}, \quad u = \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ \varphi \end{Bmatrix} \tag{21}$$

and D_{11}, D_{13}, D_{22}, D_{33}, D_{44} can be written as:

$$D_{11} = EA - \frac{EAU_3'^2}{(U_1'^2 + U_3'^2)^{3/2}} - \frac{EIU_3'}{\sqrt{U_1'^2 + U_3'^2}}\left(\frac{U_3'}{U_1'^2} - \frac{U_3'^3}{U_1'^4}\right)''$$

$$D_{13} = \frac{EAU_1'U_3'}{(U_1'^2 + U_3'^2)^{3/2}} - \frac{EIU_3'}{\sqrt{U_1'^2 + U_3'^2}}\left(\frac{U_3'^2}{U_1'^2} - \frac{1}{U_1'}\right)''$$

$$D_{22} = \left(\sqrt{U_1'^2 + U_3'^2} - 1\right)\frac{EA}{\sqrt{U_1'^2 + U_3'^2}} + EI\left(\frac{U_1''}{U_1'\sqrt{U_1'^2 + U_3'^2}}\right)'$$

$$D_{33} = EA - \frac{EAU_1'^2}{(U_1'^2 + U_3'^2)^{3/2}} - \frac{EIU_1'}{\sqrt{U_1'^2 + U_3'^2}}\left(\frac{U_3'^2}{U_1'^3} - \frac{1}{U_1'}\right)''$$

$$D_{44} = 2GI$$

where D_0 is a column vector, $[D]$ is a symmetric linear differential operator matrix, D_0 and $[D]$ are the functions of the arc coordinate s.

Substituting Eq. (20) into Eqs. (15)-(19), and neglecting structural damping, we obtain

$$[M]\ddot{u} + \frac{\partial}{\partial s}(D_0 + [D]u') = \hat{f} + \tilde{f} \tag{22}$$

where

$$[M] = \mathrm{diag}\{m, m, m, J\}, \; f = \{f_1, f_2, f_3, f_4\}^T, \;\; f = \{f_1, f_2, f_3, f_4\}^T$$

In Eq. (22), supposing $u = 0$, $\tilde{f} = 0$, we can obtain the static equilibrium equation of inclined cables

$$\frac{\partial}{\partial s}D_0 = \hat{f} \tag{23}$$

Hence we can get the linear equation of motions of cables near equilibrium position

$$[M]\ddot{u} + \frac{\partial}{\partial s}([D]u') = \tilde{f} \tag{24}$$

4. Comparison between beam model and link model

4.1 Static analysis

The integration of Eq. (23) is

$$-D_0 = -\int_0^s \hat{f} ds + F; \quad F = -D_0\big|_{s=0} \tag{25}$$

Substitute D_0 into Eq. (25) and consider the gravity only, we obtain

$$U_1' = \frac{F_1}{EA} + \frac{F_1}{\sqrt{F_1^2 + (F_3 - m_0 g s)^2}} \tag{26}$$

$$U_3' = \frac{(F_3 - m_0 g s)}{EA}\left[1 + \frac{EA}{\sqrt{F_1^2 + (F_3 - m_0 g s)^2}}\right] \tag{27}$$

With the integration of Eqs. (26)-(27) with boundary conditions $U_1 = U_2 = U_3 = 0$ at $s = 0$, we have

$$\begin{aligned} U_1 &= \frac{F_1 s}{EA} + \frac{F_1}{m_0 g}\left[\sinh^{-1}\left(\frac{F_3}{F_1}\right) - \sinh^{-1}\left(\frac{F_3 - m_0 g s}{F_1}\right)\right] \\ U_3 &= \frac{2F_3 s - m_0 g s^2}{2EA} + \frac{1}{m_0 g}\left[\sqrt{F_1^2 + F_3^2} - \sqrt{F_1^2 + (F_3 - m_0 g s)^2}\right] \end{aligned} \tag{28}$$

Comparing Eq. (28) with Eqs. (6)-(7), we can find that the static configuration obtained from beam model and Link model are the same for the shallow sagging inclined cables. So the bend stiffness of cables does not affect the static deformation.

4.2 Free vibration

The mass matrix and stiffness matrix of the arbitrary element with length l are

$$[M_e] = \int_0^l [\widetilde{N}(s)]^T [M][\widetilde{N}(s)] ds, \quad [K_e] = \int_0^l [\widetilde{N}'(s)]^T [D(s)][\widetilde{N}'(s)] ds \tag{29}$$

The deflection interpolation function of the element is

$$u = [\widetilde{N}(s)]u_i = [[\widetilde{N}_1(s)],[\widetilde{N}_2(s)]]\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} \tag{30}$$

For link model, each element has four degrees of freedom (three translational and one torsional degree of freedom), node displacement vectors at each end are represented as

$$u_1 = [\mu_1, \mu_2, \mu_3, \varphi]^T, \quad u_2 = [\mu_1, \mu_2, \mu_3, \varphi]^T \tag{31}$$

The linear interpolation function of the element is

$$u(s) = u_1 + \frac{s}{l}(u_2 - u_1) = \left[\left(l - \frac{s}{l}\right)I, \ \frac{s}{l}I\right]\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} \tag{32}$$

where I is a 4 × 4 unit matrix, and the interpolation function matrix $[\tilde{N}(s)]$ is

$$[\tilde{N}(s)] = \left[\left(1-\frac{s}{l}\right)I,\ \frac{s}{l}I\right] \in R^{4\times 8} \tag{33}$$

According to Canelas and Sensale (2010), differential operator matrix $[D(s)]$ of link model is

$$[D(s)] = \beta\begin{bmatrix} \hat{N}_1 + \beta^2 EAU_1'^2 & \beta^2 EAU_1'U_3' & 0 & 0 \\ \beta^2 EAU_1'U_3' & \hat{N}_1 + \beta^2 EAU_3'^2 & 0 & 0 \\ 0 & 0 & \hat{N} & 0 \\ 0 & 0 & 0 & 2GI \end{bmatrix},\quad \beta = \frac{1}{1+\hat{N}/EA} \tag{34}$$

where $[M] = \mathrm{diag}\{m, m, m, J\}$.

For beam model, element node displacement vectors at each end are

$$u_1 = \{u_1, u_3, u_2, \varphi, \theta_1, \theta_2\}_1^T,\quad u_2 = \{u_1, u_3, u_2, \varphi, \theta_1, \theta_2\}_2^T \tag{35}$$

Deformation interpolation function of an element can involve the beam function, longitudinal stretch and tortional degree using linear interpolation and the interpolation matrix $[\tilde{N}(s)]$ is

$$[\tilde{N}_1(s)] = \begin{bmatrix} \tilde{N}_1(s) & 0 & 0 & 0 & 0 & 0 \\ 0 & \tilde{N}_3(s) & 0 & 0 & 0 & \tilde{N}_4(s) \\ 0 & 0 & \tilde{N}_3(s) & 0 & \tilde{N}_4(s) & 0 \\ 0 & 0 & 0 & \tilde{N}_1(s) & 0 & 0 \end{bmatrix} \tag{36}$$

$$[\tilde{N}_2(s)] = \begin{bmatrix} \tilde{N}_2(s) & 0 & 0 & 0 & 0 & 0 \\ 0 & \tilde{N}_5(s) & 0 & 0 & 0 & \tilde{N}_6(s) \\ 0 & 0 & \tilde{N}_5(s) & 0 & \tilde{N}_6(s) & 0 \\ 0 & 0 & 0 & \tilde{N}_2(s) & 0 & 0 \end{bmatrix} \tag{37}$$

The differential operator matrix $[D(s)]$ of the beam model is in Eq. (21), $[M] = \mathrm{diag}\{m, m, m, J\}$.

5. Numerical examples

5.1 Problem definition

The three-span suspended cable structure is shown in Fig. 2. The mass per unit length of inclined cables is $\rho = 2.755$ kg/m, the area of the cross section of the cable $A = 6.336 \times 10^{-4}$ m^2, and the modulus of elasticity $E = 1.03 \times 10^{11}$ N/m^2. The altitude differences of those three spans are h_{ab} = 65.2 m, h_{bc} = 19.5 m and h_{cd} = 76.2 m, respectively. And the length of those three spans are l_{ab} =

563 m, l_{bc} = 1055 m and l_{cd} = 581 m, respectively. Position *A* is 0 m, position *D* is 2218 m. Cables are hinged in position *A* and *D* and O_1B and O_2C are regarded as rigid straight beam. O_1 and O_2 are hinged position for rigid straight beam, *B* and *C* can whirl around the point O_1 and O_2.

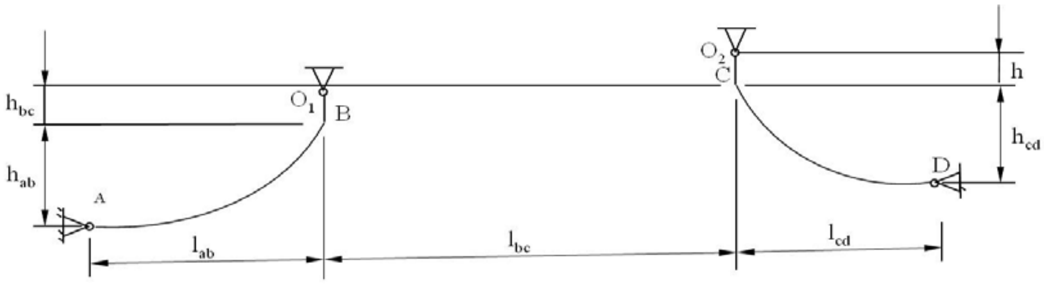

Fig. 2 Three-span suspended cable structure

5.2 Results and analysis

5.2.1 Static analysis

Fig. 3 shows the static configuration and distribution of the tension stresses of cables. Geometric parameters obtained by static analysis are listed in Table 1.

Table 1 Results of static analysis

i j	Parameters /m			Tension /N	
	h_{ij}	l_{ij}	L_{ij}	N_i	N_j
i=A, j=B	65.2	563	567.64	61614	63372
i=B, j=C	19.5	1055	1.63.42	63472	63998
i=C, j=D	76.2	581	586.94	64088	62032

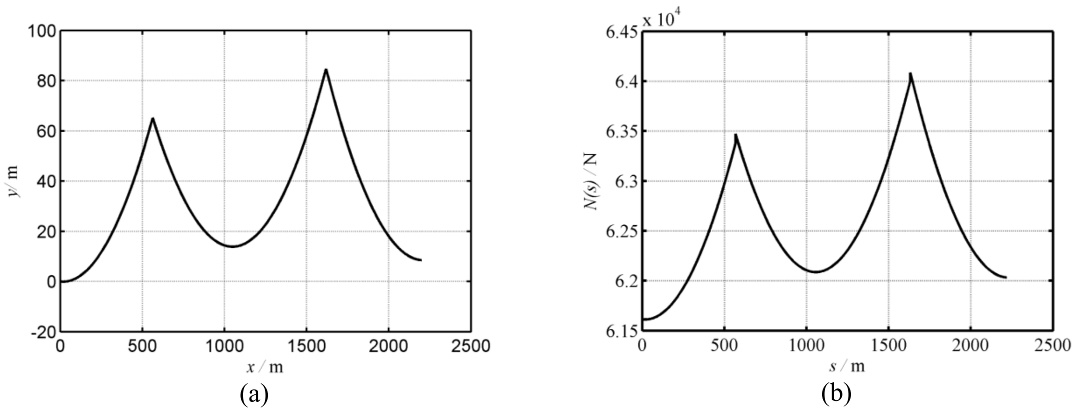

Fig. 3(a) Static configuration and (b) distribution of tension

5.2.2 The out-of-plane modes

The out-of-plane natural frequencies calculated by our method and ANSYS are listed in Tables 2 and 3. Fig. 4 shows the fourth-order out-of -plane modes.

Table 2 The out-of-plane natural frequencies calculated by proposed method /Hz

Order	1	2	3	4	5	6	7	8 (4 half wave)
Linked model	0.07107	0.12871	0.13250	0.14197	0.21315	0.25807	0.26567	0.28467
Beam model	0.07126	0.12893	0.13273	0.14227	0.21330	0.25772	0.26532	0.28440

Table 3 The out-of-plane natural frequencies calculated by ANSYS /Hz

Order	1	2	3	4	5	6	7	8
Link element	0.07073	0.12807	0.13198	0.14121	0.21170	0.25599	0.26383	0.28228
Beam element	0.07185	0.13159	0.13573	0.14343	0.21507	0.26304	0.27132	0.28672

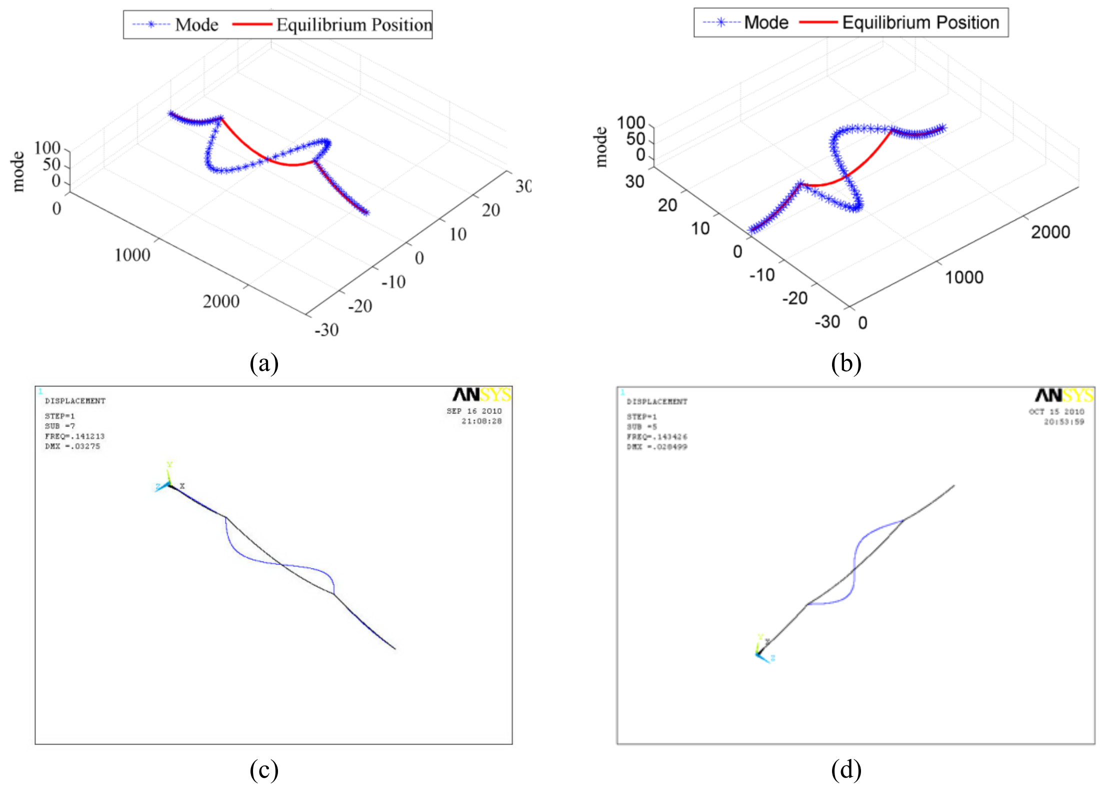

Fig. 4 The out-of-plane modes for fourth-order: (a) Link model, (b) beam model, (c) ANSYS link element, (d) ANSYS beam element

5.2.3 The in-plane modes

The in-plane natural frequencies calculated by our method and ANSYS are listed in Tables 4 and 5. Fig. 5 shows the fourth-order in-plane modes.

Table 4 The in-plane natural frequencies calculated by proposed method /Hz

Order	1	2	3	4	5	6	7 (4 half wave)
Linked model	0.14090	0.19868	0.25872	0.26634	0.30360	0.30906	0.28534
Beam model	0.15556	0.21626	0.29723	0.30171	0.31135	0.31534	0.32236

Table 5 The in-plane natural frequencies calculated by ANSYS /Hz

Order	1	2	3	4	5	6	7
Link element	0.13977	0.21059	0.25582	0.26387	-	-	0.28249
Beam element	0.14164	0.20158	0.26206	0.270361	-	-	0.28672

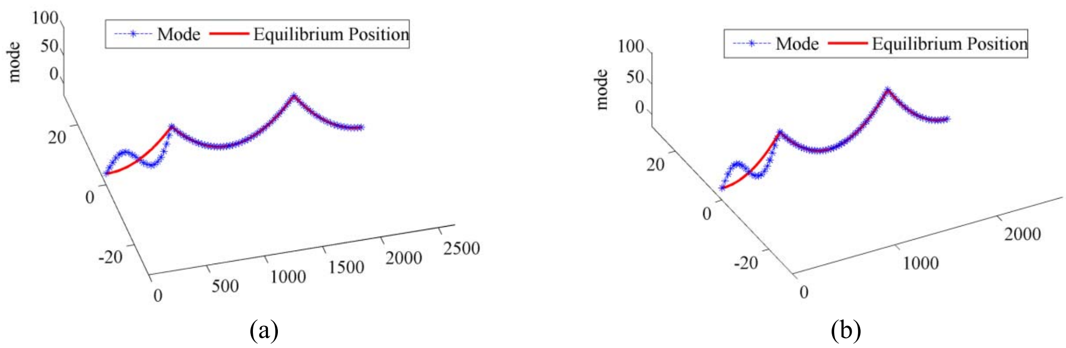

Fig. 5 The in-plane modes for fourth-order: (a) link model, (b) beam model

5.2.4 Torsional modes

Table 6 presents the natural frequencies of torsional modes calculated by proposed method. Fig. 6 shows the distribution of rotation angle along arc length.

Table 6 The torsional natural frequencies /Hz

Order	1	2	3
Link element	1.2708	1.3140	1.4028
Beam element	1.2714	1.3146	1.4035

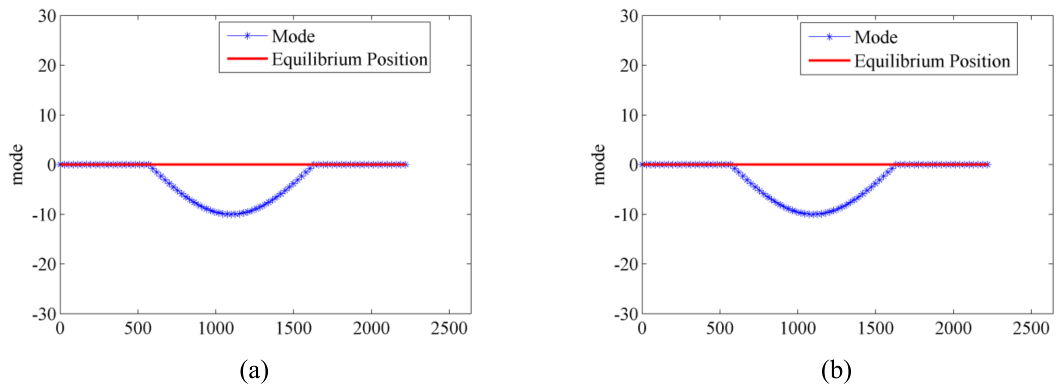

Fig. 6 The torsional mode for third-order: (a) link model; (b) beam model

6. Conclusions

The three dimensional non-linear dynamic equations of inclined cables are formulated based on Hamilton's variational principle. Using the link and beam model based on finite element method, we analyzed the vibration modal of three-span suspended cable structure, and compared the results with those obtained from ANSYS based on link and beam element. We have obtained the following conclusions:

(a) The results show the bend stiffness does not affect the linear vibration of cables near equilibrium position.
(b) The influence of bend stiffness on out-of-plane natural frequencies can be neglected. The error of natural frequencies between the link model and beam model is less than 0.267% (first-order) and the error between our method and ANSYS is less than 2.21% (third-order for beam model), which validates our method.
(c) The influence of bend stiffness on in-plane natural frequencies is greater than that of out-of-plane natural frequencies. The error of natural frequencies calculated by our method between the link model and beam model is 13.0% (third-order), and 4.47% for ANSYS results. The error of natural frequencies for link model calculated by ANSYS and proposed method is less than 5.66% (second-order), but the error of beam model is 13.4% (third-order). The influence of bend stiffness on torsional natural frequencies can be ignored since the error is less than 0.05% (third-order).

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