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Development of limit equilibrium method as optimization in slope stability analysis

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Abstract. The slope stability analysis is usually done using the methods of calculation to rupture. The problem lies in determining the critical failure surface and the corresponding factor of safety (FOS). To evaluate the slope stability by a method of limit equilibrium, there are linear and nonlinear methods. The linear methods are direct methods of calculation of FOS but nonlinear methods require an iterative process. The nonlinear simplified Bishop method's is popular because it can quickly calculate FOS for different slopes. This paper concerns the use of inverse analysis by genetic algorithm (GA) to find out the factor of safety for the slopes using the Bishop simplified method. The analysis is formulated to solve the nonlinear equilibrium equation and find the critical failure surface and the corresponding safety factor. The results obtained by this approach compared with those available in literature illustrate the effectiveness of this inverse method.

Keywords: stability; inverse analysis; genetic algorithm; safety factor

1. Introduction

The stability of slope is one of the most important problems in stability analysis of geotechnics. It has received wide attention due to its practical importance in the design of excavations, embankments, and tailing dams...

Out of various methods (finite element analysis, limit analysis), limit equilibrium method is widely used for its simplicity form and the results found to be close to that rigorous methods.

Many methods have been presented to compute the factor of safety (FOS) using limit equilibrium methods like: the vertical method of slices (Bishop 1955, Morgenstern and Price 1965, Spencer 1967, Janbu 1973, Fredlund and Krahn 1977) or the multiple wedge methods (Sarma 1979, Hoek 1987, Donald and Giam 1989a).

The problem lies in finding out the critical failure surface and its corresponding factor of safety.

Many approaches have been developed to automate the search for the critical slip surface. The traditional mathematical optimization methods that have been used include dynamic programming (Baker 1980), conjugate gradient (Arai and Tagyo 1985), random search (Siegel *et al*. 1981, Chen 1992, Greco 1996), and simplex optimization (Nguyen 1985). The main shortcoming of these optimizations is to locate the global minimum factor of safety.

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To locate the critical failure surface and to avoid the difficulty in finding out the global minimum factor of safety, evolutionary methods such as genetic algorithm (GA) is being used, which is more robust in finding out the optimal solution in many complex problems. Goh (1999) has used GA to find out the critical surface and the factor of safety using method of wedges, Sarat Kumar Das (2005) has used real coded GA to find out the critical failure surface and the corresponding factor of safety for three wedge method, Zolfaghari *et al.* (2005) have used a simple genetic algorithm search for homogenous soil layer, and heterogeneous multi soil layers slope.

The genetic algorithm used in this study is to solve the Bishop method to find out the minimum factor of safety and the corresponding critical failure surface in finite slopes.

McCombie and Wilkinson (2002) developed a simple genetic algorithm to search the minimum factor of safety of a circular failure surface in slope stability analysis. They presented a three variable parameters coding, containing the x and y coordinates of the centre of a circle and the *radius* of a circular failure surface. They also showed that replacing the radius with a tangent level or with the coordinates of a point the circle had to pass through (thus creating a four dimensional search space), would usually work better, as the formulation of the problem becomes closer to what determines the fitness of each variable parameters.

In this GA the use of three dimensional search space (the x_0 and y_0 coordinates of the centre of a circle and the horizontal coordinate of a point of the circle which pass through the slope) could work better, as the formulation of the problem becomes very closer to what determines the fitness of each variable parameters. Because we eliminate the failure surface does not hit the geometry of the natural slope, so that the search space is reduced.

The analysis of the problem can be considered in two stages:
- Presentation of objective function;
- The application of GA in solving the objective function.

2. Presentation of objective function

The slope stability analysis method used in this study to develop the objective function is the Bishop method.

The simplified Bishop method also uses the method of slices to discrete the soil mass for determining the FOS. This method satisfies vertical force equilibrium for each slice and overall moment equilibrium about the centre of the *circular* trial surface. In this method it is assumes that only shear forces acting on the sides of each slice are equal. Hence, $X_1 = X_2$ but the $E_1 \neq E_2$ (Figs. 1(a), 1(b)).

From (Huang 1983), Whitlow (1995) formulated the equation for Bishop's method as follow:

The equilibrium along the base of the slice

$$0 = W\sin\alpha - \frac{\tau_f}{F}l = W\sin\alpha - \frac{c'l + N'\tan\varphi'}{F} \tag{1}$$

The equilibrium in a vertical direction

$$0 = W - N'\cos\alpha - ul\cos\alpha - \frac{c'}{F}l\sin\alpha - \frac{N'\tan\varphi'}{F}\sin\alpha \tag{2}$$

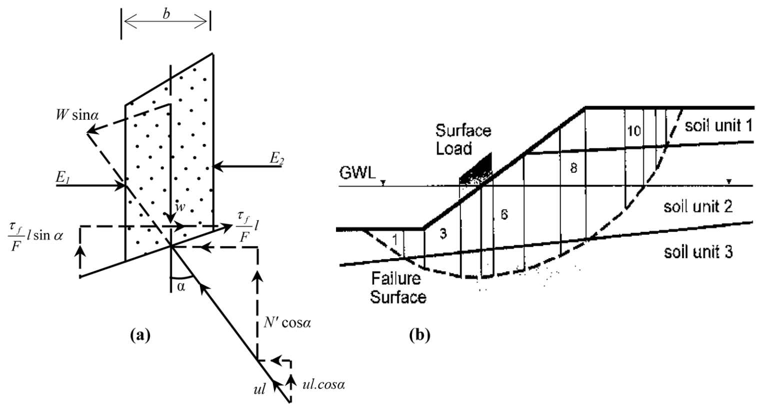

Fig. 1 (a) Bishop's simplified slice (Whitlow 1995) and (b) Division of potential sliding mass into slices

Nomenclature
α failure-line slope (slope of base of slice)
b width of slice
c' cohesion of soil
φ' internal friction of cohesion
γ bulk density of soil
N' force normal to base of slice due to the effective stress
u pore pressure
W weight of slice;
τ_f shear stress on the potential failure surface
F factor of safety
l projection of b on the failure surface

$$I = b \sec \alpha \tag{3}$$

After substituting for, l and N' (get from the Eq. (2)) into the Eq. (1)

$$F = \frac{1}{\sum W \sin\alpha} \sum \frac{[c'b + (W - ub)\tan\varphi'] \sec\alpha}{1 + \dfrac{\tan\alpha \tan\varphi'}{F}} \tag{4}$$

"The procedure is commenced by assuming a trial value for the F on the right-hand side and then, using an iterative process, to converge on the true value of F for a given trial circle. This is the routine procedure commonly used in programs designed for use on computers." (Whitlow 1995)

In this study an optimisation method was developed to found out the F minimum which satisfies Eq. (4).

3. Genetic algorithm optimization

The GA is a random search algorithm based on the concept of natural selection inherent in natural genetics, presents a robust method for search of the optimum solution to the complex problems. The algorithms are mathematically simple yet powerful in their search for improvement after each generation (Goldberg 1989). The artificial survival of better solution in GA search technique is achieved with genetic operators: selection, crossover and mutation, borrowed from natural genetics. The major difference between GA and the other classical optimization search techniques is that the GA works with a population of possible solutions; whereas the classical optimization techniques work with a single solution. Another difference is that the GA uses probabilistic transition rules instead of deterministic rules.

In the present analysis a binary-coded GA has been used, in which the GA consists of three basic operators, selection, crossover or mating, and mutation, which are discussed as follow. The first step when applying genetic algorithms is to choose an initial population consisting of members called chromosomes. The candidates of the input space are then coded using binary coding. These are called chromosomes and form an initial population. The basic procedure then becomes: choose an initial population, select the best candidates for next generation, do crossover operation, and mutate.

In the selection procedure, the chromosomes compete for survival in an elitist selection, the size of initial population is selected double that the population of the next generation, thus permitted to have a best initial exploration of the search space and have a considerable effect on the convergence rate. A fitness value associated with each chromosome is calculated. The population is then sorted in ascending order according to the fitness value, the best individuals are copied to the next generation, and then 1/3 of the population called parent population is selected for the reproduction process. A crossover process with a probability of P_c is applied to two selected parent chromosomes. A random position along the length of the chromosome is selected and the values of each binary string are exchanged or crossed by swapping all characters after this position. The two new chromosomes created are known as children of those parents. Mutation is applied to a small proportion of chromosomes, thus introducing the possibility of significant shifts away from the solutions currently being converged on, that overcomes problems associated with local maxima or minima in analyses. Each binary value in a chromosome selected for mutation is swapped with a probability of P_m. The fitness values of the new population, which include both children and parent chromosomes, are then calculated. The process of reproduction, crossover, mutation and evaluation is repeated as a cycle of generation. A number of cycles are performed until an optimal solution is determined, or some termination criterion, such as a set number of generations is reached.

The choices of the appropriate selection strategy, crossover strategy, population size, crossover rate, and mutation rate are problem dependent and generally require some experimentation. Some common strategies and guidelines for crossover and mutation rates were described earlier. In this study the choice of those parameters is referenced to (Levasseur 2007). The program of this genetic algorithm is written in Matlab following the flow chart represented by Fig. 2.

Several parameters influencing the convergence of results, among them the effects of the size and grid of search space, to minimize the number of iterations and thus the computing time it seeks to reduce the search space. Enumerative methods are simple principles. In a finite and discrete search space, enumerative algorithm evaluates the value of the function to be optimized at each point of the solution space. Through this exhaustive exploration of the search space of parameters, all possible combinations in a range of variation is limited by the user compared with each other. The

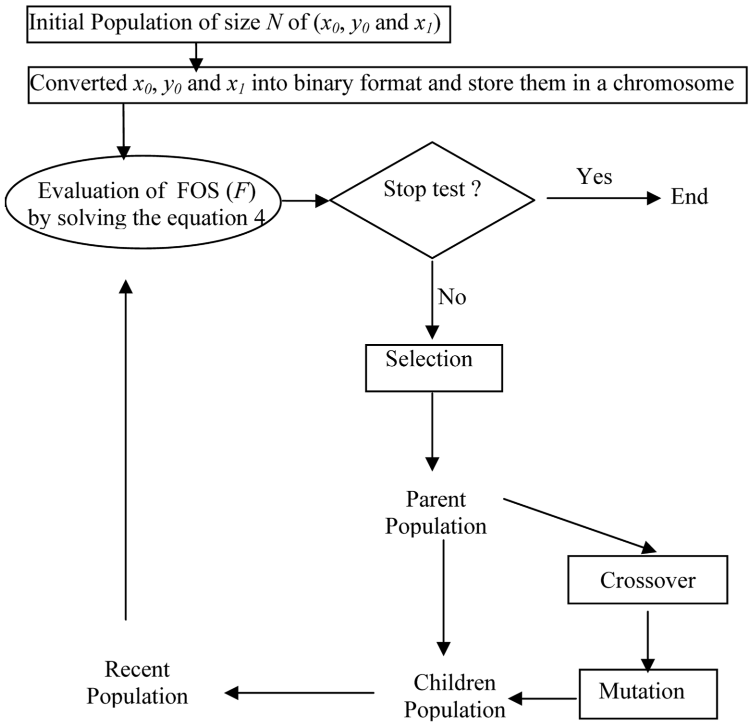

Fig. 2 Principle of optimization with a genetic algorithm

optimal solution is one for which the value of the error function is the lowest. This method is very expensive thus lacks effectiveness. It means having a clear idea of the magnitude of the parameters and not be too demanding on the accuracy of results.

In this study the chromosome (individual) is represented by three parameters (defined the circular failure surface) the coordinates of the centre of a circle (x_0, y_0), and x_1 the abscissa of a point of the circle which pass through the slope.

Also, y_1 is related to the topographic profile $s(x_1)$. $y_1 = s(x_1)$.

The GA start with a population of N individuals, real parameters such as coordinates need to be expressed as integer values with random selection, and then converted to binary coded. Each solution is created by *Nbit* binary encoding which necessitates decoding. The bounds of each parameter (P_{max}, P_{min}) are determined by the enumerative method. The restriction of the search space permitted the refining of its grid, which influences the precision of results. The generation procedure is repeated until the best chromosome representing the minimum FOS is produced or the maximum number of generation is reached (Gc_{max}).

4. Examples

Two natural slopes analysed in (Das 2005) with two others analysed in (Zolfaghari *et al*. 2005),

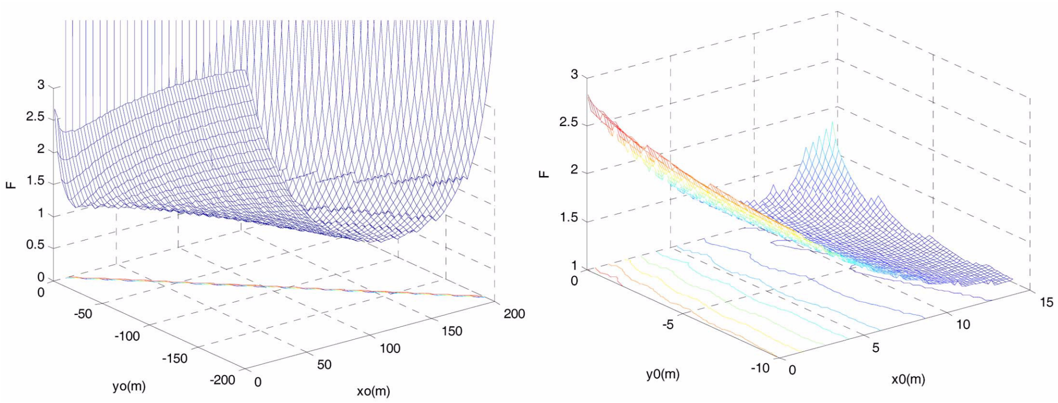

Fig. 3 Example of exhaustive exploration of the search space. Representation of the error function F on the search space (x_0, y_0, x_1)

Table 1 Slopes geometry and parameters for examples 1 and 2

	H (m)	β (°)	c' (kN/m^2)	φ' (°)	γ (kN/m^3)	$F^{(1)}$	$F^{(2)}$
Example 1	11.6	45	15	22	19.10	**1.105**	1.130
Example 2	3.8	65	7.5	28	19.04	**1.131**	1.178

(1) : Results of this study.
(2) : Results of Sarat Kumar Das.

Table 2 Slopes geometry and parameters for example 3

	H (m)	β (°)	c' (kN/m^2)	φ' (°)	γ (kN/m^3)	$F^{(1)}$	$F^{(2)}$		
							Bishop's method (circular failure)	*Morgenstern-Price's method (circular failure)*	*Morgenstern-Price's method (non circular failure)*
Example 3	8.5	26.565	15	20	19	**1.741**	1.74	1.76	1.75

(1) : Results of this study.
(2) : Results of Zolfaghari *et al.*

were considered for the present study. The four dry slopes with its geometry and soil parameters are shown in Tables 1, 2 and 3.

H and β are respectively the height and angle of the slope.

The enumerative method allowed us to choose the bounds of the search space; Fig. 3 shows the application of this method for the first example.

The GA parameters initially chosen in all the examples were as follows:

$P_c = 2 / 3$; $P_m = 0.09$; $N = 100$; $Nbit = 6 + 6 + 6$; $Gc_{max} = 50$.

The examples 1, 2 and 3 of a natural slope with a homogenous soil layer are analysed. The factor of safety is calculated for the slope using the genetic algorithm method proposed in this paper. Figs. 4, 6 and 8 shows how the fitness value (F) is changed and converged to the minimum FOS, Figs. 5,

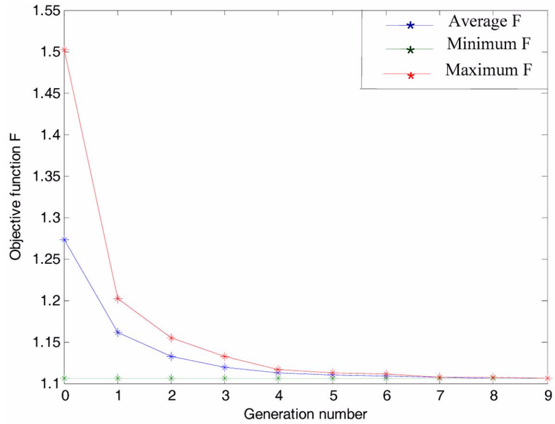

Fig. 4 Variation of the objective function with generation number for example 1

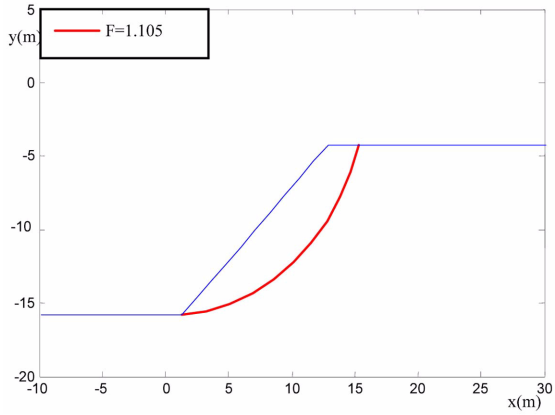

Fig. 5 Critical circular failure surface for example 1

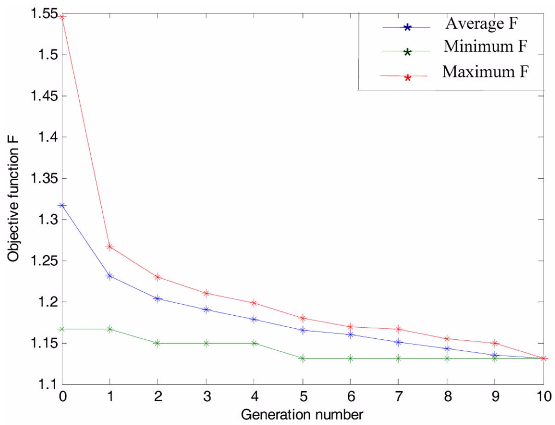

Fig. 6 Variation of the objective function with generation number for example 2

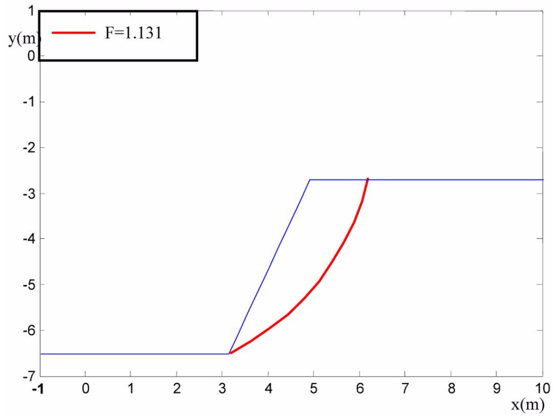

Fig. 7 Critical circular failure surface for example 2

7 and 9 shows the critical circular failure surface.

The results of the present study compared to those in literature illustrate that the FOS is similar for different methods and failure surfaces. What confirm that a simple circular failure surface method is sufficient for a slope in a homogenous soil layer.

For the example 4 of a natural slope with heterogeneous soil layers, Fig. 10 shows how the fitness value (F) is changed and converged to a minimum FOS, Fig. 11 shows the critical circular failure surface.

The FOS calculate in the present study is $F = 1.454$. The result of the present study compared to those calculates in (Zolfaghari *et al.* 2005), shows that the FOS is similar for circular failure for different methods, but it is over than that for non circular failure.

The small difference between the result in the present study and in the literature is resulted to the principle of GA that works with a population of possible solutions not a single solution.

The factor of safety obtained by the present study is less (few lower) than that obtained by three

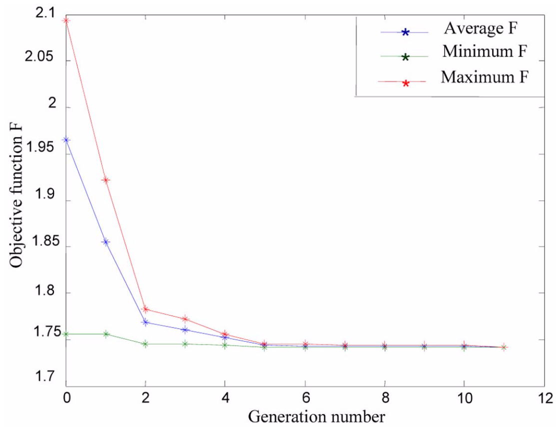

Fig. 8 Variation of the objective function with generation number for example 3

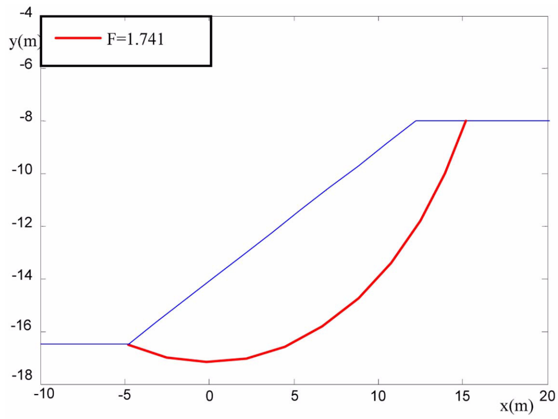

Fig. 9 Critical circular failure surface for example 3

Table 3 Slopes geometry and parameters for example 4

	H_1m	β_1 (°)	c'_1 kN/m^2	φ'_1 (°)	H_2 m	β_2 (°)	c'_2 kN/m^2	φ'_2 (°)	H_3 m	β_3 (°)	c'_3 kN/m^2	φ'_3 (°)	H_4 m
Example 4	8.5	26.565	15	20	0.5	3.18	17	21	4.4	12.17	5	10	4.7

β_4 (°)	c'_4 (kN/m^2)	φ'_4 (°)	$\gamma_1=\gamma_2=\gamma_3=\gamma_4$ (kN/m^3)	$F^{(1)}$	$F^{(2)}$ Bishop's method (circular failure)	$F^{(2)}$ Morgenstern-Price's method (circular failure)	$F^{(2)}$ Morgenstern-Price's method (non circular failure)
12.06	35	28	19	1.454	1.475	1.50	1.24

(1) : Results of this study
(2) : Results of Zolfaghari *et al*.

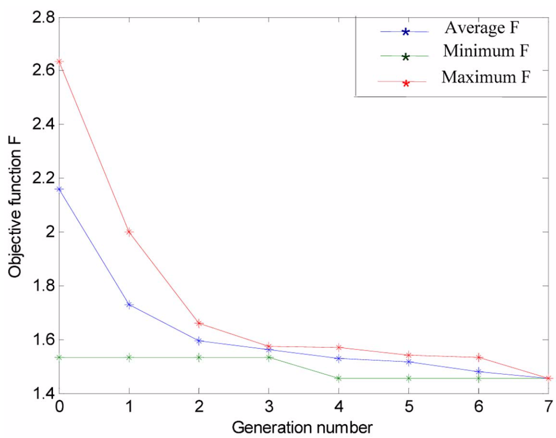

Fig. 10 Variation of the objective function with generation number for example 4

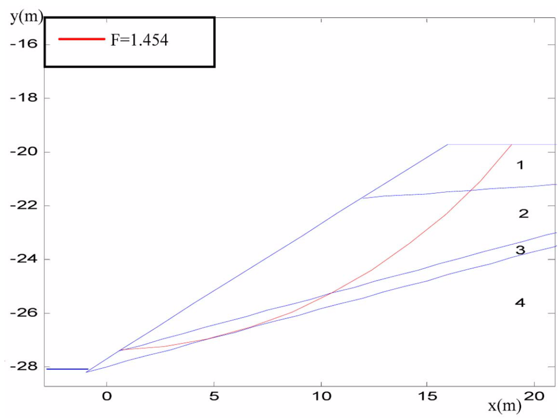

Fig. 11 Critical circular failure surface for example 4

wedge and Morgenstern-Price method's in the literature on account of the effect of GA parameters (size and grid of search space) which are optimised in this study by limited the search space in three dimensional parameters.

5. Conclusions

In this paper, application of genetic algorithm in analyzing soil slopes using Bishop's simplified method was discussed. The results obtained by this approach compared with those available in literature illustrate the efficacy of this inverse method. The optimization of the GA parameters by reduction of search space (reduction of the number of parameters and limitation of search space) minimizes the number of iterations and thus the computing time, so that the factor of safety is improved.

Also a simple circular failure surface is sufficient for a slope in a homogenous soil layer, while for a heterogeneous multi soil layers slope such as in example 4, a simple circular failure surface gives a FOS greater than that in the literature for the non circular failure.

References

Arai, K. and Tagyo, K. (1985), "*Determination of noncircular slip surface giving the minimum factor of safety in slope stability analysis*", *Soil. Found*., **25**(1), 43-51.

Baker, R. (1980), "Determination of the critical slip surface in slope stability computations", *Int. J. Numer. Anal. Meth. Geomech.*, **4**, 333-359.

Bishop, A.W. (1955), "The use of the slip circle in the stability analysis of slopes", *Géotechnique*, **5**(1), 7-17.

Chen, Z.Y. (1992), "Random trials used in determining global minimum factors of safety of slopes", *Can. Geotech. J.*, **29**, 225-233.

Das, S.K. (2005), "Slope stability analysis using genetic algorithm", Research Scholar, Department of Civil Engineering, Indian Institute of Technology Kanpur, India.

Donald, I.B. and Giam, P.S.K. (1989*a*), Improved comprehensive limit equilibrium stability analysis, Department of Civil Engineering Report No. 1/1989, Monash University, Melbourne, Australia.

Fredlund, D.G. and Krahn, J. (1977), "Comparison of slope stability methods of analysis", *Can. Geotech. J.*, **14**(3), 429-439.

Goh, A.T.C. M. (1999), "Genetic algorithm search for critical slip surface in multiple-wedge stability analysis", *Can. Geotech. J.*, **36**, 382-391.

Goldberg, D.E. (1989), *Genetic algorithm in search, optimization, and machine learning*, Addison –Wesley, Massachusetts, USA.

Greco, V.R. (1996), "Efficient Monte Carlo technique for locating critical slip surface", *J. Geotech. Eng.*, ASCE, **122**(7), 517-525.

Hoek, E. (1987), *General two-dimensional slope stability analysis*, In Analytical and computational methods in engineering rock mechanics, Ed. E.T. Brown, Allen & Unwin, London.

Huang, Y.H. (1983), *Stability analysis of slopes*, Van Nostrand Reinhold Company, USA.

Janbu, N. (1973), *Slope stability computations*, In Embankment dam engineering, Ed. R.C. Hirschfield and J. Poulos, John Wiley and Sons, New York.

Levasseur, S. (2007), "Analyse inverse en géotechnique : développement d'une méthode à base d'algorithmes génétiques", Thèse de Doctorat, Université Joseph Fourier - Grenoble I.

McCombie, P. and Wilkinson, P. (2002), "The use of the simple genetic algorithm in finding the critical factor of safety in slope stability analysis", *Comput. Geotech.*, **29**, 699-714.

Morgenstern, N.R. and Price, V.E. (1965), "The analysis of stability of general slip surface", *Géotechnique*,

15(1), 79-93.
Nguyen, V.U. (1985), “Determination of critical slope failure surfaces”, *J. Geotech. Eng.*, ASCE, **111**(2), 238-250.
Sarma, S.K. (1979), “Stability analysis of embankments and slopes”, *J. Geotech. Eng. Div.*, ASCE, **105**(12), 1511-1524.
Siegel, R.A., Kovacs, W.D. and Lovell, C.W. (1981), “Random surface generation in stability analysis”, *J. Geotech. Eng. Div.*, ASCE, **107**(7), 996-1002.
Spencer, E. (1967), “A method of analysis of the stability of embankments assuming parallel interslice forces”, *Géotechnique*, **17**(1), 11-26.
Whitlow, R. (1995), *Basic Soil Mechanics*, 3rd Edition, Longman, Harlow, Essex.
Zolfaghari, A.R., Andrew, C.H and McCombie, P.F. (2005), “Simple genetic algorithm search for critical non circular failure surface in slope stability analysis”, *Comput. Geotech.*, **32**, 139-152.