Structural Engineering and Mechanics, Vol. 41, No. 2 (2012) 183-204
DOI: http://dx.doi.org/10.12989/sem.2012.41.2.183

Effect of modeling assumptions on the seismic behavior of steel buildings with perimeter moment frames

Alfredo Reyes-Salazar*, Manuel Ernesto Soto-López[a], Eden Bojórquez-Mora[b] and Arturo López-Barraza[b]

Facultad de Ingeniería, Universidad Autónoma de Sinaloa, Culiacán, Sinaloa, México

(Received June 25, 2010, Revised May 30, 2011, Accepted December 13, 2011)

Abstract. Several issues regarding the structural idealization of steel buildings with perimeter moment resisting steel frames (MRSFs) and interior gravity frames (GFs) are studied. Results indicate that the contribution of GFs to the lateral structural resistance may be significant. The contribution increases when the stiffness of the connection of the GFs is considered and is larger for inelastic than for elastic behavior. The interstory shears generally increase when the connections stiffness is taken into account. Resultant stresses at some base columns of MRSFs also increase in some cases but to a lesser degree. For columns of the GFs, however, the increment is significant. Results also indicate that modeling the building as planes frames may result in larger interstory shears and displacements and resultant stresses than those obtained from the more realistic 3-D formulation. These differences may be much larger when semi-rigid (SR) connections are considered. The conservativism is more for resultant stresses. The differences observed in the behaviour of each structural representation are mainly due to a) the elements that contribute to strength and stiffness and b) the dynamics characteristics of each structural representation. It is concluded that, if the structural system under consideration is used, the three-dimensional model should be used in seismic analysis, the GFs should be considered as part of the lateral resistance system, and the stiffness of the connections should be included in the design of the GFs. Otherwise, the capacity of gravity frames may be overestimated while that of MRSFs may be underestimated.

Keywords: semi-rigid connections; steel buildings; perimeter moment frames; interior gravity frames; time history analysis; seismic codes

1. Introduction

Seismic provisions are specified in the codes to provide buildings with the ability to support severe ground motions without collapse, but with some structural damage. Different structural configurations, structural systems and materials are used to fulfill this purpose. For the case of steel buildings, among the different structural systems, moment resisting steel frames (MRSFs) have been the most popular because they provide maximum flexibility for space utilization and because of their high ductility capacity.

The seismic behavior of MRSFs has been a research topic of interest to the civil engineering

*Corresponding author, Professor, E-mail: reyes@uas.uasnet.mx
[a]Graduate Student
[b]Professor

profession. Lee and Foutch (2001) studied the seismic behavior of 26 post-Northridge buildings that represent typical steel MRSF buildings, subjected to sets of 20 SAC ground motions representing the 2/50 and 50/50 hazard levels. They concluded that all of the post-Northridge buildings exhibit a high confidence of performing. In another study, Lee and Foutch (2006) studied the seismic behavior of 3-, 9-, and 20-story MRSFs designed for different reductions (*R)* factors. A total of 30 different structural models and 20 ground motions were used. The results showed that the current *R* factors provide conservative designs for low-rise steel buildings but showed a low level of confidence for high buildings. Krishnan *et al.* (2006) determined the damage produced by hypothetical earthquakes on two 18-storey MRSFs, one existing and one improved according to the 1997 Uniform Building Code (UBC), located in southern California, USA. They concluded that severe damage could occur in these buildings. The redesigned building performed significantly better than the existing one, however, the design based on the 1997 UBC was still not adequate to prevent serious damage. Kazantzy *et al.* (2008) proposed a methodology for the probabilistic assessment of low-rise steel buildings and applied it to a welded MRSF, emphasizing the modeling of connections. They found that structures experiencing brittle connections fractures undergo large deformations, resulting in a low reliability in terms of achieving code-related performance parameters. Foutch and Yun (2002) investigated the accuracy of simple nonlinear as well as more detailed modeling methods used in the design of MRSFs. They showed that the model which incorporates clear length dimensions between beams and columns, panel zones and an equivalent gravity bay without composite action from the slab could be a practical model with good accuracy. Liao *et al.* (2007) developed a three-dimensional finite-element model to examine the effects of bi-axial motion and torsion on the nonlinear response of MRSFs. Effects of gravity frames, panel zones, and inelastic column deformation are also considered. Results indicated that torsional effects due to asymmetric member failures are important, that the conventional lumped-plasticity model limits the plasticity of columns and that fracture failures of the pre-Northridge connections have a severe impact on the buildings performance. More recently, Chang *et al.* (2009), by using 6- and -20 level steel office buildings, studied the role of accidental torsion in seismic reliability assessment. They concluded that ignoring the accidental torsion can lead to an unsafe evaluation for the strength of the building fragilities and that, on the other hand, the use of code accidental eccentricity may give conservative estimates.

The basic structural arrangement of MRSFs has significantly changed over the years. One of the most important changes is in the reduction of the number of fully restrained connections (FRCs) used in the buildings. These connections are expensive and their performance regarding the weak-axis bending is questionable. It was the standard practice for many years (FEMA 355C) to frame the beams to the columns by welding the beam flange to a continuity plate which in turn was welded to the web and the flanges of the column. Tests have shown (Rentschler 1980) that this type of weak-axis connection is susceptible to fracture at the weld connecting the beam flange to the continuity plate. In order to avoid this problem the preferred choice for several years has been to eliminate weak-axis moment connections. Because of this change in design practice, most of the steel buildings with MRSFs built in USA during the recent past have FRCs only on two frame lines in each direction, usually at the perimeter, and often the frames with moment-resisting connections do not extend over the full plan of the buildings. Gravity frames (GFs) are used at the interior where the beam-to-column connections are assumed to be perfectly pinned (PP). After the Northridge Earthquake of 1994, FEMA suggested structural arrangements and member sizes of some such model buildings, as will be discussed later with more detail.

In spite of the amount of research developed in the area of the seismic behavior of steel buildings with MRSFs and the important contributions of the earlier-mentioned and other studies, there are several aspects that deserve our attention regarding the idealization of this structural lateral system. The particular case of steel buildings with perimeter MRSFs and interior GFs is specifically addressed in this study. Because the number of FRCs is tremendously reduced in this structural system, its redundancy is significantly reduced too. It is well known that structures with high redundancy perform better under seismic loading than structures with low redundancy. Another issue is related to the seismic design of this structural system; the perimeter MRSFs, modeled as plane frames, are usually designed to resist the total lateral seismic loading, ignoring the contribution of the GFs. Due to the action of rigid floor diaphragms the columns of these GFs, however, will bent undergoing a similar lateral deformation than the MRSFs. Consequently, the contribution of these columns to the lateral resistance could be significant, particularly for those building with relatively few FRCs. Moreover, modeling the buildings as plane frames may not represent the actual behavior of the structure since the participation of some elements are not considered and the contribution of some vibration modes are ignored. The dynamic properties in terms of stiffness, mass distribution, natural frequencies and energy dissipation characteristics for two-dimensional (2-D) and three-dimensional (3-D) modeling of such structures are expected to be different. The corresponding structural responses are also expected to be different. However, these differences are unknown and need to be quantified.

Another simplification made in the design of steel buildings with perimeter MRSFs and interior GFs is related to the stiffness of the beam-to-column connection. Conventional analysis and design of steel frames is based on the assumption that beam-to-column connections are either fully restrained (FR) or perfectly pinned (PP). In the analysis and design of the structural system under consideration, the beam-to-column connections of the GFs are assumed to be PP, although connections type shear are used in practical design. Despite these classifications, almost all steel connections used in real buildings are essentially semi-rigid (SR) with different rigidities. It has been established in the profession, both theoretically and experimentally that these connection exhibit semi-rigid nonlinear response even if the applied loads are very small (Reyes-Salazar and Haldar 2000). The FR and PP connection consideration is nothing but an assumption made to simplify calculations and is a major weakness in current analytical procedures. These simplifications may result in erroneous values of the resultant stresses because in reality FR connections possesses some flexibility and PP connections possesses some rigidity. There is some evidence that shear connections can transmit up to 30% of the plastic moment capacity (FEMA 2000) of the beams they are connecting to. The seismic nonlinear response of steel buildings with MRSFs explicitly considering the stiffness and the dissipation of energy at the connections has been also studied (Reyes-Salazar and Haldar 2000, Reyes-Salazar *et al.* 2001, Reyes-Salazar and Haldar 2001a, b, Merhabian *et al.* 2005). These studies showed that the flexibility of the connections has an important effect on the structural response and that the dissipation of energy on the connections is comparable and even larger than those dissipated by viscous damping and hysteretic behavior at plastic hinges. The main limitation of this study is that only plane frame models were considered.

In many cases, the concrete slab rest on steel beams. When the presence of steel of the concrete slab is considered in the design of a connection, it is called a composite beam-to-column connection. It has been shown that only a nominal amount of the slab steel crossing the column lines is necessary to turn the non-composite beam-to-column connection into a rather stiffer SR connection. Thus, the contribution of the connections to the structural response is expected to be much important if the composite action of the slab is considered (Reyes-Salazar 1997, Reyes-Salazar and Haldar 1999, Liu and Astaneh-Asl 2000).

2. Objectives

Some of the issues discussed earlier are addressed in this paper. To meet the objectives of the study, the response behavior of steel buildings with perimeter MRSFs and interior GFs, need to be represented as realistically as possible, preferably in 3-D and then estimating responses by exciting them with measured seismic time histories. The implications of some of the modeling assumptions discussed earlier can then be established by comparing the results with the results obtained from simplified structural representations. The study will also provide some design guidelines to be considered by the profession. The issues specifically addressed are:

1. The level of contribution of the GFs to the lateral resistance of the overall building. The contribution is estimated for both; PP connections and SR connections.
2. The effect of the stiffness of the connections of the GFs on the structural responses of the 3-D models. The structural responses in terms of local and global response parameters are estimated for the 3-D structures with PP connections and compared to those of the structures with SR connections.
3. The accuracy of modeling the 3-D buildings with PP connections as two-dimensional frames for seismic design.
4. The accuracy of modeling the 3-D buildings with SR connections as two-dimensional frames.

To document the results numerically, the global responses in terms of base shear and interstory displacements and local responses in terms of resultant stresses at individual members are presented here. Some steel models proposed in the SAC project (FEMA, 355C) are used for this purpose. The models are analyzed in the time domain under ground motions from 20 recorded earthquakes. They are obtained from the Data Sets of the National Strong Motion Program (NSMP) of the United States Geological Surveys (USGS) and were selected to represent the characteristics of strong motion earthquakes.

3. Methods and mathematical models

An assumed stress-based finite element algorithm, developed by the authors and their associates (Gao and Haldar 1995, Reyes-Salazar 1997), is used to estimate the nonlinear seismic responses of several steel building models. The procedure estimates the responses in time domain, as accurately as possible by considering material and geometry nonlinearities and the nonlinearity introduced by SR connections (Richard 1993). In this approach, an explicit form of the tangent stiffness matrix is derived without any numerical integration. Fewer elements can be used in describing a large deformation configuration without sacrificing any accuracy, and the material nonlinearity can be incorporated without losing its basic simplicity. It gives very accurate results and is very efficient compared to the commonly used displacement-based approaches. The procedure and the algorithm have been extensively verified using available theoretical and experimental results (Reyes-Salazar and Haldar 2001a, b).

4. Structural models

Several steel model buildings were designed, as part of the SAC steel project, by three consulting firms. They considered 3-, 10- and 22- level buildings. The 10-level building has a single-level basement and the 20-level building has a 2-level basement. These buildings are supposed to satisfy all code requirements existed at the time of evaluation for the following three cities: Los Angeles (Uniform Building (UBC 1994)), Seattle (UBC 1994) and Boston (Building Officials & Code Administration (BOCA 1993)). The 3- and 10- level buildings located in the Los Angeles area are considered in this study for numerical evaluations to address the issues discussed earlier. They will be denoted hereafter as Models 1 and 2, respectively. They are considered to be bench mark models to be used by other researchers. They provide a unique opportunity to study the behavior of steel buildings with perimeter MRSFs and interior GFs.

The first four periods of Model 1 are estimated to be 1.02, 0.98, 0.30 and 0.26 sec. The corresponding periods for Model 2 are 2.34, 2.28, 0.86 and 0.80 sec. These periods, for both models, are associated to lateral vibration. The elevations of the models are given in Figs. 1(a) and 1(d) and their plans are given in Figs. 1(b) and 1(e), respectively. In these figures, the perimeter MRSFs are represented by continuous lines and the interior GFs are represented by dashed lines.

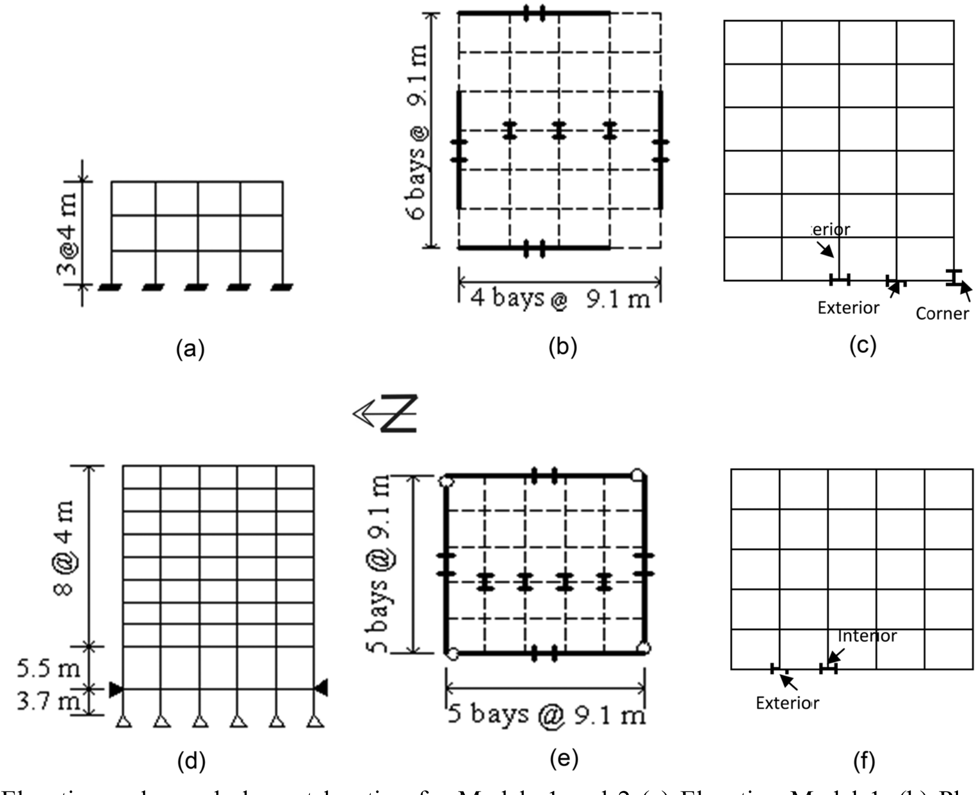

Fig. 1 Elevations, plan and element location for Models 1 and 2 (a) Elevation Model 1, (b) Plan Model 1, (c) Studied elements for Model 1, (d) Elevation Model 2, (e) Plan Model 2 and (f) Studied elements for Model 2

Table 1 Beam and columns sections for Models 1 and 2

Model	Story	Moment resisting frames			Gravity frames		
		Columns		Girders	Columns		Girders
		Exterior	Interior	Below	Penthouse	Others	
1	1\2	W14×257	W14×311	W33×118	W14×82	W14×68	W18×35
	2\3	W14×257	W14×312	W30×116	W14×82	W14×68	W18×35
	3\Roof	W14×257	W14×313	W24×68	W14×82	W14×68	W16×26
2	-1/1	W14×370	W14×500	W36×160	W14×211	W14×193	W18×44
	1/2	W14×370	W14×500	W36×160	W14×211	W14×193	W18×35
	2/3	W14×370	W14×500, W14×455	W36×160	W14×211,W14×159	W14×193,W14×145	W18×35
	3/4	W14×370	W14×455	W36×135	W14×159	W14×145	W18×35
	4/5	W14×370,W14×283	W14×455,W14×370	W36×135	W14×159,W14×120	W14×145,W14×109	W18×35
	5/6	W14×283	W14×370	W36×135	W14×120	W14×109	W18×35
	6/7	W14×283,W14×257	W14×370,W14×283	W36×135	W14×120,W14×90	W14×109,W14×82	W18×35
	7/8	W14×257	W14×283	W30×99	W14×90	W14×82	W18×35
	8/9	W14×257,W14×233	W14×283,W14×257	W27×84	W14×90,W14×61	W14×82,W14×48	W18×35
	9/Roof	W14×233	W14×257	W24×68	W14x61	W14×48	W16×26

For Model 2, the perimeter frames meet at a corner. In this case, the beam-to-column connections are considered to be pinned to eliminate weak axis bending (Fig. 1(e)). As can be seen, the buildings are essentially symmetrical in plan thus no significant torsional moments are expected to occur. Resultant stresses are estimated for some particular columns which are located at the ground floor level and are shown in Figs. 1(c) and 1(f) for Models 1 and 2, respectively. Sizes of beams and columns, as reported by FEMA (FEMA 355C), are given in Table 1 for the two models. The columns of the MRSFs of Model 1 are considered to be fixed at the base and pinned for Model 2, as considered in the FEMA report. In all these frames, the columns are made of Grade-50 steel and the girders are of A36 steel. For both models, the columns in the GFs are considered to be pinned at the base. All the columns in the perimeter MRSFs bend about the strong axis and the strong axes of the gravity columns are oriented in the N-S direction, as indicated in Figs. 1(b) and 1(e). The designs of the MRSFs in the two orthogonal directions were practically the same. The damping is considered to be 5% of the critical damping; the value used in developing the code provisions in the U.S. Additional information for the models can be obtained from the FEMA report.

The buildings are modeled as multi degree of freedom systems (MDOFs). Each column is represented by one element and each girder of the perimeter MRSFs is represented by two elements, having a node at the mid-span. The slab is modeled by near-rigid struts, as considered in the FEMA study (FEMA 355C). Each node is considered to have six degrees of freedom when the buildings are modeled in three dimensions. The GFs are assumed to have, first PP and then SR connections. An additional element is needed to represent each SR connection. These connections are considered only for bending with respect to the strong axis of the gravity columns. Therefore, they are oriented in the E-W direction.

Table 2 Earthquake models

No	Place	Year	Station	T (seg.)	ED (km)	M	PGA (mm/seg^2)
1	1317 Mich. México	1985	Paraíso	0.11	300	8.1	800
2	1634 Mammoth Lakes. USA	1980	Mammoth H. S. Gym	0.12	19	6.5	2000
3	1634 Mammoth Lakes USA	1980	Convict Creek	0.19	18	6.5	3000
4	1317 Mich. México	1985	Infiernillo N-120	0.21	67	8.1	3000
5	1317 Mich. México	1985	La Unión	0.32	121	8.1	1656
6	1733 El Salvador	2001	Relaciones Ext.	0.34	96	7.8	2500
7	1733 El Salvador	2001	Relaciones Ext.	0.41	95	7.8	1500
8	1634 Mammoth Lakes.	1980	Long Valley Dam	0.42	13	6.5	2000
9	2212 Delani Fault, AK	2000	K2-02	0.45	281	7.9	115
10	0836 Yountville CA	2000	Redwood City	0.46	95	5.2	90
11	0408 Dillon MT	2005	MT:Kalispell	0.51	338	5.6	51
12	1317 Mich. Mexico	1985	Villita	0.53	80	8.1	1225
13	1232 Northrige	1994	Hall Valley	0.54	25	6.4	2500
14	2115 Morgan Hill	1984	Hall Valley	0.61	14	6.2	2000
15	2212 Delani Fault AK	2002	K2-04	0.62	290	7.9	133
16	0836 Yountville CA	2000	Dauville F.S. Ca	0.63	73	5.2	144
17	0836 Yountville CA	2000	Pleasan Hill F.S. 1	0.71	92	5.2	74
18	0836 Yountville CA	2000	Pleasan Hill F.S. 2	0.75	58	5.2	201
19	2212 Delani Fault, AK	2002	Valdez City Hall	0.85	272	7.9	260
20	1715 Park Fiel	2004	CA: Hollister City Hall	1.01	147	6	145

5. Earthquake loading

To study the responses of the two models comprehensively and to make meaningful conclusions, they are excited by twenty recorded earthquake motions in time domain with different frequency contents, recorded at different locations. First the earthquakes are scaled to the same PGA and then scaled in such a way that the models develop a significant level of plastic deformation. The characteristics of these earthquake time histories are given in Table 2. As shown in the table, the predominant periods of the earthquakes vary from 0.11 to 1.0 sec. The predominant period for each earthquake is defined as the period where the largest peak in the elastic response spectrum occurs. The earthquake time histories were obtained from the Data Sets of the National Strong Motion Program (NSMP) of the United States Geological Surveys (USGS). Additional information on these earthquakes can be obtained from them.

6. The Richard model

Connections are structural elements that transmit resultant stresses between beams and columns. For the case of partially restrained (SR) connections, their rigidity is generally represented by the bending moment acting on them and the corresponding relative rotation. Many mathematical forms to define the bending moment-relative rotation relationship (referred as M-θ curve) for SR connections are available in the literature. They include the piecewise linear, the polynomial, the exponential, the B-spline, and the Richard model (Richard 1993, Reyes-Salazar 1997). The Richard model is a four-parameter model which was developed using actual worldwide test data. When a connection is defined in terms of member sizes, bolts and/or welds, a commercially available computer program, known as PRCONN, is available to generate the appropriate M-θ curve using the Richard model (Richard 1993). This program is used in this study to develop the required M-θ curve. According to the Richard model, the M-θ curve is given by

$$M = \frac{(k-k_p)\theta}{\left(1+\left|\frac{(k-k_p)\theta}{M_o}\right|^N\right)^{\frac{1}{N}}} + k_p\theta \tag{1}$$

where k is the initial or elastic stiffness, k_p is the plastic stiffness, M_0 is the reference moment, and N is the curve shape parameter. The loading process and the physical definition of these parameters are shown in Fig. 2(a).

Eq. (1) represents the M-θ curve when the load is increasing monotonically. When a structure is excited by dynamic or seismic loading, some of the connections are expected to be loading and others are expected to be unloading and reloading. Experimental and theoretical studies related to the unloading and reloading behavior of the M-θ curve are rare. This subject has been addressed in the literature (Colson 1991, El-Salti 1992, FEMA 355C, Chen *et al*. 1996). For the present study, the unloading and reloading behavior of the M-θ curves is essential. As in the past (Reyes-Salazar 1997, Reyes-Salazar and Haldar 1999, 2000, 2001a, b), in the present study, the monotonic loading behavior is represented by the Richard curve and the Masing rule is used to theoretically develop the unloading and reloading sections of the M-θ curves. Using the Masing rule and the Richard Model represented by Eq. (1), the mathematical representation for the unloading and reloading behavior of a connection can be expressed as

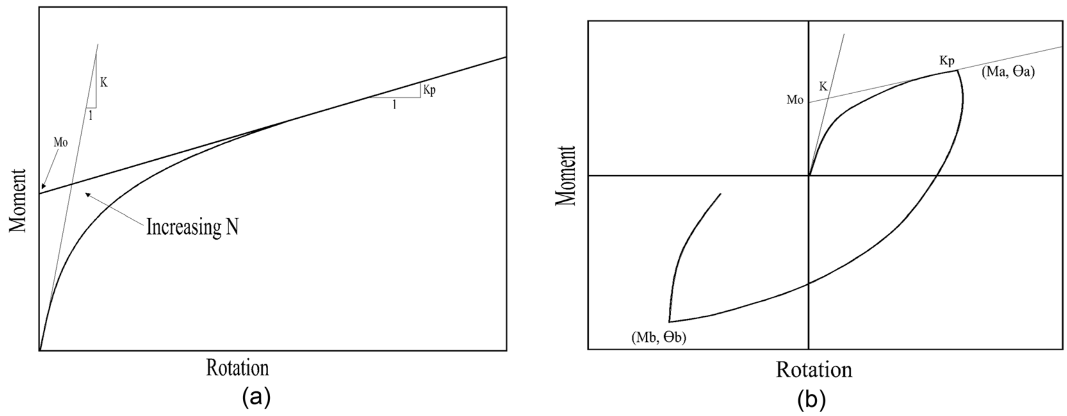

Fig. 2 The Richard's model (a) parameters of the Richard's model and (b) Loading, unloading and reloading

$$M = M_a - \frac{(K-K_p)(\theta_a-\theta)}{\left(1+\left|\frac{(K-K_p)(\theta_a-\theta)}{2M_0}\right|^N\right)^{\frac{1}{N}}} - K_p(\theta_a-\theta) \quad (2)$$

The loading, unloading and the reloading at SR connections are illustrated in Fig. 2(b). If (M_b, θ_b) is the next reversal point, as shown in the figure, the reloading relation between M and θ can be obtained by simply replacing (M_a, θ_a) with (M_b, θ_b) in Eq. (2). Thus, Eq. (1) is used if the connection is loading; if it is unloading or reloading, Eq. (2) should be used instead.

7. Contribution of GFs to the lateral resistance

7.1 Models with PP connections

7.1.1 Elastic behavior

The contribution of GFs to the lateral resistance is studied in this section of the paper. It is estimated in terms of story shears. The shear ratio V_1, defined as V_I/V_E, is introduced to represent the contribution. For a given direction and story, V_I will represent the shear resisted by all the GFs in that story and V_E will represent the shear resisted by the perimeter frames. This ratio is studied

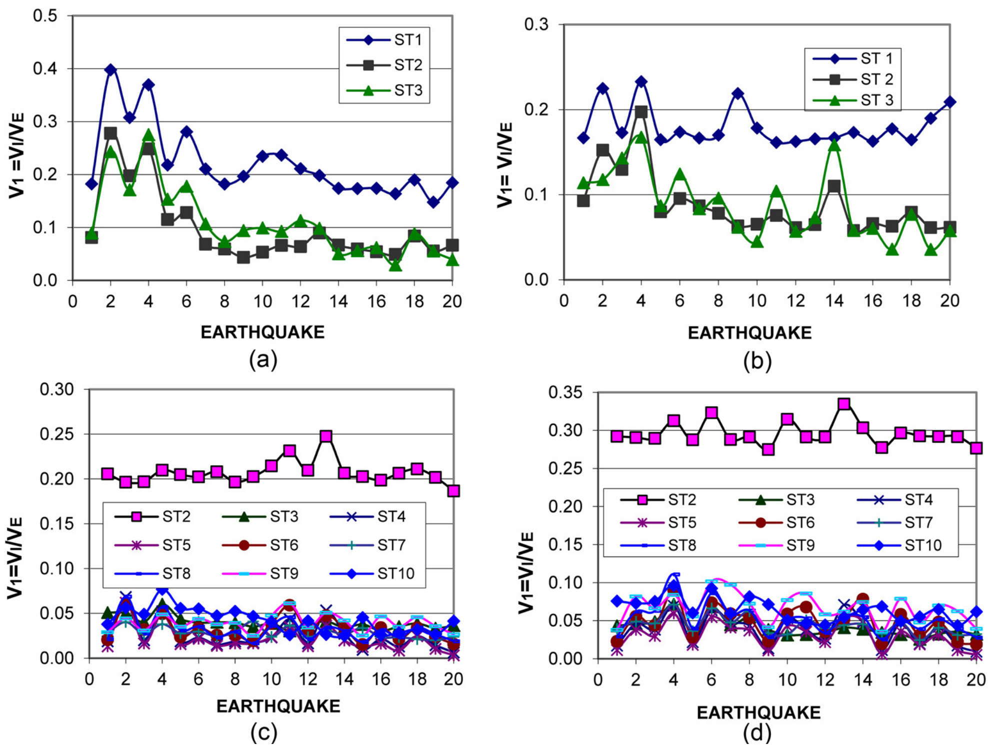

Fig. 3 Values of the V_1 parameter, elastic behavior (a) Model 1 *N-S* direction, (b) Model 1 *E-W* direction, (c) Model 2 *N-S* direction and (d) Model 2 *E-W* direction

for both horizontal directions and both of the models. All the earthquake time histories are normalized with respect to their maximum peak ground acceleration. The buildings remain essentially elastic when subjected to any of the earthquakes. The recorded seismic components are applied along the principal structural axes; the horizontal component with the major peak acceleration is applied in the *N-S* direction and the other is applied in the *W-E* direction.

Typical results of the V_1 parameter are shown in Figs. 3(a) through 3(d) for Models 1 and 2 and the *N-S* and *E-W* directions. The symbol *ST* is used in the figures to represent the word "story". It is observed that the V_1 values significantly vary from one model to another and from one story to another without showing any trend. The values are observed to be larger for stories at ground level (ST1 for Model 1 and ST2 for Model 2) than for the other stories. The most important observation that can be made is that the values of V_1 are not negligible in many cases. Values of up to 40% are obtained for Story 1 of Model 1, for the *N-S* component.

7.1.2 Inelastic behavior

The frames did not develop any plastic hinge when excited by any of the 20 recorded earthquakes. To study the effect of inelastic behavior in the V_1 parameter, the actual time histories were scaled up so that yielding was produced in all the models. Based on the past experience and for the uniformity of comparison, all the actual time histories were scaled up to develop a maximum average interstory drift of about 1.5% by the trial and error procedure, instead of tracking the total number of plastic hinges developed. It was observed that about eight to twenty four plastic hinges were formed in the

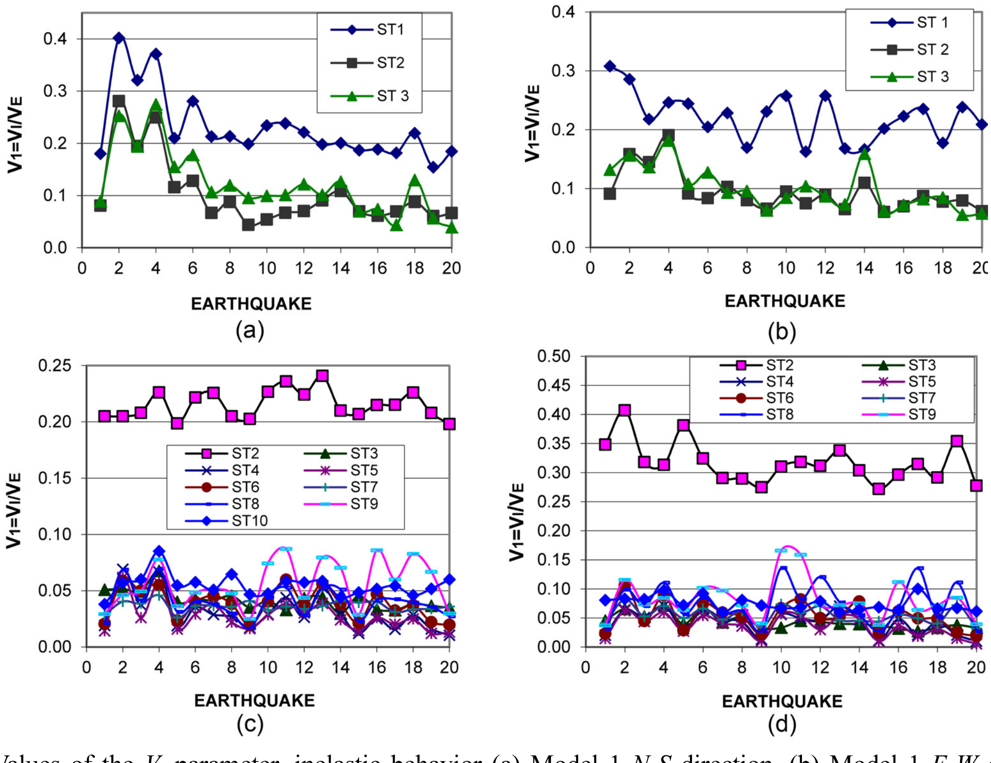

Fig. 4 Values of the V_1 parameter, inelastic behavior (a) Model 1 *N-S* direction, (b) Model 1 *E-W* direction, (c) Model 2 *N-S* direction and (d) Model 2 *E-W* direction

Table 3 Statistics of the V_1 parameter

Model		Story	*N-S* direction			*E-W* direction		
			μ	σ	δ	μ	σ	δ
1	Elastic	1	0.22	0.07	0.31	0.18	0.02	0.13
		2	0.10	0.07	0.70	0.09	0.04	0.41
		3	0.11	0.07	0.60	0.09	0.04	0.45
	Inelastic	1	0.23	0.07	0.28	0.22	0.04	0.18
		2	0.10	0.07	0.63	0.09	0.03	0.36
		3	0.12	0.06	0.52	0.10	0.04	0.36
2	Elastic	2	0.21	0.01	0.06	0.30	0.02	0.05
		3	0.04	0.01	0.18	0.04	0.01	0.30
		4	0.03	0.02	0.65	0.04	0.02	0.52
		5	0.02	0.01	0.67	0.03	0.02	0.55
		6	0.03	0.01	0.40	0.05	0.02	0.45
		7	0.03	0.01	0.22	0.04	0.01	0.28
		8	0.03	0.01	0.41	0.05	0.02	0.41
		9	0.04	0.01	0.24	0.07	0.02	0.29
		10	0.04	0.01	0.33	0.06	0.02	0.23
	Inelastic	2	0.22	0.01	0.06	0.32	0.03	0.10
		3	0.04	0.01	0.21	0.04	0.01	0.28
		4	0.03	0.02	0.54	0.04	0.02	0.48
		5	0.03	0.01	0.50	0.04	0.02	0.50
		6	0.04	0.01	0.36	0.06	0.03	0.46
		7	0.04	0.01	0.13	0.06	0.01	0.24
		8	0.04	0.01	0.36	0.08	0.03	0.45
		9	0.06	0.02	0.37	0.08	0.03	0.40
		10	0.06	0.01	0.18	0.08	0.01	0.16

models when they developed the desired drift. Plots similar to those previously discussed are then developed for both models and both component. They are shown in Figs. 4(a) through 4(d). As for the elastic behavior case, it is observed that, the V_1 values are not negligible in many cases. It is observed that for the *N-S* direction the results are similar for both levels of deformation (elastic and inelastic behavior). For the *E-W* direction, however, the V_1 values are larger for inelastic behavior. The reason for this is that more plastic hinges were formed in the perimeter MRSFs oriented in this direction in such a way that the redistribution of shear toward the GFs was more significant.

The statistics of V_1 are summarized in Table 3. As observed from individual values of V_1, the statistics also indicate that the GFs significantly contribute to the lateral resistance. It is also observed from the table that the uncertainty associated with the estimation, in terms of coefficient of variation (δ), is relatively high. Based on the above results, it is concluded that the contribution of the GFs to the lateral resistance could be significant and consequently should not be overlooked in the design of the structural systems under consideration.

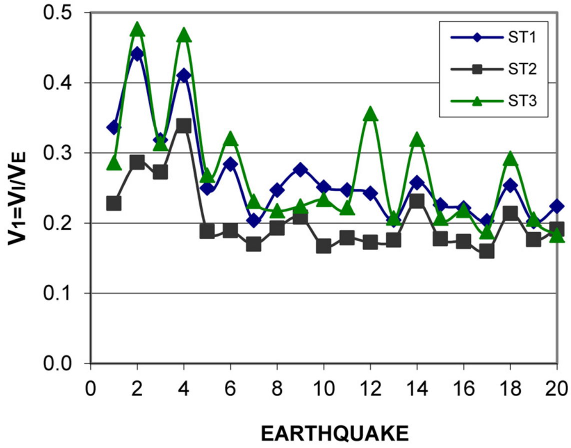

Fig. 5 Values of V_1, Model 1, SR connections

7.2 Models with SR connections

The magnitude of the V_1 parameter is now estimated considering the stiffness of the beam-to-column connection of the GFs. Only Model 1, the *E-W* direction and inelastic behavior are considered. Since no information is given in the FEMA report (355C) regarding the properties of the connections, a typical "double web angle connections" was assumed. Then, by using the PRCONN computer program (Richard 1993), the Richard parameters were generated. When the beam size is W16 × 26 these parameters are k = 18,084 kN-m, k_p = 220 kN-m, Mo = 78 kN-m and N = 1.4. The corresponding values of these parameters for the other beam section (W18 × 35) are k = 30510 kN-m, k_p = 65 kN-m, Mo = 95 kN-m and N = 1.2. The results of V_1 are shown in Fig. 5. It is observed that the contribution of the GFs to the lateral resistance significantly increases when the stiffness of the connections is considered. The increment is particularly important for the upper stories. For example, for Story 3, V_1 was smaller than 0.12 in most of the cases for the frames with PP connections. For SR connections however, this parameter takes values larger than 0.20 in most of the cases. Values close to 0.50 are observed for Story 3 for two cases.

The earlier results indicate that the contribution of the GFs to the lateral resistance is not negligible and cannot be ignored in the analysis and design of members, particularly the columns, of the GFs. The major implication is that the members in the GFs may not be able to carry this unexpected load effects due to non-negligible lateral load.

8. Seismic responses of 3-D models with PP and SR connections

To study the effect of the stiffness of the beam-to-column connections of the GFs on the overall structural response, the ratio of the responses of the buildings with PP connections to that of the building with SR connections is discussed in this part of the paper. The responses are estimated in terms of average interstory shears and displacements, individual interstory shears and displacements, and resultant stresses on individual members. Only Model 1, inelastic behavior and *E-W* direction are considered.

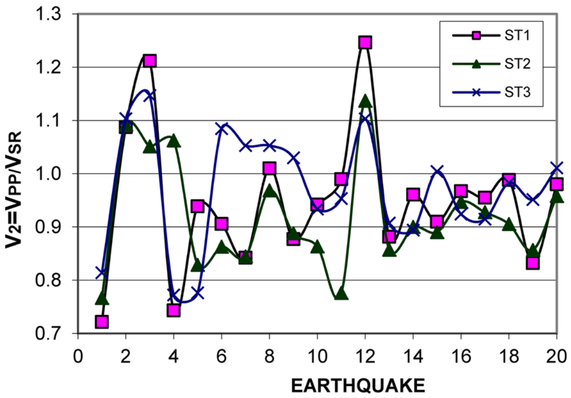

Fig. 6 Values of V_2 for average interstory shears

8.1 Average interstory shears

Results for interstory shears, averaged over all the plane frames, are first discussed. The V_2 parameter, defined as V_{PP}/V_{SR} is used for this purpose. For a given story, V_{PP} will represent the average shear on that story when PP connections are considered in the GFs. V_{SR} will represent the same, except that SR connections are used instead. Results are shown in Fig. 6. It is observed that the values of V_2 are smaller than unity in most of the cases, indicating that, in general, the interstory shears increase when the stiffness of the connection is considered. For some other cases, however, V_2 is larger than unity. Unlike the case of lateral static load application, where the interstory shears are expected to always increase with the stiffness of the connection, the response due to dynamic loading depends on several parameters which are not significant for static analysis. The effect of higher mode effects, energy dissipation and frequency content of ground motions is clearly illustrated in Fig. 6; the values of V_2 may significantly vary from one earthquake to another even though the maximum deformation is approximately the same for all the earthquakes (1.5%). The implication of this is that the seismic behavior of a steel building modeled with PP connections can be quite different from that of the more realistic representation obtained when the stiffness of the connections is considered.

8.2 Interstory shears for individual frames

The values of the V_2 parameter are next discussed for individual frames. The results are presented in Fig. 7(a) for an exterior (MRSF) and in Fig. 7(b) for an interior (GF) frame. As stated earlier, both frames are located in the *E-W* direction. The results of Fig. 7(a) show that the V_2 values are, in general, larger than unity indicating that the interstory shears of the exterior frame are larger for the model with PP connections in comparison with those of the model with SR connections. This is expected; the contribution to the lateral resistance of the MGs of the model with PP connections is relatively small since it is provided only by the exterior columns (V_{PP}) which are part of the transversal MRSFs located in the *N-S* direction. Therefore, the lateral overall resistance is mostly provided by the exterior frames. On the other hand, the contribution of the MGs to the lateral resistance is significantly increased when the SR connections are considered, therefore the

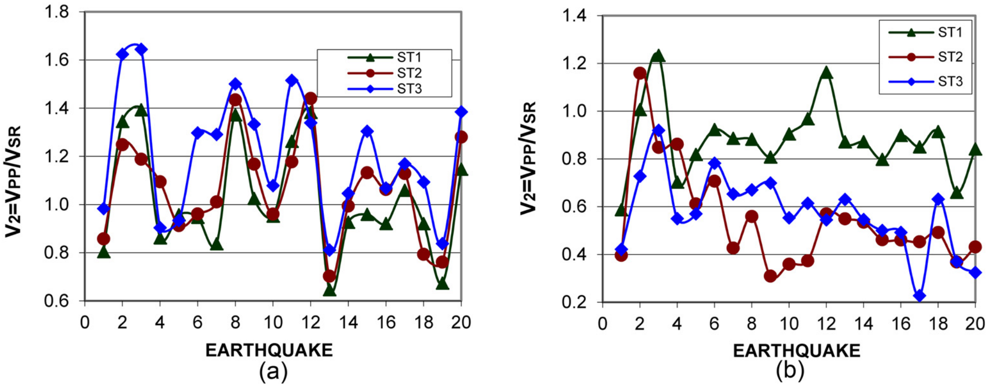

Fig. 7 Values of V_2 for individual frames (a) Interior frame and (b) Exterior frame

contribution of the exterior frames (V_{SR}) decreases. Results of Fig. 7b indicate that the V_2 values for the interior frames are smaller than unity practically in all the cases. As discussed earlier, the contribution to the lateral resistance (V_{PP}) of the interior frames of the model with PP connections is smaller than that (V_{SR}) of the model with SR connections.

8.3 Resultant stresses

Similar ratios to those of shears are also estimated for axial loads and moments on some columns of the base of the MRSFs (see Fig. 1(c)). The A_1 and M_1 parameters defined as A_{PP}/A_{SR} and M_{PP}/M_{SR}, respectively, are used for this purpose. The A_1 values for an exterior and an interior column of MRSFs as well as for a corner column of GFs are shown in Fig. 8. It is observed that, for the columns of MRSFs, the values of this parameter can be larger or smaller than unity. For the corner column, however, the A_1 parameter is smaller than unity in all the cases, indicating that, as expected, axial loads on gravity columns increase when the stiffness of the connection is considered. Similar plots to those of axial loads are also estimated for bending moments but are not presented. The major observations made for axial loads apply to the case of moments.

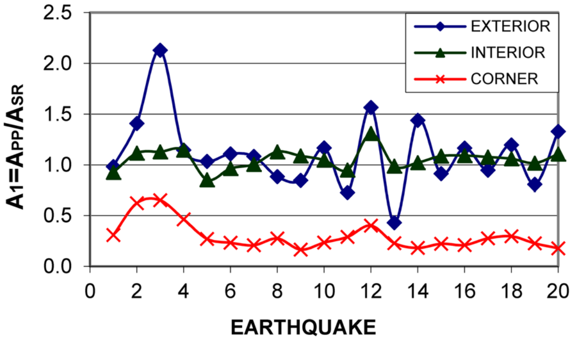

Fig. 8 Values of the A_1 parameter

8.4 Displacements

A similar ratio to that of interstory shears is also estimated for both; average interstory displacements and interstory displacements for individual frames, but the results are not shown. The results indicate, however, that, the ratios are quite similar for average and individual displacements. As for the case of shears, these ratio values vary from one earthquake to another and from one story to another without showing any trend. Values smaller or larger than unity are observed. The mean values for Interstories 1, 2 and 3 are 1.02, 1.01 and 0.95, respectively. It indicates that, on an average basis, the displacements of the frames with PP connections are similar to those of the frame with SR connections.

9. Seismic response of 3-D with PP connections vs 2-D models

The seismic responses of the buildings modeled as 2-D frames are compared to those of the buildings modeled as 3-D structures with PP connections. The responses are expressed in terms of global and local response parameters. Both of the models are considered.

9.1 Elastic behavior

Results for interstory shears are studied first. The shear ratio V_3, defined as V_{2D}/V_{3D}, is introduced for this purpose. For a given direction and interstory, V_{2D} will represent the maximum shear resisted by all the columns of the interstory when the building is modeled as a plane frame and V_{3D} will represent the same but the shear is now estimated for the corresponding columns of the building modeled as three-dimensional structure. The V_3 ratio is estimated for both horizontal directions.

Typical results of the V_3 parameter are shown in Figs. 9(a) and 9(b) for Models 1 and 2, respectively, for the *N-S* direction. It is observed that the V_3 values significantly vary from one model to another and from one story to another without showing any trend. In most of the cases the values of V_3 are larger than unity indicating that the interstory shears are larger for the 2-D than for the 3-D model. Values larger than 1.8 are observed in some cases. The V_3 values are larger, in general, for the story at ground level. The differences between the responses of the 2-D and 3-D

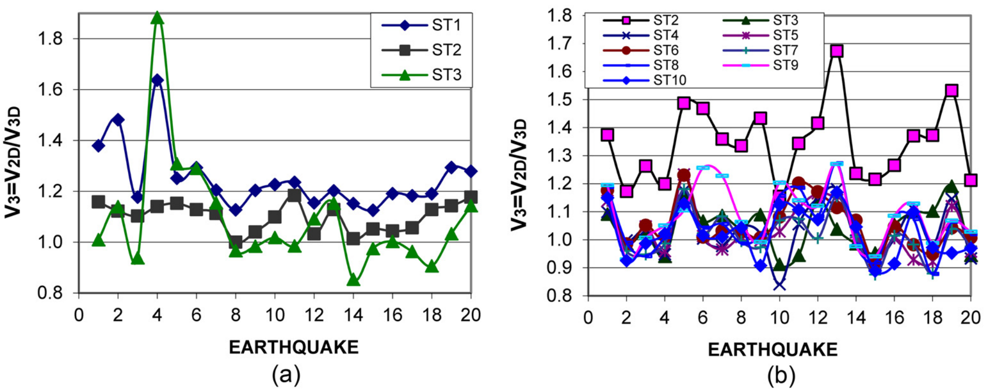

Fig. 9 Values of the V_3 parameter, elastic behavior, *N-S* direction (a) Model 1 (b) Model 2

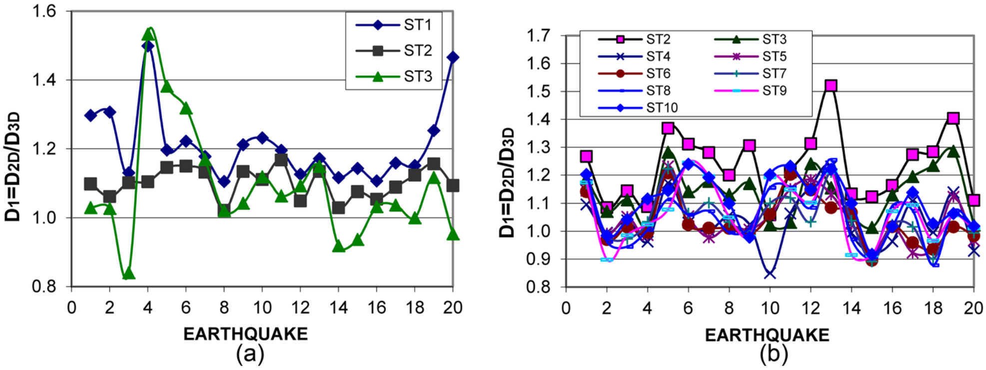

Fig. 10 Values of the D_1 parameter, elastic behavior (a) Model 1 and (b) Model 2

models are due to the strength and stiffness contribution of some elements that are considered in the 3-D model but not in the 2-D model. This also points out that the dynamic characteristics of 2-D and 3-D representations are different and they just cannot be overlooked. It is well known that the response of three-dimensional buildings when subjected to strong motions depends on many factors, specifically on the spatial distribution of strength, stiffness and mass, the frequency content of the excitation, and the energy dissipation characteristics (damping) in the linear and nonlinear responses. It is important to emphasize that a building modeled a 3-D frame is expected to have different natural frequencies than that modeled as a 2-D frame and will respond differently when subjected to the same excitation.

A Similar ratio (D_1) to that of interstory shears (V_3) is also estimated for interstory displacements. The results are presented in Figs. 10(a) and 10(b) for Models 1 and 2 when excited by the *N-S* excitation. As for the case of the V_3 parameter, the values of D_1 are, in general, larger than unity in most of the cases varying from one model and one story to another without showing any trend. A high correlation is observed between V_3 and D_1. The magnitude and range of variation of these two parameters is quite similar.

To study the 2-D and 3-D modeling effect at the local element level, similar ratios to that of shear and displacements are developed for axial loads (A_2) and moments (M_2) at some columns at the

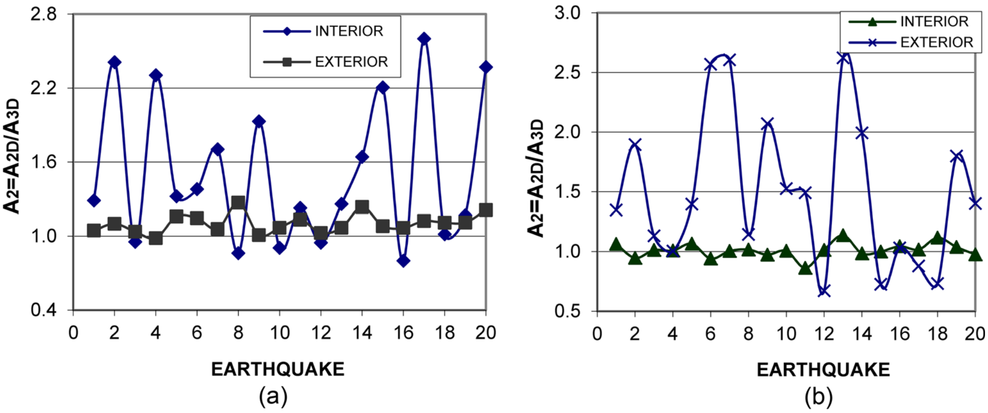

Fig. 11 Values of the A_2 parameter, elastic behavior (a) Model 1 and (b) Model 2

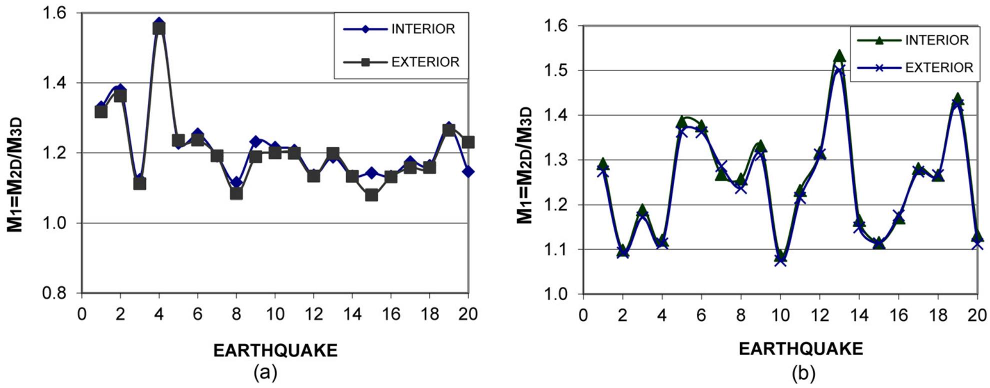

Fig. 12 Values of the M_1 parameter, elastic behavior (a) Model 1 and (b) Model 2

base. As for the case of interstory shears and displacements, plots are developed for the two models, the two directions and the twenty earthquake excitations. The results for A_2 are presented in Figs. 11(a) and 11(b) for Models 1 and 2 respectively, for columns located in planes oriented in the N-S direction (Figs. 1(c) and 1(f)). As for the case of global response parameters, it is shown that the axial loads are in general larger for the 2-D model. The values of A_2 are much larger for interior than for exterior columns. The values of the M_2 parameter are shown in Fig. 12. It is observed that the values of this parameter are also larger than unity in most of the cases. However, for the case of M_2, the values for interior and exterior columns are highly correlated because of the lateral displacements of the two columns are quite similar. Based on these results it is concluded that the 2-D modeling will introduce significant more stresses in the members than the 3-D modeling.

9.2 Inelastic behavior

Plots similar to those of elastic analysis (Figs. 9 through 12) for the V_3, D_1, A_2 and M_2 parameters are also developed for inelastic behavior, but are not presented because of lack of space, only their statistics are given (Tables 4 and 5). It is observed, however, that for global response parameters (V_3 and D_1) the results are slightly larger for inelastic behavior for some particular earthquakes, but on an average basis their values are quite similar. For local parameters (A_2 and M_2) the elastic and inelastic responses can be quite different.

From a comparison of Tables 4 and 5, it is observed that the mean values of A_2 and M_2 are, in general, larger than those of V_1 and D_1. Moreover, in general, the record-to-record variability in A_2 and M_2 is much larger than that of V_1 and D_1. Large values of coefficients of variation, as high as 0.51, can be observed in some members at the local level. It is expected; in the estimation of the interstory shears and displacements there is an averaging effect which is not present in the estimation of resultant stresses. In other words, global responses have an averaging effect since they depend on contributions of many members reducing the overall level of uncertainty; however, local responses are estimated at a point in a member producing a higher level of uncertainty. Based on these results, it is concluded that, modeling the structural systems under consideration as a plane frames may significantly overestimate the seismic response, in other words, it will produce more conservative design.

Table 4 Statistics for the V_3 and D_1 parameters

Model		Story	Statistics of V_3						Statistics of D_1					
			N-S direction			*E-W* direction			*N-S* direction			*E-W* direction		
			μ	σ	δ	μ	σ	δ	μ	σ	δ	μ	σ	δ
1	Elastic	1	1.25	0.13	0.10	1.06	0.04	0.04	1.21	0.11	0.09	1.08	0.06	0.05
		2	1.10	0.06	0.05	1.05	0.04	0.04	1.10	0.04	0.04	1.02	0.05	0.05
		3	1.09	0.22	0.20	1.02	0.06	0.06	1.09	0.16	0.15	0.98	0.09	0.09
	Inelastic	1	1.26	0.13	0.10	1.17	0.08	0.07	1.26	0.13	0.10	1.03	0.08	0.08
		2	1.12	0.05	0.04	1.13	0.06	0.05	1.14	0.06	0.05	1.06	0.07	0.07
		3	1.12	0.19	0.17	1.06	0.08	0.07	1.10	0.18	0.16	1.03	0.11	0.11
2	Elastic	2	1.19	0.07	0.06	1.34	0.13	0.10	1.12	0.07	0.07	1.23	0.12	0.10
		3	0.99	0.06	0.06	1.04	0.08	0.08	1.04	0.07	0.07	1.13	0.09	0.08
		4	1.00	0.06	0.06	1.03	0.09	0.09	0.98	0.07	0.07	1.04	0.09	0.09
		5	1.02	0.07	0.07	1.03	0.09	0.09	1.01	0.09	0.09	1.05	0.09	0.09
		6	1.03	0.06	0.06	1.06	0.08	0.08	1.01	0.07	0.07	1.04	0.08	0.08
		7	1.00	0.07	0.07	1.02	0.08	0.08	1.01	0.08	0.08	1.05	0.09	0.08
		8	1.02	0.07	0.07	1.05	0.10	0.09	1.00	0.07	0.07	1.05	0.10	0.09
		9	1.03	0.08	0.08	1.09	0.10	0.09	1.01	0.08	0.07	1.07	0.11	0.10
		10	1.03	0.08	0.08	1.03	0.09	0.08	1.05	0.07	0.07	1.10	0.10	0.09
	Inelastic	2	1.19	0.06	0.05	1.35	0.12	0.09	1.15	0.08	0.07	1.22	0.11	0.09
		3	1.00	0.05	0.05	1.02	0.06	0.06	1.09	0.08	0.07	1.16	0.12	0.10
		4	1.02	0.06	0.06	1.03	0.07	0.06	1.01	0.08	0.08	1.06	0.12	0.11
		5	1.02	0.05	0.04	1.02	0.07	0.07	1.00	0.08	0.08	1.04	0.12	0.11
		6	1.03	0.04	0.04	1.06	0.07	0.07	0.99	0.07	0.07	1.02	0.11	0.11
		7	1.00	0.05	0.05	1.01	0.07	0.07	1.00	0.09	0.09	1.03	0.09	0.08
		8	1.04	0.06	0.06	1.06	0.09	0.08	1.03	0.09	0.09	1.03	0.10	0.09
		9	1.06	0.06	0.06	1.11	0.10	0.09	1.02	0.06	0.06	1.06	0.11	0.10
		10	1.02	0.06	0.06	1.02	0.02	0.02	1.04	0.07	0.07	1.10	0.11	0.10

10. Seismic responses of 3-D models with SR connections vs 2-D models

The ratios of the responses of the 2-D model representation to those of the 3-D buildings with SR connections are discussed in this part of the paper. The ratios are estimated in terms of interstory shears (V_4), interstory displacements (D_2), axial loads (A_3) and moments (M_3) at some columns of the base. Only Model 1, inelastic behavior and the *E-W* direction are considered.

The results for V_4, D_2, A_3 y M_3, are presented in Figs. 13(a), 13(b), 13(c) and 13(d), respectively. It is observed that the values of these parameters are, in general, larger than unity indicating that the responses are larger for the 2-D model. Results also indicate that the values of A_3 are larger than the others. As concluded from the comparison between the 2-D model and the 3-D model with PP connections, it is observed that the 2-D modeling will introduce significant more stresses at global and local levels than the 3-D modeling with SR connections.

Table 5 Statistics for the A_2 and M_2 parameters

Model		Location Member	Statistics of A_2						Statistics of M_2					
			N-S direction			*E-W* direction			*N-S* direction			*E-W* direction		
			μ	σ	δ	μ	σ	δ	μ	σ	δ	μ	σ	δ
1	Elastic	Interior	1.35	0.10	0.07	1.23	0.10	0.08	1.22	0.11	0.09	1.15	0.11	0.10
		Exterior	1.11	0.06	0.05	1.13	0.11	0.10	1.21	0.11	0.09	1.09	0.06	0.06
	Inelastic	Interior	1.54	0.65	0.42	1.29	0.51	0.40	1.23	0.11	0.09	1.44	0.24	0.17
		Exterior	1.10	0.08	0.07	1.07	0.04	0.04	1.22	0.11	0.09	1.27	0.16	0.13
2	Elastic	Interior	1.31	0.21	0.16	1.21	0.08	0.07	1.10	0.07	0.06	1.24	0.12	0.10
		Exterior	1.08	0.10	0.09	1.09	0.09	0.08	1.11	0.07	0.06	1.25	0.12	0.10
	Inelastic	Interior	1.50	0.63	0.42	1.86	0.94	0.51	1.12	0.06	0.05	1.29	0.14	0.11
		Exterior	1.08	0.06	0.06	1.12	0.07	0.06	1.12	0.07	0.06	1.26	0.12	0.10

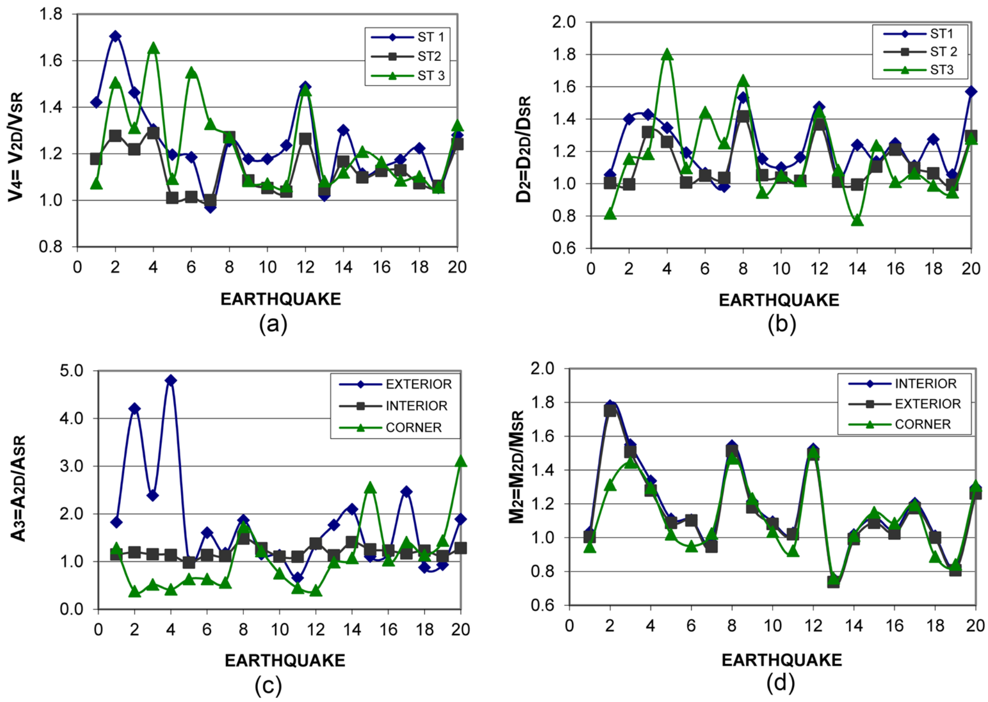

Fig. 13 Comparison for 3-D model with SR connections and the 2-D model (a) Values of the V_4 parameter (b) Values of the D_2 parameter (c) Values of the A_3 parameter and (d) Values of the M_3 parameter

11. Conclusions

Several issues regarding the structural idealization of steel buildings with perimeter moment resisting steel frames (MRSFs) and interior gravity frames (GFs) are addressed in this paper. The contribution of GFs to the lateral resistance is evaluated. The GFs are assumed to have, first perfectly pinned (PP) connections, and then semi-rigid (SR) connections. Then, the seismic responses, in terms of global (shear and interstory displacements) and local (resultant stresses at individual members of the base) response parameters, for the buildings with PP connections are compared to those of the buildings with SR connections. Finally, the accuracy of modeling the three-dimensional (3-D) buildings as plane (2-D) frames for seismic design is estimated. The comparison is made for both, the 3-D models with PP connections and the 3-D models with SR connections. Some steel model buildings used in the SAC steel project are used for this purpose. The models are excited in the time domain by twenty recorded earthquake motions. They are obtained from the Data Sets of the National Strong Motion Program (NSMP) of the United States Geological Surveys (USGS) and were selected to represent the characteristics of strong motion earthquakes.

The numerical study indicates that the contribution of GFs to the lateral structural resistance may be significant. This contribution is larger for lower stories for the buildings with PP connections. The contribution increases when the stiffness of the beam-to-column connection of the GFs is considered, particularly for upper stories, and is larger for inelastic than for elastic behavior. Since the imposed forces resulting from this contribution are not considered in their design, their strength capacity may be lower than that assumed, in other words their capacity may be overestimated. From a comparison of the results of the 3-D model with PP connections and the results of the 3-D model with SR connections it is observed that the interstory shears generally increase when the connections stiffness is taken into account. For some particular cases, however, it decreases. Unlike the case of lateral static load application, where the interstory shears are expected to always increase with the stiffness of the connection, the response due to dynamic loading depends on several parameters which are not significant for static analysis. Among them we can mention the distribution of mass and stiffness, energy dissipation characteristics, distribution of inelastic deformations through the structure, higher mode effects and frequency characteristics of the earthquakes. The average interstory displacements are similar for the models with PP and SR connections. Resultant stresses, in terms of axial loads and moments, at some base columns of the MRSFs can increase or decrease but to a lesser degree. For columns of the GFs, however, the decrement is significant.

Results also indicate that modeling the building as planes frames may result in larger interstory shears and displacements and resultant stresses than those obtained from the more realistic 3-D representation. These differences may be much larger when SR connections of the GFs are considered in the 3-D model. The implication of this is that perimeter MRSFs may be designed for larger forces, consequently their capacity is larger than that assumed in design and thus underestimated. In other words the design may result conservative. The conservativism is more for resultant stresses.

In general, the differences observed in the behaviour of each structural representation are mainly due to a) the elements that contribute to strength and stiffness and b) the dynamics characteristics of each structural representation. Based on the results of this study, it is concluded that, if the structural system under consideration is used, the three-dimensional model should be used in seismic analysis, that the GFs should be considered as part of the lateral resistance system, and that the stiffness of the connections should be included in the design of the GFs. Otherwise, the capacity of gravity frames may be overestimated while that of MRSFs may be underestimated.

Acknowledgements

This paper is based on work supported by La Universidad Autónoma de Sinaloa under grant

PROFAPI 2009/146 and by El Consejo Nacional de Ciencia y Tecnología (CONACyT) under grant 50298-J. Any opinions, findings, conclusions, or recommendations expressed in this publication are those of the authors and do not necessarily reflect the views of the sponsors.

References

BOCA (1993), 12th Edition Building Officials & Code Administration International Inc., National Building Code.

Chen, W.F., Goto, Y. and Richard Liew, J.Y. (1996), *Stability Designs of Semi-Rigid Frames*, John Wiley and Sons.

Chang, H.Y., Lin, Jay C.C., Lin, K.C., and Chen J.Y. (2009), “Role of accidental torsion in seismic reliability assessment for steel buildings”, *Steel Compos. Struct.,* **9**(5), 457-471.

Colson, A. (1991), “Theoretical Modeling of Semi-rigid Connections Behavior”, *J. Constr. Steel Res.*, **19**, 213-224.

El-Salti, M.K. (1992), “Design of frames with partially restrained connections”, PhD Thesis*,* Depart. Civ. Eng. Eng. Mech., University of Arizona, USA.

Foutch, A. and Yun S.Y. (2002), “Modeling of steel moment frames for seismic loads”, *J. Constr. Steel Res.*, **58**, 529-564.

FEMA (2000), “State of the art report on systems performance of steel moment frames subjected to earthquake ground shaking”, SAC Steel Project, Report FEMA 355C, Federal Emergency Management Agency.

Gao, L. and Haldar, A. (1995), “Nonlinear seismic response of space structures with PR connections”, *Int. J. Micro. Civil Eng.*, **10**, 27-37.

Krishnan, S., Ji, C., Komatitsch, D. and Tromp, J. (2006), “Performance of two 18-storey steel moment-frame building in southern California during two large simulated San Andres Earthquakes”*, Earthq. Spectra*, **22**(4), 1035-1061.

Kazantzi, A.K., Righiniotis T.D. and Chryssanthopoulos, M.K. (2008), “Fragility and hazard analysis of a welded steel moment resisting frame”, *J. Earthq. Eng.*, **12**, 596-615.

Liao, K.W., Wen Y.K., and Foutch D.A. (2007), “Evaluation of 3D steel moment frames under earthquake excitations. I: modeling”, *J. Struct. Eng.*, ASCE, **133**(3), 462-470.

Liu, J. and Astaneh-Asl, A. (2000), “Cyclic tests on simple connections including effects of the slab”, Report SAC/BD-00/03, SAC Joint Venture.

Lee, K. and Foutch, D.A. (2001), “Performance evaluation of new steel frame buildings for seismic loads”, *Earthq. Eng. Struct. D.*, **31**(3), 653-670.

Lee, K. and Foutch, D.A. (2006), “Seismic evaluation of steel moment frames buildings designed using different R-Values”, *J. Struct. Eng. Divi.,* ASCE, **132**(9), 1461-1472.

Mehrabian, A., Haldar, A. and Reyes-Salazar, A. (2005), “Seismic response analysis of steel frames with post-northridge connections”, *Steel Compos. Struct.,* **5**(4), 271-287.

Rentschler, G.P., Chen, W.F., and Driscoll, G.C. (1980), “Tests of beam-to-column web moment connections”, *J. Struct. Div.,* ASCE, **106**(5),1005-1022.

Reyes-Salazar, A., Velazquez-Dimas, J.I., and López-Barraza, A. (2001), “Respuesta Sísmica Inelástica de Marcos de Acero Resistentes a Momento con Conexiones Rígidas y Semi-rígidas”, *Revista de Ingenieria Sismica de la Sociedad Mexicana de Ingenieria Sismica A.C.,* **64**, 45-68.

Reyes-Salazar, A. (1997), “Inelastic seismic response and ductility evaluation of steel frames with fully, partially restrained and composite connections”, PhD Thesis, Dept. Civ. Eng. Eng. Mech., University of Arizona, USA.

Reyes-Salazar A. and Haldar, A. (1999), “Nonlinear seismic response of steel structures with semi-rigid and composite connections”, *J. Constr. Steel Res.*, **5**1, 37-59.

Reyes-Salazar, A. and Haldar, A. (2000), “Dissipation of energy in steel frames with PR connections”, *Struct. Eng. Mech.,* **9**(3), 241-256.

Reyes-Salazar, A. and Haldar, A. (2001a), “Energy dissipation at PR frames under seismic loading”, *J, Struct. Eng.* ASCE, **127**(5), 588-593.

Reyes-Salazar, A. and Haldar, A. (2001b), "Seismic response and energy dissipation in partially restrained and fully restrained steel frames: an analytical study", *Steel Compos. Struct.,* **1**(4), 459-480.
Richard, R.M. (1993), "Moment-rotation curves for partially restrained connections", *PRCONN, RMR Design Group,* Tucson, Arizona.
UBC (1994), "Structural engineering design provisions", Uniform Building Code, Volume 2, International Conference of Building Officials.