*Structural Engineering and Mechanics, Vol. 45, No.1 (2013) 111 – 127*
DOI: http://dx.doi.org/10.12989/sem.2013.45.1.111


# Efficient gravitational search algorithm for optimum design of retaining walls

Mohammad Khajehzadeh*[1], Mohd Raihan Taha[1] and Mahdiyeh Eslami[2]

[1]*Department of Civil and Structural Engineering, National University of Malaysia, Bangi, Selangor, Malaysia*
[2]*Department of Electrical Engineering, Science and Research Branch, Islamic Azad University, Kerman, Iran*

*(Received August 17, 2012, Revised November 15, 2012, Accepted December 1, 2012)*

**Abstract.** In this paper, a new version of gravitational search algorithm based on opposition-based learning (OBGSA) is introduced and applied for optimum design of reinforced concrete retaining walls. The new algorithm employs the opposition-based learning concept to generate initial population and updating agents' position during the optimization process. This algorithm is applied to minimize three objective functions include weight, cost and $CO_2$ emissions of retaining structure subjected to geotechnical and structural requirements. The optimization problem involves five geometric variables and three variables for reinforcement setups. The performance comparison of the new OBGSA and classical GSA algorithms on a suite of five well-known benchmark functions illustrate a faster convergence speed and better search ability of OBGSA for numerical optimization. In addition, the reliability and efficiency of the proposed algorithm for optimization of retaining structures are investigated by considering two design examples of retaining walls. The numerical experiments demonstrate that the new algorithm has high viability, accuracy and stability and significantly outperforms the original algorithm and some other methods in the literature.

**Keywords:** retaining wall; minimum weight; minimum cost; minimum $CO_2$ emissions; gravitational search algorithm

## 1. Introduction

Reinforced concrete cantilever (RCC) retaining wall, which is one of the most common and utilized types of geotechnical retaining structures, constitute an integral part of the infrastructure and are frequently constructed for a variety of applications. In the analysis and design of retaining structures, the structure must safely and reliably support the backfill soil; provide stability against the possibility of overturning and sliding; limit stresses in both the soil and the structure; and provide acceptable safety factors for all failure modes. In addition to these design objectives, there are many requirements that a reinforced concrete wall must satisfy. It must have sufficient shear and moment capacities in the stem, toe and heel of the wall; the bearing capacity of the foundation cannot be exceeded or allowed to be in tensile stress; and the configuration of the steel reinforcement must meet all building code requirements (Camp and Akin 2012).

*Corresponding author, Ph.D., E-mail: mohammad.khajehzadeh@gmail.com

The traditional goals of engineers in the field of structural optimization design were minimize the objective function, which is usually the cost or the weight of the structure rather than environmental factors. Nowadays, the objective of structural design becomes to optimize the consumption of materials not only from an economic point of view, but also environmental. This study deals with the optimization of RCC retaining walls in terms of minimum weight, cost and $CO_2$ emissions. The $CO_2$ objective function quantifies the total amount of carbon dioxide emissions resulting from the use of materials and minimization of embedded $CO_2$ emissions seems necessary to include design criteria.

Generally, the structural optimization problem can be executed using either conventional deterministic or modern heuristic methods. In deterministic methods, the objective function must be differentiable or continuous or the reasonable region must be convex. This requirement indicates that the efficiency of these methods is limited to problems with a few design variables. Conversely, the second main category of the optimization methods is heuristic methods, which are not restricted in the aforementioned manner. Specifically, heuristics are techniques which provide acceptable (near optimal) solutions at a reasonable computational cost for solving hard and complex problems. These algorithms mimic physical or biological processes. The heuristic optimization algorithms do not require the objective function to be derivable or even continuous and can be employed directly on the fitness function to be optimized. This category includes a large number of search algorithms based on iterations in which the objective function is evaluated and the structural constraints are checked.

In the last decade, these algorithms have been broadly implemented to solve various structural optimization problems and have occasionally overcome several deficiencies of conventional deterministic methods. Some of these include application of genetic algorithm (GA) (Camp *et al.* 1998, Aguilar Madeira *et al.* 2005), harmony search (HS) (Lee and Geem 2004; Togan *et al.* 2011), particle swarm optimization (PSO) (Perez and Behdinan 2007, Doğan and Saka 2012), simulated annealing (SA) (Hasançebi and Erbatur 2002, Yepes *et al.* 2008), artificial bee colony (ABC) (Sonmez 2011, Degertekin 2012), ant colony optimization (ACO) (Camp *et al.* 2005, Aydogdu and Saka 2012) and many others. However, because of complexity and nonlinearity of the objective function and constraints in the field of structural engineering problems, many researchers tried to improve the performance and efficiency of the original heuristic algorithms in some ways and applied them for a specific application. Salajegheh and Gholizadeh (2005) applied improved GA for optimum design of large-scale structures, Wang *et al*. (2010) employed modified ACO for optimization of a laminated composite plate, Hasancebi *et al*. (2010) applied improved SA for optimization of steel structures, Degertekin (2011) implemented improved HS for optimization of truss structures, Khajehzadeh *et al*. (2011) employed modified PSO for optimization of geotechnical structures, etc.

Gravitational search algorithm (GSA) is one of the latest heuristic optimization algorithms, motivated by the gravitational law and laws of motion (Rashedi *et al.* 2009). This approach provides an iterative method that simulates mass interactions, and moves through a multi-dimensional search space under the influence of gravitation. The GSA is characterized as a simple concept that is both easy to implement and computationally efficient. In this study, a new version of GSA (OBGSA) is introduced and applied for minimization of weight, cost and $CO_2$ emissions of RCC retaining walls. The proposed OBGSA introduces a novel scheme for population initialization and updating the agents' positions by applying opposition-based learning concept.

## 2. Retaining wall optimization

A general RCC retaining wall optimization problem can be stated as follows

$$\begin{aligned} &\text{minimize} \quad f(\mathbf{X}) \qquad \mathbf{X} = [x_1, x_2, \ldots, x_n]^T \\ &\text{subject} \quad g_i(\mathbf{X}) \le 0 \qquad i = 1, 2, \ldots, m \\ &\qquad \mathbf{X}^L \le \mathbf{X} \le \mathbf{X}^U \end{aligned} \tag{1}$$

where $\mathbf{X}$ is a vector of length $n$ containing the design variables, $f(\mathbf{X})$ is the objective function, which returns a scalar value to be minimized, $g(\mathbf{X})$ is a vector of length $m$ containing the values of the inequality constraints evaluated at $\mathbf{X}$. $\mathbf{X}^L$ and $\mathbf{X}^U$ are two vectors of length $n$ containing the lower and upper bounds of the design variables, respectively. The above mathematical formulation contains only inequality constraints, as equality constraints are usually not the case in retaining wall optimization.

### 2.1. Objective functions

In this study, the problem of RCC retaining wall optimization consists of three objective functions; the embedded $CO_2$ emissions, total cost and total weight of the structure. Therefore, the optimization aims to minimize one of these three objective functions.

The first objective function measures the total amount of $CO_2$ emissions resulting from the use of materials that involve emissions at the different stages of production and placement. Mathematically, the $CO_2$ emissions objective function can be presented in the following form

$$f_1(\mathbf{X}) = e_c V_c + e_e V_e + e_b V_b + e_f A_f + e_s W_s \tag{2}$$

where $e_c$, $e_e$, $e_b$, $e_f$ and $e_s$ are the $CO_2$ unit emissions of concrete, excavation, backfill, formwork, and reinforcement, respectively. In addition, $V_c$, $V_e$ and $V_b$ denote the volume of concrete, excavation and backfill per unit length of the wall. $A_f$ shows the area of formwork and $W_s$ indicates the weight of steel per unit length of the structure. In the current study, the $CO_2$ unit emissions considered for the optimization are given in Table 1 and are obtained from the BEDEC PR/PCT ITEC materials database of the Catalonia Institute of Construction Technology (2009).

The second objective function quantifies the total cost of RCC retaining wall. This objective function includes the cost of the materials and costs associated with labor and installation. The cost function can be expressed in the following form

$$f_2(\mathbf{X}) = C_c V_c + C_e V_e + C_b V_b + C_f A_f + C_s W_s \tag{3}$$

In Eq. (3), $C_c$, $C_e$, $C_b$, $C_f$ and $C_s$ are the unit cost of concrete, excavation, backfill, formwork, and reinforcement, respectively. The unit costs considered here are presented in Table 1 and are obtained from the BEDEC PR/PCT ITEC materials database of the Catalonia Institute of Construction Technology (2009).

The last objective function is based solely on the weight of the materials, which is expressed as follows

$$f_3(\mathbf{X}) = 100 V_c \gamma_c + W_s \tag{4}$$

Table 1 Basic prices and $CO_2$ emission considered in the analysis

| Item | Unit | Unit Emission (kg/m) | Unit Cost (USD/m) |
|---|---|---|---|
| Earth removal | $m^3$ | 13.16 | 11.41 |
| Foundation formwork | $m^2$ | 14.55 | 36.82 |
| Stem formwork | $m^2$ | 31.66 | 37.08 |
| Reinforcement | kg | 2.82 | 1.51 |
| Concrete in foundations | $m^3$ | 224.94 | 104.51 |
| Concrete in stem | $m^3$ | 265.28 | 118.05 |
| Earth fill-in | $m^3$ | 27.20 | 38.10 |

where $\gamma_c$ is the unit weight of concrete and a factor of 100 is used for consistency of units (Saribas and Erbatur 1996).

### 2.2. Design variables

Fig. 1 shows the cross section and design variables for the retaining wall model. The design variables are divided into two categories: those that describe the geometric dimensions of wall cross-section, and those that model the steel reinforcement. As it is shown in Fig. 1, there are five geometric design variables representing the dimensions of the retaining wall: $X_1$ is width of the heel, $X_2$ is stem thickness at the top, $X_3$ is stem thickness at the bottom, $X_4$ is width of the toe and $X_5$ is thickness of the base slab. There are three additional design variables related to the steel reinforcement of the various sections of the retaining wall: $X_6$ is the vertical steel reinforcement in the stem, $X_7$ is the horizontal steel reinforcement in the toe and $X_8$ is the horizontal steel reinforcement in the heel. $B$ is the base width of the wall's foundation, $H$ is total height of the wall and $H'$ is height of the stem.

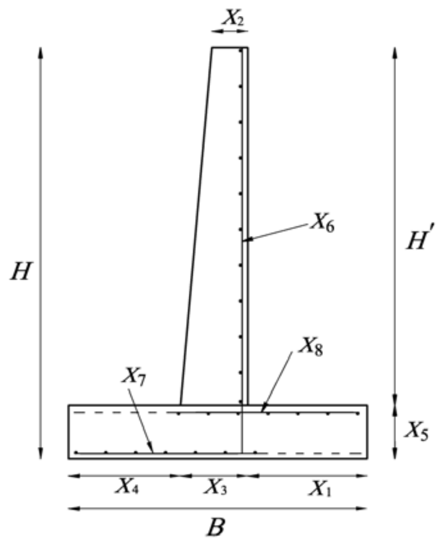


Fig. 1 Design variables of the RCC retaining wall

### *2.3. Design Constraints*

Fig. 2 shows the general forces acting on the retaining wall. In this figure, $\phi'_1$ and $\gamma_1$ are the effective friction angle and unit weight of retained soil; $\phi'_2$, $\gamma_2$ and $c_2$ are the effective friction angle, unit weight and cohesion of base soil; $\beta$ is the backfill slop angle; $D$ is the depth of soil in front of the wall and $q$ is the distributed surcharge load. In addition, $P_a$ is the active earth pressure; $P_p$ is the passive earth pressure; $W_W$ is the combined weight of all the sections of the structure; $W_S$ is the weight of backfill acting on the heel and $Q$ is the centralized surcharge load. The active and passive earth pressure can be evaluated by Rankine or Coulomb theory (Bowles 1982). $q_{\min}$ and $q_{\max}$ are the minimum and maximum bearing stresses on the base of the foundation, respectively, based on the following equation

$$q_{\substack{\min \\ \max}} = \frac{\sum V}{B}\left(1 \mp \frac{6e}{B}\right) \tag{5}$$

In Eq. (5), $\sum V$ is sum of the vertical forces due to weight of wall, soil above the base, and surcharge load. $e$ is the eccentricity which is ratio of the summation of overturning moments about the toe to the sum of vertical forces and can be calculated by

$$e = \frac{B}{2} - \frac{\sum M_R - \sum M_O}{\sum V} \tag{6}$$

where $\sum M_R$ is sum of the moments of forces that tends to resist overturning about the toe and $\sum M_O$ is sum of the moments of forces that tends to overturn the structure about the toe. $\sum M_R$ and $\sum M_O$ can be evaluated from the following equations

$$\sum M_R = M_W + M_S + M_Q + M_{Pav} \tag{7}$$

$$\sum M_O = M_{Pah} \tag{8}$$

In the above equations, $M_W$, $M_S$, $M_Q$, and $M_{Pav}$ are moments about the toe due to $W_W$, $W_S$, $Q$ and vertical components of the active earth pressure ($P_a$), respectively (see Fig. 2). $M_{Pah}$ is moments due to the horizontal components of the active earth pressure ($P_a$).

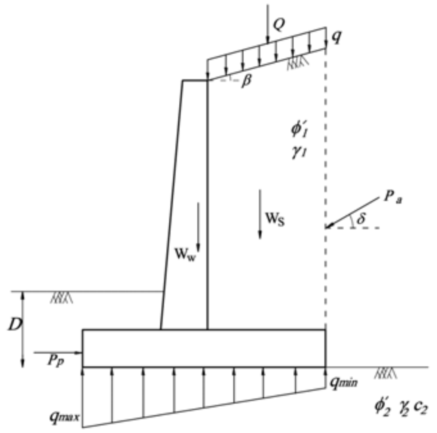


Fig. 2 Forces acting on a retaining wall

Table 2 Failure modes of retaining wall

| Inequality constraints | Failure mode | Constraints |
|---|---|---|
| $g_1(X)$ | Sliding stability | $FS_S \leq (\Sigma F_R / \Sigma F_d)$ |
| $g_2(X)$ | Overturning stability | $FS_O \leq (\Sigma M_R / \Sigma M_O)$ |
| $g_3(X)$ | Bearing capacity | $FS_b \leq (q_{ult} / q_{max})$ |
| $g_4(X)$ | Eccentricity failure | $e \leq (B/6)$ |
| $g_5(X)$ | Toe shear | $V_{ut} \leq V_{nt}$ |
| $g_6(X)$ | Toe moment | $M_{ut} \leq M_{nt}$ |
| $g_7(X)$ | Heel shear | $V_{uh} \leq V_{nh}$ |
| $g_8(X)$ | Heel moment | $M_{uh} \leq M_{nh}$ |
| $g_9(X)$ | Shear at bottom of stem | $V_{us} \leq V_{ns}$ |
| $g_{10}(X)$ | Moment at bottom of stem | $M_{us} \leq M_{ns}$ |
| $g_{11}(X)$ | Deflection at top of the stem | $(1/150) \times H' \leq \delta_{max}$ |

The typical design philosophy of retaining walls seeks designs that provide safety and stability against failure modes and comply with concrete building code requirements. The various design constraints shall be considered in the optimization of the RCC retaining wall are summarized and presented in Table 2.

In Table 2, $FS_S$ = required factor of safety against sliding; $FS_O$ = required factor of safety against overturning; $FS_b$ = required factor of safety against bearing capacity; $\sum F_R$ = sum of the horizontal resisting forces; $\sum F_d$ = sum of the horizontal driving forces; $V_{ut}$, $V_{uh}$ and $V_{us}$ = ultimate shearing force of toe, heel and stem; $V_{nt}$, $V_{nh}$ and $V_{ns}$ = nominal shear strength of concrete; $M_{ut}$, $M_{uh}$ and $M_{us}$ = ultimate bending moment of toe, heel and stem; $M_{nt}$, $M_{nh}$ and $M_{ns}$ = nominal flexural strength of concrete; $\delta_{max}$ = maximum deflection at the top of the stem.

According to ACI (2005) the nominal shear strength and nominal flexural strength of concrete can be evaluated by Eqs. (9) and (10), respectively.

$$V_n = \frac{1}{6} \phi_V \sqrt{f'_c}\, b d \tag{9}$$

where $\phi_V$ is the shear strength reduction factor equal to 0.75 (ACI 2005), $f'_c$ is the compression strength of concrete and $b$ is the width of the section.

$$M_n = \phi_M A_S f_y \left( d - \frac{a}{2} \right) \tag{10}$$

where $\phi_M$ is the flexure strength reduction factor equal to 0.9 (ACI 2005), $A_s$ is the cross-sectional area of steel reinforcement, $f_y$ is the yield strength of steel, $d$ is the distance from compression surface to the centroid of tension steel and $a$ is the depth of stress block.

In addition to the above mentioned constraints, the design variables have practical minimum and maximum value (Bowles 1982). The lower and upper bounds of the design variables are summarized in Table 3.

Table 3 Upper bound and lower bound for design variables of retaining wall

| Description | Lower bound | Upper bound |
|---|---|---|
| Width of footing | $B_{\min} = 0.4\,H$ | $B_{\max} = 0.7\,H$ |
| Thickness of base slab | $X_{5\min} = H/12$ | $X_{5\max} = H/10$ |
| Width of toe | $X_{4\min} = 0.4\,H/3$ | $X_{4\max} = 0.7\,H/3$ |
| Stem thickness at the top | $X_{2\min} = 20$ cm | - |
| Steel reinforcement ratio | $\rho_{\min} = \max\left\{\frac{1.4}{f_y}, 0.25\frac{\sqrt{f_c'}}{f_y}\right\}$ | $\rho_{\max} = 0.85\beta_1\frac{f_c'}{f_y}\left(\frac{600}{600+f_y}\right)$ |

## 3. Gravitational search algorithm

Gravitational search algorithm (GSA) is one of the newest heuristic population based search algorithms. The GSA could be considered as a small artificial world of masses obeying the Newtonian laws of gravitation and motion (Rashedi *et al.* 2009). In this approach, all the individuals (search agents) can be viewed as objects and their performances are evaluated by their masses. All these objects attract each other by a gravity force, and this force causes the movement of all objects globally towards objects with heavier masses. The heavy masses correspond to good solutions of the problem. The position of the agent represents a potential answer of the problem, and its mass is determined using a fitness function. By lapse of time, masses are attracted by the heaviest mass, which correspond an optimum solution in the search space. In the following, the formulation of GSA is presented in short.

In GSA, an agent status on the search space is characterized by two factors: its position ($\boldsymbol{X}_i$) and velocity ($\boldsymbol{V}_i$). In this approach, the new velocity and position of agent $i$ will be updated according to the following equations

$$v_i^d(t+1) = rand_i \times v_i^d(t) + a_i^d(t) \tag{11}$$

$$x_i^d(t+1) = x_i^d(t) + v_i^d(t+1) \tag{12}$$

where $v_i^d$ is the velocity of agent $i$ in dimension $d$, which represents the distance to be traveled by this agent from its current position, $x_i^d$ represents the position of agent $i$ and $t$ is the iteration number. $rand_i$ is a uniform random variable in the interval [0, 1]. This random number is applied to give a randomized characteristic to the search and to increase diversity and the probability of finding the global optimum. In Eq. (11), $a_i^d$ is the acceleration of agent $i$ in dimension $d$ and can be calculated as follows

$$a_i^d(t) = \sum_{j \in kbest,\, j \neq i} rand_j G(t) \frac{M_j(t)}{R_{i,j}(t) + \varepsilon}(x_j^d(t) - x_i^d(t)) \tag{13}$$

where $rand_j$ is a random number in the interval [0, 1]; $G(t)$ is the gravitational constant at time $t$ based on Eq. (14); $M_j$ is mass of agent $j$ based on Eq. (15); $\varepsilon$ is a small value to avoid division by zero and $R_{i,j}(t)$ is the Euclidean distance between two agents, $i$ and $j$ defined as $R_{i,j}(t) = \| X_i(t), X_j(t)\|_2$. It is worth to mention that we use here $R$ instead of $R^2$ in Eq. (13), because, according to the experiments presented in (Rashedi *et al.* 2009) $R$ provides better results than $R^2$. *kbest* is the set of first $K$ agents with the best fitness value and biggest mass. *Kbest* is a function of time, initialized to $K_0$ at the beginning and decreased with time to improve the performance of GSA by controlling exploration and exploitation (Rashedi *et al.* 2009). Here $K_0$ is set to $N$ (total number of agents) and is decreased linearly to 1. In Eq. (13), the gravitational constant, $G(t)$, is a decreasing function of time where it is set to $G_0$ at the beginning and will be reduced exponentially to control the search accuracy of the algorithm based on the following equation

$$G(t) = G_0 \times \exp\left(-\beta \times t / t_{\max}\right) \tag{14}$$

where $\beta$ is a constant, $t$ is the current iterations and $t_{max}$ is the maximum iteration number.

In addition, the mass of each agent in Eq. (13) is evaluated using the following equation

$$M_i(t) = \frac{m_i(t)}{\sum_{j=1}^{N} m_j(t)} \tag{15}$$

in which

$$m_i(t) = \frac{\mathrm{fit}_i(t) - \mathrm{worst}(t)}{\mathrm{best}(t) - \mathrm{worst}(t)} \tag{16}$$

where $\mathrm{fit}_i(t)$ represent the fitness value of the agent $i$ at time $t$. best($t$) and worst($t$) is the best and worst fitness of all agents, respectively.

Even though GSA is faster in finding quality solutions, compared to other evolutionary computation techniques (Rashedi *et al.* 2009), it faces some difficulty in obtaining better quality solutions and may face premature convergence while exploring complex functions.

## 4. Opposition-based gravitational search algorithm (OBGSA)

In order to overcome the problem mentioned above and to improve the performance, efficiency and accuracy of the classical GSA, this paper presents a novel approach of the algorithm by applying the concept of opposition-based learning (OBL). The opposition-based learning theory was first introduced by Tizhoosh (2005). Before concentrating on opposition-based gravitational search algorithm (OBGSA), we need to define opposite numbers.

For a real number $x \in [x^L, x^U]$, the opposite number of $x$, which denoted by $ox$ is

$$ox = x^L + x^U - x \tag{17}$$

where $x^L$ and $x^U$ are the lower and upper bounds of $x$.

Similarly, if $\boldsymbol{X} = (x_1, x_2, ..., x_n)$ be an $n$-dimensional vector, where $x_i \in [x^L{}_i, x^U{}_i]$ and $i = 1, 2, \ldots, n$, the opposite point of $x_i$, $ox_i$, is defined by

$$ox_i = x^L{}_i + x^U{}_i - x_i \tag{18}$$

The proposed OBGSA employed OBL concept during two stages of the optimization procedure. The first stage is during the population initialization and the second one is during updating agents' positions.

As a member of population based optimization algorithm, GSA starts with some initial solutions (initial population) and try to improve performance toward some optimal solutions. Generally speaking, population initialization is a very important task in GSA because it can affect the algorithm performance, convergence speed and the quality of the final solution. In the absence of a priori information about the solution, classical GSA uses random initialization in order to generate candidate solutions (initial population). In the proposed OBGSA, the OBL theory is employed to generate initial population and make GSA faster. In this approach, the random agent (candidate solution) and its opposite are considered at the same time in order to achieve a better approximation for current candidate solution. A mathematical proof to show that, in general, opposite numbers are more likely to be closer to the optimal solution than purely random ones has been proposed by Rahnamayan *et al.* (2008).

Let $x_i$ be an agent in $n$-dimensional space (i.e., candidate solution). Assume $f(\cdot)$ is a fitness function, which is used to measure the agent's fitness. According to the definition of the opposite point, $ox_i$ is the opposite of $x_i$. Now, in the OBGSA, if $f(ox_i)$ is better than $f(x_i)$, then agent $x_i$ can be replaced with $ox_i$; otherwise, we continue with $x_i$. Hence, the agent and its opposite are evaluated simultaneously in order to obtain fitter starting candidate solutions even when there is no a priori knowledge. The following is the proposed opposition-based population initialization algorithm, which can be used instead of a pure random initialization.

1. Generating uniformly distributed random population, $\boldsymbol{X}$,
2. Calculating opposite population, $O\boldsymbol{X}$,
3. Selecting $N$ fittest individuals from $\{\boldsymbol{X} \cup O\boldsymbol{X}\}$ as initial population.

In addition, the proposed OBGSA employed OBL concept for updating the agents' positions. During the search process, occasionally some agents fallen into a local minimum and do not move for several iterations. Therefore, measures must be taken to overcome this problem and prevent premature convergence. In the OBGSA, based on OBL theory, $m$ worst agents yielding the largest fitness values replace with their opposite at each iteration of the optimization process.

In order to keep balance between global exploration and local exploitation, $m$ should be a variable. At the beginning stage of optimization, $m$ should be a large value to provide an effective global exploration of the search space. Over the iterations, the value of $m$ should reduce gradually to provide a local exploitation. Therefore, the following time varying equation is introduced

$$m = Round\left(\frac{N}{5}(1-\frac{t}{t_{\max}})\right) \tag{19}$$

where $Round(x)$ rounds the value of $x$ to the nearest integer. This process tries to improve the solution, by maintaining diversity in the population and explores new regions across the search space. The new strategy replaces the position vectors of a predefined number of least ranked agents with their opposite in each iteration.

The whole workflow of the proposed OBGSA is shown as a flowchart in Fig. 3.

## 5. Model Verification

In this section, the efficiency and robustness of the proposed OBGSA for numerical optimization will be investigated. In order to prove that an algorithm is able to perform sufficiently well over a wide range of feasible functions, the most commonly used strategy is the application of benchmark test comprising several functions. These functions are specially designed and routinely used to evaluate the performance of the global optimization algorithms. In this study, a set of five well-known standard benchmark functions are employed. Although these functions may not necessarily give an accurate indication of the performance of an algorithm on real world problems, they can be used to investigate certain aspects of the algorithms under consideration. The functions, dimension, admissible range of the variable and the optimum to be obtained are summarized in Table 4. All the functions are to be minimized. The first three functions are unimodal functions whereas the next two functions are multimodal optimization problems with a considerable amount of local minima.

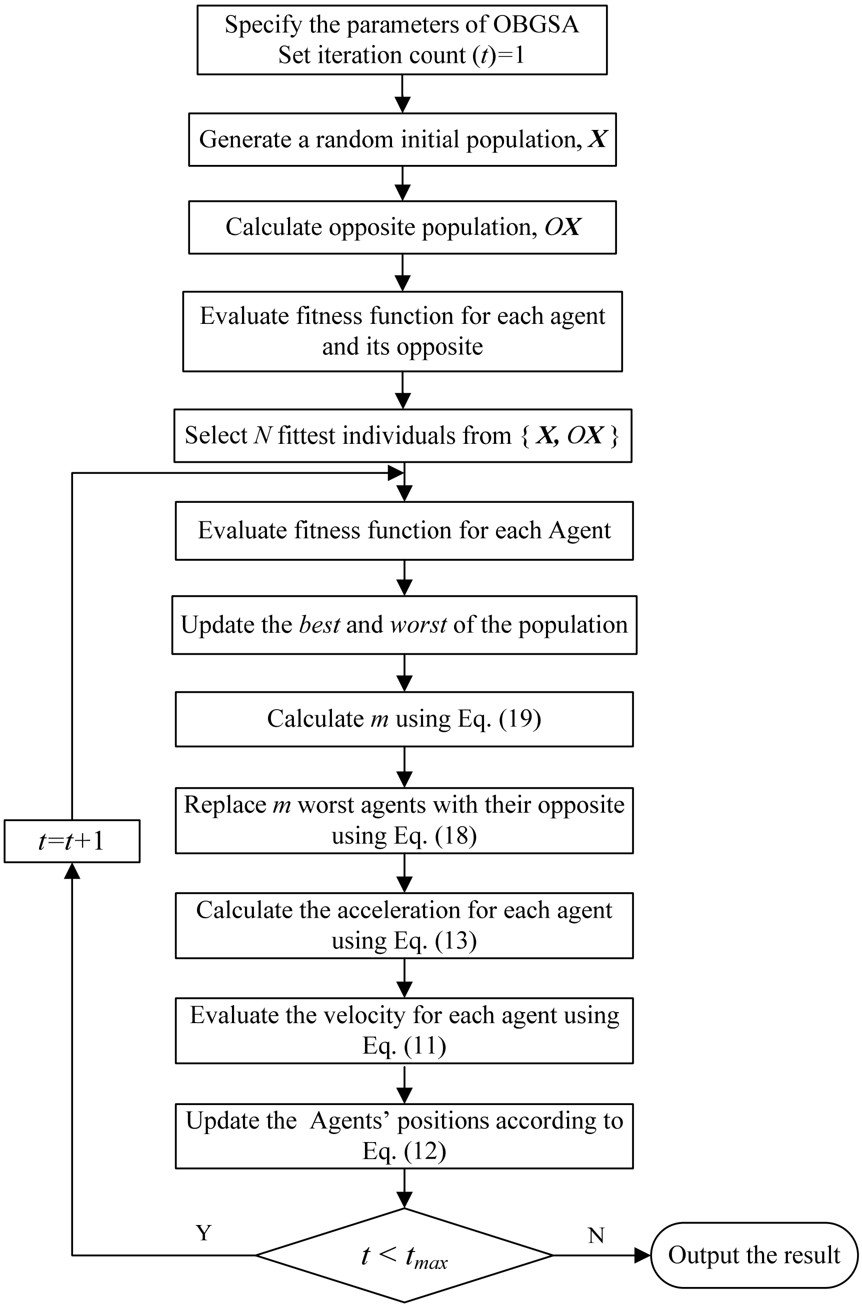


Fig. 3 Flowchart of the opposition-based gravitational search algorithm

Table 4 Standard benchmark functions

| Test Function | Dimension ($n$) | Range | Optimum |
|---|---|---|---|
| $F_1(X)=\sum_{i=1}^{n}\left(\sum_{j=1}^{i}x_j\right)^2$ | 30 | $[-100,100]^n$ | 0 |
| $F_2(X)=\sum_{i=1}^{n}ix_i^4+random[0,1)$ | 30 | $[-1.28,1.28]^n$ | 0 |
| $F_3(X)=\max\left\{\lvert x_i\rvert,1\le i\le n\right\}$ | 30 | $[-100,100]^n$ | 0 |
| $F_4(X)=\sum_{i=1}^{n}(x_i^2-10\cos(2\pi x_i)+10)$ | 30 | $[-5.12,5.12]^n$ | 0 |
| $F_5(x)=\frac{\pi}{n}\left\{10\sin^2(\pi y_1)+\sum_{i=1}^{n-1}(y_i-1)^2[1+10\sin^2(\pi y_{i+1})]+(y_n-1)^2\right\}$ $+\sum_{i=1}^{n}u(x_i,5,100,4)$ $y_i=1+\frac{x_i+1}{4},\ u(x_i,a,k,m)=\begin{cases}k(x_i-a)^m, & x_i>a\\ 0, & -a<x_i<a\\ k(-x_i-a)^m, & x_i<-a\end{cases}$ | 30 | $[-50,50]^n$ | 0 |

Table 5 Parameters of GSA and OBGSA algorithms

| Parameter | Description | GSA | OBGSA |
|---|---|---|---|
| $N$ | Population size | 50 | 50 |
| $G_0$ | Initial value of gravitational constant | 100 | 150 |
| $\beta$ | | 20 | 20 |
| $\varepsilon$ | | 2.22e-16 | 2.22e-16 |
| $t_{max}$ | Maximum iteration number | 500 | 500 |

Table 6 Minimization result of benchmark functions

| Function | Method | Worst | Mean | Median | Best | Standard deviation |
|---|---|---|---|---|---|---|
| $F_1$ | GSA | 1120.8 | 486.03 | 418.38 | 261.32 | 189.9 |
| | OBGSA | 7.84 e-7 | 5.33 e-8 | 1.6 e-9 | 4.54 e-15 | 1.55 e-7 |
| $F_2$ | GSA | 2.304 | 0.151 | 0.0418 | 0.014 | 0.428 |
| | OBGSA | 3.2 e-4 | 7.05 e-5 | 3.89 e-5 | 4.56 e-7 | 7.68 e-5 |
| $F_3$ | GSA | 7.34 | 3.73 | 3.51 | 0.086 | 1.88 |
| | OBGSA | 8.56 e-9 | 5.11 e-9 | 4.89 e-9 | 3.36 e-9 | 1.46 e-9 |
| $F_4$ | GSA | 29.85 | 19.43 | 19.4 | 11.94 | 5.085 |
| | OBGSA | 2.27 e-13 | 5.3 e-14 | 5.68 e-14 | 0.00 | 6.14 e-14 |
| $F_5$ | GSA | 2.88 | 0.736 | 0.374 | 0.0111 | 0.946 |
| | OBGSA | 0.0099 | 0.0017 | 6.24 e-4 | 2.76 e-5 | 0.003 |

The presented benchmark functions are solved using both GSA and OBGSA algorithms. In all experiments, the parameters of each algorithm are selected utilizing several experimental studies examining the effect of each parameter on the final solution, convergence and overall performance of the algorithms. Table 5 presents the best-selected parameters of each algorithm.

The algorithms are simulated 50 times independently and the results are recorded. Then the statistical analyses are carried out and for each method, worst, mean, median, best and standard deviation are calculated. The performance comparison between two algorithms on five functions is presented in Table 6.

Table 6 shows that OBGSA converged to a more significantly accurate final solution than GSA for all test functions. In terms of mean and best fitness values, the new algorithm could provide a significantly better solution for all functions. At the same time, the standard deviation of the results obtained by OBGSA for all functions are smaller than those computed by GSA indicating the superior stability of the new method.

Fig.s 4-8 demonstrate the convergence rate comparison among the proposed OBGSA and classical GSA. In these figures, the representative variations of the mean best fitness in the form of logarithm values over the number of iterations are depicted.

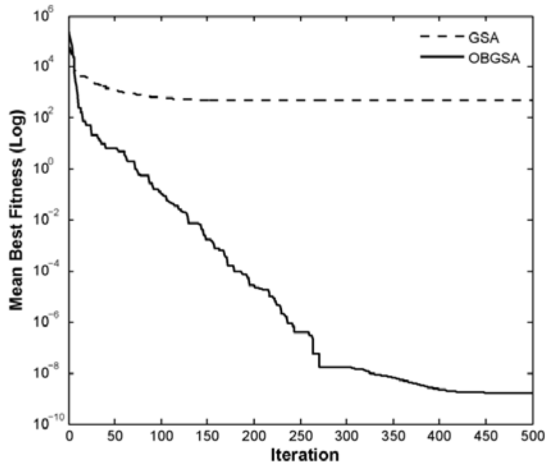


Fig. 4 Performance comparison of GSA and OBGSA for minimization of $F_1$

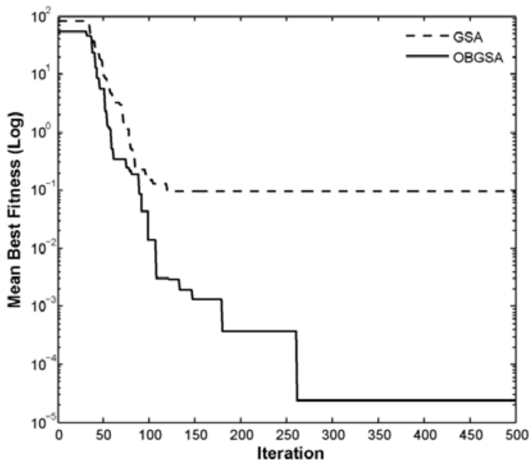


Fig. 5 Performance comparison of GSA and OBGSA for minimization of $F_2$

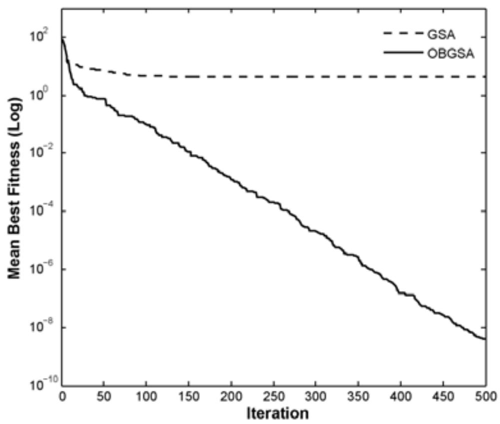


Fig. 6 Performance comparison of GSA and OBGSA for minimization of $F_3$

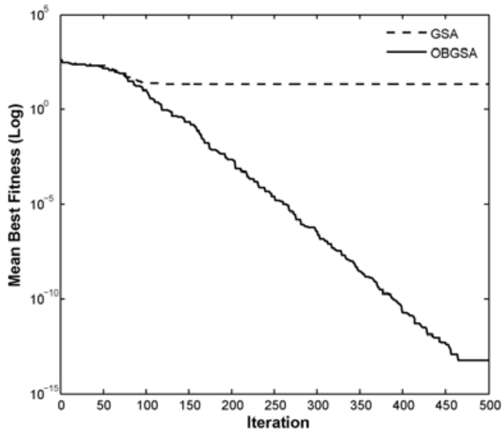


Fig. 7 Performance comparison of GSA and OBGSA for minimization of $F_4$

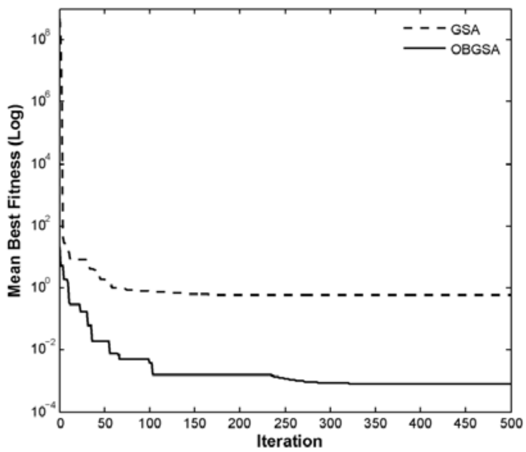


Fig. 8 Performance comparison of GSA and OBGSA for minimization of $F_5$

From Figs. 4-8, it can be seen that the varying curves of fitness values obtained by OBGSA descend much faster to a lower level than those of GSA. In addition, as shown in these figures the resulting history converges very quickly by GSA, within the first 100 iterations, but does not improve after the initial convergence. In other words, after becoming converged, GSA loses its ability to explore and then becomes inactive. However, the new algorithm is more successful in exploring the search space. The obtained results indicate that OBGSA significantly improves the solution quality and surpasses GSA for all test functions.

## 6. Illustrative Examples

In this section, the efficiency and robustness of the proposed OBGSA for optimization of RCC retaining walls will be investigated. In order to demonstrate, compare and analyze the effectiveness and performance of the new method, two illustrative examples will be presented. The implementation of OBGSA for retaining wall optimization has been carried out using a computer program developed in MATLAB R2009a. In the following cases, the considered parameters of the algorithms are specified in the previous section.

### *6.1. Design Example 1*

The first example is concern with the optimum design of a RCC retaining structure with height of 3m. Other input parameters for this example are given in Table 7.

The problem is solved by the presented procedure using OBGSA algorithm. The results of analyses for minimum weight, cost and $CO_2$ emissions are presented in Table 8.

The results of Table 8 show that OBGSA could provide a better solution by evaluating a lower value of objective functions compared with classical GSA. In addition, the findings presented in Table 8 show the dependence of the design variables to the objective function. The results of Table 8 show the dependence of the design variables to the objective function. We can evaluate the cost of the structure at optimum values found with the $CO_2$ objective function and vice versa. By doing this, the cost of the structure with respect to the $CO_2$ objective function is 554.5 USD/m and it means to have an environmentally friendly structure the structure cost increases only 0.76%. Alternatively, the $CO_2$ emissions at the best cost solutions is 708.7 kg/m, which is only 0.80% is higher than the best $CO_2$ emissions solutions. This indicates both cost and $CO_2$ objective functions are closely related and yield almost similar solutions. However, the best weight solutions are significantly different with the solutions of other target functions.

This problem originally presented by Saribas and Erbatur (1996) and solved using nonlinear programming for minimum weight and minimum cost only. The minimum weight of the structure evaluated in their study was 2498.77 kg/m, which is almost 3% heavier than that calculated by the proposed method. For minimum cost, Saribas and Erbatur (1996) used different unit prices and did not measure the cost of excavation, formwork and backfill. The best price achieved in their research was 82.47 USD/m. However, the optimum cost design is dependent on the unit price and it changes with variation of the unit prices. For further verification of the new method, the problem is solved under the same conditions with same unit prices of Saribas and Erbatur (1996) study and the best price computed by OBGSA is 60.9 USD/m, which is 26% cheaper than that computed in their study.

Table 7 Input parameters for design examples 1 and 2

| Parameter | Unit | Value for example 1 | Value for example 2 |
|---|---|---|---|
| Height of stem | m | 3.0 | 4.5 |
| Internal friction angle of retained soil | degree | 36 | 36 |
| Internal friction angle of base soil | degree | 0.0 | 34 |
| Unit weight of retained soil | $kN/m^3$ | 17.5 | 17.5 |
| Unit weight of base soil | $kN/m^3$ | 18.5 | 18.5 |
| Unit weight of concrete | $kN/m^3$ | 23.5 | 23.5 |
| Cohesion of base soil | kPa | 125 | 100 |
| Depth of soil in front of wall | m | 0.5 | 0.75 |
| Surcharge load | kPa | 20 | 30 |
| Backfill slop | degree | 10 | 15 |
| Concrete cover | cm | 7.0 | 7.0 |
| Yield strength of reinforcing steel | MPa | 400 | 400 |
| Compressive strength of concrete | MPa | 21 | 21 |
| Shrinkage and temporary reinforcement percent | - | 0.002 | 0.002 |
| Factor of safety for overturning stability | - | 1.5 | 1.5 |
| Factor of safety against sliding | - | 1.5 | 1.5 |
| Factor of safety for bearing capacity | - | 3.0 | 3.0 |

### *6.2. Design Example 2*

Optimum design of a RCC retaining structure with height of 4.5m is investigated in the second example. This problem is also originally presented and solved by Saribas and Erbatur (1996) for minimum weight and minimum cost only. Other input parameters for this example are given in Table 7.

This example is solved using OBGSA and the results of analyses for optimum weight, optimum cost and optimum $CO_2$ emissions are presented in Table 9.

From the results of Table 9, it can be observed that the minimum weight, cost and $CO_2$ emissions obtained by OBGSA are lower than those evaluated by classical algorithm. Similar to the firs case, the results of Table 9 also show that both cost and $CO_2$ objectives yield similar solutions and are rather coincident. In the other words, solutions that are acceptable in terms of $CO_2$ emissions are also viable in terms of cost while good solutions in terms of cost are also acceptable in terms of $CO_2$ emissions. However, the $CO_2$ objective function appears more environmentally friendly and robust, as prices are more sensitive to variations in market values, while emissions are more rigid since they depend on manufacturing processes.

The findings in Table 9 are also comparable with Saribas and Erbatur (1996) study. The minimum weight of the structure evaluated by Saribas and Erbatur (1996) was 5280.96 kg/m which is almost 13% more than that calculated by the proposed method. For minimum cost, the best price achieved in their research was 189.55 USD/m. Under the same conditions, the best price computed by OBGSA is 133.6 USD/m.

Table 8 Optimization result for design example 1

| Design variable | Unit | Optimum values for minimum weight | Optimum values for minimum cost | Optimum values for minimum $CO_2$ emissions |
|---|---|---|---|---|
| Width of heel ($X_1$) | m | 0.80 | 0.517 | 0.561 |
| Stem thickness at the top ($X_2$) | m | 0.20 | 0.2 | 0.2 |
| Stem thickness at the bottom ($X_3$) | m | 0.20 | 0.26 | 0.209 |
| Width of toe ($X_4$) | m | 0.50 | 0.778 | 0.778 |
| Thickness of base slab ($X_5$) | m | 0.272 | 0.272 | 0.272 |
| Vertical steel area of the stem ($X_6$) | $cm^2/m$ | 11 | 7.0 | 9.0 |
| Horizontal steel area of the toe ($X_7$) | $cm^2/m$ | 7.0 | 7.0 | 7.0 |
| Horizontal steel area of the heel ($X_8$) | $cm^2/m$ | 7.0 | 7.0 | 7.0 |
| Objective function value (OBGSA) | | 2416.8 kg/m | 550.34 USD/m | 703.1 kg/m |
| Objective function value (GSA) | | 2419.8 kg/m | 550.92 USD/m | 706.9 kg/m |

Table 9 Optimization result for design example 2

| Design variable | Unit | Optimum values for minimum weight | Optimum values for minimum cost | Optimum values for minimum $CO_2$ emissions |
|---|---|---|---|---|
| Width of heel ($X_1$) | m | 1.136 | 0.694 | 0.773 |
| Stem thickness at the top ($X_2$) | m | 0.216 | 0.216 | 0.216 |
| Stem thickness at the bottom ($X_3$) | m | 0.222 | 0.451 | 0.354 |
| Width of toe ($X_4$) | m | 0.850 | 1.167 | 1.167 |
| Thickness of base slab ($X_5$) | m | 0.409 | 0.409 | 0.409 |
| Vertical steel area of the stem ($X_6$) | $cm^2/m$ | 32 | 12 | 16 |
| Horizontal steel area of the toe ($X_7$) | $cm^2/m$ | 11 | 11 | 11 |
| Horizontal steel area of the heel ($X_8$) | $cm^2/m$ | 11 | 11 | 11 |
| Objective function value (OBGSA) | | 4596.3 kg/m | 1039.8 USD/m | 1410.6 kg/m |
| Objective function value (GSA) | | 4601.4 kg/m | 1062.6 USD/m | 1448.2 kg/m |

## 7. Conclusion

In this paper, a new version of gravitational search algorithm based on opposition-based learning concept (OBGSA) is introduced and applied for optimum design of RCC retaining structures. In the proposed OBGSA, initial random population and its opposite are considered simultaneously to generate fittest initial population. In addition, OBGSA replaces some of the worst agents yielding the largest fitness values with their opposite at each iteration of the optimization process. Compared with classical GSA on five well-known unimodal/multimodal benchmark functions, our proposed algorithm has been testified to possess excellent performance in terms of accuracy, convergence rate, stability and robustness. For RCC retaining wall optimization, three objective functions include weight, cost and amount of embedded $CO_2$ emissions are considered. The results comparison between presented method, classical GSA, and selected other methods employed in previous studies demonstrate better performance of OBGSA in terms of computational efficiency and robustness. In terms of objective function type, the findings indicates that $CO_2$ emissions and cost target functions are almost related, in which the acceptable solutions in terms of $CO_2$ emissions are also viable in terms of cost while good solutions in terms of cost are also acceptable in terms of $CO_2$ emissions.

## References

ACI (2005), "318-05, Building Code Requirements for Structural Concrete and Commentary", American Concrete Institute International.

Aguilar Madeira, J., Rodrigues, H. and Pina, H. (2005), "Multi-objective optimization of structures topology by genetic algorithms", *Adv. Eng. Softw*., 36(1), 21-28.

Aydogdu, I. and Saka, M. (2012), "Ant colony optimization of irregular steel frames including elemental warping effect", *Adv. Eng. Softw*., 44(1), 150–169.

Bowles, J. (1982), Foundation analysis and design, McGraw-Hill, New York.

Camp, C., Pezeshk, S. and Cao, G. (1998), "Optimized design of two-dimensional structures using a genetic algorithm", *J. Struct. Eng*., 124(5), 551-559.

Camp, C.V. and Akin, A. (2012), "Design of Retaining Walls Using Big Bang-Big Crunch Optimization", *J. Struct. Eng*., 138(3), 438–448.

Camp, C.V., Bichon, B.J. and Stovall, S.P. (2005), "Design of steel frames using ant colony optimization", *J. Struct. Eng*., 131(3), 369-379.

Degertekin, S. (2011), "Improved harmony search algorithms for sizing optimization of truss structures", *Comput. Struct*., (92-93), 229–241.

Degertekin, S. (2012), "Optimum design of geometrically non-linear steel frames using artificial bee colony algorithm", *Steel Compos. Struct,* 12(6), 505-522.

Doğan, E. and Saka, M. (2012), "Optimum design of unbraced steel frames to LRFD-AISC using particle swarm optimization", *Adv. Eng. Softw*., 46(1), 27-34.

Hasançebi, O., Çarbaş, S. and Saka, M.P. (2010), "Improving the performance of simulated annealing in structural optimization", *Struct. Multidiscip. Optim*., 41(2), 189-203.

Hasançebi, O. and Erbatur, F. (2002), "Layout optimisation of trusses using simulated annealing", *Adv. Eng. Softw.*, 33(7), 681-696.

Khajehzadeh, M., Taha, M.R., El-Shafie, A. and Eslami, M. (2011), "Modified particle swarm optimization for optimum design of spread footing and retaining wall", *J. Zhejiang Univ. Sci A*, 12(6), 415-427.

Lee, K.S. and Geem, Z.W. (2004), "A new structural optimization method based on the harmony search algorithm", *Comput. Struct*., 82(9), 781-798.

Perez, R. and Behdinan, K. (2007), "Particle swarm approach for structural design optimization", *Comput.*

*Struct.*, 85(19-20), 1579-1588.

Rahnamayan, S., Tizhoosh, H.R. and Salama, M. (2008), "Opposition versus randomness in soft computing techniques", *Appl. Soft Comput.*, 8(2), 906-918.

Rashedi, E., Nezamabadi-pour, H. and Saryazdi, S. (2009), "GSA: a gravitational search algorithm", *Inform. Sci.*, 179(13), 2232-2248.

Salajegheh, E. and Gholizadeh, S. (2005), "Optimum design of structures by an improved genetic algorithm using neural networks", *Adv. Eng. Softw.*, 36(11), 757-767.

Saribas, A. and Erbatur, F. (1996), "Optimization and sensitivity of retaining structures", *J. Geotech. Eng.*, 122(8), 649-656.

Sonmez, M. (2011), "Discrete optimum design of truss structures using artificial bee colony algorithm", *Struct. Multidiscip. Optim.*, 43(1), 85-97.

Technology, C.I.o.C. (2009), BEDEC PR/PCT ITEC materials database, Barcelona, Spain.

Tizhoosh, H.R. (2005). "Opposition-based learning: A new scheme for machine intelligence", *International Conference on Computational Intelligence for Modelling Control and Automation*, CIMCA 2005, Vienna, Austria.

Togan, V., Daloglu, A.T. and Karadeniz, H. (2011), "Optimization of trusses under uncertainties with harmony search", *Struct. Eng. Mech.*, 37(5), 543-560.

Wang, W., Guo, S., Chang, N., Zhao, F. and Yang, W. (2010), "A modified ant colony algorithm for the stacking sequence optimisation of a rectangular laminate", *Struct. Multidiscip. Optim.*, 41(5), 711-720.

Yepes, V., Alcala, J., Perea, C. and González-Vidosa, F. (2008), "A parametric study of optimum earth-retaining walls by simulated annealing", *Eng. Struct.*, 30(3), 821-830.