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A simple procedure to simulate the failure evolution

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Abstract. To simulate the large-scale failure evolution with current computational facilities, a simple approach, that catches the essential feature of failure mechanisms, must be available so that the routine use of failure analysis is feasible. Based on the previous research results, a simple analysis procedure is described in this paper for failure simulation. In this procedure, the evolution of localization is represented by a moving surface of discontinuity, and the transition between continuous and discontinuous failure modes are described via the moving jump forms of conservation laws. As a result, local plasticity and damage models, that are formulated based on thermodynamic restrictions, are still valid without invoking higher order terms, and simple integration schemes can be designed for the rate forms of constitutive models. To resolve localized large deformations and subsequent cracking, an efficient structural solution scheme is given for static and dynamic problems.

Key words: localization; jump and moving surfaces; failure analysis.

1. Introduction

For many engineering structures, the limit design methodology based on elastic or yield strength might not be suitable from an economical viewpoint. In dynamic cases such as impact or seismic disturbances, the loads are of very short duration so that collapse may not occur even if the limit point is reached. In situations involving stress corrosion, creep and relaxation, it is still questionable how to define the limit point. In addition, many structural members are designed to be statically indeterminate which also helps to preclude collapse. Since a significant part of energy dissipation associated with the material failure evolution is missing in the conventional analyses, much research has been conducted worldwide to investigate the experimental, analytical and computational aspects of failure simulation, as reviewed recently by Chen and Schreyer (1994), and Xie *et al.* (1994). Two different approaches, smeared and discrete models, have been employed separately to predict the failure evolution. Among the smeared approaches proposed are nonlocal plasticity and damage models, rate-dependent models and Cosserat continuum models, which aim at the evolution of continuously distributed microcracking process with the use of additional terms in space and/or time as compared with local models. In the context of discrete approaches, a discontinuity in spatial variables is assumed in advance or introduced whenever a macrocrack occurs, and the subsequent crack propagation is traced based on the theory of fracture mechanics. No single existing approach enables the prediction of a complete

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failure process with a correct energy dissipation, in addition to some pressing limitations that prohibit the applications of proposed models to a general case. One of the fundamental problems to be solved is how to predict the essential feature of failure mechanisms with the least computational cost. For the analyst with a small computer, the failure analysis tool must use as least as possible the higher order terms inside a very refined mesh or the remeshing steps, because of the small storage capacity and low computational speed. Even if a supercomputer is available, it is still too expensive to use an existing model for the large-scale simulation of failure evolution.

Based on the previous research results, a simple procedure is described in this paper to simulate the evolution of material failure. To catch the essential feature of failure mechanisms, a newly proposed approach (Chen 1996, Chen and Sulsky 1995) is employed to establish a material failure criterion, and to describe the evolution of failure and the transition between continuous and discontinuous failure modes. As a result, local plasticity and damage models, that are formulated based on thermodynamic restrictions, are still valid inside and outside the localization zone, and simple integration schemes can be designed for the rate forms of constitutive models. To resolve localized large deformations and subsequent cracking, an efficient structural solution scheme is given for static and dynamic problems. With the use of this simple procedure, hence, a complete failure process might be simulated without invoking higher order terms or powerful mesh generators.

2. Material failure criteria

Since Drucker proposed his material stability criterion (Drucker 1949), and Hill derived his uniqueness and stability theory (Hill 1958), much research has been performed to identify the initiation of material failure, representative references of which are, but not limited to, the papers by Rudnicki and Rice (1975), by Valanis (1989), and by Neilsen and Schreyer (1993). Recent work by Simo and Oliver (1994) illustrates that strain softening models must lead to the appearance of strong discontinuities, and the discontinuous solutions for rate-independent softening solids can be obtained based on the kinematics of strong discontinuities. It is now clear that the material failure is an evolving process that involves jumps in certain field variables. However, attention has not been paid in the research community to the use of the moving jump forms of conservation laws in analyzing failure mechanisms. Since every physical phenomenon must follow the conservation laws no matter how its constitutive mechanism is, the moving jump forms of conservation laws in mass density and linear momentum are employed here to develop a simple failure criterion, with an emphasis on the transition between different failure states. For the purely mechanical problems without energy sources or sinks, the conservation in mass and linear momentum implies that in energy. A direct notation is employed to describe the continuum equations, with bold-faced letters denoting tensors of first or higher orders.

Consider a continuum body V subjected to essential and natural boundary conditions S^e and S^n, as shown in Fig. 1. In a three-dimensional framework, the spatial forms of conservation in mass and linear momentum can be written as

$$\dot{\rho}+\rho(\nabla\cdot\boldsymbol{v})=0 \tag{1}$$

and

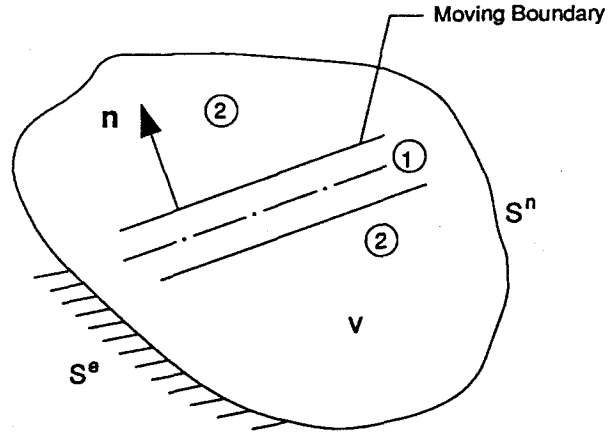

Fig. 1 A continuum body with discontinuous failure.

$$\rho a - \nabla \cdot \sigma = 0 \tag{2}$$

respectively. In Eqs. (1) and (2), ρ denotes the mass density, ∇ gradient operator, a particle velocity vector, v particle acceleration vector, and σ Cauchy stress tensor, with body forces omitted. If there exist jumps in certain field variables across a material boundary moving with a velocity v_b in the space, the jump conditions corresponding to Eqs. (1) and (2) take the forms of

$$\rho_1 (\boldsymbol{v}_1 - \boldsymbol{v}_b) \cdot \boldsymbol{n} = \rho_2 (\boldsymbol{v}_2 - \boldsymbol{v}_b) \cdot \boldsymbol{n} \tag{3}$$

and

$$\rho_1 [(\boldsymbol{v}_1 - \boldsymbol{v}_b) \cdot \boldsymbol{n}] \boldsymbol{v}_1 - \rho_2 [(\boldsymbol{v}_2 - \boldsymbol{v}_b) \cdot \boldsymbol{n}] \boldsymbol{v}_2 = (\sigma_1 - \sigma_2) \cdot \boldsymbol{n} \tag{4}$$

in which the subscripts 1 and 2 denote field variables on the two sides of the moving boundary, and $\boldsymbol{n}$ the unit normal to the boundary. The rearrangement of the terms in Eqs. (3) and (4) then yields

$$\boldsymbol{v}_b \cdot \boldsymbol{n} = \frac{\rho_1 \boldsymbol{v}_1 \cdot \boldsymbol{n} - \rho_2 \boldsymbol{v}_2 \cdot \boldsymbol{n}}{\rho_1 - \rho_2} \tag{5}$$

and

$$\left[- \frac{\rho_1 \rho_2 \boldsymbol{n} \cdot (\boldsymbol{v}_1 - \boldsymbol{v}_2)}{\rho_1 - \rho_2} \right] (\boldsymbol{v}_1 - \boldsymbol{v}_2) = (\sigma_1 - \sigma_2) \cdot \boldsymbol{n} \tag{6}$$

if there is a jump in mass density.

Introduce a local coordinate system with coordinate x_1' perpendicular and coordinate x_2' parallel to $\boldsymbol{n}$, as depicted in Fig. 2. The unit vector $\boldsymbol{t}$ is along the x_1'-axis. Suppose the origin of the coordinate system is the point from which material failure evolves. During a time increment Δt in the evolution process, the half-band width, b, increases with its final dimension being material-dependent. From a smeared viewpoint, b represents a required finite range in which a cluster of microcracks can interact on each other to form macrocracks. Hence, b must be nonzero for a nonzero energy dissipation associated with the failure evolution.

Since the material failure is an evolving process that involves jumps in certain field variables,

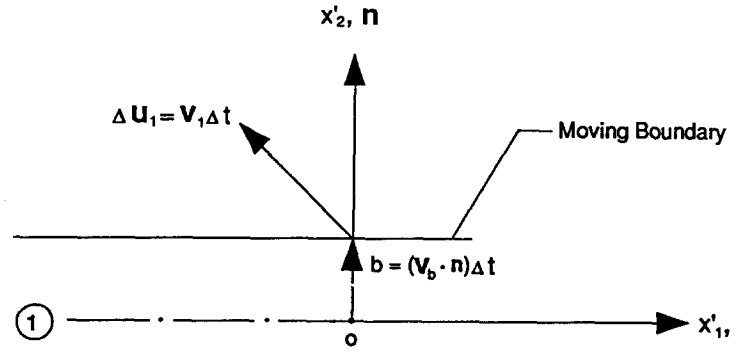

Fig. 2 A local coordinate system parallel and perpendicular to n.

a simple failure criterion can be formulated based on the jump degree in the kinematic field variables, as follows:

$$\begin{aligned} &\text{Diffuse Failure:=} && v_1=v_2 \text{ and } \dot{\varepsilon}_1=\dot{\varepsilon}_2 \\ &\text{Localized Failure:=} && v_1=v_2 \text{ and } \dot{\varepsilon}_1\neq\dot{\varepsilon}_2 \\ &\text{Discrete Failure:=} && v_1\neq v_2 \text{ and } \dot{\varepsilon}_1\neq\dot{\varepsilon}_2 \end{aligned}$$

with $\dot{\varepsilon}_1$ and $\dot{\varepsilon}_2$ being the total strain rates on the side 1 and 2, respectively. In other words, the jump degree in the kinematic field variables identifies the initiation of different failure states, and different constitutive models can be used for different states, as illustrated later.

As can be seen, the transition between continuous and discontinuous failure states is characterized by the condition of localized failure. The mechanisms involved can be made clear by invoking the jump forms of conservation laws together with the conditions of different failure states. The substitution of the condition for diffuse failure into Eqs. (3) and (4) results in no jumps in any field variables across the moving boundary. Hence, the diffuse failure represents a prior state, such as hardening, before a real critical state occurs.

The case for localized failure needs a careful reasoning before a conclusion can be made. Since the condition of $v_1=v_2$ still holds for localized failure, the result similar to the case of diffuse failure follows if there is no jump in mass density. Based on Eqs. (5) and (6), however, the occurrence of a jump in mass density would yield

$$v_1\cdot n=v_2\cdot n=v_b\cdot n \tag{7}$$

and the continuity of the traction across the moving boundary, i.e.,

$$(\sigma_1-\sigma_2)\cdot n=0 \tag{8}$$

Because there is a jump in the strain rate for localized failure, it makes sense to claim that a corresponding jump must exist in mass density. Thus, Eqs. (7) and (8) together with a jump in mass density represent the essential feature of localized failure.

To examine how the jump in the strain rate is derivable from Eq. (8), assume that during a time increment the stress tensor in side 1 is related to that in side 2 by

$$\sigma_1=\sigma_2+T_1:\dot{\varepsilon}_1^k\,\Delta t \tag{9}$$

in which $\boldsymbol{T}_1$ denotes a fourth-order tangent stiffness tensor with minor symmetries (Chen and Schreyer 1990b), and $\dot{\varepsilon}_1^k$ the jump in the strain rate field. According to Maxwell's compatibility conditions, the jump $\dot{\varepsilon}_1^k$ must be a rank-one tensor of the form (Neilsen and Schreyer 1993)

$$\dot{\varepsilon}_1^k = \frac{1}{2b}\,(\dot{m}\otimes\boldsymbol{n}+\boldsymbol{n}\otimes\dot{m}) \tag{10}$$

To be consistent with the definition of localized failure, the vector $\dot{\boldsymbol{m}}$ in Eq. (10) is taken to be $\boldsymbol{v}_1$. Thus, it follows that $\boldsymbol{n}\cdot\dot{\varepsilon}_1^k\cdot\boldsymbol{n}=\frac{\boldsymbol{v}_1\cdot\boldsymbol{n}}{b}$ represents Mode I failure, and $\boldsymbol{n}\cdot\dot{\varepsilon}_1^k\cdot\boldsymbol{t}=\frac{\boldsymbol{v}_1\cdot\boldsymbol{t}}{2b}$ mode II failure. The total displacement increment at the material point of concern is given by $\Delta\boldsymbol{u}_1=\boldsymbol{v}_1\Delta t$, as shown in Fig. 2. The use of Eqs. (8)-(10) then yields the classical necessary condition for a discontinuous bifurcation:

$$\boldsymbol{Q}\cdot\boldsymbol{m}=0 \tag{11}$$

with $\boldsymbol{Q}=\boldsymbol{n}\cdot\boldsymbol{T}_1\cdot\boldsymbol{n}$ being the acoustic tensor. In other words, the arguments based on the moving jump forms of conservation laws produce the same condition Eq. (11) as derived from classical approaches based on certain assumptions (Neilsen and Schreyer 1993). An eigen analysis can be performed to find out the orientation of the localized failure mode. However, the magnitude of failure is undetermined by Eq. (11).

Since both the orientation and magnitude of failure evolution must be known for a complete failure analysis, a one-dimensional problem is used here to illustrate a simple approach. For a bar under an external load, a linear velocity field occurs for a diffuse state. As long as a localized failure state is initiated at some weak material point, a moving material boundary evolves with time such that $v_1=v_2$ and $\dot{\varepsilon}_1\neq\dot{\varepsilon}_2$ hold across the boundary. Since the width of localization zone is usually very small as compared with the size of engineering structures, it is reasonable to assume that the velocity field is linear inside the zone. With the use of Eq. (7), it follows that $v_1=v_2=v_b$. Hence, the zone width can be determined if $v_1=v_2$ is known at the moving boundary. Conversely, V_1, V_2, $\dot{\varepsilon}_1$ and $\dot{\varepsilon}_2$ can be found if v_b is prescribed. This simple illustration indicates that a partitioned-modeling approach (Chen 1993a) combined with the moving jump forms of conservation laws might be a promising alternative to predict the evolution of localized failure without invoking higher order terms in space and/or time. The basic ingredients of the partitioned-modeling method are that different simple constitutive models are used in different sub-domains with a moving material boundary being defined between two sub-domains. As a result, simplified governing differential equations can be formulated in the partitioned domains for given boundary and initial conditions (Chen and Sulsky 1995), which is similar to turbulence modeling (Chen and Clark 1995). In next section, creep damage will be considered to further demonstrate this point.

Because a discrete failure state is characterized by $\boldsymbol{v}_1\neq\boldsymbol{v}_2$ and $\dot{\varepsilon}_1\neq\dot{\varepsilon}_2$, it follows from Eq. (6) that the continuity of the traction across the moving boundary might not hold in general for discrete failure. And also, the speed of the moving boundary depends not only on the velocity jump but on the mass density jump, as described by Eq. (5), which should be related to the crack tip opening speed.

It appears from the above discussions that a complete failure evolution process, including continuous and discontinuous failure states, might be simulated by tracing the moving material

boundary associated with different degrees of jump. From a kinematical viewpoint, this moving boundary might be related to the concept of failure wave that has been observed in some dynamic experiments (Grady 1994).

3. Constitutive modeling and integration schemes

After different failure states are identified based on the jump conditions, a simple stress-strain relation must be formulated for structural analyses. Nonlinear material behaviors arise from two distinct modes of microstructural changes: one is plastic flow and the other is the degradation of material properties. Plastic flow, which is reflected through permanent deformation, is the consequence of a dislocation process along preferred slip planes as in metals, or particle motion and rearrangement as in geologic materials. Because the number of bonds between material points is hardly altered during the flow process, the material stiffness remains insensitive to this mode of microstructural motion, and change of strength is reflected through plastic strain hardening and apparent softening. On the other hand, the nucleation, crushing and coalescence of microcracks and microvoids result in debonding, which is reflected through the damaging of material stiffness and strength. In general, both modes are present and interacting although some mode might dominate at some stage of the evolution process. Final rupture occurs when macrocracks form and propagate from the cluster of microcracks. If no special treatment is taken, the evolution of inhomogeneous interactions among material points can not be predicted by local constitutive models in which the stress at a material point is related to the strain only at that point. With the use of the partitioned-modeling approach mentioned above, however, the essential feature of failure evolution can be predicted without invoking enhanced constitutive models. Here, a systematic procedure, which satisfies thermodynamic restrictions, is adopted to formulate simple local models for the partitioned-modeling approach.

For a stress-based formulation, a general form of the Gibb's free energy per unit volume can be defined as

$$\bar{G}=\bar{G}(\sigma, I) \tag{12}$$

where I represents a set of internal state variables. Within the context of a purely mechanical case, the Clausius-Duhem inequality is given by

$$\dot{\bar{G}}(\sigma, I)-\varepsilon:\dot{\sigma}\geq 0 \tag{13}$$

for any admissible process. The strain-based counterpart follows the use of the internal energy per unit volume (Chen and Schreyer 1990b and 1994). Based on Eq. (12), an alternative form of Eq. (13) can be written as

$$\left(\frac{\partial \bar{G}}{\partial \sigma}-\varepsilon\right):\dot{\sigma}+\frac{\partial \bar{G}}{\partial I}\dot{I}\geq 0 \tag{14}$$

Since the stress rate $\dot{\sigma}$ is arbitrary and independent of I, it follows that

$$\varepsilon=\frac{\partial \bar{G}}{\partial \sigma} \tag{15}$$

and the remaining term in Eq. (14) results in the dissipative inequality

$$\frac{\partial \bar{G}}{\partial I}\dot{I} \geq 0 \tag{16}$$

which must be satisfied by any constitutive model that dissipates energy.

For small deformations, the total strain rate can be assumed to be the sum of plastic and damage parts as follows:

$$\varepsilon = \frac{\partial \bar{G}}{\partial \sigma} = \varepsilon^{ed} + \varepsilon^{p} = C^{ed} : \sigma + \varepsilon^{p} \tag{17}$$

with

$$\bar{G} = \frac{1}{2}\sigma : C^{ed} : \sigma + \sigma : \varepsilon^{p} \tag{18}$$

where C^{ed} is the secant elastodamage compliance tensor with major and minor symmetries. Thus, the inequality Eq. (16) is satisfied if

$$\frac{1}{2}\sigma : \dot{C}^{ed} : \sigma \geq 0 \tag{19.1}$$

and

$$\sigma : \dot{\varepsilon}^{p} \geq 0 \tag{19.2}$$

which must be imposed on the constitutive models, as shown next for a rate-dependent case.

Rate-dependent models have been proposed by some researchers for localization problems. However, the slow loading rates or the rate-independent limit can not be well-specified by existing rate-dependent models. In addition, it is still not clear how to determine the failure state associated with long-term rate-dependent problems such as creep and relaxation, although an attempt has been made to derive an algorithmic tangent operator for elastic-viscoplastic materials (Willam *et al.* 1993). Recently, the creep response of rock salt has been a research topic of considerable interest, because the natural salt deposits might be used as a repository for waste disposal. Based on the previous work (Chan *et al.* 1992, Chen and Wang 1993a and b), an isotropic plasticity-damage model is developed within the partitioned-modeling framework to predict the complete creep process of rock salt.

In this simple model, the total strain rate is taken to be the sum of elastodamage part, $\dot{\varepsilon}^{ed}$, and viscoplastic part, $\dot{\varepsilon}^{vp}$. To find the evolution equation for the viscoplastic flow, a rate-dependent yield surface for the response beyond the primary stage of creep is defined to be

$$f^{vp} = \frac{\bar{s}}{G} - \sin h^{-1}\left[\frac{\dot{\bar{e}}^{vp}}{A}\exp\left(\frac{Q}{RT}\right)\right]^{1/n} = 0 \tag{20}$$

in which $\bar{s} = \left(\frac{3}{2}\sigma^{d} : \sigma^{d}\right)^{1/2}$ with the superscript d indicating the deviatoric part of stress, and $\dot{\bar{e}}^{vp} = \left(\frac{2}{3}\dot{\varepsilon}^{vp} : \dot{\varepsilon}^{vp}\right)^{1/2}$ are used to represent the isotropic feature. G is the shear modulus, Q is the activation energy, T is the absolute temperature, R is the universal gas constant, and A and n are model parameters. With the use of an associated flow rule, it follows that

$$\dot{\varepsilon}^{vp}=\dot{\lambda}\frac{\partial f^{vp}}{\partial \sigma}=\dot{\lambda}\frac{3}{2G\bar{s}}\sigma^{d} \tag{21}$$

with $\dot{\lambda}$ being a positive loading parameter to parameterize the irreversible evolution of viscoplastic flow. Based on the loading/unloading criterion (Chen and Schreyer 1994), it can be found that $\dot{\bar{e}}^{vp}=\frac{\dot{\lambda}}{G}$, and

$$\dot{\lambda}=AG\exp\left(-\frac{Q}{RT}\right)\left[\sin h\left(\frac{\bar{s}}{G}\right)\right]^{n} \tag{22}$$

If the primary creep is to be included in the model, a simple approach results in

$$\dot{\varepsilon}^{vp}=F^{T}\dot{\lambda}\frac{\partial f^{vp}}{\partial \sigma}=\frac{3A}{2\bar{s}}F^{T}\exp\left(\frac{Q}{RT}\right)\left[\sin h\frac{\bar{s}}{G}\right]^{n}\sigma^{d} \tag{23}$$

with the transient function, $F^{T}>0$, being given by Eq. (26) in (Chan *et al.* 1992). Because of

$$\sigma:\dot{\varepsilon}^{vp}=A\bar{s}F^{T}\exp\left(-\frac{Q}{RT}\right)\left[\sin h\left(\frac{\bar{s}}{G}\right)\right]^{n}\geq 0 \tag{24}$$

the thermodynamic restriction, Eq. (19.2), is satisfied.

Eq. (23) is the evolution equation for the viscoplastic part of the creep model, which yields the constant strain rate beyond the primary creep if no damage occurs. Based on experimental observations, it is assumed that the transition between the secondary and tertiary stages of creep is due to the initiation of localized damage, and that the damage evolution increases the effective stress in Eq. (23) which accelerates the creep process and results in the final rupture. The macro- and micro-experiments conducted at the University of New Mexico indicated that the orientation of localized failure is not rate-sensitive under a conventional range of loading rates, and that a failure surface formulated in the strain space is a better measure than that in the stress space. Hence, a simple rate-independent damage surface is defined to be

$$f^{d}(\varepsilon,\ P,\ \omega)=\bar{e}-[\bar{e}_{L}(1+aP)+b\omega]=0 \tag{25}$$

in which $\bar{e}=\left(\frac{3}{2}\varepsilon^{d}:\varepsilon^{d}\right)^{1/2}$ with the superscript d indicating the deviatoric part of strain, P is the mean pressure, a and b and denote model parameters. The limit strain, $\bar{e}_{L}$, represents the critical state at which the transition between the secondary and tertiary stages of creep occurs. For a given total strain, it follows from Eq. (25) that the damage parameter ω can be determined by

$$\omega=\frac{\bar{e}-\bar{e}_{L}(1+aP)}{b} \tag{26}$$

A simple formulation, $C^{ed}=\frac{1}{1-\omega}C^{e}$, can be used to calculate the damage strain, which satisfies the thermodynamic restriction, Eq. (19.1). It should be pointed that the creep model is still rate-dependent because the effective stress is used in the viscoplastic part. However, the rate-independent damage surface enables the formulation of a tangent stiffness tensor for the eigen analysis of Eq. (11). Because of the simplicity of the model, a forward integration scheme

can be used to solve the rate forms of viscoplastic flow and damage evolution.

To illustrate the solution procedure for the partitioned-modeling approach, a bar under a constant load is considered, through which the applicability of the solution procedure to a general case can be made clear. To predict the essential feature of localized creep damage, it is assumed that a moving material boundary is initiated if the damage surface is activated at some weak material point. Inside the damage zone, i.e., in zone 1, the total strain rate consists of both viscoplastic and damage parts, and outside the damage zone, i.e., in zone 2, the total strain rate is only due to the viscoplastic flow. Because the damage term is active only inside the damage zone, the evolution of localized creep damage can be predicted without invoking enhanced constitutive models as long as the moving boundary is determined. It should be pointed out that conventional rate-dependent models do not introduce a length scale into creep problems, and hence, do not yield mesh-independent solutions. Assuming that the fixed end of bar initiates damage and the total strain inside the damage zone is constant, the incremental solution for the location of moving boundary under given total strain increments $\Delta\varepsilon_1^n$ can be found to be (Chen and Clark 1995)

$$b^n = b^{n-1}(1+\Delta\varepsilon_1^n) \tag{27}$$

with the superscript n denoting the time step, and b^0 being the length of initial imperfection. By using the condition of localized failure, the deformation response outside the damage zone can be easily calculated at any time because the displacement continuity, $u_1 = u_2 = u_b$, holds and $\dot{\varepsilon}$ is known. As can be seen from the above analysis, a partitioned-modeling approach can be used to predict localized creep damage with less computational cost.

4. Structural solution schemes

There exist two major computational difficulties in simulating the structural response of failure evolution. One is the occurrence of an ill-conditioned tangent stiffness matrix around critical points, namely, limit and bifurcation points. The other is the selection of a suitable constraint on the solution path such that the post-critical response can be traced. Since a robust and efficient solution scheme is necessary to make failure simulation available in a routine manner, several procedures have been proposed to circumvent the difficulties associated with critical points, as reviewed by Chen and Schreyer (1994). The standard arc-length control is still commonly employed in geometrically nonlinear cases, and with some modifications in materially nonlinear cases. Because the material failure zone is localized into a small region, the arc-length constraint formulated in the global deformation field is insensitive to the evolution of the localized deformation mode. As a result, a suitable constraint should be constructed in terms of a localized kinematical field if localized failure needs to be simulated. Due to the fact that the localization zone is evolving and there is a sign change of the load increment at a critical point, the control point (element) should also vary in position with a suitable measure of failure, and the localized constraint parameter that reflects the extent of irreversible energy dissipation should be constrained to increase monotonically. Preliminary results obtained for plasticity and damage problems indicate that the use of an evolving localized control is a reasonable choice for localization problems including snap-back or snap-through (Chen 1993b, Chen and Schreyer 1990a and 1991). As a remedy to avoid the use of an ill-conditioned tangent stiffness matrix around critical points,

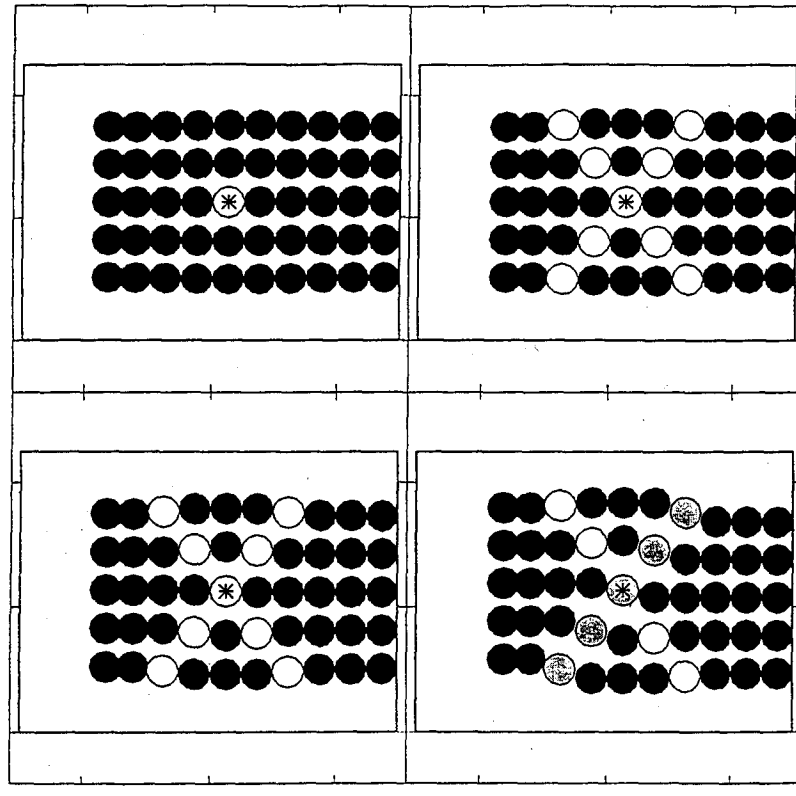

Fig. 3 The evolution of shear banding with the use of the particle method and evolving-localization constraint.

a secant stiffness matrix based on continuum damage mechanics has been used with a dramatic increase in the rate of convergence with respect to the rate obtained using a tangent stiffness matrix. In order to incorporate both damage and plasticity models into one computer code, it has been proposed that an incremental-iterative solution scheme be constructed through the use of an initial elasticity stiffness matrix together with an evolving-localization constraint (Chen and Schreyer 1994). Thus, only one inverse calculation is required, and the amount of computation involved in the iterative loop is dramatically reduced. It can be found that the numerical procedure for tracing the post-bifurcation path depends mainly on the choice of a suitable constraint instead of a stiffness matrix. Different constraints generally yield different solution paths. In other words, the constraint imposed on the solution path plays a crucial role whether or not the stiffness matrix is ill-conditioned. Thus, the dependence of the evolving-localization constraint on the location of initial imperfections and on the evolution history of localization detects the solution path following the critical point, while the use of a well-conditioned stiffness matrix guarantees that a numerical solution can be obtained.

To resolve localized large deformations and to predict the subsequent transition from continuous to discrete failure, a material particle method (Sulsky *et al.* 1994) might be an efficient semi-discretization procedure in space. Briefly speaking, the particle method uses both a fixed set of cell nodes, and a finite set of material particles which move during the deformation history.

Thus, it combines the advantages of Eulerian and Lagrangian schemes while avoiding the shortcomings of each. The equations of motion are solved on cell nodes using standard integration procedures. Convection is modeled by moving the material particles in the computed velocity field. Each material particle can carry its material properties with accuracy while it is convected. If all properties of the continuum body are assigned to the finite set of material particles, then the information carried by these particles characterizes the material flow, and the cell points just serve as a "temporary channel" without carrying permanent information. Hence, the cell grid can be discarded and reconstructed for computational convenience at each time step, which allows the use of adaptive meshes for the evolution of failure. One important feature of the material particle method is the ease of treating the moving material boundary with jumps, and large deformations with history dependence. Because the material particle method was originally formulated within a dynamic framework for penetration problems, a dynamic relaxation procedure has been utilized to simulate quasi-static failure problems with the use of an evolving-localization constraint (Zhou *et al.* 1995), as shown in Fig. 3 for the evolution of two-dimensional shear banding.

4. Conclusions

A simple procedure to simulate the failure evolution has been described based on the research results up to date. The essence of the procedure is to use the moving jump forms of conservation laws to fill the gap between smeared and discrete models so that a complete failure process can be simulated with different simple models being used in different states. As a result, the large-scale failure simulation might be feasible with current computational facilities. Further research is required to quantitatively measure the moving material surface of discontinuity so that quantitative comparisons with experiments can be made to verify the procedure.

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