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Fuzzy control for geometrically nonlinear vibration of piezoelectric flexible plates

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Abstract. This paper presents a LMI(linear matrix inequality)-based fuzzy approach of modeling and active vibration control of geometrically nonlinear flexible plates with piezoelectric materials as actuators and sensors. The large-amplitude vibration characteristics and dynamic partial differential equation of a piezoelectric flexible rectangular thin plate structure are obtained by using generalized Fourier series and numerical integral. Takagi-Sugeno (T-S) fuzzy model is employed to approximate the nonlinear structural system, which combines the fuzzy inference rule with the local linear state space model. A robust fuzzy dynamic output feedback control law based on the T-S fuzzy model is designed by the parallel distributed compensation (PDC) technique, and stability analysis and disturbance rejection problems are guaranteed by LMI method. The simulation result shows that the fuzzy dynamic output feedback controller based on a two-rule T-S fuzzy model performs well, and the vibration of plate structure with geometrical non-linearity is suppressed, which is less complex in computation and can be practically implemented.

Keywords: geometrical nonlinearity; piezoelectric flexible plate; T-S fuzzy model; output feedback control; LMI

1. Introduction

The lightweight and flexible structures are extensively used in aerospace engineering, civil and mechanical engineering, such as large space structures, flexible manipulators, tall buildings and so on. The flexible structure is less stiff and therefore more susceptible to the vibration which may last for a long time. Since piezoelectric materials can be well integrated into lightweight flexible structures and efficiently transform mechanical energy into electrical energy and vice versa , one approach to control the undesired vibrations is to employ a control system with piezoelectric sensors and actuators which have good characteristics of lightweight, electromechanical coupling effects and broad bandwidth. In literatures (Gao and Chen 2003, Narayanan Balamurugan 2003, Kusculuoglu and Fallahi 2004, Song and Sethi 2006, Hu and Ma 2004, Samuel and Vicente 2006, Qiu and Wu 2009), active vibration control of flexible structures with piezoelectric effect has been studied.

However, most of works has been devoted to the linear dynamics and vibration control of flexible structures, and investigations on the control of large-amplitude nonlinearity vibration due to the

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structural flexibility are limited in number. Nonlinear systems can exhibit extremely complex behaviors which linear systems can not, such as jumps, bifurcations, saturation, subharmonic, superharmonic and internal resonances, resonance captures, limit cycles, modal interactions and chaos. So the vibration analysis and control design work of nonlinear flexible structures become harder and prominent in the research of flexible structures (Zhou and Wang 2004, Belouettar and Azrar 2008, Dash and Singh 2009).

Since control design of nonlinear systems is a difficult process and many nonlinear control methods for nonlinear systems are so complicated that they are not suitable for practical application, linear control methods are still employed in the active control of nonlinear plate structure. Zhou and Wang (2004) have suggested a control law of negative feedback of the identified deflection and velocity, which are obtained by using a wavelet-based approach, for piezoelectric beam-type plates with geometrically nonlinear deformation. Panda and Ray (2009) have developed a static velocity feed back control based on a finite element nonlinear dynamics of the functionally graded laminated composite plates by using the first-order shear deformation theory. According to the previous studies, the robustness and stability of controlled system using linear control method based on the traditional nonlinear model can not be strictly guaranteed. In addition, it is worth to mention that most of the available works focus on the state feedback controller or observer-based feedback controllers of structural systems (Zhou and Wang 2004, Belouettar and Azrar 2008, Panda and Ray 2009). In practice, the measurement of the modal displacement and velocity is not practical for vibration control of engineering structure and an observer-based controller may require a significant amount of online computational effort.

The research of fuzzy logic control systems has drawn a great deal of attention because of the universal approximation ability of fuzzy logic systems in nonlinear problems. As a knowledge-based approach, the fuzzy logic control systems usually depend on linguistics-based reasoning in design. Due to the complexity and nonlinearity of the fuzzy rules, it was extremely difficult to develop a general stability analysis and design theory for the fuzzy control when Takagi and Sugeno (1985) proposed their fuzzy inference system, now known as the Takagi-Sugeno (T-S) fuzzy model. In this type of fuzzy model, the dynamics of nonlinear system can be approximately by a set of fuzzy models ,which are locally linear time-invariant models interconnected by IF-THEN rules with nonlinear fuzzy membership functions. For the reason that it employs linear model in the consequent part, the conventional linear system theory can be applied to the stability analysis and control synthesis of nonlinear system accordingly. The T-S fuzzy models are becoming powerful engineering tools for modeling and control of complex nonlinear dynamic systems (Zheng and Sun 2004, Chen 2006, FaruqueAli and Ramaswamy 2009, Liu and Wang 2010), but few application to active vibration control of nonlinear flexible structures have been realized.

In this paper, our aim is to design a T-S fuzzy robust dynamical output feedback vibration control law for the geometrically nonlinear piezoelectric flexible plate structure by LMI method, in which stability analysis and control design work can be reduced to a few standard convex optimization problems involving the LMIs (Lu and Tsai 2003, Chen and Guo 2005, Samuel and Vicente 2006, Xu and Chen 2008). The remainder of this paper is organized as follows. First, the dynamic vibration equation for piezoelectric flexible plate with geometric non-linearity subjected to external disturbance is constructed by using generalized Fourier series and numerical integral, and the obtained nonlinear model is approximated by the T-S fuzzy model, which combines the fuzzy inference rule and the local linear state model. Then, the dynamic T-S fuzzy output feedback control design is carried out on the basis of the fuzzy model via the parallel-distributed-compensation

(PDC) technique, and stability analysis and disturbance rejection problems is guaranteed by the linear convex optimization using LMI method. Finally, Simulation results are given to demonstrate the effectiveness of the presented method.

2. Modeling of nonlinear piezoelectric flexible plate

Consider the transverse vibration of a rectangular flexible plate with K discretely distributed piezoelectric actuator patches bonded to it. The plate material is assumed to be homogeneous and isotropic, and has the dimensions of $a \times b \times h$. Cartesian x, y and z coordinates are used to specify the geometry of the plate. The x-y plane is coincident with the middle plane of the plate, and the origin is at the center of the middle plane as shown in the Fig. 1. In pure transverse bending cases, the mid-plane experiences no longitudinal strain or stress. As a consequent, it can be conveniently used as the reference plane for stress-strain calculation purposes. The displacements of an arbitrary point of coordinates (x, y) on the middle surface of the plate are denoted by u, v and w in the x, y and z directions. The jth ($j = 1, 2, \ldots, K$) rectangular actuating patch has the dimensions of $a_{aj} \times b_{aj} \times h_{aj}$ located at (x_{1j}, y_{1j}) and (x_{2j}, y_{2j}) as shown in the Fig. 1. The influence of the piezoelectric materials on the plate structural dynamics will be ignored.

According to the classical thin plate theory, the Green strain components $\varepsilon_x, \varepsilon_y, \varepsilon_{xy}$ at an arbitrary point of the plate are related to the middle surface strains $\varepsilon_x^0, \varepsilon_y^0, \varepsilon_{xy}^0$ and to the changes in the curvature and torsion of the middle surface k_x, k_y, k_{xy} by the following three relationships

$$\varepsilon_x = \varepsilon_x^0 + zk_x, \quad \varepsilon_y = \varepsilon_y^0 + zk_y, \quad \varepsilon_{xy} = \varepsilon_{xy}^0 + zk_{xy} \tag{1}$$

where z is the distance of the arbitrary point of the plate from the middle surface. The middle surface strain-displacement relationships and changes in the curvature and torsion are given as follows

$$\begin{gathered}\varepsilon_x^0 = \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2, \quad \varepsilon_y^0 = \frac{\partial v}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2, \quad \varepsilon_{xy}^0 = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\right) \\ k_x = -\frac{\partial^2 w}{\partial x^2}, \quad k_y = -\frac{\partial^2 w}{\partial y^2}, \quad k_{xy} = -2\frac{\partial^2 w}{\partial x \partial y}\end{gathered} \tag{2}$$

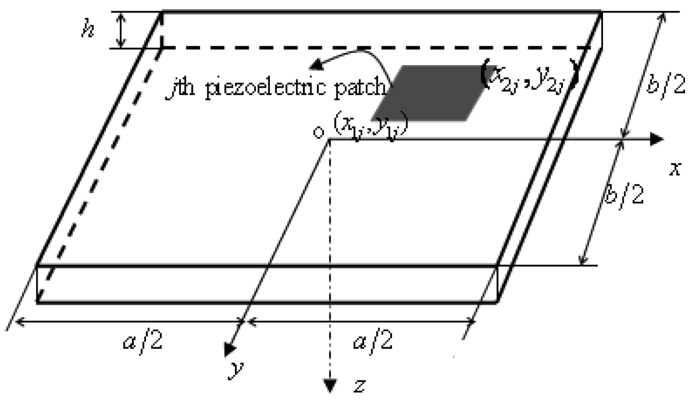

Fig. 1 Schematic diagram of a piezoelectric rectangular flexible plate

From Hooke's law, the Kirchhoff stress components $\sigma_x, \sigma_y, \sigma_{xy}$ at an arbitrary point of the plate related to the strain $\varepsilon_x, \varepsilon_y, \varepsilon_{xy}$ for homogeneous and isotropic material by

$$\sigma_x = \frac{E}{1-\nu^2}(\varepsilon_x + \nu\varepsilon_y), \quad \sigma_y = \frac{E}{1-\nu^2}(\varepsilon_y + \nu\varepsilon_x), \quad \sigma_{xy} = \frac{1}{G}\varepsilon_{xy} = \frac{E}{2(1+\nu)}\varepsilon_{xy} \tag{3}$$

Where E is Young's modulus of elasticity, ν is Poisson ratio of the plate Material and G is the shear modulus which can be related to E and ν as described above.

The stress components $\sigma_x, \sigma_y, \sigma_{xy}$ at an arbitrary point of the plate are related to the middle surface stress $\sigma_x^0, \sigma_y^0, \sigma_{xy}^0$ and the transverse displacements by the following three relationships

$$\sigma_x = \sigma_x^0 - z\left(\frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{\partial y^2}\right), \quad \sigma_y = \sigma_y^0 - z\left(\frac{\partial^2 w}{\partial y^2} + \nu\frac{\partial^2 w}{\partial x^2}\right), \quad \sigma_{xy} = \sigma_{xy}^0 - z(1-\nu)\frac{\partial^2 w}{\partial x \partial y} \tag{4}$$

So the mid-surface membrane forces N_x, N_y, N_{xy} and the bending and twisting moments M_x, M_y, M_{xy} can be calculated by integrating the stress couples through the plate thickness respectively

$$[N_x, N_y, N_{xy}] = \int_{-h/2}^{h/2} [\sigma_x, \sigma_y, \sigma_{xy}]dz = [\sigma_x^0, \sigma_y^0, \sigma_{xy}^0]h \tag{5}$$

$$\begin{aligned}[M_x, M_y, M_{xy}] &= \int_{-h/2}^{h/2} [\sigma_x, \sigma_y, \sigma_{xy}]zdz \\ &= \left[-D\left(\frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{\partial y^2}\right), -D\left(\frac{\partial^2 w}{\partial y^2} + \nu\frac{\partial^2 w}{\partial x^2}\right), -D(1-\nu)\frac{\partial^2 w}{\partial x \partial y}\right]\end{aligned} \tag{6}$$

where $D = \frac{E}{1-\nu^2}\frac{h^3}{12}$ is the flexural rigidity of the plate.

The elastic strain energy U of the plate, neglecting $\sigma_z, \sigma_{xz}, \sigma_{yz}$ under Kirchhoff's hypotheses, is given by

$$\begin{aligned}U = \frac{h}{2}\int_{-a/2}^{a/2}\int_{-b/2}^{b/2}\int_{-h/2}^{h/2}(\sigma_x\varepsilon_x + \sigma_y\varepsilon_y + \sigma_{xy}\varepsilon_{xy})dzdydx = \frac{h}{2}\int_{-a/2}^{a/2}\int_{-b/2}^{b/2}(\sigma_x^0\varepsilon_x^0 + \sigma_y^0\varepsilon_y^0 + \sigma_{xy}^0\varepsilon_{xy}^0)dydx \\ + \frac{h^3}{24}\int_{-a/2}^{a/2}\int_{-b/2}^{b/2}\left[-k_x\left(\frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{\partial y^2}\right) - k_y\left(\frac{\partial^2 w}{\partial y^2} + \nu\frac{\partial^2 w}{\partial x^2}\right) - (1-\nu)k_{xy}\frac{\partial^2 w}{\partial x \partial y}\right]dydx\end{aligned} \tag{7}$$

where h is the plate thickness, a and b are the in-plane dimensions in x and y directions, respectively. The first term is the membrane energy and the second one is the bending energy.

The kinetic energy T for the transverse vibration of a rectangular plate (neglecting longitudinal inertia and rotary inertia), is given by

$$T = \frac{1}{2}\rho h\int_{-a/2}^{a/2}\int_{-b/2}^{b/2}\left(\frac{\partial w}{\partial t}\right)^2 dydx \tag{8}$$

Where ρ is the mass density of plate.

The virtual work W done by the external forces is written as

$$\delta W = \int_{-a/2}^{a/2}\int_{-b/2}^{b/2}\left(\sum_{j=1}^{k}\left(\frac{\partial^2 M_{axj}}{\partial x^2} + \frac{\partial^2 M_{ayj}}{\partial y^2}\right) + f(x,y)q(t)\right)\delta w dydx \tag{9}$$

Where $f(x,y)q(t)$ is out-of-plane distributed external disturbance load of intensity $q(t)$ on the plate. M_{axj}, M_{ayj} are the bending moments coming from the jth ($j = 1,2,\ldots,K$) piezoelectric actuator patches which can be obtained by Fuller *et al.* (1996)

$$M_{axj} = K_{xj}[H(x-x_{1j})-H(x-x_{2j})]\times[H(y-y_{1j})-H(y-y_{2j})]v_{aj}(t)$$
$$M_{ayj} = K_{yj}[H(x-x_{1j})-H(x-x_{2j})]\times[H(y-y_{1j})-H(y-y_{2j})]v_{aj}(t) \tag{10}$$

Where $H(\bullet)$ is the Heaviside step function. Here, $H(x-x_{ij})$ ($i = 1,2$) is zero for $x<x_{ij}$ and one for $x>x_{ij}$, and $H(y-y_{ij})$ ($i = 1,2$) is zero for $y<y_{ij}$ and one for $y>y_{ij}$, $x_{1j}, x_{2j}, y_{1j}, y_{2j}$ are the location of the ends of the jth actuating patch respectively. v_{aj} is the applied voltage to the jth actuator. K_{xj}, K_{yj} depend on the performance and dimensions of plate and piezoelectric actuator patches which can be obtained by

$$K_{xj} = \frac{\alpha_j d_{31} D(1+\upsilon)}{h_{aj}}, \quad K_{yj} = \frac{\alpha_j d_{32} D(1+\upsilon)}{h_{aj}}$$

$$\alpha_j = \frac{12E_{aj}h_{aj}(h_{aj}+h)}{24D(1-\upsilon_{aj}^2)+E_{aj}[(h+2h_{aj})^3-h^3]} \tag{11}$$

Where d_{31j} and d_{32j} are the piezoelectric charge constants along x and y respectively. E_{aj} and υ_{aj} are the Young's modulus and Poisson ratio of piezoelectric material respectively. Hence, we have

$$\frac{\partial^2 M_{axj}}{\partial^2 x}+\frac{\partial^2 M_{ayj}}{\partial^2 y} = K_{xj}\left[\frac{d\delta(x-x_{1j})}{dx}-\frac{d\delta(x-x_{2j})}{dx}\right]\times[H(y-y_{1j})-H(y-y_{2j})]v_{aj}(t)$$
$$+K_{yj}[H(x-x_{1j})-H(y-y_{2j})]\times\left[\frac{d\delta(y-y_{1j})}{dy}-\frac{d\delta(y-y_{1j})}{dy}\right]v_{aj}(t) \tag{12}$$

Where $\delta(\bullet)$ is Kronecker delta function.

Using the principle of Hamilton

$$\delta\int_{t_1}^{t_2}(T-U)dt+\int_{t_1}^{t_2}\delta w dt = 0 \tag{13}$$

Hence, the equation of mid-surface membrane force equilibrium and transverse vibration for the large amplitude behavior of the piezoelectric rectangular flexible plate under out-of-plane distributed external disturbance of intensity $q(t)$ can be respectively written as

$$\frac{\partial N_x}{\partial x}+\frac{\partial N_{xy}}{\partial y} = 0, \quad \frac{\partial N_{xy}}{\partial x}+\frac{\partial N_y}{\partial y} = 0 \tag{14}$$

$$\frac{\partial^2 M_x}{\partial x^2}+2\frac{\partial^2 M_{xy}}{\partial x\partial y}+\frac{\partial^2 M_y}{\partial y^2}+N_x\frac{\partial^2 w}{\partial x^2}+2N_{xy}\frac{\partial^2 w}{\partial x\partial y}+N_y\frac{\partial^2 w}{\partial y^2}$$
$$+\sum_{j=1}^{k}\left(\frac{\partial^2 M_{axj}}{\partial x^2}+\frac{\partial^2 M_{ayj}}{\partial y^2}\right)+f(x,y)q(t)-\rho h\frac{\partial^2 w}{\partial t^2} = 0 \tag{15}$$

Where w denotes the mid-plane transverse displacement response of point (x, y) either on the plate or on the piezoelectric patches.

Introducing the stress function ϕ which is defined by

$$N_x = \frac{\partial^2 \phi}{\partial y^2}, \quad N_y = \frac{\partial^2 \phi}{\partial x^2}, \quad N_{xy} = \frac{\partial^2 \phi}{\partial x \partial y} \tag{16}$$

With Eq. (6), Eq. (15) and Eq. (16), the transverse vibration governing equation in terms of stress function can be represented as

$$\rho h \frac{\partial^2 w}{\partial t^2} + D\nabla^4 w = L(w, \phi) + \sum_{j=1}^{k}\left(\frac{\partial^2 M_{axj}}{\partial x^2} + \frac{\partial^2 M_{ayj}}{\partial y^2}\right) + f(x, y)q(t)$$

$$L(w, \phi) = \frac{\partial^2 \phi}{\partial x^2}\frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 \phi}{\partial y^2}\frac{\partial^2 w}{\partial x^2} - 2\frac{\partial^2 \phi}{\partial x \partial y}\frac{\partial^2 w}{\partial x \partial y}, \quad \nabla^4 w = \frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} \tag{17}$$

According to the stress-strain relationship (3), equilibrium Eq. (14) and definition (16), the stress function can be chosen by satisfying the following compatibility

$$\nabla^4 \phi = Eh\left[\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 - \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}\right], \quad \nabla^4 \phi = \frac{\partial^4 \phi}{\partial x^4} + 2\frac{\partial^4 \phi}{\partial x^2 \partial y^2} + \frac{\partial^4 \phi}{\partial y^4} \tag{18}$$

The transverse displacement and the stress function can be expanded in a generalized double Fourier series satisfying the boundary conditions and compatibility condition

$$w = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} T_{mn}(t) X_m(x) Y_n(y), \quad \phi = \sum_{p=1}^{\infty}\sum_{q=1}^{\infty} F_{pq}(t) \zeta_m(x) \eta_q(y) \tag{19}$$

Where $T_{mn}(t)$ is the generalized coordinates that are unknown functions of t. $X_m(x), Y_n(y)$, $\xi_p(x), \eta_q(y)$ satisfy the orthogonality as follows

$$\int_{-a/2}^{a/2} X_i X_j dx = \begin{cases} 0 & i \neq j \\ \|X_i\|^2 & i = j \end{cases}, \quad \int_{-b/2}^{b/2} Y_i Y_j dy = \begin{cases} 0 & i \neq j \\ \|Y_i\|^2 & i = j \end{cases}$$

$$\int_{-a/2}^{a/2} \zeta_i \zeta_j dx = \begin{cases} 0 & i \neq j \\ \|\zeta_i\|^2 & i = j \end{cases}, \quad \int_{-b/2}^{b/2} \eta_i \eta_j dy = \begin{cases} 0 & i \neq j \\ \|\eta_i\|^2 & i = j \end{cases} \tag{20}$$

Substituting Eq. (19) into Eq. (17) and Eq. (18), multiplying the two equations by $X_m(x) Y_n(y)$ and $\zeta_s(x) \eta_t(y)$ respectively and integrating as follows

$$\int_{-a/2}^{a/2}\int_{-b/2}^{b/2}\left\{\rho h \frac{\partial^2 w}{\partial t^2} + D\nabla^4 w - \left[L(w, \phi) + \sum_{j=1}^{k}\left(\frac{\partial^2 M_{axj}}{\partial x^2} + \frac{\partial^2 M_{ayj}}{\partial y^2}\right) + f(x, y)q(t)\right]\right\} X_m(x) Y_n(y) dx dy = 0$$

$$\int_{-a/2}^{a/2}\int_{-b/2}^{b/2} \nabla^4 \phi \zeta_s(x) \eta_t(y) dx dy = \int_{-a/2}^{a/2}\int_{-b/2}^{b/2} Eh\left[\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 - \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}\right] \zeta_s(x) \eta_t(y) dx dy \tag{21}$$

Using the orthogonality of Eq. (20), the dynamic equation for piezoelectric flexible plate with non-linearity can be obtained by the Galerkin approximate method and numerical integral (Chia 1980)

$$\hat{a}_{mn}T''_{mn}(t)+\hat{b}_{mn}T_{mn}(t)+\sum_{s,t}\hat{c}^{st}_{mn}F_{st}(t)T_{mn}(t)=\sum_{j=1}^{k}b_{jmn}v_{aj}(t)+\tilde{b}_{mn}q(t)$$

$$\hat{a}_{mn}=\rho h\left\|X_m\right\|^2\left\|Y_n\right\|^2$$

$$\hat{b}_{mn}=D\int_{-a/2}^{a/2}\int_{-b/2}^{b/2}(X''''_m(x)Y_n(x)+2X''_m(x)Y''_m(y)+X_m(x)Y''''_m(y))X_m(x)Y_n(y)dxdy$$

$$\hat{c}^{st}_{mn}=\int_{-a/2}^{a/2}\int_{-b/2}^{b/2}-(\zeta''_s(x)\eta_t(y)X_m(x)Y''_n(y)+\zeta_s(x)\eta''_t(x)X''_m(x)Y_n(y)-$$

$$2\zeta'_s(x)\eta'_t(y)X'_m(x)Y'_n(y))X_m(x)Y_n(y)dxdy$$

$$b_{jmn}=\int_{-a/2}^{a/2}\int_{-b/2}^{b/2}\left(\frac{\partial^2 M_{axj}}{\partial x^2}+\frac{\partial^2 M_{ayj}}{\partial y^2}\right)X_m(x)Y_n(y)dxdy\ ,\quad \tilde{b}_{mn}=\int_{-a/2}^{a/2}\int_{-b/2}^{b/2}f(x,y)X_m(x)Y_n(y)dxdy$$

$$\hat{d}_{st}F_{st}(t)=\sum_{g,h}\sum_{a,b}\hat{e}^{ghab}_{st}T_{gh}(t)T_{ab}(t)$$

$$\hat{d}_{st}=\int_{-a/2}^{a/2}\int_{-b/2}^{b/2}(\zeta''''_s(x)\eta_t(x)+2\zeta''_s(x)\eta''_t(y)+\zeta_s(x)\eta''''_t(y))\zeta_s(x)\eta_t(y)dxdy$$

$$\hat{e}^{ghab}_{st}=Eh\int_{-a/2}^{a/2}\int_{-b/2}^{b/2}(X'_g(x)Y'_h(y)X'_a(x)Y_b(y)-X''_g(x)Y_h(y)X_a(x)Y''_b(y))$$

$$\zeta_s(x)\eta_t(y)dxdy,\qquad m,n,s,t,g,h,a,b=1,2,\ldots,\infty \tag{22}$$

The dynamic equations of plates with cubic non-linearity can be further rearranged as follows

$$T''_{mn}(t)+\alpha_{mn}T_{mn}(t)+\sum_{g,h}\sum_{a,b}\beta^{ghab}_{mn}T_{gh}(t)T_{ab}(t)T_{mn}(t)=\sum_{j=1}^{k}\bar{b}_{jmn}v_{aj}(t)+\bar{\bar{b}}_{mn}q(t)$$

$$\alpha_{mn}=\frac{\hat{b}_{mn}}{\hat{a}_{mn}},\ \beta^{ghab}_{mn}=\sum_{s,t}\frac{\hat{c}^{st}_{mn}\hat{e}^{ghab}_{st}}{\hat{a}_{mn}\hat{d}_{st}},\ \bar{b}_{jmn}=\frac{b_{jmn}}{\hat{a}_{mn}},\ \ \bar{\bar{b}}_{mn}=\frac{\tilde{b}_{mn}}{\hat{a}_{mn}}$$

$$w=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}T_{mn}(t)X_m(x)Y_n(y) \tag{23}$$

3. T-S fuzzy modeling of structural system

Using a finite subset of the infinite set which is needed to represent w,ϕ in the Eq. (23) and defining $\mathbf{x}_o=[T_{11}(t)\ \ T'_{11}(t)\ \ldots\ T_{mn}(t)\ \ T'_{mn}(t)\ \ldots\ T_{MN}(t)\ \ T'_{MN}(t)]^T$ as state variables, the state space model of the piezoelectric flexible plate structure with geometrical non-linearity can be written as

$$\mathbf{x}'_o(t)=\mathbf{A}\mathbf{x}_o(t)+\mathbf{B}\mathbf{V}_a(t)+\tilde{\mathbf{B}}q(t)$$

$$\mathbf{A}=\mathrm{diag}\left[\hat{\mathbf{A}}_{11}\ \ \cdots\ \ \hat{\mathbf{A}}_{mn}\ \ \cdots\ \ \hat{\mathbf{A}}_{MN}\right],\ \hat{\mathbf{A}}_{mn}=\begin{bmatrix}0 & 1\\ -\alpha_{mn}-\sum_{g,h}\sum_{a,b}\hat{\beta}^{ghab}_{mn}T_{gh}(t)T_{ab}(t) & 0\end{bmatrix}$$

$$\mathbf{B}=\left[\mathbf{B}_{11}\ \ \cdots\ \ \mathbf{B}_{mn}\ \ \cdots\ \ \mathbf{B}_{MN}\right]^T,\ \mathbf{B}_{mn}=\begin{bmatrix}0 & \cdots & 0\\ \bar{b}_{1mn} & \cdots & \bar{b}_{kmn}\end{bmatrix},$$

$$\tilde{\mathbf{B}} = \begin{bmatrix} \tilde{\mathbf{B}}_{11} & \ldots & \tilde{\mathbf{B}}_{mn} & \ldots & \tilde{\mathbf{B}}_{MN} \end{bmatrix}^T, \ \tilde{\mathbf{B}}_{mn} = \begin{bmatrix} 0 \\ \tilde{\bar{b}}_{mn} \end{bmatrix}, \ \mathbf{V}_a(t) = [v_{a1}, \cdots, v_{ak}]^T \tag{24}$$

With the displacement coming from a point (x_0, y_0) along the plate as the performance displacement, we have

$$y(t) = \mathbf{C}\mathbf{x}_o(t)$$
$$\mathbf{C} = \begin{bmatrix} X_1(x_0)Y_1(y_0) & 0 & \ldots & X_m(x_0)Y_n(y_0) & 0 & \ldots & X_M(x_0)Y_N(y_0) & 0 \end{bmatrix} \tag{25}$$

By decomposing the whole nonlinear state space into several linear subspaces interconnected by IF-THEN inference rules with nonlinear fuzzy membership functions, structural system Eq. (24) can be approximated by T-S fuzzy model. The ith model rule is given by the following IF-THEN fuzzy rules

IF $T_{11}(t)$ is M_{i11} and … and $T_{mn}(t)$ is M_{imn} and … and $T_{MN}(t)$ is M_{iMN} THEN

$$\mathbf{x}_o(t) = \mathbf{A}_i\mathbf{x}_o(t) + \mathbf{B}\mathbf{V}_a(t) + \tilde{\mathbf{B}}\mathbf{q}(t) \tag{26}$$

Where $M_{imn}(i = 1, 2, \ldots, r;\ m = 1, 2, \ldots, M;\ n = 1, 2, \ldots, N)$ is the fuzzy set, r is the rule number of model rules, $\mathbf{x}_o$ is the state vector, $T_{11}, \ldots, T_{mn}, \ldots, T_{MN}$ are the premise variables, $\mathbf{A}_i$ is the local system matrix. $\mathbf{B}$, $\tilde{\mathbf{B}}$ are the same as that in the Eq. (24).

The final dynamic fuzzy model for the system Eq. (24) can be inferred as follows

$$x_o'(t) = \sum_{i=1}^{r} h_i(T(t))\mathbf{A}_i\mathbf{x}_o(t) + \mathbf{B}\mathbf{V}_a(t) + \tilde{\mathbf{B}}\mathbf{q}(t)$$
$$y(t) = \mathbf{C}\mathbf{x}_o(t), \quad \mathbf{V}_s(t) = \tilde{\mathbf{C}}\mathbf{x}_o(t)$$
$$h_i(T(t)) = \frac{\lambda_i(T(t))}{\sum_{i=1}^{r}\lambda_i(t)}, \quad \lambda_i(T(t)) = \prod_{m,n}\lambda_i(T_{mn}(t)) \tag{27}$$

Where $\lambda_i(T_{mn}(t))(i = 1, 2, \ldots, r;\ m = 1, 2, \ldots, M;\ n = 1, 2, \ldots, N)$ is the grade of membership of $T_{mn}(t)$ in the fuzzy set M_{imn}, r is the rule number. Since

$$\begin{cases} \sum_{i=1}^{r}\lambda_i(T(t)) > 0 \\ \lambda_i(T(t)) \geq 0 \end{cases} \qquad i = 1, 2, \ldots, r \tag{28}$$

We have

$$\begin{cases} \sum_{i=1}^{r} h_i(T(t)) = 1 \\ h_i(T(t)) \geq 0 \end{cases} \qquad i = 1, 2, \ldots, r \tag{29}$$

$\mathbf{V}_s = [v_{s1}, \ldots, v_{sl}, \ldots, v_{sJ}]^T$ is the measured output voltage of the piezoelectric sensor patches, and the output matrix $\tilde{\mathbf{C}}$ can be obtained by

$$\tilde{\mathbf{C}} = \begin{bmatrix} \tilde{c}_{111} & 0 & \dots & \tilde{c}_{112} & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \tilde{c}_{J11} & 0 & \dots & \tilde{c}_{J12} & 0 \end{bmatrix}, \; \tilde{c}_{lmn} = \Omega_{xl}\left(\int_{y_{1l}}^{y_{2l}} \left[\frac{\partial X_m(x_{2l})}{\partial x} - \frac{\partial X_m(x_{1l})}{\partial x} \right] Y_n(y)dy \right) + \Omega_{yl}$$

$$\left(\int_{x_{1l}}^{x_{2l}} \left[\frac{\partial Y_n(y_{2l})}{\partial y} - \frac{\partial Y_n(y_{1l})}{\partial y} \right] X_m(x)dx \right), \; l = 1,2,\cdots,J; \; m = 1,2....,M; \; n = 1,2,...,N \tag{30}$$

Ω_{xl}, Ω_{yl} depend on the performance and dimensions of plate and lth piezoelectric sensor patches, which can be obtained by

$$\Omega_{xl} = \frac{h + h_{sl}}{2c_{sl}} \frac{k_{31l}^2}{g_{31l}}, \; \Omega_{yl} = \frac{h + h_{sl}}{2c_{sl}} \frac{k_{32l}^2}{g_{32l}} \tag{31}$$

Where h_{sl} is the thickness, c_{sl} is the piezoelectric capacitance. g_{31l}, g_{32l} are the voltage constants in x and y directions respectively. k_{31l}, k_{32l} are the electromechanical coupling factors in x and y directions respectively. $x_{1l}, x_{2l}, y_{1l}, y_{2l}$ are the location of the ends of the lth sensing patch respectively.

4. Fuzzy dynamic output feedback control law

In order to design a global controller for the T-S fuzzy model, the parallel distributed compensation (PDC) technique is adopted in this section. In the PDC technique, each control rule is designed from the corresponding rule of the T-S fuzzy model, and the designed fuzzy controller shares the same fuzzy sets with the fuzzy model in the premise parts.

For the fuzzy model Eq. (27), we construct the following fuzzy controller via the PDC. The th rule of the fuzzy logic controller is defined as follows

IF $T_{11}(t)$ is M_{i11} and … and $T_{mn}(t)$ is M_{imn} and … and $T_{MN}(t)$ is M_{iMN} THEN

$$\begin{aligned} \mathbf{x}_c'(t) &= \mathbf{A}_{ci}\mathbf{x}_c(t) + \mathbf{B}_{ci}\mathbf{V}_s(t) \\ \mathbf{V}_a(t) &= \mathbf{C}_{ci}\mathbf{x}_c(t) + \mathbf{D}_{ci}\mathbf{V}_s(t) \end{aligned} \tag{32}$$

Where $\mathbf{x}_c(t)$ is the state variable vector of the controller, $\mathbf{A}_{ci}, \mathbf{B}_{ci}, \mathbf{C}_{ci}, \mathbf{D}_{ci}$ are unknown parameters of the local controller, $\mathbf{D}_{ci} = \mathbf{0}$ means that the control law is strictly real.

The overall model-based fuzzy dynamic output feedback law is analytically represented by

$$\begin{aligned} \mathbf{x}_c'(t) &= \sum_{i=1}^{r} h_i(T(t))(\mathbf{A}_{ci}\mathbf{x}_c(t) + \mathbf{B}_{ci}\mathbf{V}_s(t)) \\ \mathbf{V}_a(t) &= \sum_{i=1}^{r} h_i(T(t))(\mathbf{C}_{ci}\mathbf{x}_c(t) + \mathbf{D}_{ci}\mathbf{V}_s(t)) \end{aligned} \tag{33}$$

With controller Eq. (33) and structure Eq. (27), the state space realization of the overall closed-loop fuzzy system can be obtained

$$\mathbf{x}'(t) = \mathbf{A}_h\mathbf{x}(t) + \mathbf{B}_h q(t)$$
$$y(t) = \mathbf{C}_h\mathbf{x}(t)$$
$$\mathbf{x}(t) = \begin{Bmatrix} \mathbf{x}_o(t) \\ \mathbf{x}_c(t) \end{Bmatrix},\ \mathbf{A}_h = \begin{bmatrix} \sum_{i=1}^{r} h_i(\mathbf{A}_i + \mathbf{B}\mathbf{D}_{ci}\mathbf{C}) & \sum_{i=1}^{r} h_i\mathbf{B}\mathbf{C}_{ci} \\ \sum_{i=1}^{r} h_i\mathbf{B}_{ci}\mathbf{C} & \sum_{i=1}^{r} \mathbf{A}_{ci} \end{bmatrix},\ \mathbf{B}_h = \begin{bmatrix} \mathbf{B} \\ 0 \end{bmatrix},\ \mathbf{C}_h = \begin{bmatrix} \mathbf{C} \\ \mathbf{0} \end{bmatrix}^T \tag{34}$$

Our aim is to design the fuzzy dynamic output controller Eq. (33) to suppress the geometrically nonlinear vibration of the plate structure due to any unknown external disturbance with finite energy, which can be implemented by minimize a measure of energy transfer from the external disturbance $q(t)$ entering the flexible structure at the known fixed locations $f(x,y)$ to vibration output $y(t)$ of the closed-loop system Eq. (34). Because of the robustness consideration, the H_∞ norm is chosen as the performance index, which relates to the maximum magnitude of the frequency response for system and the worst situation taking place in the vibration such as the resonance excited by the external disturbance whose frequency equals to the fundamental frequency of system. In the following, we will reformulate the H_∞ fuzzy controller design problem into solving an LMI problem based on Lyapunov direct method. As a result, the stability analysis and disturbance rejection control design problems can be reduced to linear matrix inequality problems.

Robust stability and disturbance rejection (an upper bound γ on the H_∞ norm from the disturbance $q(t)$ to the output $y(t)$) of the T-S fuzzy system Eq. (34) can be guaranteed (Chen and Guo 2005, Samuel and Vicente 2006, Xu and Chen 2008) if there exists $\mathbf{P} = \mathbf{P}^T > 0$ and Lyapunov function $V(x(t)) = \mathbf{x}^T(t)\mathbf{P}\mathbf{x}(t) > 0$ is taken such that

$$\frac{dV(\mathbf{x}(t))}{dt} + y^T(t)y(t) - \gamma^2 q^T(t)q(t) < 0 \tag{35}$$

Substituting Eq. (34) into Eq. (35) and applying S-procedure, Eq. (35) will hold if there exits $\mathbf{P} = \mathbf{P}^T > 0$, such that (Assawinchaichote *et al.* 2008)

$$\begin{bmatrix} (\mathbf{A}_i + \mathbf{B}\mathbf{D}_{ci}\tilde{\mathbf{C}})^T\mathbf{P} + \mathbf{P}(\mathbf{A}_i + \mathbf{B}\mathbf{D}_{ci}\tilde{\mathbf{C}}) & \tilde{\mathbf{C}}^T\mathbf{B}_{ci}^T\mathbf{P} + \mathbf{P}\mathbf{B}_{ci}\tilde{\mathbf{C}} & \mathbf{P}\tilde{\mathbf{B}} & \mathbf{C}^T \\ & \mathbf{A}_{ci}^T\mathbf{P} + \mathbf{P}\mathbf{A}_{ci} & \mathbf{0} & \mathbf{0} \\ & & -\gamma^2\mathbf{I} & \mathbf{0} \\ sym & & & -\mathbf{I} \end{bmatrix} < 0 \tag{36}$$

$$i = 1, 2, \cdots, r$$

Hence, the stability and disturbance rejection of the T-S fuzzy model Eq. (34) are achieved by finding a common symmetric positive definite matrix $\mathbf{P}$ for r subsystems Eq. (36). The matrix inequalities Eq. (36) are nonlinear in the unknown parameters $\mathbf{P}, \mathbf{A}_{ci}, \mathbf{B}_{ci}, \mathbf{C}_{ci}, \mathbf{D}_{ci}$. Partitioning $\mathbf{P}$ as $\mathbf{P} = \begin{bmatrix} \mathbf{p}_1 & \mathbf{p}_2 \\ \mathbf{p}_2^T & \mathbf{P}_3 \end{bmatrix}$ and $\mathbf{P}^{-1}$ as $\mathbf{P}^{-1} = \begin{bmatrix} \hat{\mathbf{p}}_1 & \hat{\mathbf{p}}_2 \\ \hat{\mathbf{p}}_2^T & \hat{\mathbf{P}}_3 \end{bmatrix}$ $(\mathbf{p}_1, \hat{\mathbf{p}}_1 \in R^{2M\times 2M})$, we can change Eq. (36) into the linear matrix inequalities as follows (Tanaka and Wang 2001)

$$\begin{bmatrix} \mathbf{A}_i\hat{\mathbf{p}}_1 + \hat{\mathbf{p}}_1\mathbf{A}_i^T + \mathbf{B}\mathbf{C}_{Mi} + \mathbf{C}_{Mi}^T\mathbf{B}^T & \mathbf{A}_i + \mathbf{B}\mathbf{D}_{Mi}\tilde{\mathbf{C}} + \mathbf{A}_{Mi}^T & \tilde{\mathbf{B}} & \hat{\mathbf{p}}_1\mathbf{C}^T \\ & \mathbf{A}_i^T\mathbf{p}_1 + \mathbf{p}_1\mathbf{A}_i + \tilde{\mathbf{C}}^T\mathbf{B}_{Mi}^T + \mathbf{B}_{Mi}\tilde{\mathbf{C}} & \mathbf{p}_1\tilde{\mathbf{B}} & \mathbf{C}^T \\ & & -\gamma^2\mathbf{I} & \mathbf{0} \\ sym & & & -\mathbf{I} \end{bmatrix} < 0 \tag{37}$$

$$i = 1,2,\ldots,r$$

Where

$$\begin{bmatrix} \hat{\mathbf{p}}_1 & \mathbf{I} \\ \mathbf{I} & \mathbf{p}_1 \end{bmatrix} > 0, \ \mathbf{p}_2\hat{\mathbf{p}}_2 = \mathbf{I} - \mathbf{p}_1\hat{\mathbf{p}}_1$$

$$\begin{aligned} \mathbf{A}_{Mi} &= \mathbf{p}_1(\mathbf{A}_i + \mathbf{B}\ \mathbf{D}_{ci}\tilde{\mathbf{C}})\hat{\mathbf{p}}_1 + \mathbf{p}_2\mathbf{B}_{ci}\tilde{\mathbf{C}}\ \hat{\mathbf{p}}_1 + \mathbf{p}_1\mathbf{B}\mathbf{C}_{ci}\hat{\mathbf{p}}_2^T + \mathbf{p}_2\mathbf{A}_{ci}\hat{\mathbf{p}}_2^T \\ \mathbf{B}_{Mi} &= \mathbf{p}_1\mathbf{B}\ \mathbf{D}_{ci} + \mathbf{p}_2\mathbf{B}_{ci} \\ \mathbf{C}_{Mi} &= \mathbf{D}_{ci}\tilde{\mathbf{C}}\hat{\mathbf{p}}_1 + \mathbf{C}_{ci}\hat{\mathbf{p}}_2^T \\ \mathbf{D}_{Mi} &= \mathbf{D}_{ci}, \quad i = 1,2,\ldots,r \end{aligned} \tag{38}$$

The fuzzy vibration controller design problem of the piezoelectric flexible plate structure with geometrical non-linearity can be converted into a linear convex optimization problem as follows

$$\begin{aligned} &\min \quad \gamma^2 \\ &s.t. \quad \text{Eq. (37)} \end{aligned} \tag{39}$$

With the solution $\hat{\mathbf{p}}_1, \mathbf{p}_1, \mathbf{A}_{Mi}, \mathbf{C}_{Mi}, \mathbf{B}_{Mi}, \mathbf{D}_{Mi}$ to Eq. (39), the parameters of the fuzzy controller Eq. (33) can be obtained by

$$\begin{aligned} \mathbf{D}_{ci} &= \mathbf{D}_{\mathrm{M}i} \\ \mathbf{C}_{ci} &= (\mathbf{C}_{Mi} - \mathbf{D}_{ci}\tilde{\mathbf{C}}\hat{\mathbf{p}}_1)(\hat{\mathbf{p}}_2^T)^{-1} \\ \mathbf{B}_{ci} &= \mathbf{p}_2^{-1}(\mathbf{B}_{Mi} - \mathbf{p}_1\mathbf{B}\mathbf{D}_{ci}) \\ \mathbf{A}_{ci} &= \mathbf{p}_2^{-1}(\mathbf{A}_{Mi} - \mathbf{p}_1(\mathbf{A}_i + \mathbf{B}\mathbf{D}_{ci}\tilde{\mathbf{C}})\hat{\mathbf{p}}_1 - \mathbf{p}_2\mathbf{B}_{ci}\tilde{\mathbf{C}}\hat{\mathbf{p}}_1 - \mathbf{p}_1\mathbf{B}\mathbf{C}_{ci}\hat{\mathbf{p}}_2^T)(\hat{\mathbf{p}}_2^T)^{-1} \\ & i = 1,2,\ldots,r \end{aligned} \tag{40}$$

5. Simulation results

Here consider the transverse vibration control of a simply supported thin plate with collocated piezoelectric actuator/sensor patches bonded to it. The plate has the dimension of length $a = 0.7$ m, width $b = 0.7$ m and thickness $h = 3 \times 10^{-3}$ m. The properties of plate material are $E = 7.0 \times 10^{10}$ N/m^2, $\rho h = 2500$ kg/m^2, $\upsilon = 0.3$. Two identical piezoelectric patches are used as an actuator and a sensor respectively, which are located at $0 \to 0.1a, 0 \to 0.1b$ on either side of the flexible plate, and have the thickness of 2×10^{-4} m. The piezoelectric actuator/sensor pair has similar properties in x and y directions, i.e., $d_{31} = d_{32} = 3.2 \times 10^{-10}$ m/V, $g_{31} = g_{32} = 9.5 \times 10^{-3}$ Vm/N, $C_p = 4.5 \times 10^{-7}$ F, $k_{31} = k_{32} = 0.44$, $E_a = 6.2 \times 10^{10}$ N/m^2. The performance displacement output is the displacement coming from a center point of the plate. The damping of the structure is ignored.

For the simply supported plate, the following boundary condition is introduced as

$$w = \frac{\partial^2 w}{\partial x^2} = \frac{\partial^2 \phi}{\partial y^2} = \frac{\partial^2 \phi}{\partial x \partial y} = 0, \quad x = \pm\frac{a}{2}$$

$$w = \frac{\partial^2 w}{\partial y^2} = \frac{\partial^2 \phi}{\partial x^2} = \frac{\partial^2 \phi}{\partial x \partial y} = 0, \quad y = \pm\frac{b}{2} \tag{41}$$

In order to simplify the calculation, the middle surface transverse displacements w is approximated by using the following functions with one-term fundamental mode satisfying geometric boundary conditions

$$w = hT(t)\cos(\pi x/a)\cos(\pi y/b) \tag{42}$$

Substituting Eq. (42) into Eq. (18) gives

$$\nabla^4 \varphi = -\frac{E\pi^4 h^3}{2a^2 b^2} T^2(t)[\cos(2\pi x/a) + \cos(2\pi y/b)] \tag{43}$$

For the simply supported plate with boundary conditions Eq. (41), the stress function is determined as follows

$$\phi = T^2(t)\sum_{p=0}^{\infty}\sum_{q=0}^{\infty} \varphi_{pq}\zeta_p(x)\eta_q(y) \tag{44}$$

Where

$$\zeta_p(x) = \cos(2p\pi x/a), \quad \eta_q(y) = \cos(2q\pi y/b), \quad \varphi_{10} = -\frac{a^2 E h^3}{32 b^2}, \quad \varphi_{01} = -\frac{b^2 E h^3}{32 a^2} \tag{45}$$

The maximum transverse displacement $w_{\max}$ is taken as the order of the thickness of plate, and $T(t) = w/w_{\max} \in [-1, 1]$. Since the number of rules for the overall control system is basically the combination of the model rules and control rules, and the number of model rules is directly related to complexity of analysis and design LMI conditions, we attempt to construct a two-rule fuzzy model by local approximation in the fuzzy partition spaces. M_1 is the fuzzy set for small amplitude and M_2 is the fuzzy set for large amplitude . The grades of membership for M_1 and M_2 are chosen

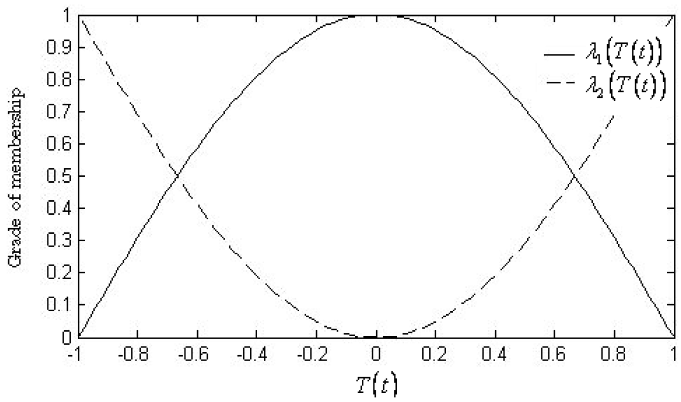

Fig. 2 Grades of membership of fuzzy sets M_1 and M_2

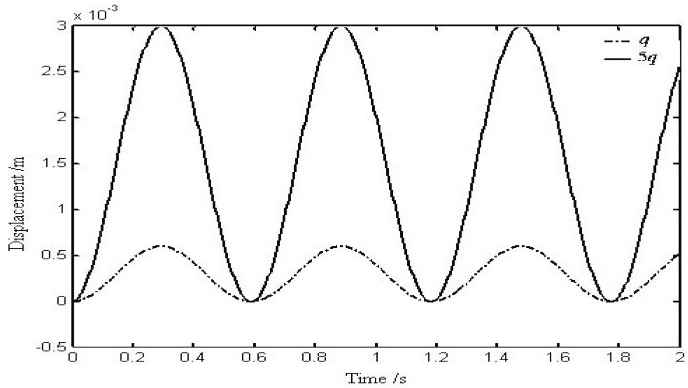

Fig. 3 Displacement responses for linear system under different forcing amplitudes

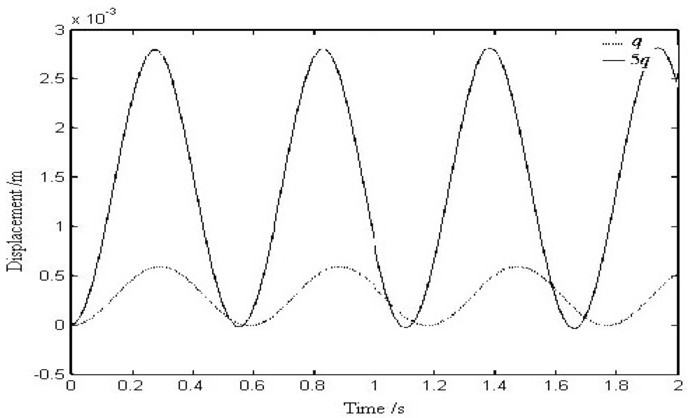

Fig. 4 Displacement responses for nonlinear system under different forcing amplitudes

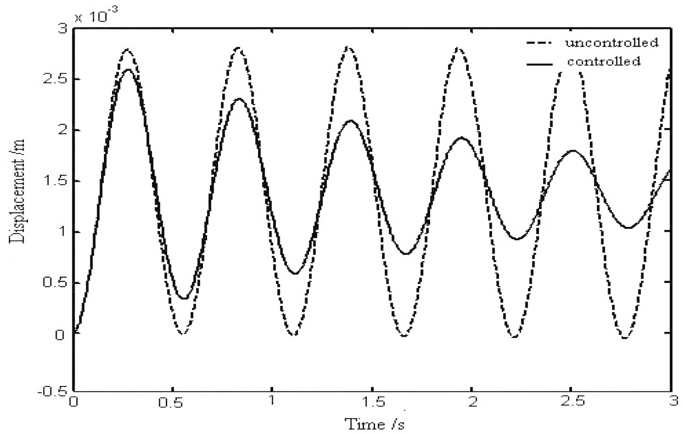

Fig. 5 Displacement responses for uncontrolled and controlled nonlinear systems

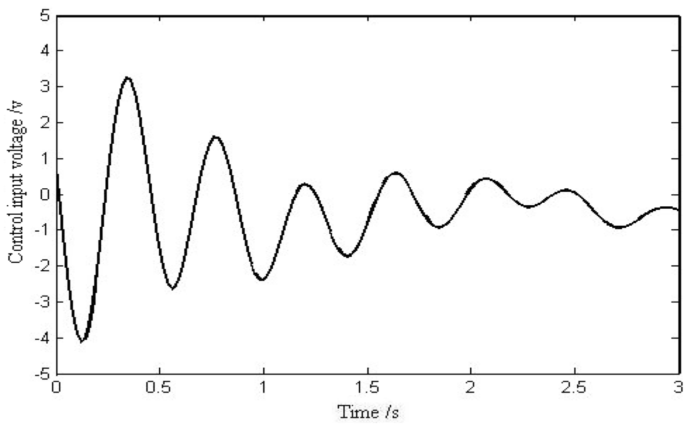

Fig. 6 Control input voltage for actuator of controlled nonlinear system

as shown in Fig. 2. The time interval is taken as 0.005 to obtain reasonably accurate results.

The displacement responses for the uncontrolled linear and nonlinear system with different forcing amplitudes (q and $5q$) are shown in the Fig. 3 and Fig. 4 respectively, which show the typical dynamical feature of nonlinear systems that is the highly amplitude dependence of nonlinear frequency. From the Fig. 3 and Fig. 4, It is also shown that the smaller amplitude occurs in the nonlinear system compared to linear system. The displacement responses for the uncontrolled and controlled nonlinear system are shown in Fig. 5, and the control input voltage for the actuator of controlled system is shown in Fig. 6. It is shown that the fuzzy controller based on the two-rule fuzzy model, which is only an approximation to the original structure, performs well when applied to the original nonlinear plate structures, and the vibration of plate structures with geometrical non-linearity is suppressed.

6. Conclusions

The T-S fuzzy model-based control approach and the LMI approach are combined to obtain a robust vibration control scheme for piezoelectric flexible thin rectangular plate with geometrical non-linearity. The T-S fuzzy model is developed to approximate the large-amplitude vibration dynamic characteristics of structure which is obtained by using generalized Fourier series and numerical integral. Based on the T-S fuzzy model, a global fuzzy dynamic output feedback control law is designed to suppress the vibration due to the external disturbance by using the PDC technique. Each fuzzy control rule is designed from the corresponding rule of the T-S fuzzy model, and the designed fuzzy controller shares the same fuzzy sets with the T-S fuzzy model in the premise parts. The stability analysis and disturbance rejection control design problems are converted into a linear convex optimization problem by using linear matrix inequalities. The numerical results show that the designed fuzzy controller of a simply supported piezoelectric flexible rectangular plate based on the two-rule fuzzy model, which is only an approximation to the original plate structure, performs well when applied to the original nonlinear plate structure. Moreover, the control law is less complex in computation and can be practically implemented.

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