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Investigation on electromagnetothermoelastic interaction of functionally graded piezoelectric hollow spheres

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Abstract. An analytical method is presented to investigate electromagnetothermoelastic behaviors of a hollow sphere composed of functionally graded piezoelectric material (FGPM), placed in a uniform magnetic field, subjected to electric, thermal and mechanical loads. For the case that material properties obey an identical power law in the radial direction of the FGPM hollow sphere, exact solutions for electric displacement, stresses, electric potential and perturbation of magnetic field vector in the FGPM hollow sphere are determined by using the infinitesimal theory of electromagnetothermoelasticity. Some useful discussion and numerical examples are presented to show the significant influence of material inhomogeneity. The aim of this research is to understand the effect of composition on electromagnetothermoelastic stresses and to design optimum FGPM hollow spheres.

Keywords: functionally graded piezoelectric material (FGPM); hollow sphere; analytical method; electromagnetothermoelastic; perturbation of magnetic field vector

1. Introduction

Functionally graded piezoelectric material (FGPM) is a kind of piezoelectric material with material composition and properties varying continuously along certain directions. FGPM is the composite material intentionally designed so that they possess desirable properties for some specific applications. The advantage of this new kind of material can improve the reliability of life span of devices. In recent years, the applications for spherical structures have continuously increased in some engineering areas, including aerospace, offshore and submarine structures, chemical vessel and civil engineering structures.

This research subject is so new that only a few results can be found in the literatures. Previous studies on the subject have considered FGM spherical structures including those, Obata and Noda (1995) used the perturbation technique to derive the thermal stress equations of the thick hollow spheres made of functionally graded materials under different temperature distributions. Using the method of Frobenius series, Lutz and Zimmerman (1996) gave the analytical solution for the

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stresses in FGM spheres. Using the infinitesimal theory of elasticity, Naki and Murat (2001) obtained the closed-form solutions for stresses and displacements in functionally graded spherical vessels subjected to internal pressure. Jabbari *et al.* (2002) investigated mechanical and thermal stresses in a functionally graded hollow sphere due to radially symmetric loads. Wu *et al.* (2003) presented an exact solution for functionally graded piezothermoelastic spherical shell as sensors or actuators. By means of an analytical-numerical method, Han and Liu (2003) studied elastic wave propagation in a functionally graded piezoelectric sphere. Chen *et al.* (2004) analyzed 3D free vibration of a functionally graded piezoelectric hollow sphere filled with compressible fluid. Eslami *et al.* (2005) gave the analytical solution for the one-dimensional steady-state thermoelastic stresses in a hollow sphere made of functionally graded material. Ootao and Tanigawa (2007) studied the transient piezothermoelastic problem of a functionally graded thermopiezoelectric hollow sphere due to uniform heat supply. Ganapathi (2007) studied the dynamic stability behavior of a clamped functionally graded material spherical shell structural element subjected to external pressure loading. By means of the infinitesimal theory, Dai *et al.* (2007) investigated electromagneto- elastic interactions for functionally graded piezoelectric hollow and solid sphere. By means of using the Legendre polynomials and the system of the functionally graded energy equation to solve the Navier equations, Poultangari *et al.* (2008) developed an analytic method to obtain the solution for the dimensional steady state thermal and mechanical stresses in a hollow thick sphere made of functionally graded material. Using the Hankel and Laplace transform techniques, Arani *et al.* (2009) developed an analytical method to obtain the response of magnetothermoelastic stress and perturbation of the magnetic field vector for a thickwalled spherical vessel. However, so far, investigation on electromagnetothermoelastic interaction for a FGPM hollow sphere placed in a uniform magnetic field has not been found in the literatures.

In this paper, by means of employing simplifying assumptions, the colsed-form solutions for the electric displacement, stresses, electric potential and perturbation of magnetic field vector distributions in the FGPM hollow sphere are obtained. The aim of the work is to understand the influence of the volumetric ratio of constituents on electric displacement, electromagnetothermoelastic stresses, electric potential and perturbation of magnetic field vector of the FGPM hollow sphere and to design the optimum FGPM spherical structures for engineering applications.

2. Basic formulations of the problem

2.1 Derivation of equations

In spherical coordinates (r, θ, ψ), considering a FGPM hollow sphere with internal radius a and external radius b (as shown in Fig. 1). The FGPM hollow sphere with perfect conductivity placed in a uniform magnetic field $\vec{H}(0, 0, H_\psi)$. The components of displacement, stresses, electric displacement and electric potential are, respectively, expressed as $u(r)$, $\sigma_i(i = r, \theta)$, D_r and $\varphi(r)$. The constitutive relations for the FGPM hollow sphere subjected to a rapid change in temperature $T(r)$ are expressed as (Dai and Fu 2006)

$$\sigma_r = c_{11}\frac{\partial u}{\partial r} + 2c_{12}\frac{u}{r} + e_{11}\frac{\partial \varphi}{\partial r} - \lambda_1 T(r) \tag{1a}$$

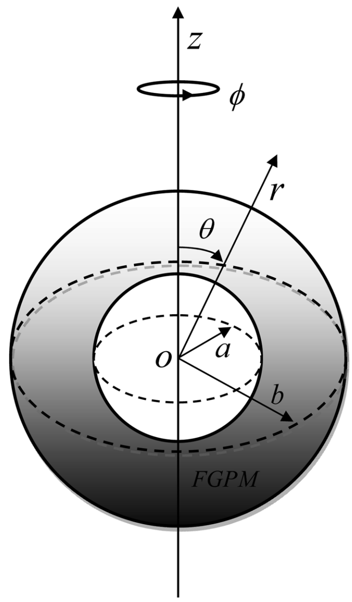

Fig. 1 The geometry of a FGPM hollow sphere placed in a uniform magnetic field

$$\sigma_\theta = c_{12}\frac{\partial u}{\partial r} + (c_{22}+c_{23})\frac{u}{r} + e_{12}\frac{\partial \varphi}{\partial r} - \lambda_2 T(r) \tag{1b}$$

$$D_r = e_{11}\frac{\partial u}{\partial r} + 2e_{12}\frac{u}{r} - g_{11}\frac{\partial \varphi}{\partial r} + p_{11}T(r) \tag{1c}$$

where c_{ij} $(i=1,2;\, j=1,2,3)$, e_{1i} $(i=1,2,3)$, g_{11} and p_{11} are elastic constants, thermal expansion, respectively, and

$$\lambda_1 = c_{11}\alpha_1 + 2c_{12}\alpha_2, \quad \lambda_2 = c_{12}\alpha_1 + (c_{22}+c_{23})\alpha_2 \tag{2}$$

It is now considered that all material coefficients and piezoelectric constants have the same power-law function along the radial direction, i.e.

$$c_{ij}(R) = c_{ij}^0\left(\frac{r}{b}\right)^\beta \ (i=1,2;\, j=1,2,3), \quad e_{1i}(r) = e_{1i}^0\left(\frac{r}{b}\right)^\beta (i=1,2), \quad g_{11}(r) = g_{11}^0\left(\frac{r}{b}\right)^\beta$$
$$p_{11}(r) = p_{11}^0\left(\frac{r}{b}\right)^\beta, \quad \alpha_i(r) = \alpha_i^0\left(\frac{r}{b}\right)^\beta (i=1,2), \quad \mu(r) = \mu^0\left(\frac{r}{b}\right)^\beta \tag{3}$$

Here, superscript zero denotes corresponding value at the outer surface ($r = b$) of the FGPM hollow sphere, and β is the inhomogeneous constant determined empirically. However, these values for β do not necessarily represent a certain material, various β values are used to demonstrate the effect of inhomogeneity on the electric displacement, stresses, electric potential and perturbation of magnetic field vector distributions.

The mechanical and electric boundary conditions are expressed as

$$\sigma_r\big|_{r=a} = -P_a, \quad \sigma_r\big|_{r=b} = -P_b, \quad \phi_r\big|_{r=a} = \phi_a, \quad \phi_r\big|_{r=b} = \phi_b \tag{4}$$

Omitting displacement electric currents, the governing electrodynamic Maxwell equations (Kraus 1984, Dai and Wang 2004) for a perfectly conducting, elastic body are given by

$$\vec{J} = \nabla \times \vec{h}, \quad \nabla \times \vec{e} = -\mu(r)\frac{\partial \vec{h}}{\partial t}, \quad div\vec{h} = 0, \quad \vec{e} = -\mu(r)\left(\frac{\partial \vec{U}}{\partial t} \times \vec{H}\right), \quad \vec{h} = \nabla \times (\vec{U} \times \vec{H}) \tag{5}$$

Applying an initial magnetic field vector $\vec{H}(0, 0, H_\psi)$ in the spherical coordinate (r, θ, ψ) system to Eq. (5), yields

$$\vec{U} = (u, 0, 0), \quad \vec{e} = -\mu(r)\left(0, H_\psi\frac{\partial u}{\partial t}, 0\right), \quad \vec{h} = (0, 0, h_z)$$

$$\vec{J} = \left(0, -\frac{\partial h_\psi}{\partial r}, 0\right), \quad h_z = -H_\psi\left(\frac{\partial u}{\partial r} + \frac{2u}{r}\right) \tag{6}$$

The equilibrium equation of the FGPM sphere, in absence of body forces, is expressed as

$$\frac{\partial \sigma_r}{\partial r} + \frac{2(\sigma_r - \sigma_\theta)}{r} + f_\psi = 0 \tag{7}$$

where f_ψ is defined as Lorentz's force (Arani *et al.* 2009, Kraus 1984, Paul and Nasar 1987), which may be written as

$$f_\psi = \mu(r)(\vec{J} \times \vec{H}) = \mu^0 r^\beta H_\psi^2 \frac{\partial}{\partial r}\left(\frac{\partial u}{\partial r} + \frac{2u}{r}\right) \tag{8}$$

In absence of free charge density, the charge equation of electrostatics (Heyliger 1996) is expressed as

$$\frac{\partial D_r}{\partial r} + \frac{2D_r}{r} = 0, \quad (a \le r \le b) \tag{9}$$

Solving Eq. (9), yields

$$D_r = \frac{A_1}{r^2} \tag{10}$$

Thus, Eq. (1c) may be rewritten as

$$\frac{\partial \phi(r)}{\partial r} = \frac{e_{11}^0}{g_{11}^0}\frac{\partial u}{\partial r} + \frac{2e_{12}^0}{g_{11}^0}\frac{u}{r} + \frac{p_{11}^0}{g_{11}^0}T(r) - \frac{A_1}{g_{11}^0}\frac{1}{r^{\beta+1}} \tag{11}$$

Substituting Eq. (11) into Eqs. (1a,b) and utilizing Eq. (3), yields

$$\sigma_r = C_1 r^\beta \frac{\partial u}{\partial r} + C_2 r^\beta \frac{u}{r} + C_3 r^\beta T(r) - C_4 \frac{A_1}{r^2} - \lambda_1^0 r^{2\beta} T(r) \tag{12a}$$

$$\sigma_\theta = C_5 r^\beta \frac{\partial u}{\partial r} + C_6 r^\beta \frac{u}{r} + C_7 r^\beta T(r) - C_8 \frac{A_1}{r^2} - \lambda_2^0 r^{2\beta} T(r) \tag{12b}$$

where

$$C_1 = c_{11}^0 + \frac{(e_{11}^0)^2}{g_{11}^0}, \quad C_2 = 2c_{12}^0 + \frac{2e_{11}^0 e_{12}^0}{g_{11}^0}, \quad C_3 = \frac{e_{11}^0 p_{11}^0}{g_{11}^0}, \quad C_4 = \frac{e_{11}^0}{g_{11}^0}$$

$$C_5 = c_{12}^0 + \frac{e_{11}^0 e_{12}^0}{g_{11}^0}, \quad C_6 = c_{22}^0 + c_{23}^0 + \frac{2(e_{12}^0)^2}{g_{11}^0}, \quad C_7 = \frac{e_{12}^0 p_{11}^0}{g_{11}^0}, \quad C_8 = \frac{e_{12}^0}{g_{11}^0} \tag{13}$$

Substituting Eqs. (12), (8) into Eq. (7), the equilibrium equation is expressed as

$$\frac{\partial^2 u}{\partial r^2} + (2+\beta)\frac{1}{r}\frac{\partial u}{\partial r} + W_1 \frac{u}{r^2}$$
$$= W_2 T(r) r^{\beta-1} + W_3 T(r) r^{-1} + W_4 \frac{\partial T(r)}{\partial r} r^{\beta} + W_5 \frac{\partial T(r)}{\partial r} + W_6 A_1 r^{-\beta-3} \tag{14}$$

where

$$W_1 = \frac{2\mu^0 H_\psi^2(\beta+1) + C_2\beta + C_2 - 2C_6}{C_1 + \mu^0 H_\psi^2}, \quad W_2 = \frac{2\lambda_1^0\beta + 2\lambda_1^0 - 2\lambda_2^0}{C_1 + \mu^0 H_\psi^2}$$
$$W_3 = \frac{-C_3\beta - 2C_3 + 2C_7}{C_1 + \mu^0 H_\psi^2}, \quad W_4 = \frac{\lambda_1^0}{C_1 + \mu^0 H_\psi^2}$$
$$W_5 = \frac{-C_3}{C_1 + \mu^0 H_\psi^2}, \quad W_6 = \frac{-2C_8}{C_1 + \mu^0 H_\psi^2} \tag{15}$$

2.2 Heat conduction problem

The heat conduction equation in the steady-state condition for the one-dimension problem in polar coordinates and the thermal boundary conditions for the FGPM hollow sphere is given (Eslami *et al.* 2005), respectively, as

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2 k(r)\frac{\partial T(r)}{\partial r}\right] = 0, \quad (a \le r \le b) \tag{16a}$$

$$T(r)\big|_{r=a} = T_0 \tag{16b}$$

$$\left[\frac{\partial T(r)}{\partial r} + hT(r)\right]\Bigg|_{r=b} = 0 \tag{16c}$$

where $k = k(r)$ is the thermal conduction coefficient of the FGPM hollow sphere in the r direction, h is the ratio of the convective heat-transfer coefficient of the FGPM hollow sphere and the surrounding medium.

It is assumed that the nonhomogeneous thermal conduction coefficient $k(r)$ is power function of r as

$$k(r) = k^0\left(\frac{r}{b}\right)^{\beta} \tag{17}$$

where k^0 is the thermal conduction coefficient at the external surface ($r = b$). Utilizing Eq. (17), the heat conduction Eq. (16a) becomes

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left[r^{\beta+2}\frac{\partial T(r)}{\partial r}\right] = 0 \tag{18}$$

Integrating the Eq. (18) twice, yields

$$T(r) = W_7 r^{-(\beta+1)} + W_8 \tag{19}$$

Using the boundary conditions (16b,c) to determine the constants W_7 and W_8, yields

$$W_7 = \frac{hT_0}{\beta b^{-(\beta+1)} + h(a^{-\beta} - b^{-\beta})}, \quad W_8 = \frac{T_0[\beta b^{-(\beta+1)} - hb^{-\beta}]}{\beta b^{-(\beta+1)} + h(a^{-\beta} - b^{-\beta})} \tag{20}$$

3. Solution of the problem

Substituting Eq. (19) into Eq. (14), yields

$$\frac{\partial^2 u}{\partial r^2} + (2+\beta)\frac{1}{r}\frac{\partial u}{\partial r} + W_1 \frac{u}{r^2} = W_9 r^{-1} + W_{10} r^{\beta-1} + W_{11} r^{-\beta-1} + W_6 A_1 r^{-\beta-3} \tag{21}$$

where

$$W_9 = W_2 W_7 + W_3 W_8 - W_4 W_7 \beta, \quad W_{10} = W_2 W_8, \quad W_{11} = W_3 W_7 - W_5 W_7 \beta \tag{22}$$

Assume that the complete solution of the Eq. (21) may be expressed in the following form

$$u(r) = \mu^g(r) + \mu^p(r) \tag{23}$$

It is obvious that the homogeneous solution to Eq. (21) can be obtained by assuming

$$u^g(r) = Qr^m \tag{24}$$

Where Q is an arbitrary constant, substituting Eq. (24) into Eq. (21), one obtains

$$m^2 + (\beta+1)m + W_1 = 0 \tag{25}$$

The characteristic Eq. (25) has two real roots m_1 and m_2 as follows

$$m_1 = \frac{1}{2}[-(\beta+1) + \sqrt{(\beta+1)^2 - 4W_1}], \quad m_2 = \frac{1}{2}[-(\beta+1) - \sqrt{(\beta+1)^2 - 4W_1}] \tag{26}$$

Thus, the homogeneous solution is

$$u^g(r) = B_1 r^{m_1} + B_2 r^{m_2} \tag{27}$$

where B_1 and B_2 are unknown constants, and determined by the given boundary conditions. The particular solution $u^p(r)$ is assumed to be of the following form

$$u^p(r) = B_3 r + B_4 r^{\beta+1} + B_5 r^{-\beta+1} + B_6 r^{-\beta-1} \tag{28}$$

Substituting Eq. (28) into Eq. (21), yields

$$\begin{aligned}&\frac{\partial^2 u^p}{\partial r^2}+(2+\beta)\frac{1}{r}\frac{\partial u^p}{\partial r}+W_1\frac{u^p}{r^2}\\&=[B_3(\beta+2)+W_1B_3]r^{-1}+[B_4(\beta+1)\beta+B_4(\beta+1)(\beta+2)+W_1B_4]r^{\beta-1}\\&\quad+[B_5(-\beta+1)(-\beta)+B_5(-\beta+1)(\beta+2)+W_1B_5]r^{-\beta-1}\\&\quad+[B_6(-\beta-2)(-\beta-1)+B_6(-\beta-1)(\beta+2)+W_1B_6]r^{-\beta-3}\\&=W_9r^{-1}+W_{10}r^{\beta-1}+W_{11}r^{-\beta-1}+W_6A_1r^{-\beta-3}\end{aligned}\tag{29}$$

According to the coefficients of the identical powers, from Eq. (29), one obtains

$$B_3=\frac{W_9}{\beta+2+W_1},\quad B_4=\frac{W_{10}}{2\beta^2+4\beta+2+W_1},\quad B_5=\frac{W_{11}}{2-2\beta+W_1},\quad B_6=\frac{W_6A_1}{W_1}\tag{30}$$

Thus, the complete solution for $u(r)$ is expressed as

$$u(r)=u^g(r)+u^p(r)=B_1r^{m_1}+B_2r^{m_2}+B_3r+B_4r^{\beta+1}+B_5r^{-\beta+1}+B_6r^{-\beta-1}\tag{31}$$

Substituting Eq. (31) into Eq. (11), yields

$$\begin{aligned}\frac{\partial\phi}{\partial r}=&\frac{e_{11}^0}{g_{11}^0}\left[B_1m_1r^{m_1-1}+B_2m_2r^{m_2-1}+B_3+B_4(1+\beta)r^{\beta}+B_5(-\beta+1)r^{-\beta}+\frac{A_6A_1}{W_1}(-\beta-1)r^{-\beta-2}\right]\\&+\frac{2e_{12}^0}{g_{11}^0}\left[B_1r^{m_1-1}+B_2r^{m_2-1}+B_3+B_4r^{\beta}+B_5r^{-\beta}+\frac{W_6A_1}{W_1}r^{-\beta-2}\right]\\&+\frac{p_{11}^0}{g_{11}^0}(W_7r^{-\beta}+W_8)-\frac{A_1}{g_{11}^0}r^{-\beta-2}\end{aligned}\tag{32}$$

Integrating Eq. (32), one have

$$\begin{aligned}\phi=&\frac{e_{11}^0}{g_{11}^0}\left[B_1r^{m_1}+B_2r^{m_2}+B_3r+B_4r^{\beta+1}+B_5r^{-\beta+1}+\frac{W_6A_1}{W_1}r^{-\beta-1}\right]\\&+\frac{2e_{12}^0}{g_{11}^0}\left[\frac{B_1}{m_1}r^{m_1}+\frac{B_2}{m_2}r^{m_2}+B_3r+\frac{B_4}{\beta+1}r^{\beta+1}+\frac{B_5}{-\beta+1}r^{-\beta+1}+\frac{W_6A_1}{W_1(-\beta-1)}r^{-\beta-1}\right]\\&+\frac{p_{11}^0}{g_{11}^0}\left(\frac{W_7}{-\beta+1}r^{-\beta+1}+W_8r\right)+\frac{A_1}{g_{11}^0(\beta+1)}r^{-\beta-1}+A_2\end{aligned}\tag{33}$$

where A_1 and A_2 are unknown constants, and determined by the given boundary conditions.

Substituting Eq. (31) into Eq. (12a), Eq. (12b), Eq. (12c), and the final item of Eq. (6), the radial stress, circumferential stress and perturbation of magnetic field vector of the FGPM hollow sphere are obtained as

$$\sigma_r = C_1 r^{\beta}\left[B_1 m_1 r^{m_1-1} + B_2 m_2 r^{m_2-1} + B_3 + B_4(1+\beta)r^{\beta} + B_5(-\beta+1)r^{-\beta} + \frac{W_6 A_1}{W_1}(-\beta-1)r^{-\beta-2}\right]$$
$$+ C_2 r^{\beta}\left[B_1 r^{m_1-1} + B_2 r^{m_2-1} + B_3 + B_4 r^{\beta} + B_5 r^{-\beta} + \frac{W_6 A_1}{W_1} r^{-\beta-2}\right]$$
$$+ C_3 r^{\beta}(W_7 r^{-\beta} + W_8) - \lambda_1^0 r^{2\beta}(W_7 r^{-\beta} + W_8) - C_4 \frac{A_1}{r_2} \tag{34a}$$

$$\sigma_\theta = C_5 r^{\beta}\left[B_1 m_1 r^{m_1-1} + B_2 m_2 r^{m_2-1} + B_3 + B_4(1+\beta)r^{\beta} + B_5(-\beta+1)r^{-\beta} + \frac{W_6 A_1}{W_1}(-\beta-1)r^{-\beta-2}\right]$$
$$+ C_6 r^{\beta}\left[B_1 r^{m_1-1} + B_2 r^{m_2-1} + B_3 + B_4 r^{\beta} + B_5 r^{-\beta} + \frac{W_6 A_1}{W_1} r^{-\beta-2}\right]$$
$$+ C_7 r^{\beta}(W_7 r^{-\beta} + W_8) - \lambda_2^0 r^{2\beta}(W_7 r^{-\beta} + W_8) - C_8 \frac{A_1}{r_2} \tag{34b}$$

$$h_\psi = -H_\psi\left(\frac{\partial u}{\partial r} + \frac{2u}{r}\right) = -H_\psi[(2+m_1)B_1 r^{m_1-1} + (2+m_2)B_2 r^{m_2-1} + 3B_3$$
$$+ (\beta+3)B_4 r^{\beta} + (-\beta+3)B_5 r^{-\beta} + (1-\beta)\frac{W_6 A_1}{W_1} r^{-\beta-2}\Big] \tag{34c}$$

To determine the unknown constants A_1, A_2, B_1 and B_2, utilizing the mechanical and electric boundary conditions (4), yields

$$\phi|_{r=a} = W_{12}B_1 + W_{13}B_2 + W_{14}A_1 + A_2 - W_{15} = \phi_a \tag{35a}$$
$$\phi|_{r=b} = W_{16}B_1 + W_{17}B_2 + W_{18}A_1 + A_2 - W_{19} = \phi_b \tag{35b}$$
$$\sigma_r|_{r=a} = W_{20}B_1 + W_{21}B_2 + W_{22}B_1 - W_{23} = -P_a \tag{35c}$$
$$\sigma_r|_{r=b} = W_{24}B_1 + W_{25}B_2 + W_{26}B_1 - W_{27} = -P_b \tag{35d}$$

Where W_i $(i = 12, 13, \ldots, 27)$ are shown in appendix A.
Solving Eq. (35), yields

$$B_2 = \frac{[(W_{23}-P_a)W_{26}-(W_{27}-P_B)W_{22}][(W_{14}-W_{18})W_{20}-(W_{12}-W_{16})W_{22}]}{(W_{21}W_{26}-W_{22}W_{25})[(W_{14}-W_{18})W_{20}-(W_{12}-W_{16})W_{22}]-(W_{20}W_{26}-W_{22}W_{24})[(W_{14}-W_{18})W_{21}-(W_{13}-W_{17})W_{22}]}$$
$$-\frac{(W_{20}W_{26}-W_{22}W_{24})[(W_{14}-W_{18})(W_{23}-P_a)-(W_{19}+\phi_b)W_{22}]}{(W_{21}W_{26}-W_{22}W_{25})[(W_{14}-W_{18})W_{20}-(W_{12}-W_{16})W_{22}]-(W_{20}W_{26}-W_{22}W_{24})[(W_{14}-W_{18})W_{21}-(W_{13}-W_{17})W_{22}]}$$
$$B_1 = \frac{[(W_{23}-P_a)W_{26}-(W_{27}-P_b)W_{22}]-(W_{12}W_{26}-W_{22}W_{25})B_2]}{(W_{20}W_{26}-W_{22}W_{24})}$$
$$A_1 = \frac{(W_{23}-P_a)-W_{20}B_1-W_{21}B_2}{W_{22}}$$
$$A_2 = (W_{15}+\phi_a) - W_{12}B_1 - W_{13}B_2 - W_{14}A_1 \tag{36}$$

4. Numerical results and discussions

Electomagnetothermoelastic interactions in a FGPM hollow sphere subjected to electric, thermal and mechanical loads are considered. In numerical calculations,material constants for the FGPM hollow sphere are taken as (Jabbari *et al.* 2002, Dunn and Taya 1994)

$$c_{11}^0 = 111(\mathrm{GPa}),\quad c_{12}^0 = 77.8(\mathrm{GPa}),\quad c_{22}^0 = 220(\mathrm{GPa}),\quad c_{23}^0 = 115(\mathrm{GPa})$$

$$e_{11}^0 = 15.1(\mathrm{C/m^2}),\quad e_{12}^0 = -5.2(\mathrm{C/m^2}),\quad \alpha_1^0 = 0.0001(1/\mathrm{K}),\quad \alpha_2^0 = 0.00001(1/\mathrm{K})$$

$$g_{11}^0 = 5.62\times 10^{-9}(\mathrm{C^2/Nm^2}),\quad p_{11}^0 = -2.5\times 10^{-5}(\mathrm{C^2/m^2K}),\quad \mu^0 = 4\pi\times 10^{-7}(\mathrm{H/m})$$

$$\mu^0 = 4\pi\times 10^{-7}(\mathrm{H/m}),\quad h = 0.72(\mathrm{W/mK})$$

Example 1. In this example, considering that the FGPM hollow sphere of internal radius $a = 0.1$ m and external radius $b = 0.2$ m, the temperature of internal boundary T_0 is taken as $0K$, and the corresponding boundary conditions are expressed as

$$P_a = 1\times 10^7(\mathrm{Pa}),\quad P_b = 0(\mathrm{Pa}),\quad \phi_a = 1\times 10^3(\mathrm{W}/A),\quad \phi_b = 0(\mathrm{W}/A) \tag{37}$$

The non-dimensional $R = \dfrac{r-a}{b-a}$, $T^* = \dfrac{T(R)}{T_0}$, $\sigma_i^* = \dfrac{\sigma_i}{P_a}(i = r, \theta)$, $\phi^* = \dfrac{\phi}{\phi_a}$ and $h_\psi^* = \dfrac{h_\psi}{H_\psi}$ are introduced in numerical results.

Fig. 2 and Fig. 3 depict the temperature and electric displacement distributions along the radial direction of the FGPM hollow sphere with different β, respectively. It is seen easily from Fig. 2 that the temperature at the internal boundary equals one, which satisfies the prescribed thermal boundary condition, and the temperature decreases as the power law index β increasing. In Fig. 3, when β is taken as a negative value, it is seen easily from the curves that the electric displacement decreases from the internal wall to external wall, while a positive β gives a contrary result. It is also seen

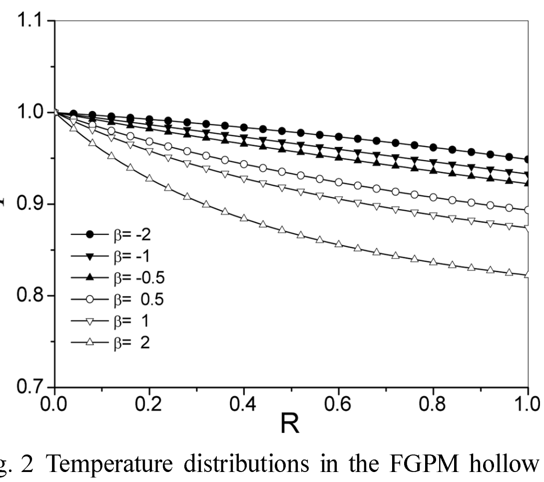

Fig. 2 Temperature distributions in the FGPM hollow sphere with different β, where $a = 0.1$ m, $b = 0.2$ m and $T_0 = 0\ K$

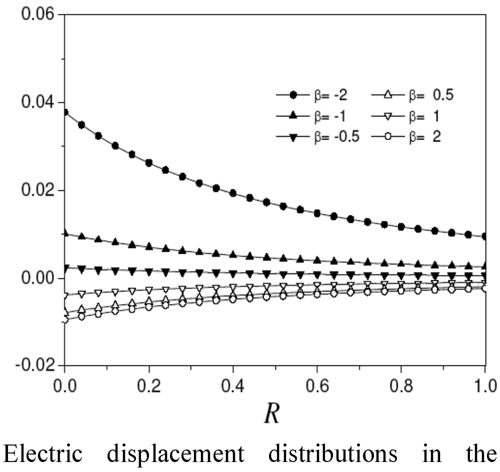

Fig. 3 Electric displacement distributions in the FGPM hollow sphere with different β, where $a = 0.1$ m, $b = 0.2$ m and $T_0 = 0\ K$

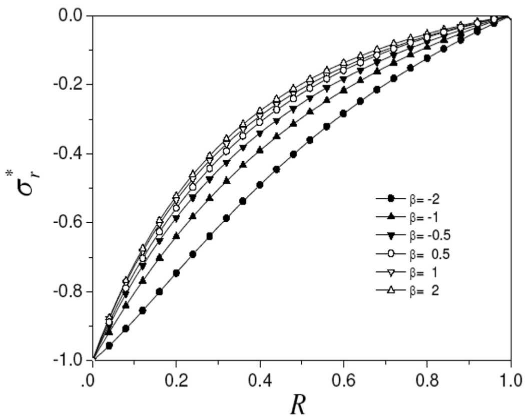

Fig. 4 Radial stress distributions in the FGPM hollow sphere with different β, where a = 0.1 m, b = 0.2 m and T_0 = 0 K

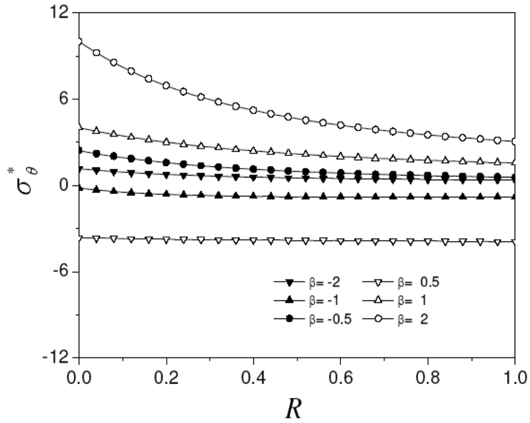

Fig. 5 Circumferential stress distributions in the FGPM hollow sphere with different β, where a = 0.1 m, b = 0.2 m and T_0 = 0 K

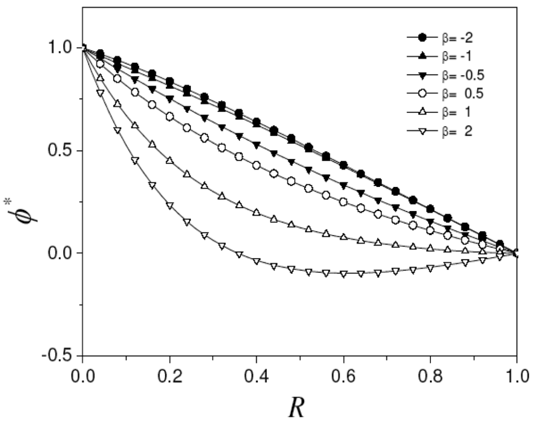

Fig. 6 Electric potential distributions in the FGPM hollow sphere with different β, where a = 0.1 m, b = 0.2 m and T_0 = 0 K

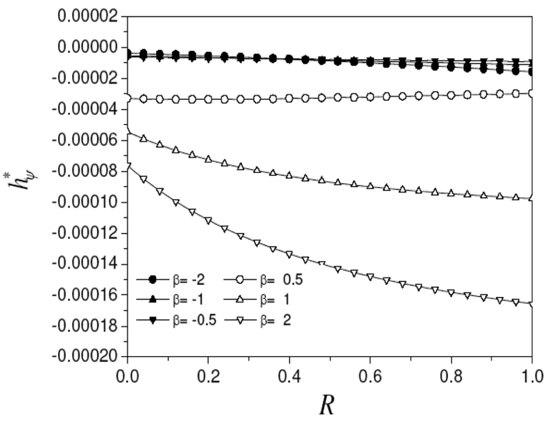

Fig. 7 The perturbation of magnetic field vector distributions in the FGPM hollow sphere with different β, where a = 0.1 m, b = 0.2 m and T_0 = 0 K

from the curves that the electric displacement at the same radial point decreases as index β increasing.

Fig. 4 and Fig. 5 show the radial stress and circumferential stress distributions in the FGPM hollow sphere with different β, respectively. From the curve of Fig. 4, one knows, the radial stress satisfies fully the mechanical boundary conditions, and the increased trend of the radial stress becomes slowly as the power index β increasing. Comparing Fig. 5 and with Fig. 4, the change trend of circumferential stresses are different with the change trend of radial stresses as the power index β increasing. To our knowledge, it can be also seen from the curves of the Fig. 5 that the circumferential stress is influenced greatly by the coupling of mechanical and electric loads.

Fig. 6 and Fig. 7 depict the electric potential and perturbation of magnetic field vector distributions in the FGPM hollow sphere, respectively. From the curves of Fig. 6, one knows, the electric potential satisfies fully the electric boundary conditions, and the electric potential decreases as the power law index β increasing. In Fig. 7, it is seen from the curve that the similar distribution happens to the perturbation of the magnetic field vector as the circumferential stress.

Example 2. In this example, considering that electric displacement, electomagnetothermo- elastic stresses, electric potential and perturbation of magnetic field vector of the FGPM hollow sphere at different thermal boundary T_0, all other conditions as example1. Figs. 8-12 show the electric displacement, radial stress, circumferential stress, electric potential and perturbation of magnetic field vector distributions in the FGPM hollow sphere with T_0 = 0 K, 20 K, 100 K, 200 K and 500 K, respectively. From the Fig. 8, it is seen easily that electric displacement increases from inner wall to out wall, and the electric displacement at the same radial point decreases as the temperature T_0 increasing. In Fig. 9, it is seen that radius stresses satisfy fully the given mechanical boundary

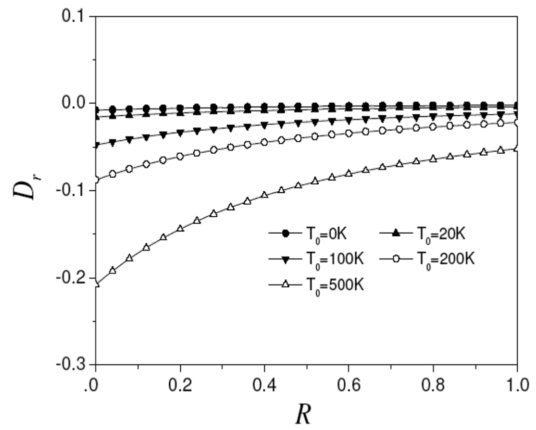

Fig. 8 Electric displacement distributions in the FGPM hollow sphere with different T_0, where a = 0.1 m, b = 0.2 m and β = 1

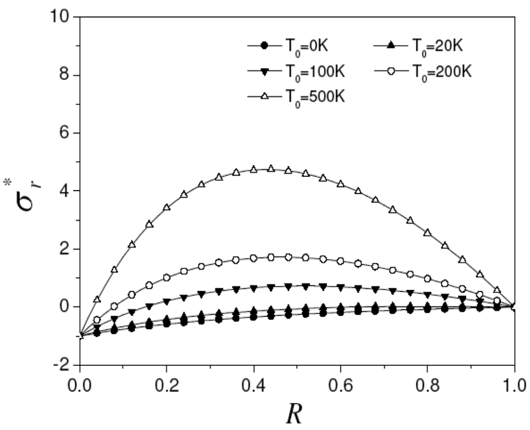

Fig. 9 Radial stress distributions in the FGPM hollow sphere with different T_0, where a = 0.1 m, b = 0.2 m and β = 1

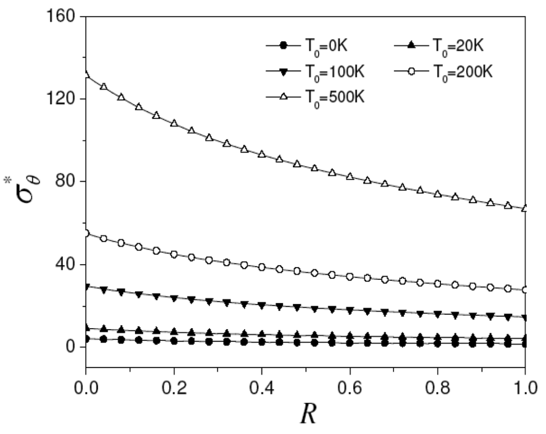

Fig. 10 Circumferential stress distributions in the FGPM hollow sphere with different T_0, where a = 0.1 m, b = 0.2 m and β = 1

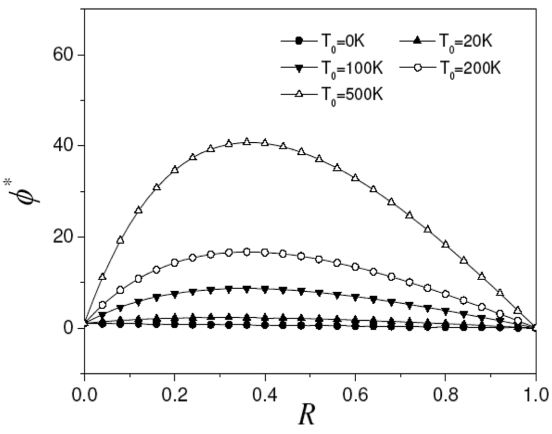

Fig. 11 Electric potential distributions in the FGPM hollow sphere with different T_0, where a = 0.1 m, b = 0.2 m and β = 1

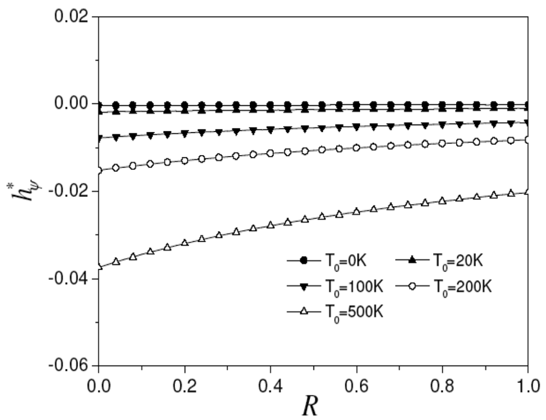

Fig. 12 The perturbation of magnetic field vector distributions in the FGPM hollow sphere with different T_0, where $a = 0.1$ m, $b = 0.2$ m and $\beta = 1$

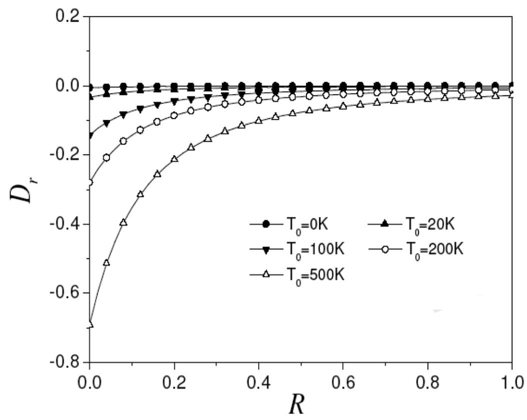

Fig. 13 Electric displacement distributions in the FGPM hollow sphere with different T_0, where $a = 0.1$ m, $b = 0.5$ m and $\beta = 1$

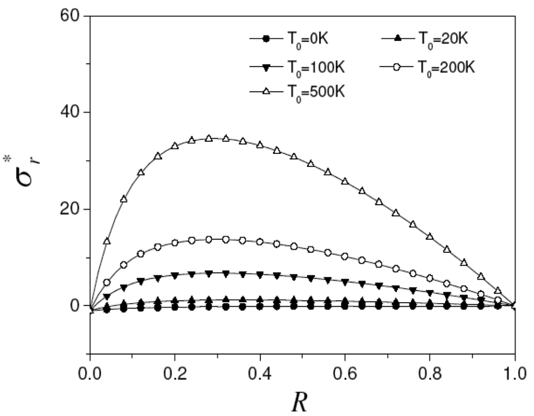

Fig. 14 Radial stress distributions in the FGPM hollow sphere with different T_0, where $a = 0.1$ m, $b = 0.5$ m and $\beta = 1$

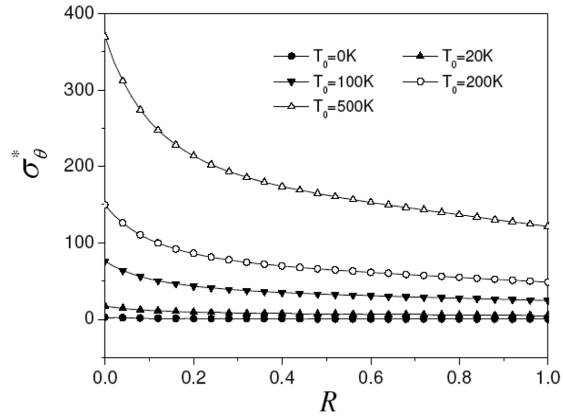

Fig. 15 Circumferential stress distributions in the FGPM hollow sphere with different T_0, where $a = 0.1$ m, $b = 0.5$ m and $\beta = 1$

conditions and its values increases as the temperature T_0 increasing. It is seen from curves of the Fig. 10 that the values of the circumferential stresses at the same radial point increases as the temperature T_0 increasing, and it shows nearly linear decreases from the inner wall to outer wall of the FGPM hollow sphere. In Fig. 11, one knows, the electric potential satisfies fully the electric boundary condition, and its change trend is similar that of Fig. 9. In Fig. 12, one knows, the distribution of the perturbation of the magnetic field vector is similar that of Fig. 8.

Example 3. In this example, consider a FGM hollow sphere of internal radius $a = 0.1$ m and external radius $b = 0.5$ m, and all other conditions are the same as example 2.

Figs. 13-17 show the electric displacement, radial stress, circumferential stress, electric potential and perturbation of magnetic field vector distributions along the radial direction of the FGPM

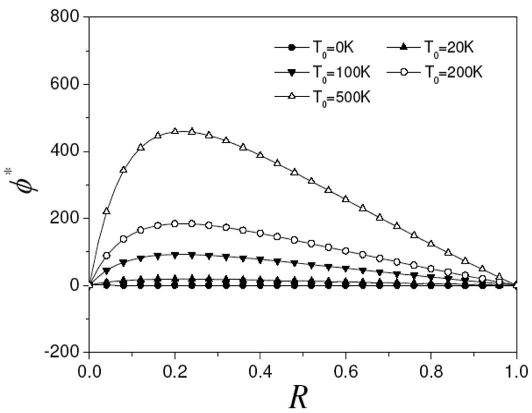

Fig. 16 Electric potential distributions in the FGPM hollow sphere with different T_0, where a = 0.1 m, b = 0.5 m and β = 1

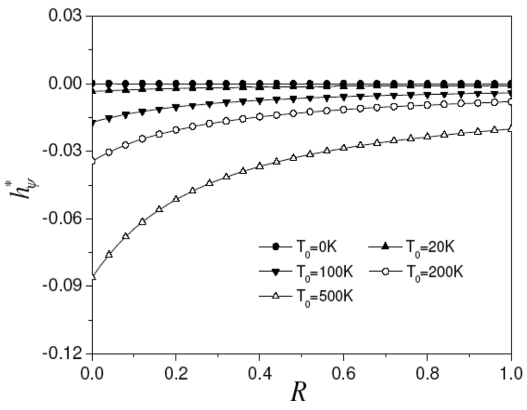

Fig. 17 The perturbation of magnetic field vector distributions in the FGPM hollow sphere with different T_0, where a = 0.1 m, b = 0.5 m and β = 1

hollow sphere, respectively. Comparing example 3 and example 2, the trend of all curves is similar as example 2, and its values is become larger as the wall thickness increasing. Thereby, it can be concluded that it is possible to control the distributions of physical parameters by selecting suitable wall thickness.

4. Conclusions

1. By means of the infinitesimal theory of electromagnetothermoelasticity, the paper presents an exact solution for a FGPM hollow sphere, placed in a uniform magnetic field, subjected to electric, thermal and mechanical loads. All material parameters are assumed to have the same exponent-law along the radial direction of the FGPM hollow sphere. The obtained solution is valid for arbitrary electric, thermal and mechanical loads applied on the FGPM hollow sphere.
2. The work enriches the solution method for electromagnetothermoelastic problem for FGPM hollow spherical structures, which is very useful for carrying out active control of hollow spherical structures using functionally graded piezoelectric materials.
3. Numerical results show that the gradient index β has a great effect on the electric displacement, stresses, electric potential and perturbation of magnetic field vector of the FGPM hollow sphere, and the distributions of all physical parameters are concerned with applying electric, thermal and mechanical loads and selecting the size of hollow spherical structures. Thus by selecting a proper value of β, structural size and suitable loads, it is possible for engineers to design the FGPM hollow sphere that can meet some special requirements.

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Appendix A

$$W_{12} = C_4 a^{m_1} + C_7 \frac{a^{m_1}}{m_1}, \quad W_{13} = C_4 a^{m_2} + C_7 \frac{a^{m_2}}{m_2}$$

$$W_{14} = C_4 \frac{W_6}{W_1} a^{-\beta} + C_7 \frac{W_6}{W_1(-\beta)} a^{-\beta} + \frac{1}{g_{11}^0 \beta} a^{-\beta}$$

$$W_{15} = -C_4(B_3 a + B_4 a^{\beta+1} + B_5 a^{-\beta+1}) - C_7\left(B_3 a + \frac{B_4}{\beta+1} a^{\beta+1} + \frac{B_5}{-\beta+1} a^{-\beta+1}\right) - \frac{p_{11}^0}{g_{11}^0}\left(\frac{W_7}{-\beta+1} a^{-\beta+1} + W_8 a\right)$$

$$W_{16} = C_4 b^{m_1} + C_7 \frac{b^{m_1}}{m_1}, \quad W_{17} = C_4 b^{m_2} + C_7 \frac{b^{m_2}}{m_2}$$

$$W_{18} = C_4 \frac{W_6}{W_1} b^{-\beta} + C_7 \frac{W_6}{W_1(-\beta)} b^{-\beta} + \frac{1}{g_{11}^0 \beta} b^{-\beta}$$

$$W_{19} = -C_4(B_3 b + B_4 b^{\beta+1} + B_5 b^{-\beta+1}) - C_7\left(B_3 b + \frac{B_4}{\beta+1} b^{\beta+1} + \frac{B_5}{-\beta+1} b^{-\beta+1}\right) - \frac{p_{11}^0}{g_{11}^0}\left(\frac{W_7}{-\beta+1} b^{-\beta+1} + W_8 b\right)$$

$$W_{20} = C_1 m_1 a^{\beta+m_1-1} + C_2 a^{\beta+m_1-1}, \quad W_{21} = C_1 m_2 a^{\beta+m_2-1} + C_2 a^{\beta+m_2-1}$$

$$W_{22} = C_1 \frac{W_6(-\beta)}{W_1} a^{-1} + C_2 \frac{W_6}{W_1} a^{-1} - \frac{C_4}{a}$$

$$W_{23} = -C_1 a^{\beta}[B_3 + B_4(1+\beta)a^{\beta} + B_5(1-\beta)a^{-\beta}] - C_7 a^{\beta}(B_3 + B_4 a^{\beta} + B_5 a^{-\beta}) - \lambda_1^0 a^{2\beta}(W_7 a^{-\beta} + W_8) - C_3 a^{\beta}(W_7 a^{-\beta} + W_8)$$

$$W_{24} = C_1 m_1 b^{\beta+m_1-1} + C_2 b^{\beta+m_1-1}, \quad W_{25} = C_1 m_2 b^{\beta+m_2-1} + C_2 b^{\beta+m_2-1}$$

$$W_{26} = C_1 \frac{W_6(-\beta)}{W_1} b^{-1} + C_2 \frac{W_6}{W_1} b^{-1} - \frac{C_4}{b}$$

$$W_{27} = -C_1 b^{\beta}[B_3 + B_4(1+\beta)b^{\beta} + B_5(1-\beta)b^{-\beta}] - C_7 b^{\beta}(B_3 + B_4 b^{\beta} + B_5 b^{-\beta}) - \lambda_1^0 b^{2\beta}(W_7 b^{-\beta} + W_8) - C_3 b^{\beta}(W_7 b^{-\beta} + W_8)$$

Notations

$\vec{U}, u$	: displacement vector and radial displacement [m]
r	: radial variable [m]
a, b	: inner and outer radii of the FGPM hollow sphere [m]
$\sigma_i(i = r, \theta)$	: components of stresses [N/m^2]
$T(r)$	: temperature distribution [K]
$\phi(r)$	: electric potential [W/A]
D_r	: radial electric displacement [C/m^2]
$c_{ij}(i = 1,2; j = 1,2,3)$	: elastic constant [N/m^2]
$e_{1i}(i = 1,2)$	: piezoelectric constant [C/m^2]
g_{11}	: dielectric constant [C^2/Nm^2]
p_{11}	: pyroelectric coefficient [C/m^{2}K]
$\alpha_j \lambda_j$ $(j = 1,2)$	: thermal constants [1/K] and thermal modulus [N/m^{2}K]
$\vec{J}$	: electric current density vector
$\vec{h}, h_2$	: perturbation of magnetic field vector
μ_0	: magnetic permeability [H/m]
$\vec{e}$	: perturbation of electric field vector
$\vec{H}, H_Z$	: magnetic intensity vector
f_ψ	: Lorentz's force [kg/m^{2}s^2]
κ	: thermal conduction coefficient [W/mK]
h	: ratio of the convective heat-transfer coefficient [W/K]
T_0	: temperature [K]

Non-dimensional quantities

$R = (r-a)/(b-a)$, $T^* = T(r)/T_0$, $\sigma_i^* = \sigma_i/P_a(i = r, \theta)$, $h_\psi^* = h_\psi/H_\psi$, $\phi^* = \phi/\phi_a$