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Concurrent flexural strength and deformability design of high-performance concrete beams

J.C.M. Ho* and K.J.H. Zhou[a]

Department of Civil Engineering, The University of Hong Kong, Pokfulam Road, Hong Kong

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Abstract. In the design of earthquake resistant reinforced concrete (RC) structures, both flexural strength and deformability need to be considered. However, in almost all existing RC design codes, the design of flexural strength and deformability of RC beams are separated and independent on each other. Therefore, the pros and cons of using high-performance materials on the flexural performance of RC beams are not revealed. From the theoretical results obtained in a previous study on flexural deformability of RC beams, it is seen that the critical design factors such as degree of reinforcement, concrete/steel yield strength and confining pressure would simultaneously affect the flexural strength and deformability. To study the effects of these factors, the previous theoretical results are presented in various charts plotting flexural strength against deformability. Using these charts, a "concurrent flexural strength and deformability design" that would allow structural engineers to consider simultaneously both strength and deformability requirements is developed. For application in real construction practice where concrete strength is usually prescribed, a simpler method of determining the maximum and minimum limits of degree of reinforcement for a particular pair of strength and deformability demand is proposed. Numerical examples are presented to illustrate the application of both design methods.

Keywords: beams; curvature; deformability; high-strength concrete; reinforced concrete; rotation capacity

1. Introduction

In the design of reinforced concrete (RC) beams, it is a common belief that both the flexural strength and ductility need to be considered. Adequate flexural ductility design would prevent the beam from immediate collapse when it is attacked by accidental impact or severe earthquake (Inel *et al*. 2008, Lam *et al*. 2008, Weerheijm *et al*. 2009, Yagob *et al*. 2009). This generally requires the beams to dissipate the enormous energy induced by impact/earthquake attack through the following methods: installing dampers (Chen and Ding 2008, Ghoso and Ghosh 2008, Li and Xiong 2008, Chung *et al*. 2009, Fu 2009, Kavianipour and Sadati 2009, Wu and Cai 2009), adopting seismic isolation design (Hino *et al*. 2008, Kim *et al*. 2008a, b, Ates *et al*. 2009) or careful detailing of reinforcement such that plastic hinges could be formed at designated location(s) to dissipate excessive energy through inelastic deformation (Lu and Zhou 2007, Bindhu *et al*. 2008, Bechtoula

*Corresponding author, Assistant Professor, E-mail: johnny.ho@hku.hk
[a]PhD Student

et al. 2009, Zhang *et al*. 2009, Sadjadi and Kianoush 2010). Comparing with the installation of dampers and adopting base isolation method, the method of reinforcement detailing is less costly and is applicable to all types of structures. However, in the existing RC design codes (ECS 2004, ACI 318 2008, Hawileh *et al*. 2009), much more attention is paid to the flexural strength design and only some deemed-to-satisfy rules of reinforcement detailing are provided to ensure a certain level of ductility. For beams constructed of normal-strength concrete (NSC) and/or normal-strength steel (NSS), these rules can generally provide a consistent level of ductility (Kwan *et al*. 2002). However, the same set of rules, which are concrete strength and steel yield strength independent, cannot provide the same level of ductility provided in beams with high-performance materials.

In a series of theoretical studies carried out by the authors (Pam *et al*. 2001, Ho *et al*. 2003, 2005) on flexural ductility of RC beams using nonlinear moment-curvature analysis taken into account the stress-strain curve of the constitutive materials and stress-path dependence of steel reinforcement, it was found that the critical factors affecting the ductility are the degree of reinforcement, concrete strength, steel yield strength and confining pressure. From the results, a formula for ductility evaluation of RC beams was developed (Kwan *et al*. 2004). Some design charts were developed for designing strength and ductility simultaneously (Kwan *et al*. 2002). Guidelines were also proposed for designing RC beams with a prescribed minimum level of ductility even without the risk of earthquake attack (Ho *et al*. 2004).

Generally speaking, RC beams provided with sufficient flexural strength and ductility are able to sustain large inelastic deformation without immediate collapse by fully developed plastic hinge mechanism and redistribution of moment to adjacent structural members (Maghsoudi and Bengar 2006, Chen *et al*. 2009). However, ductility is an indication of deformability (rotation capacity) at ultimate state relative to that at first yield (Watson and Park 1994). Therefore, it may not be a good indicator to reveal the deformability of beams in absolute magnitude at ultimate state, which should not be smaller than the ultimate deformability demand (Kim and Kim 2007, Hong *et al*. 2008, Tsang *et al*. 2009). Particularly for beams constructed of high performance materials such as high-strength concrete (HSC) and/or high-strength steel (HSS), where the yield displacement is smaller than that of beams constructed of NSC/NSS due to larger stiffness, the design of beam based solely on ductility may overestimate the deformability of beams at ultimate state. This is not desirable because the flexural strength of beams may have already decreased to a fairly low level at ultimate state. From performance-based design point of view (Rubinstein *et al*. 2007, Challamel 2009), the beams should be provided with adequate ductility and deformability, which can cater for large inelastic displacement demand without causing collapse (Wu *et al*. 2004).

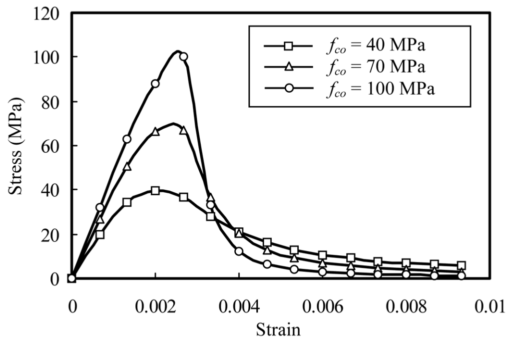
The authors have previously carried out a comprehensive parametric study on the deformability of RC beams using nonlinear moment-curvature analysis (Ho *et al*. 2010a, Zhou *et al*. 2010). The deformability of RC beams was studied by “normalised rotation capacity” defined as the product of ultimate beam curvature and effective depth. According to this definition, the normalised rotation capacity gives the ultimate rotation of concrete beams with plastic hinge length equal to its effective depth. For beams with other plastic hinge length (Mendis 2001, Pam and Ho 2009), the ultimate rotation could be obtained by multiplying the normalised rotation capacity with the ratio of plastic hinge length to effective depth. From the results of the parametric study, it was found that the most critical factors affecting the deformability of RC beams are similar to those affecting ductility. In general, the deformability increases as the degree of reinforcement decreases and confining pressure increases. Nonetheless, the effects of concrete strength and steel yield strength are dependent on the degree of reinforcement and steel ratio. Two empirical formulas were developed for rapid evaluation of deformability of NSC and HSC beams, which applicability was verified by comparing with available experimental results.

While it is important to understand how these critical factors affecting the deformability of RC beams, it should be noted that they would also influence the flexural strength. For instance, the use of HSS as tension steel at the same steel ratio would decrease significantly the deformability of RC beams. However, the strength is at the same time substantially improved. Therefore, it is not very clear whether the use of HSS at the same flexural strength requirement would be beneficial to the deformability performance. To illustrate the pros and cons of using high-performance materials in RC beams design, it is more appropriate to consider the simultaneous effects on flexural strength and deformability. In connection with this, the authors would in this paper carry out a concurrent analysis of flexural strength and deformability for RC beams. Using the results of the parametric study carried out by the authors (Zhou *et al*. 2010), the effects of those critical parameters on the both flexural strength and deformability can be clearly revealed. As an application, these charts can also be adopted to determine various possible combinations of concrete strength, degree of reinforcement, steel yield strength and confining pressure for a pair of flexural strength and deformability requirement. To cater for real design situation where the concrete strength is usually pre-determined, a simpler method of designing the steel content to satisfy both flexural strength and deformability requirements has also been developed.

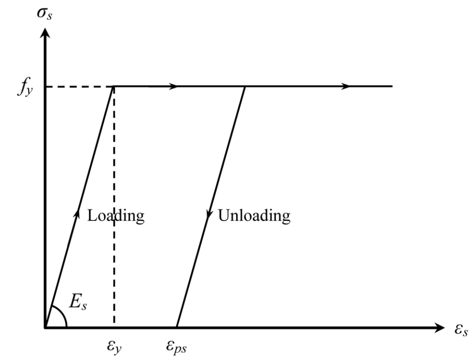
2. Nonlinear moment-curvature analysis

The method of nonlinear moment-curvature analysis developed previously by the authors (Pam *et al.* 2001, Ho *et al.* 2003) has been adopted for a parametric study regarding the deformability analysis of RC beams. It incorporates the stress-strain curve of concrete developed by Attard and Setunge (1996) and steel (ECS 2004) as well as the stress-path dependence of steel. When steel is unloaded in the post-peak stage of moment-curvature curves where the beam starts to soften, the path of unloading is with the same initial elastic modulus until it reaches zero steel stress. The stress-strain curves of concrete and steel are shown in Fig. 1.

Five basic assumptions are made in the analysis: (1) Plane sections before bending remain plane after bending. (2) The tensile strength of the concrete may be neglected. (3) There is no relative slip between concrete and steel reinforcement. (4) The concrete core is confined while the concrete cover is unconfined. (5) The confining pressure provided to the concrete core by confinement is assumed to be constant throughout the concrete compression zone. Assumptions (1) to (4) are commonly accepted and have been adopted by various researchers (Park *et al*. 2007, Au *et al*. 2009, Bai and Au 2009, Lam *et al*. 2008, Kwak and Kim 2010). Assumption (5) is not exact because the confining pressure varies in the concrete compression zone with strain gradient. However, it is a fairly reasonable assumption (Ho *et al*. 2010b). In the analysis, the moment-curvature behaviour of the beam section is analysed by applying prescribed curvatures incrementally starting from zero. At a prescribed curvature, the stresses developed in the concrete and the steel are determined from the strain profile and their respective stress-strain curves. Then, the neutral axis depth and resisting moment are evaluated from the axial and moment equilibrium conditions, respectively. The above procedure is repeated until the curvature is large enough for the resisting moment to increase to the peak and then decrease to 50% of the peak moment. Fig. 2 describes a typical beam sections adopted in the nonlinear moment-curvature analysis.

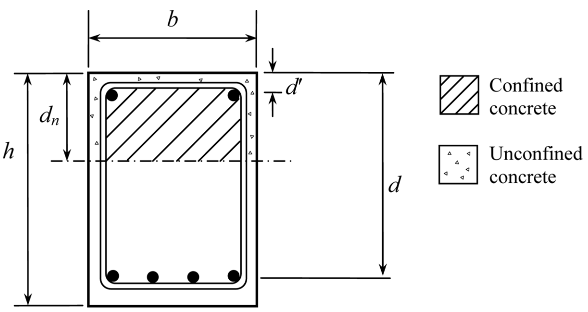


(a) Stress-strain curves of concrete



(b) Stress-strain curve of steel with stress-path dependence considered

Fig. 1 Stress-strain curves of concrete and steel reinforcement



Section properties

- b = 300 mm
- h = 600 mm
- d' = 50 mm
- d = 550 mm
- f_{co} = 40 to 100 MPa
- f_y = 400 to 800 MPa
- f_r = 0 to 4 MPa
- $\rho_t = A_{st}/bd$ = 0.4 to $2\rho_b$
- $\rho_c = A_{sc}/bd$ = 0 to 2%

Fig. 2 Beam sections analysed

3. Parametric study for deformability

3.1 Flexural deformability analysis

The flexural deformability of beam sections are expressed in terms of normalised rotation capacity θ_{pl} defined as follows

$$\theta_{pl} = \phi_u d \tag{1}$$

where ϕ_u is the ultimate curvature, d is the effective depth of the beam section. The ultimate curvature ϕ_u is taken as the curvature when the resisting moment has dropped to $0.8M_p$ after reaching M_p, where M_p is the peak moment. The value of θ_{pl} evaluated from Eq. (1) would give the rotation capacity of the beam with plastic hinge length equal to the effective depth. However, the actual rotation capacity of concrete beams should be obtained by multiplying θ_{pl} with ℓ_p/d, where ℓ_p is the actual plastic hinge length which is dependent on concrete strength, section geometry and loading condition (Mendis 2001, Bayrak and Sheikh 2001, Bae and Bayrak 2008, Bernardo and Lopes 2009, Haskett *et al*. 2009, Pam and Ho 2009, Ho and Pam 2010).

3.2 Failure modes and balanced steel ratio

Three failure modes of beam section were observed: (1) Tension failure – the maximum tension steel strain that can be reached is larger than the yield strain; (2) Compression failure – the maximum tension steel strain that can be reached is smaller than the yield strain, and (3) Balanced failure – the maximum tension steel strain that can be reached is equal to the yield strain. The tension steel ratio of a singly-reinforced beam section having balanced failure is defined as the balanced steel ratio denoted by $\rho_{bo} = A_{sb}/bd$, where A_{sb} is the balanced steel area. For beam sections with non-zero compression steel ratio ρ_c and unequal tension f_{yt} and compression f_{yc} steel yield strengths, ρ_b is equal to

$$\rho_b = \rho_{bo} + (f_{yc}/f_{yt})\rho_c \tag{2}$$

The values of ρ_{bo} for various concrete strengths and confining pressure have been evaluated (Ho

Table 1(a) Balanced steel ratios rrrbo for tension steel yield strength f_{yt} = 400 MPa

f_{co} (MPa)	Balanced steel ratios without compression steel ρ_{bo}(%)				
	f_r = 0 MPa	f_r = 1 MPa	f_r = 2 MPa	f_r = 3 MPa	f_r = 4 MPa
40	4.74	5.98	6.90	7.73	8.56
50	5.63	6.91	7.86	8.78	9.60
60	6.46	7.79	8.77	9.70	10.59
70	7.29	8.62	9.61	10.54	11.50
80	8.06	9.38	10.37	11.35	12.29
90	8.77	10.11	11.13	12.11	13.03
100	9.42	10.80	11.82	12.78	13.76

Table 1(b) Balanced steel ratios ρ_{bo} for tension steel yield strength f_{yt} = 600 MPa

f_{co} (MPa)	Balanced steel ratios ρ_{bo}(%) for f_{yt} = 600 MPa				
	f_r = 0 MPa	f_r = 1 MPa	f_r = 2 MPa	f_r = 3 MPa	f_r = 4 MPa
40	2.74	3.60	4.23	4.83	5.37
50	3.23	4.12	4.78	5.40	6.00
60	3.69	4.61	5.29	5.93	6.55
70	4.13	5.06	5.76	6.41	7.04
80	4.56	5.50	6.19	6.85	7.49
90	4.94	5.90	6.59	7.28	7.91
100	5.29	6.27	6.97	7.67	8.29

Table 1(c) Balanced steel ratios ρ_{bo} for tension steel yield strength f_{yt} = 800 MPa

f_{co} (MPa)	Balanced steel ratios ρ_{bo} (%) for f_{yt} = 800 MPa				
	f_r = 0 MPa	f_r = 1 MPa	f_r = 2 MPa	f_r = 3 MPa	f_r = 4 MPa
40	1.82	2.48	2.96	3.42	3.84
50	2.13	2.82	3.33	3.80	4.25
60	2.43	3.14	3.66	4.14	4.61
70	2.70	3.43	3.96	4.45	4.93
80	2.97	3.69	4.22	4.75	5.21
90	3.22	3.95	4.50	5.00	5.49
100	3.44	4.19	4.74	5.22	5.74

et al. 2003) and are listed in Table 1 for different yield strength of tension steel. To facilitate practical design application, the following empirical equation was derived using regression analysis

$$\rho_{bo} = 0.005(f_{co})^{0.58}(1+1.2f_r)^{0.3}(f_{yt}/460)^{-1.35} \tag{3}$$

All strengths are in MPa, 400 MPa $\le f_{yt} \le$ 800 MPa and 0 $\le f_r \le$ 4 MPa.

3.3 Degree of reinforcement

From previous theoretical studies on ductility and deformability of RC beams, it is found that one of the major influencing factors is the degree of reinforcement denoted by λ. It can be expressed in the following form

$$\lambda = \frac{f_{yt}\rho_t - f_{yc}\rho_c}{f_{yt}\rho_{bo}} \tag{4}$$

where ρ_c and ρ_t are the compression and tension steel ratios respectively, f_{yc} and f_{yt} are the yield strength of compression and tension steel respectively. Generally in real practical construction, the yield strength of tension and compression steel is the same and thus $f_{yt} = f_{yc}$. The beam section is classified as under-reinforced, balanced and over-reinforced sections when λ is less than, equal to and larger than 1.0 respectively.

3.4 Parametric study

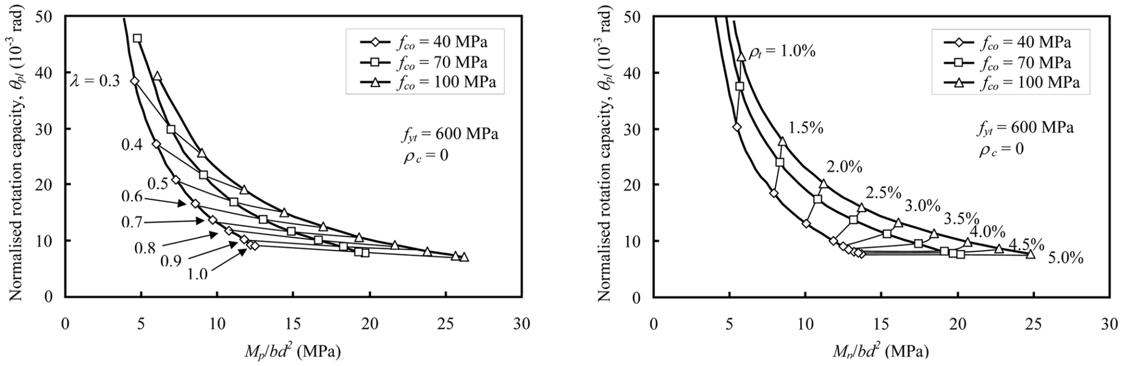
Based on the above definition, a comprehensive parametric study on the effects of various factors on the normalised rotation capacity has been conducted previously (Ho *et al*. 2010a, Zhou *et al*. 2010). The studied factors are: (1) Degree of reinforcement; (2) Concrete strength; (3) Steel yield strength; and (4) Confining pressure. The beam sections analysed has been shown in Fig. 2. The concrete strength f_{co} was varied from 40 to 100 MPa, the confining pressure f_r was varied from 0 to 4 MPa, the tension steel ratio ρ_t was varied from 0.4λ to 2λ, the compression steel ratio ρ_c was varied from 0 to 2%, and the steel yield strength f_y was varied from 400 to 800 MPa. The upper bound yield strength of steel reinforcement is adopted from the maximum value of f_y permitted by the New Zealand Code (NZS3101 2006). It should be note that, in order to have a full range comprehensive study of the flexural strength and ductility of concrete beams with different failure modes, the range of ρ_t studied in this paper is larger than the respective allowable limit of 4% stipulated in most of the current concrete design codes, which should be avoided in real design practice.

4. Interrelation of flexural strength and deformability

It is now evident that the major factors affecting the flexural strength and deformability are the concrete strength, degree of reinforcement, steel yield strength, steel ratio and confining pressure. In the case of a singly reinforced section, the use of high-performance materials, such as HSS, at the same concrete strength and tension steel ratio would increase the flexural strength but decrease the deformability. Hence, the increase in flexural strength is achieved at the expense of a lower deformability. However, at the same flexural strength requirement, the deformability may or may not be reduced. On the other hand, the use of HSS at the same degree of reinforcement would increase the deformability but decrease the flexural strength. Hence, the increase in deformability is achieved at the expense of a lower flexural strength. However, at the same deformability requirement, the flexural strength may or may not be reduced. To assess whether the use of high-performance materials would have beneficial or adverse effects on the flexural performance of RC beams, it is proposed in this study to investigate the effects of each parameters on both the flexural strength and deformability.

4.1 Effects of concrete strength, degree of reinforcement and tension steel ratio

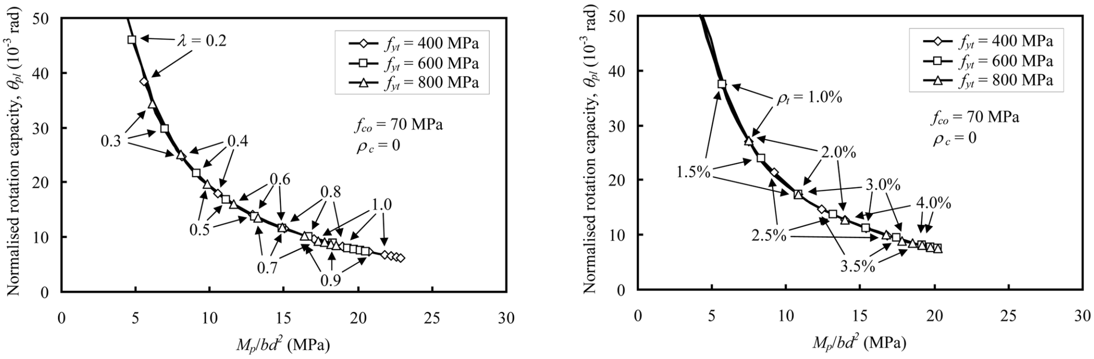
The effects of different concrete strength f_{co} = 40, 70 and 100 MPa on both flexural strength and deformability are shown in Figs. 3(a) and 3(b) at constant degree of reinforcement λ and tension steel ratio ρ_t respectively. Generally, it can be seen from these graphs that after using HSC to replace NSC, the curve will shift upwards and to the right hand side provided that a suitable tension steel ratio or degree of reinforcement is selected. Therefore, it is possible that the use of HSC will increase both flexural strength and deformability simultaneously, notwithstanding that HSC is stiffer and less deformable *per se*. In particular, it is seen in Fig. 3(b) that if HSC is used at the same ρ_t,



(a) For different levels of λ

(b) For different levels of ρ_t

Fig. 3 Maximum limits of flexural strength and deformability for beams with different concrete strength



(a) For different levels of λ (b) For different levels of ρ_t

Fig. 4 Maximum limits of flexural strength and deformability for beams with different tension steel yield strength

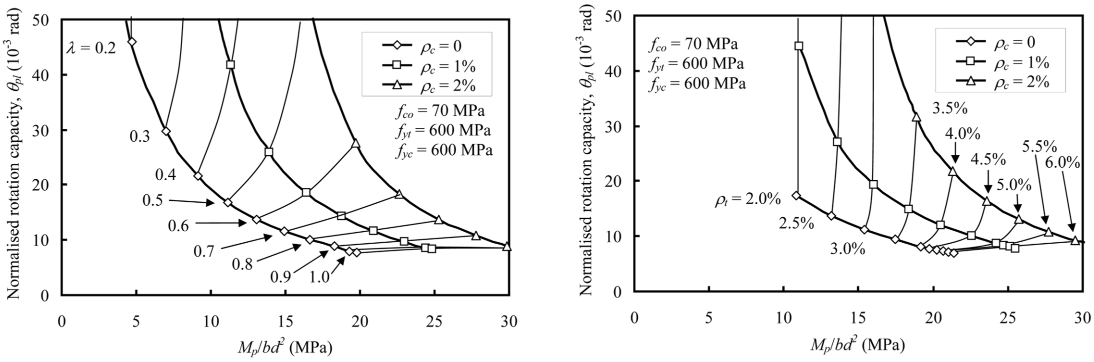
there is a significant increases in both flexural strength and deformability until $\rho_t = \rho_{bo}$, where using HSC would only increase flexural strength but not deformability. This is because after using HSC, λ decreases at constant ρ_t, thereby increasing the deformability. On the contrary, if HSC is used at the same λ, it can be seen from Fig. 3(a) that the flexural strength increases but the deformability decreases until λ = 1.0, after which the flexural strength increases at constant deformability. On the whole, it is seen that the use of HSC could increase the deformability at the same flexural strength, increase the flexural strength at the same deformability, or increase at the same time both flexural strength and deformability.

4.2 Effects of tension steel yield strength

The effects of different tension steel yield strength f_{yt} = 400, 600 and 800 MPa on both flexural strength and deformability are shown in Figs. 4(a) and 4(b) at constant degree of reinforcement λ and tension steel ratio ρ_t respectively. Generally, it can be seen from these graphs that three curves representing different yield strengths of tension steel overlap each other. It therefore implies that using HSS as tension steel could not improve both flexural strength and deformability at the same time. In particular, it is seen in Fig. 4(a) that if HSS is used as tension steel at the same λ, there is an increases in deformability but an reduction in flexural strength. This is because the balanced steel ratio and hence tension steel ratio decrease as tension steel yield strength increases at constant λ. On the contrary, if HSS is used at the same ρ_t, it can be seen from Fig. 4(b) that the flexural strength increases but deformability decreases. This is because λ increases as f_{yt} increases at constant ρ_t, which reduces the deformability.

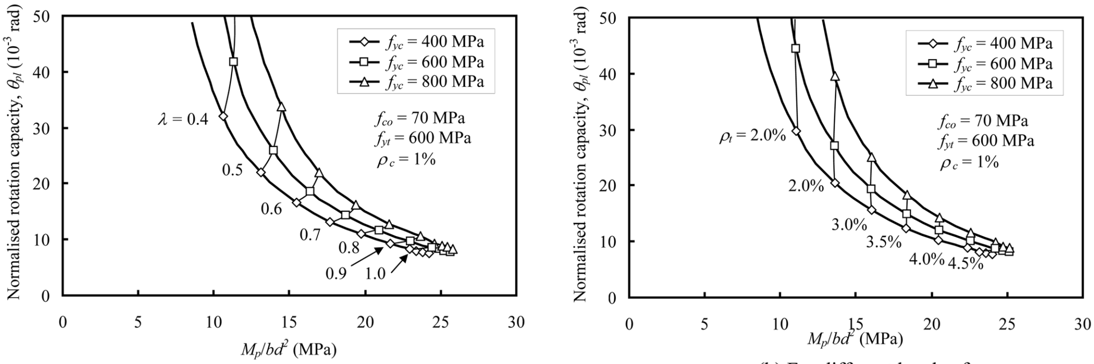
4.3 Effects of compression steel

The effects of different compression steel ratio ρ_c = 0, 1% and 2% on both flexural strength and deformability are shown in Figs. 5(a) and 5(b) at constant degree of reinforcement λ and tension steel ratio ρ_t respectively. Generally, it can be seen from these graphs that increasing the compression steel would shift the curve upwards and to the right hand side. It therefore implies that adding compression steel would significantly improve both the flexural strength and deformability



(a) For different levels of λ (b) For different levels of ρ_t

Fig. 5 Maximum limits of flexural strength and deformability for beams with different compression steel ratios



(a) For different levels of λ (b) For different levels of ρ_t

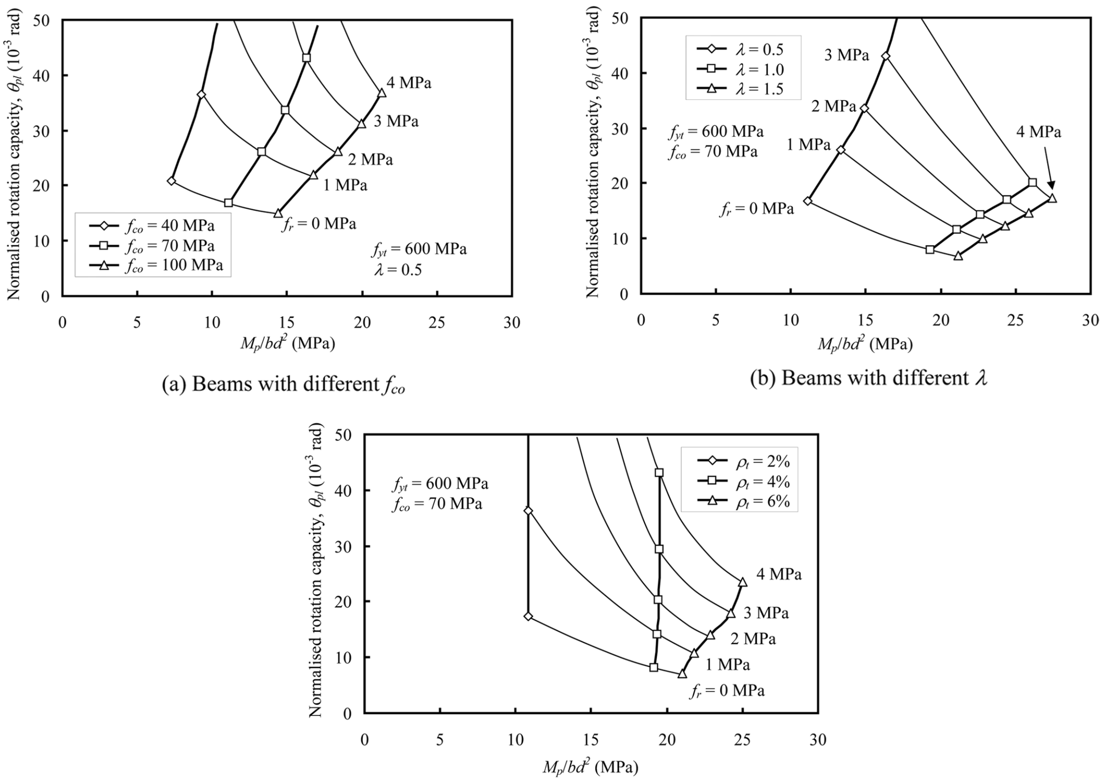
Fig. 6 Maximum limits of flexural strength and deformability for beams with different compression steel yield strength

of RC beams simultaneously. In particular, it is seen from both figures that adding compression steel at constant λ and ρ_t would substantially increase the deformability initially, but the deformability improvement due to addition of compression decreases as λ or ρ_t increases until λ = 1.0 or $\rho_t = \rho_{bo}$, after which the deformability improvement becomes insignificant. This is because the deformability of RC beams decreases as λ or ρ_t increases. However, adding compression steel at constant λ or ρ_t would always increase the flexural strength of RC beams.

The effects of different compression steel yield strength f_{yc} = 400, 600 and 800 MPa on both flexural strength and deformability are shown in Figs. 6(a) and 6(b) at constant degree of reinforcement λ and tension steel ratio ρ_t respectively. Generally, it can be seen from these graphs that increasing the yield strength of compression steel would shift the curve upwards and to the right hand side. It therefore implies that increasing the yield strength of compression steel would improve simultaneously both the flexural strength and deformability of RC beams. In particular, it is seen from both figures that adding compression steel at constant λ and ρ_t would substantially increase the deformability initially, but the deformability improvement due to addition of compression decreases as λ or ρ_t increases until λ = 1.0 or $\rho_t = \rho_{bo}$, after which the deformability improvement becomes insignificant. It is also evident from Fig. 6(b) that the flexural strength is virtually not improved if higher strength compression steel is adopted at the same ρ_t. The increase in flexural strength becomes more prominent when the beam section is more heavily reinforced.

4.4 Effects of confining pressure

The effects of different confining pressure f_r = 0 to 4 MPa on both flexural strength and deformability are shown in Figs. 7(a), 7(b) and 7(c) for RC beam sections with different concrete strength f_{co}, degree of reinforcement λ and tension steel ratio ρ_t respectively. The confining pressure is usually provided by various types of confinement in forms of closely-spaced transverse reinforcement (Ho and Pam 2003, Yeh and Chang 2007, Belarbi *et al*. 2009, Kwan and Ho 2010), external steel plates (Zhu *et al*. 2007, Altin *et al*. 2008, Su *et al*. 2009, Zhu and Su 2010), concrete-filled steel tube (Wang *et al*. 2007, Feng and Young 2009, Zhou and Young 2009), steel-concrete composite section (Park and Kim 2008, Li *et al*. 2009); FRP wraps (Wei *et al*. 2007, Hashemi *et al*. 2008, Mahini and Ronagh 2009, Wu and Wei 2010), FRP confinement (Galal 2007), fibre reinforced concrete (Leung 1996, Ramadoss and Nagamani 2009) or other methods (Papakonstantinou and Katakalos 2009, Zhou *et al*. 2010). It can be observed from these figures that the curves for beam sections with different f_{co}, λ or ρ_t shift downwards and to the right hand sides as f_{co}, λ or ρ_t increases. This is because increasing f_{co} at constant λ, increasing λ at constant f_{co} or



(a) Beams with different f_{co}

(b) Beams with different λ

(c) Beams with different ρ_t

Fig. 7 Maximum limits of flexural strength and deformability for beams with different confining pressure (I)

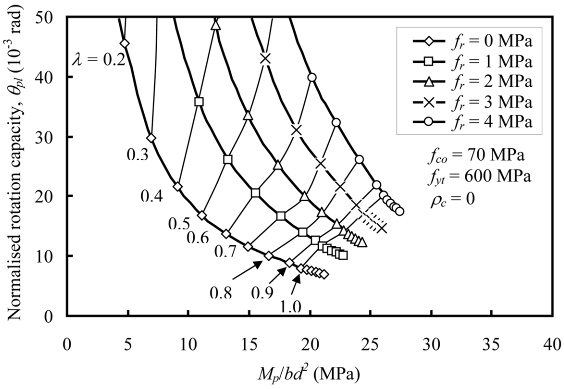


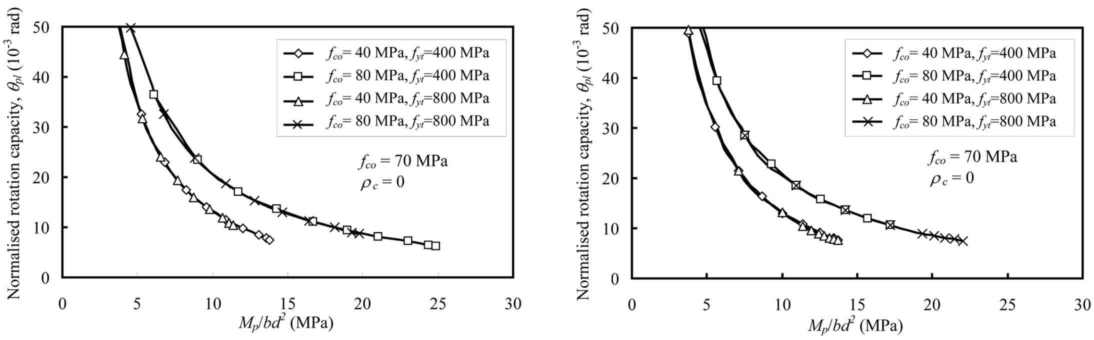
Fig. 8 Maximum limits of flexural strength and deformability for beams with different confining pressure (II)

increasing ρ_t at constant f_{co} would enhance the flexural strength at the expense of deformability (see also Figs. 3(a) and 3(b)).

The effect of different confining pressure f_r = 0 to 4 MPa on both flexural strength and deformability at the same concrete strength f_{co} is shown in Fig. 8 in another format. From the figure, it is clear that adding confinement at a given concrete strength f_{co} could increases both the flexural strength and deformability at all values of λ.

4.5 Effects of using high-performance materials

Figs. 3 and 4 show the effects on the maximum limits of flexural strength and deformability that can be achieved when the concrete strength f_{co} is increased at a fixed tension steel yield strength f_{yt}, or vice versa. To study the effects when both concrete and tension steel yield strength vary, the maximum limits of flexural strength and deformability that could be achieved simultaneously for beam sections having f_{co} = 40 and 80 MPa as well as f_{yt} = 400 and 800 MPa are plotted in Fig. 9. From the figure, it can be observed that the maximum flexural strength capacity and deformability that could be achieved by HSC beams containing NSS or HSS is higher than those of NSC beams containing NSS or HSS. This implies that the adoption of HSC would always increase the beam



(a) For different levels of λ

(b) For different levels of ρ_t

Fig. 9 Maximum limits of flexural strength and deformability for beams incorporating high-performance materials performance in terms of the maximum capacity of flexural strength and deformability, albeit that HSC and HSS are stiffer and less deformable *per se*. For large beam section constructed of HSC, because of the aggravated tensile straining of the reinforcement placed closed to the tension surface of the beam due to reduced neutral axis depth, it may be subjected to an increased risk of low cycle fatigue. This additional failure criterion should also be checked in practical design if the tension steel strain is found too large at the ultimate limit state. However, the use of HSS as tension reinforcement will improve flexural performance of the beam only if HSC is adopted at the same time. It should also be noted in some cases that the serviceability may control the minimum amount of reinforcement. Under such a circumstance, the use of HSS will not improve the flexural performance of RC beams.

5. Design charts and formula

In the design of RC beams, it is necessary to design the beam with sufficient flexural strength and deformability. From the above discussions, it is seen that the critical factors affecting deformability would also influence the flexural strength at the same time. Therefore, in the current beam design where the flexural strength and deformability are separated, it would be an iterative process to achieve a specified pair of flexural strength and deformability. However, the design would become much more simplified by using the strength-deformability graphs presented earlier for beams with different materials strength and properties. In practical design situation where the concrete strength is usually pre-determined, a simple method of evaluating the maximum and minimum limits of degree of reinforcement λ is developed.

5.1 Graphical method of concurrent strength and deformability design

Figs. 3, 5 and 8 can be used as design charts for a more convenient design method for RC beams that could satisfy both flexural strength and deformability requirement in a single step. This method is named hereinafter as "Concurrent strength and deformability design". The advantages of this method are that, apart from allowing an one-step design for both strength and deformability, it would provide different feasible design options, such as using HSC, HSS and confinement. Then, the most economical or appropriate design option, taking into account other restraints such as architectural requirements, can be easily selected. For this application, Figs. 3, 5 and 8 are combined and further elaborated to form a series of design charts, as shown in Figs. 10 and 11. For given flexural strength and deformability requirements in terms of $M_p/(bd^2)$ and θ_{pl}, the concrete strength and steel ratios that would meet these requirements can be obtained directly from reading the charts, which are shown in Figs. 10 and 11. In the event that adding compression steel is not a preferred option due to steel congestion problem at joints, more confinement can be added to the section to increase the flexural strength and deformability performance. The required confining pressure can be read from Fig. 11 for beams with different concrete strength without compression steel.

The following illustrates the application of the design charts. For a given set of flexural strength and deformability requirements, there are several design options as can be seen from Fig. 10 if the addition of compression steel is preferred to adding confinement. There could be many different combinations of concrete strength and compression steel ratio that would meet a given set of strength and deformability requirements. Since the addition of compression steel is generally quite

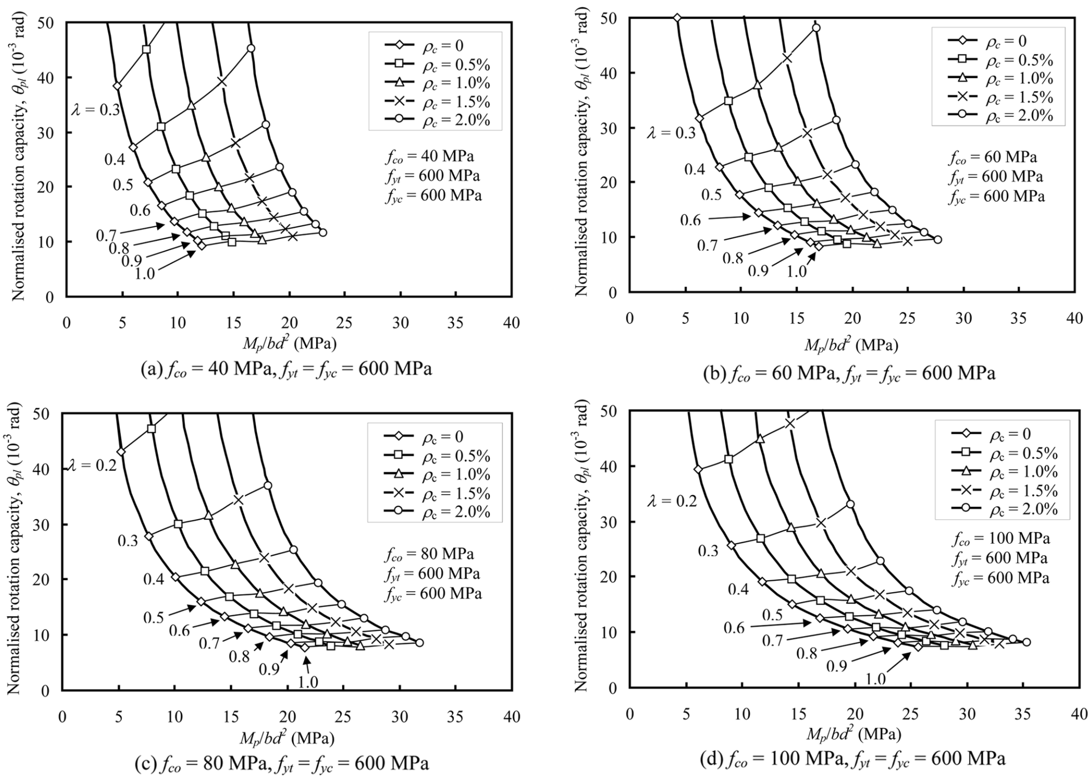


Fig. 10 Design charts for beams with different compression steel ratios

costly, it is recommended that ρ_c = 0% in each of the charts should be tried first. If the flexural strength and deformability requirements could not be simultaneously satisfied despite the use of HSC of 100 MPa, then the size of the beam section should be enlarged or some compression steel should be added. If it is decided that the size of the beam section is to remain unchanged and compression steel is to be added, the required compression steel ratio can be determined by using successively ρ_c = 0.5%, ρ_c = 1.0%, ρ_c = 1.5% and ρ_c = 2.0%. If the flexural strength and deformability requirements could not be met even when a compression steel ratio of 2.0% is used, then there is no other option apart from increasing the size of the beam section compression steel ratios. The use of compression steel greater than 2.0% is generally not recommended (ECS 2004).

On the other hand, if the addition of confinement is preferred to compression steel, Fig. 11 should be used for the concurrent flexural strength and deformability design. The use of all charts in Fig. 11 would produce beam design with provision of significant confinement. Similar to Fig. 10, there are many different combinations of concrete strength and confining pressure that would meet a given set of strength and deformability requirements. It is recommended that f_r = 1 MPa in each of the charts should be tried first. If the flexural strength and deformability requirements could not be simultaneously satisfied despite the use of HSC of 100 MPa and f_r = 1 MPa, then the required confining pressure can be determined by using successively f_r = 2 MPa, f_r = 3 MPa and f_r = 4 MPa. If the flexural strength and deformability requirements could not be met even when f_{co} = 100 MPa and f_r = 4 MPa are used, then there is no other option apart from increasing the size of the beam section. This is because when the beam is too heavily confined, the spacing of confinement may be

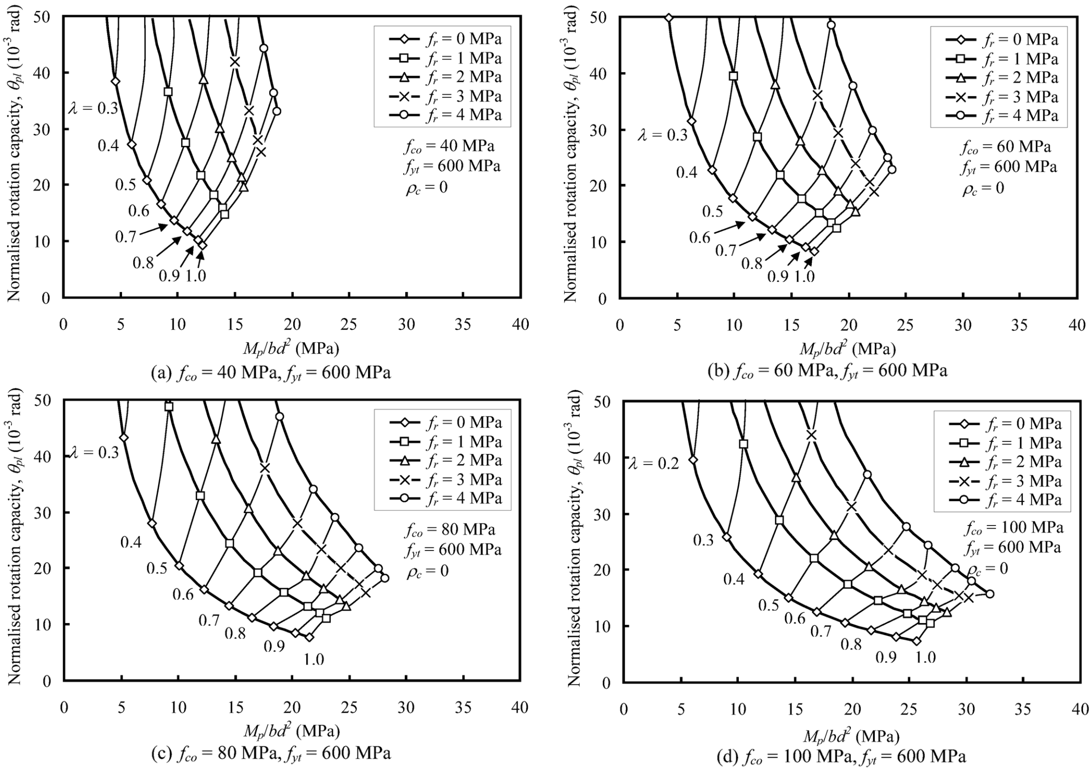


Fig. 11 Design charts for beams with different confining pressure

too small that would affect the quality of concrete placing.

5.2 Simple design method for beams with prescribed concrete strength

The specified flexural strength and deformability requirements are actually the minimum requirements in member design. However, the design of flexural strength and flexural ductility should not be treated in the same manner. The design of flexural strength should be just sufficient to meet the requirement because the provision of an excessive amount of flexural strength to the beam would violate the “strong column - weak beam” design philosophy or increase the risk of having brittle shear failure. Also, it would increase the cost of construction because more tension steel is required. On the other hand, the provision of a more than sufficient flexural deformability is always recommended as long as it does not require additional compression or confining steel. Therefore, the general recommended design strategy is to provide just sufficient flexural strength and, as long as compression and confining steel are not required, as much deformability as possible.

An equation for rapid flexural deformability evaluation for NSC and HSC beams has been previously proposed by the authors (Zhou *et al*. 2010), which is rewritten in Eq. (5). The validity of the equation has been compared with available experimental results on NSC and HSC beams (Nawy *et al*. 1968, Pecce and Fabbrocino 1999, Ko *et al*. 2001, Debernardi and Taliano 2002, Lopes and Bernardo 2003, Haskett *et al*. 2009), which are shown in Tables 2 and 3 respectively.

Table 2 Comparison with experimental results on rotation capacities of NSC beams

Code	f_c' (MPa)	f_r (Mpa)	f_{yt} (Mpa)	ρ_t (%)	ρ_c (%)	θ_{pl} by Eq. (5) (rad) [1]	θ_{pl} by others (rad) [2]	θ_{pl} by EC2 (rad) [3]	[1]/[2]	[3]/[2]
					Nawy *et al*. (1968)					
P9G1	33.6	0.00	328	1.73	0.71	0.0870	0.0650	0.0330	1.34	0.51
P11G3	35.1	0.50	328	1.73	0.71	0.1536	0.1110	0.0320	1.38	0.29
P3G4	37.5	1.30	452	1.73	0.71	0.1232	0.1340	0.0260	0.92	0.19
P4G5	39.1	1.30	452	1.73	0.71	0.1217	0.1360	0.0265	0.89	0.19
					Pecce and Fabbocino (1999)					
A	41.3	0.98	471	2.60	0.05	0.0255	0.0220	0.0100	1.16	0.45
B	41.3	0.94	454	1.10	0.05	0.0736	0.1220	0.0265	0.60	0.22
					Debernardi and Taliano (2002)					
T1A1	27.7	0.46	587	0.67	0.30	0.1433	0.1035	0.0310	1.38	0.30
T3A1	27.7	0.46	587	2.00	0.59	0.0270	0.0290	0.0080	0.93	0.28
T5A1	27.7	0.35	587	0.63	0.22	0.0978	0.1130	0.0300	0.87	0.27
T6A1	27.7	0.35	587	1.28	0.22	0.0311	0.0245	0.0160	1.27	0.65
					Haskett *et al*. (2009)					
A1	38.2	0.67	315	1.47	0.0	0.0313	0.0360	0.0269	0.87	0.75
A2	42.3	0.32	318	1.47	0.0	0.0226	0.0205	0.0280	1.10	1.37
A3	41.0	0.31	336	1.47	0.0	0.0209	0.0168	0.0270	1.24	1.61
A4	42.9	1.29	315	2.95	0.0	0.0222	0.0305	0.0172	0.73	0.56
A5	39.6	0.59	314	2.95	0.0	0.0136	0.0207	0.0154	0.66	0.74
A6	41.1	0.31	328	2.95	0.0	0.0103	0.0118	0.0153	0.87	1.30
B1	43.0	0.65	329	1.47	0.0	0.0293	0.0277	0.0278	1.06	1.00
B2	41.8	0.31	322	1.47	0.0	0.0222	0.0152	0.0277	1.46	1.82
B3	42.9	1.29	321	2.95	0.0	0.0217	0.0218	0.0168	1.00	0.77
B4	42.9	0.64	323	2.95	0.0	0.0138	0.0120	0.0166	1.15	1.38
C2	26.0	0.39	329	1.47	0.0	0.0219	0.0258	0.0203	0.85	0.79
C3	25.6	0.32	330	1.47	0.0	0.0201	0.0187	0.0200	1.07	1.07
C4	25.9	1.23	325	2.95	0.0	0.0205	0.0297	0.0080	0.69	0.27
C5	23.4	0.64	328	2.95	0.0	0.0126	0.0130	0.0080	0.97	0.62
C6	27.4	0.34	319	2.95	0.0	0.0102	0.0125	0.0080	0.82	0.64
					Average				1.01	0.72
					Standard deviation				0.24	0.47

$$\theta_{pl} = 0.03m(f_{co})^{-0.3}(\lambda)^{-1.0n}\left(1+110(f_{co})^{-1.1}\left(\frac{f_{yc}\rho_c}{f_{yt}\rho_t}\right)^3\right)\left(\frac{f_{yt}}{460}\right)^{0.3} \tag{5a}$$

$$m = 1+4f_{co}^{0.4}(f_r/f_{co}) \tag{5b}$$

$$n = 1+3f_{co}^{0.2}(f_r/f_{co}) \tag{5c}$$

Table 3 Comparison with experimental results on rotation capacities of HSC beams

Code	f_c' (MPa)	f_r (Mpa)	f_{yt} (Mpa)	ρ_t (%)	ρ_c (%)	θ_{pl} by Eq. (5) (rad) [1]	θ_{pl} by others (rad) [2]	θ_{pl} by EC2 (rad) [3]	$\frac{[1]}{[2]}$	$\frac{[3]}{[2]}$
Pecce and Fabbocino (1999)										
AH	93.8	0.98	471	2.60	0.05	0.0271	0.0220	0.0170	1.23	0.77
CH	95.4	1.11	534	2.20	0.04	0.0300	0.0380	0.0170	0.79	0.45
Ko *et al.* (2001)										
6-65-1	66.6	2.26	415	3.59	0.79	0.0547	0.0472	0.0150	1.16	0.32
6-75-1	66.6	2.33	427	4.27	0.77	0.0399	0.0412	0.0100	0.97	0.24
8-50-1	82.1	2.42	443	3.35	0.80	0.0580	0.0482	0.0160	1.20	0.33
8-65-1	82.1	2.33	427	4.27	0.77	0.0398	0.0450	0.0100	0.88	0.22
8-75-1	82.1	2.15	394	4.97	0.79	0.0338	0.0484	0.0080	0.70	0.17
7-62^{00}-1	70.8	1.91	408	3.16	0.00	0.0403	0.0530	0.0135	0.76	0.25
7-62^{15}-1	70.8	1.91	408	3.16	0.79	0.0587	0.0510	0.0160	1.15	0.31
Lopes and Bernardo (2003)										
A(64.9-2.04)	64.9	0.59	555	2.04	0.20	0.0248	0.0200	0.0210	1.24	1.05
A(63.2-2.86)	63.2	0.62	575	2.86	0.20	0.0161	0.0180	0.0110	0.89	0.61
A(65.1-2.86)	65.1	0.62	575	2.86	0.20	0.0161	0.0150	0.0110	1.07	0.73
B(82.9-2.11)	82.9	0.59	555	2.11	0.20	0.0243	0.0210	0.0180	1.16	0.86
B(83.9-2.16)	83.9	0.59	555	2.16	0.20	0.0237	0.0200	0.0180	1.19	0.90
B(83.6-2.69)	83.6	0.62	575	2.69	0.20	0.0178	0.0210	0.0150	0.85	0.71
B(83.4-2.70)	83.4	0.62	575	2.70	0.20	0.0177	0.0200	0.0150	0.89	0.75
Average									1.01	0.54
Standard deviation									0.18	0.28

In real construction practice, where the concrete strength of different structural members is usually prescribed, Eq. (5) can be used to check whether a particular minimum requirement of flexural deformability could be met by a beam section. Suppose the minimum deformability requirement is $\theta_{pl,\min}$, the respective maximum limit of λ for a beam section without compression and confining steel can be calculated by the following inequalities

$$\theta_{pl,\min} \le 0.03(f_{co})^{-0.3}(\lambda)^{-1.0}\left(\frac{f_{yt}}{460}\right)^{0.3} \tag{6a}$$

$$\lambda \le 0.03(\theta_{pl,\min})^{-1.0}\left(\frac{f_{yt}/460}{f_{co}}\right)^{0.3} \tag{6b}$$

To satisfy the minimum flexural strength requirement, the respective minimum limit of λ for a beam section without compression and confining steel can be calculated by the following inequalities using the equivalent rectangular concrete stress block of NSC or HSC as stipulated in the common RC design codes, e.g., Eurocode 2 (ECS 2004).

$$\lambda \geq \frac{-f_{co} + f_{co}\sqrt{1+\left(\frac{2}{f_{co}}\right)\left(\frac{M_p}{bd^2}\right)}}{f_{yt}\rho_{bo}} \tag{7}$$

Combining Inequalities (6b) and (7), the suitable range of λ for designing a singly reinforced unconfined beam section with prescribed concrete strength f_{co} to satisfy a pair of deformability and strength requirements (i.e., $\theta_{pl,\min}$ and M/bd^2) can be derived

$$\frac{-f_{co} + f_{co}\sqrt{1+\left(\frac{2}{f_{co}}\right)\left(\frac{M_p}{bd^2}\right)}}{f_{yt}\rho_{bo}} \leq \lambda \leq 0.03(\theta_{pl,\min})^{-1.0}\left(\frac{f_{yt}/460}{f_{co}}\right)^{0.3} \tag{8}$$

If Inequality (8) gives a proper range of λ, the reinforcement details of the beam section can then be designed using the lower bound value of λ. However, if Inequality (8) does not give a proper range of λ, either the section should be enlarged or some compression steel or confining steel should be added.

5.3 Numerical examples

Design the steel reinforcement of a beam section with the following minimum strength and deformability requirements: M_p/bd^2 = 6.0 and $\theta_{pl,\min}$ = 0.03 rad. Given that f_{co} = 60 MPa and f_{yt} = f_{yc} = 600 MPa. As a first attempt, use Fig. 10(b) and set ρ_c = 0%. Plotting the point (6.0, 0.03) on the graph, it is found that the required strength and deformability can be simultaneously achieved by section having λ = 0.30 or ρ_t = 1.1% and ρ_c = 0%. If an addition of 1% compression steel is permitted, the same minimum deformability requirement can be achieved with a larger strength, i.e., M_p/bd^2 can be increased to 10, or equivalently, the effective depth of the beam could be reduced by about 30% while maintaining the same moment capacity. Similarly, if 1 MPa of confining pressure is provided to the section, the same deformability requirement can be achieved with a larger strength (see Fig. 11(b)), i.e., M_p/bd^2 can be increased to about 11.5, or equivalently, the effective depth of the beam could be reduced by about 38% while maintaining the same moment capacity. The choice between them is a matter of engineering judgment taking into consideration the economy and simplicity of the overall design.

Alternatively, the required amount of tension steel can be worked out using Inequality (8) for singly-reinforced section. Based on the minimum deformability requirement $\theta_{pl,\min}$ = 0.03 rad, the permissible range of λ can be calculated by

$$\lambda \leq 0.03(\theta_{pl,\min})^{-1.0}\left(\frac{f_{yt}/460}{f_{co}}\right)^{0.3} = \frac{0.03}{0.03}\times\left(\frac{600/400}{60}\right)^{0.3} = 0.317$$

Based on the strength requirement M_p/bd^2 = 6.0, the permissible range of λ can be calculated by

$$\rho_{bo} = 0.005(f_{co})^{0.58}(1+1.2f_r)^{0.3}(f_{yt}/460)^{-1.35} = 0.005\times 60^{0.58}\times(600/460)^{-1.35} = 0.0375$$

$$\lambda \geq \frac{-f_{co} + f_{co}\sqrt{1+\left(\frac{2}{f_{co}}\right)\left(\frac{M_p}{bd^2}\right)}}{f_{yt}\rho_{bo}} = \frac{-60+60\sqrt{1+\left(\frac{2}{60}\right)\times 6}}{600\times 0.0375} = 0.255$$

Therefore, the permissible range of λ is $0.255 \le \lambda \le 0.317$. Select $\lambda = 0.255$ to provide more than adequate deformability and just enough flexural strength, the required tension steel ratio is given by $\rho_t = \lambda \times \rho_{bo} = 0.255 \times 3.75\% = 1.0\%$.

6. Conclusions

The flexural performance of RC beams constructed of HSC and/or HSS is assessed by the maximum limits of flexural strength and deformability that can be achieved simultaneously in this study. The flexural strength and deformability of RC beams are obtained from nonlinear moment-curvature analysis taking into account the stress-strain curve of the constitutive materials. The deformability is measured in terms of normalised rotation capacity, which represents the beam rotation with plastic hinge length equal to its effective depth. A series of graphs were plotted showing the maximum limits of flexural strength and deformability that could be achieved for NSC and HSC beams with/without HSS. From the graphs, it is evident that the use of HSC can always improve the maximum limits of flexural strength and deformability that can be achieved simultaneously, albeit that HSC is stiffer and less deformable *per se*. However, the use of HSS would not increase the maximum limits of strength and deformability by its own, unless HSC is also adopted at the same time.

Two methods for designing RC beams satisfying a pair of flexural strength and deformability requirement are proposed. The first method consists of a series of design charts plotting the flexural strength and deformability of RC beams consisting of different concrete strength, compression steel ratios and confining pressure. These charts would allow structural designers to consider both the strength and deformability requirements before deciding whether to use HSC, compression steel or confining steel. For practical design application where the concrete strength is usually prescribed, a simpler design method of adopting the authors' previously proposed equation is developed. This method determines the permissible range of λ that satisfies both flexural strength and deformability requirements for a singly-reinforced beam section with prescribed concrete strength. Lastly, a numerical example has been given to illustrate the application of these developed methods.

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Notations

A_{sb} : Balanced steel area
A_{sc} : Area of compression steel
A_{st} : Area of tension steel
b : Breadth of beam or column section
d : Effective depth of beam or column section
E_s : Elastic modulus of steel reinforcement
f_{co} : Peak stress on stress-strain curve of unconfined concrete
f_r : Confining pressure
f_y : Yield strength of steel reinforcement
f_{yc} : Yield strength of compression steel
f_{yt} : Yield strength of tension steel
h : Total depth of the beam section
ℓ_p : Plastic hinge length
M_p : Peak moment
ε_{ps} : Residual plastic strain in steel reinforcement
ε_s : Strain in steel
θ_{pl} : Normalised rotation capacity of beam
$\theta_{pl,\min}$: Minimum required normalised rotation capacity of beam
λ : Degree of reinforcement
ϕ_u : Ultimate curvature
ρ_b : Balanced steel ratio ($= A_{sb}/bd$)
ρ_{bo} : Balanced steel ratio for beam section with no compression steel
ρ_c : Compression steel ratio ($= A_{sc}/bd$)
ρ_t : Tension steel ratio ($= A_{st}/bd$)
σ_s : Stress in steel reinforcement